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www.thp.uni-koeln.de/gravitation/courses/rci12.html

3rd exercise sheet on Relativity and Cosmology I

Winter term 2012/13

Deadline for delivery: Exercises **7**, **8.1** and **9**: Wednesday, 31st October 2012 at the end of the lecture.
Exercises **6**, **8.2** and **8.3**: Thursday, 8th November 2012 during the exercise class.

Exercise 6 (5 credit points): *Clocks*

Two atomic clocks are transported in two airplanes once around the Earth in either eastern or western direction. Calculate the respective time dilations the clocks exhibit after the landing of the airplanes compared to a clock which stayed on the ground. For this purpose, assume for reasons of simplicity that the airplanes fly directly above the equator, where the rotation velocity of the Earth is about $v_{\mathrm{E}} \approx 1667$ km/h. Furthermore, use an average cruising speed of $v_{\mathrm{F}} \approx 800$ km/h and a mean flying altitude of 10 km.

Hint: Take into account the separate contributions due to the gravitational and the special relativistic velocity effect, respectively.

Exercise 7 (5 credit points): *Inertial frames*

A rocket with a rest length L_0 moves with constant velocity radially away from Earth. From Earth a light pulse is emitted, which is then reflected by mirrors at the front point as well as at the end of the rocket. The first signal is received after a time t_{A}, the second after a time t_{B}.

Calculate the velocity at which the rocket moves.

Determine at which distance from Earth the rocket is located when the first signal reaches Earth.

Exercise 8 (5 credit points): *Accelerated frames of reference*

8.1 Show that the equations

$$
\begin{aligned}
t &= \frac{c}{g}\,\sinh\!\left(\frac{g\,t'}{c}\right) + \frac{x'}{c}\,\sinh\!\left(\frac{g\,t'}{c}\right)\,,\\
x &= \frac{c^2}{g}\,\left[\cosh\!\left(\frac{g\,t'}{c}\right) - 1\right] + x'\,\cosh\!\left(\frac{g\,t'}{c}\right)\,,\\
y &= y'\,,\\
z &= z'\,,
\end{aligned}
$$

describe a transformation from an inertial frame to an accelerated frame of reference ($g = \text{const.}$). Calculate the components of the metric with respect to the frame (t', x', y', z').

8.2 Consider a frame of reference which is accelerated in x-direction as given in 8.1. In this frame a photon is emitted from a source at the height x_1 and is absorbed at the height x_2.

Calculate the frequency shift compared to a non-accelerated reference frame with $g = 0$. Perform an expansion up to the first order in gx'/c^2.

8.3 A photon "falls" in a homogeneous gravitational field $\mathbf{g} = g\,\mathbf{e}_x$ for a distance Δx along $\mathbf{e}_x$.

Use the equivalence of mass and energy (Planck–Einstein relation) and the formula for the potential energy known from the Newtonian theory of gravity to derive the frequency shift of the photon. Discuss your results with regard to the equivalence principle.

See reverse.

Exercise 9 (5 credit points): *Rindler coordinates*

Consider a two-dimensional metric

$$ds^2 = -\,v^2\,du^2 + dv^2\,.$$

At which point in space do the components of the metric tensor exhibit a singularity?

Find a coordinate transformation which shows that this so-called Rindler space is only a part of the two-dimensional Minkowski space, which is usually represented by $ds^2 = -\,dt^2 + dx^2$.

Compare the Rindler coordinates with the coordinates from exercise **8.1**.

Give an illustrative interpretation of the Rindler coordinates (consider $u =$ const. and $v =$ const.).

Determine the proper acceleration along the curve $v =$ const.