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www.thp.uni-koeln.de/gravitation/courses/rci1516.html

5th exercise sheet on Relativity and Cosmology I

Winter term 2015/16

Deadline for delivery: Thursday, 26th November 2015 during the exercise class.

Exercise 12 (7 credit points): *Motion in the gravitational field*

The equation of motion for a test particle in a gravitational field is given by

$$\ddot{x}^\mu + \Gamma^\mu{}_{\nu\kappa}\,\dot{x}^\nu \dot{x}^\kappa = 0\,, \tag{1}$$

where $\dot{x}^\mu = \mathrm{d}x^\mu/\mathrm{d}\tau$, τ is the proper time and $\Gamma^\mu{}_{\nu\kappa} = \frac{1}{2}\,g^{\mu\sigma}\,(\partial_\kappa\, g_{\sigma\nu} + \partial_\nu\, g_{\sigma\kappa} - \partial_\sigma\, g_{\nu\kappa})$.

11.1 Repeat briefly the derivation of (1) from the variational principle $\delta\int \mathrm{d}\tau = 0$ as presented in the lecture. Why can the derivation not be used for photons?

11.2 Derive (1) from the alternative variational principle

$$\delta \int g_{\mu\nu}\,\dot{x}^\mu \dot{x}^\nu\,\mathrm{d}\lambda \equiv \delta \int \mathcal{K}\,\mathrm{d}\lambda = 0\,,$$

where λ is an affine parameter and $\dot{x}^\mu = \mathrm{d}x^\mu/\mathrm{d}\lambda$.
Show that this derivation also holds for photons and determine $\mathcal{K}$ for the solution of (1).

Exercise 13 (8 credit points): *Rotating reference frame*

Calculate the Christoffel symbols for a system which rotates with constant angular velocity ω around the z-axis in the Newtonian approximation and formulate the geodesic equation for this case.
Identify the centrifugal force and the Coriolis force in the resulting equation of motion.

Exercise 14 (5 credit points): *Curvature*

The ratio of the Scharzschild radius of a body to its radius is a heuristic measure for the deviation of the geometry in the neighbourhood of the considered body from a flat Minkowski spacetime.

15.1 Compare this ratio for a globular cluster of stars ($M \approx 10^6 M_\odot, R \approx 20$ pc), the Sun, the Earth, a neutron star ($M \approx M_\odot$, $R \approx 10$ km), a White Dwarf ($M \approx M_\odot$, $R \approx 10^4$ km) as well as for a proton and an electron. For the latter two, use their Compton wavelengths $\hbar/mc$ as the (effective) radius.

15.2 Which mass would an elementary particle need to have, such that its Compton wavelength would be as large as its Schwarzschild radius? What size would its Schwarzschild radius then be?

15.3 The quantities appearing in these considerations are often expressed in terms of the so-called Planck units, which result from a unique combination of the natural constants G, c and $\hbar$. Calculate the Planck mass, the Planck length, the Planck time and the Planck energy in CGS units or SI units.