

## Third exercise sheet on Relativity and Cosmology II

Summer term 2021

**Release:** Mon, May 3<sup>rd</sup>

**Submit:** Mon, May 10<sup>th</sup> on ILIAS

**Discuss:** Thu, May 13<sup>th</sup>

### Exercise 41 (14 points): *Redshift in the Schwarzschild spacetime*

Consider a stationary\* observer  $\mathcal{A}$  at  $r = R$ ,  $R \geq 2GM$  in the Schwarzschild spacetime of mass  $M$  and an observer  $\mathcal{B}$  at infinity. The timelike Killing vector shall be denoted by  $\zeta^\mu = (1, 0, 0, 0)$ . Furthermore, we define the quantity  $V^2 := -\zeta_\mu \zeta^\mu$ . Observer  $\mathcal{A}$  emits energy with frequency  $\omega_R$  (measured in her/his rest frame) which is measured by observer  $\mathcal{B}$  as being  $\omega_\infty$ .

- 41.1** Express the four-velocity  $u^\mu$  of observer  $\mathcal{A}$  in terms of  $\zeta^\mu$  and  $V$  and use this to derive the relation between the frequencies  $\omega_R$  and  $\omega_\infty$ .
- 41.2** What does observer  $\mathcal{B}$  measure when observer  $\mathcal{A}$  reaches the Schwarzschild radius  $r = 2GM$ ? What does this mean for the redshift?

### Exercise 42 (6 points): *Time dilation in the Schwarzschild spacetime*

Show that the proper time  $d\tau$  on a circular geodesic in the Schwarzschild geometry of mass  $M$  obeys the relation:

$$d\tau = \sqrt{1 - \frac{3GM}{r}} dt.$$

Use this to give an estimate for the time dilation of a satellite flying in a low orbit around the Earth.

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\*A *stationary* observer is an observer in a stationary spacetime whose 4-velocity  $u^\mu$  is proportional to the given timelike Killing vector.