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www.thp.uni-koeln.de/gravitation/courses/rcii19.html

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9th exercise sheet on Relativity and Cosmology II

Summer term 2019

Release: Mon, June 3rd **Submit**: Mon, June 17th in lecture **Discuss**: June 27th/21st

Exercise 55 (7 credit points): *Hawking temperature*

In the lecture it was mentioned that a Schwarzschild black hole radiates with the so-called *Hawking temperature*

$$T_{\mathrm{H}} = \frac{\hbar c^3}{8\pi k_{\mathrm{B}} G M}\,.$$

Assume that only photons are emitted and that they have a perfect Planck spectrum. Find a relation between the initial mass of the black hole and its lifetime and analyse this relation for several interesting masses and time intervals.

Exercise 56 (6 credit points): *Accretion disks*

Give an estimate for the characteristic energy that is emitted by an accretion disk with radius R around a compact spherically symmetric object. For simplicity (even though this is not totally realistic), assume that the luminosity is that of a black body of radius R and temperature T and that it amounts to a given fraction ε of the Eddington luminosity. (At the end, use $\varepsilon \approx 0.5$.)

Exercise 57 (7 credit points): *Redshift in case of a gravitational collapse*

Consider an observer on the surface of a collapsing spherical star who emits radial light signals in short proper time intervals Δs, i. e. with a constant frequency $\omega_* = 2\pi/\Delta s$. These signals are received by a stationary observer at large distance $r = r_{\mathrm{R}}$, i.e. with a frequency $\omega_{\mathrm{R}} = 2\pi/\Delta t_{\mathrm{R}}$, where Δt_{R} refers to the Schwarzschild time.

Calculate the dependence of the frequency ratio $\omega_{\mathrm{R}}/\omega_*$ on t_{R}. Indicate the time scale of the redshift in terms of seconds if you measure M in solar masses.

Hint: Use Eddington–Finkelstein coordinates (as discussed in the lecture) and assume that the emitting observer is already located near the Schwarzschild radius.