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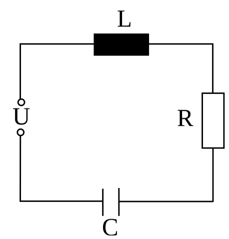
Exercise 8 of Theoretische Physik II: Elektrodynamik
radiation field, antenna

Abgabe: 06/23/2004

Problem 1 (3 points): *circuit*
Consider the following circuit with inductance L, resistance R, and capacitance C, powered by an AC voltage $U = U_0 e^{i\omega t}$.



Calculate the current $I(t)$ as well as the voltage drop across the capacitor and the inductor (neglect the ...). Determine the impedance $|Z|$ as well as the phase factor φ of the complex resistance $Z := U/I = |Z|e^{i\varphi}$. Use the following data: $C = 7$pF, $L = 2$mH, $R = 2$kΩ, $U_0 = 220\sqrt{2}$V and $\omega = 7$MHz.

Problem 2 (6 points): *radiation field*
1. In the lecture, the retarded potentials were derived in the radiation field dipole approximation. Calculate the corresponding fields $\mathbf{B}(\mathbf{x}, t)$ and $\mathbf{E}(\mathbf{x}, t)$ up to terms $\mathcal{O}(1/r^2)$ (cf. the results of the lecture). (3 points)

2. Calculate the energy density e of the electromagnetic field, and show that e integrated over the surface of a spherical shell is constant. What is the physical meaning of this? (3 points)

Problem 3 (11 points): *antenna in dipole approximation*
An AC current

$$\mathbf{j}(\mathbf{x}, t) = I_0 cos(\pi z/d) cos(\omega t)\delta(x)\delta(y)\theta(d/2 - |z|)\mathbf{e}_z,$$

is flowing in a thin, linear antenna on the z-axis between $-d/2$ and $d/2$ ($\omega = \pi c/d$).

1. Determine the radiation field of the antenna without the dipole approximation. As usual in the radiation field, approximate $|\mathbf{x} - \mathbf{x}'| \approx r - (\mathbf{x} \cdot \mathbf{x}')/r$, and calculate the function $\dot{\tilde{\mathbf{p}}}(t) = \int d^3x' e^{-ik\hat{\mathbf{n}}\cdot\mathbf{x}'} \mathbf{j}(\mathbf{x}', t)$ (analogous to $\dot{\mathbf{p}}(t)$ in the dipole approximation). Using this, determine the vector potential $\mathbf{A}(\mathbf{x}, t)$ and the fields $\mathbf{B}(\mathbf{x}, t)$ and $\mathbf{E}(\mathbf{x}, t)$. Calculate the Poynting vector $\mathbf{S}(\mathbf{x}, t)$ as well as the average power emitted per solid angle dΩ, given by $\mathrm{dP/d}\Omega = \mathrm{r}^2 \hat{\mathbf{n}} \cdot \overline{\mathbf{S}(\mathbf{x}, \mathrm{t})}$. (6 points)

2. Calculate the vector potential $\mathbf{A}(\mathbf{x}, t)$, the fields $\mathbf{B}(\mathbf{x}, t)$ and $\mathbf{E}(\mathbf{x}, t)$, $\mathbf{S}(\mathbf{x}, t)$ as well as dP/dΩ for the antenna in dipole approximation. (4 points)

3. Make sketches of dP/dΩ from parts 1 and 2 and compare the two. (1 point)