University of Cologne
Institute for Theoretical Physics
Prof. Dr. Claus Kiefer
Branislav Nikolic and Patrick Wong

20th January 2016

www.thp.uni-koeln.de/gravitation/courses/rci1616.html

11th exercise sheet on Relativity and Cosmology I

Winter term 2015/16

Deadline for delivery: Thursday, 28th January 2016 during the exercise class.

Exercise 27 (10 credit points): *Fierz–Pauli Lagrange density*

Consider the following Lagrange density *(Fierz and Pauli 1939)*:

$$\mathcal{L}_{\mathrm{FP}} = \frac{1}{8\pi G}\left(\psi^{\mu\nu,\sigma}\,\psi_{\mu\nu,\sigma} - 2\,\psi^{\mu\nu,\sigma}\,\psi_{\sigma\nu,\mu} - \psi^{\mu}{}_{\mu,\nu}\,\psi^{\rho}{}_{\rho,}{}^{\nu} + 2\,\psi^{\rho\nu}{}_{,\nu}\,\psi^{\sigma}{}_{\sigma,\rho}\right) + 2\,T_{\mu\nu}\,\psi^{\mu\nu}\,.$$

27.1 Show that the Lagrangian equations of motion following from this are equivalent to the linearized Einstein equations.

27.2 Calculate the canonical energy–momentum tensor

$$t_{\mu\nu} = \frac{\partial \mathcal{L}_{\mathrm{FP}}}{\partial \psi_{\alpha\beta,}{}^{\nu}}\,\psi_{\alpha\beta,\mu} - \eta_{\mu\nu}\,\mathcal{L}_{\mathrm{FP}}\,.$$

Exercise 28 (10 credit points): *Quadrupole formula*

In the lecture, the following expression for the energy flux was given for propagation in x-direction:

$$f_x = \frac{1}{4\pi G}\left[\frac{1}{4}\left(\dot{\psi}_{22} - \dot{\psi}_{33}\right)^2 + \dot{\psi}_{23}^2\right].$$

Repeat the steps that lead to the quadrupole formula and give the calculational details. Show, in particular, the following relations for the components n^i of a unit vector $\hat{n}$:

$$\frac{1}{4\pi}\int_{S^2} n^l n^m \,\mathrm{d}\Omega = \frac{1}{3}\,\delta_{lm}\,,$$

$$\frac{1}{4\pi}\int_{S^2} n^k n^l n^m n^r \,\mathrm{d}\Omega = \frac{1}{15}\left(\delta_{kl}\delta_{mr} + \delta_{km}\delta_{lr} + \delta_{kr}\delta_{lm}\right).$$