University of Cologne
Institute for Theoretical Physics
Prof. Dr. Claus Kiefer
Branislav Nikolic and Patrick Wong

4th November 2015

www.thp.uni-koeln.de/gravitation/courses/rci1516.html

3rd exercise sheet on Relativity and Cosmology I

Winter term 2015/16

Deadline for delivery: Thursday, 12th November 2015 at the beginning of exercise class.

Exercise 6 (5 credit points): *Inertial frames*

A rocket with a rest length L_0 moves with constant velocity radially away from Earth. From Earth a light pulse is emitted, which is then reflected by mirrors at the front as well as at the rear of the rocket. The first signal is received after the time t_A, the second after the time t_B.

6.1 Calculate the velocity at which the rocket moves in terms of L_0, t_A and t_B.

6.2 Determine at which distance from Earth the rocket is located when the first signal reaches Earth.

Exercise 7 (9 credit points): *Covariant Maxwell Equations*

Recall from classical electromagnetism the Maxwell equations:*

$$\vec{\nabla}\cdot\vec{E} = 4\pi\rho \qquad \vec{\nabla}\times\vec{B} - \frac{\partial\vec{E}}{\partial t} = 4\pi\vec{J} \tag{1a}$$

$$\vec{\nabla}\cdot\vec{B} = 0 \qquad \vec{\nabla}\times\vec{E} + \frac{\partial\vec{B}}{\partial t} = 0 \tag{1b}$$

In terms of the scalar (Φ) and vector ($\vec{A}$) potentials, the electric and magnetic fields are $\vec{E} = -\vec{\nabla}\Phi - \frac{\partial\vec{A}}{\partial t}$ and $\vec{B} = \vec{\nabla}\times\vec{A}$. Let $A^\mu = (\Phi, \vec{A})$ be the 4-potential, $j^\mu = (\rho, \vec{J})$ the 4-current, and define the new covariant tensor

$$F_{\mu\nu} := \partial_\mu A_\nu - \partial_\nu A_\mu\,. \tag{2}$$

7.1 Show that the expressions

$$\partial_\nu F^{\mu\nu} = 4\pi j^\mu \tag{3a}$$

$$\partial_{[\rho} F_{\mu\nu]} = 0 \tag{3b}$$

correspond to equations (1a) and (1b) respectively. Note that the notation $T_{[\rho\mu\nu]} = T_{\rho\mu\nu} + T_{\nu\rho\mu} + T_{\mu\nu\rho}$ has been used.

7.2 Show that (3a) leads to continuity equation $\frac{\partial\rho}{\partial t} = -\vec{\nabla}\cdot\vec{J}$. How does the continuity equation look like in a Lorentz-boosted reference frame?

Exercise 8 (6 credit points): *Covariant Lorentz Force*

Let $P^\mu = (E, \vec{p})$ be the 4-momentum, $u^\nu = (\gamma, \gamma\vec{v})$, the 4-velocity, τ proper time, and t coordinate time. Using (2), show that the space component of manifestly covariant 4-dimensional Lorentz force,

$$\frac{dP^\mu}{d\tau} = f^\mu = qF^{\mu\nu}u_\nu \tag{4}$$

gives for small velocities the well-known Lorentz force:

$$\frac{d\vec{p}}{dt} = \vec{F} = q\left(\vec{E} + \vec{v}\times\vec{B}\right)\,. \tag{5}$$

What is the physical meaning of the time component f^0 of the covariant 4-force (4)?

*In units where $c = \mu_0 = \epsilon_0 = 1$