

2nd exercise sheet on Relativity and Cosmology I

Winter term 2013/14

Deadline for delivery: Thursday, 31st October 2013 at the beginning of the exercise class.

Exercise 4 (12 credit points): *Addition of velocities*

Consider a mass point moving with velocity \vec{w}' with respect to the inertial system \mathcal{I}' .

What is its velocity \vec{u} with respect to an inertial system \mathcal{I} if \mathcal{I}' moves with velocity \vec{v} against \mathcal{I} ? (Set $c = 1$.)
Show that the result can be written as

$$\vec{u} = \frac{\vec{v} + \vec{w}'_{\parallel} + \frac{\vec{w}'_{\perp}}{\gamma}}{1 + \vec{v} \vec{w}'},$$

where \vec{w}'_{\parallel} and \vec{w}'_{\perp} denote the parallel and orthogonal components of \vec{w}' with respect to \vec{v} , respectively.

Discuss the special cases $\vec{v} \parallel \vec{w}'$ and $\vec{v} \perp \vec{w}'$.

Show that

$$\vec{u}^2 = 1 - \frac{(1 - \vec{w}'^2)(1 - \vec{v}^2)}{(1 + \vec{v} \vec{w}')^2} \leq 1$$

and discuss the limiting case $|\vec{w}'| \rightarrow 1$.

Exercise 5 (8 credit points): *Aberration*

Consider an inertial system \mathcal{I}' that moves with velocity \vec{v} against an inertial system \mathcal{I} . Consider a ray of light which arrives in \mathcal{I} at an angle θ with respect to \vec{v} .

Under which angle θ' does this light ray arrive in \mathcal{I}' ? Show that this relation can be written in the form

$$\tan\left(\frac{\theta}{2}\right) = \sqrt{\frac{1+v}{1-v}} \tan\left(\frac{\theta'}{2}\right)$$

Hint: Use the law for the addition of velocities from exercise 4.