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www.thp.uni-koeln.de/gravitation/courses/rci13.html

2nd exercise sheet on Relativity and Cosmology I

Winter term 2015/16

Deadline for delivery: Thursday, 5th November 2015 at the beginning of the exercise class.

Exercise 4 (12 credit points): *Addition of velocities*

Consider a mass point moving with velocity $\vec{w}'$ with respect to the inertial system $\mathcal{I}'$.

What is its velocity $\vec{u}$ with respect to an inertial system $\mathcal{I}$ if $\mathcal{I}'$ moves with velocity $\vec{v}$ against $\mathcal{I}$? (Set $c = 1$.) Show that the result can be written as

$$\vec{u} = \frac{\vec{v} + \vec{w}'_{\parallel} + \frac{\vec{w}'_{\perp}}{\gamma}}{1 + \vec{v}\,\vec{w}'}\,,$$

where $\vec{w}'_{\parallel}$ and $\vec{w}'_{\perp}$ denote the parallel and orthogonal components of $\vec{w}'$ with respect to $\vec{v}$, respectively.

Discuss the special cases $\vec{v} \parallel \vec{w}'$ and $\vec{v} \perp \vec{w}'$.

Show that

$$\vec{u}^2 = 1 - \frac{\left(1 - \vec{w}'^2\right)\left(1 - \vec{v}^2\right)}{\left(1 + \vec{v}\,\vec{w}'\right)^2} \leq 1$$

and discuss the limiting case $|\vec{w}'| \to 1$.

Exercise 5 (8 credit points): *Aberration*

Consider an inertial system $\mathcal{I}'$ that moves with velocity $\vec{v}$ against an inertial system $\mathcal{I}$. Consider a ray of light which arrives in $\mathcal{I}$ at an angle θ with respect to $\vec{v}$.

Under which angle θ' does this light ray arrive in $\mathcal{I}'$? Show that this relation can be written in the form

$$\tan\left(\frac{\theta}{2}\right) = \sqrt{\frac{1+v}{1-v}}\,\tan\left(\frac{\theta'}{2}\right)$$

Hint: Use the law for the addition of velocities from exercise 4.