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www.thp.uni-koeln.de/gravitation/courses/rci1616.html

6th exercise sheet on Relativity and Cosmology I

Winter term 2015/16

Deadline for delivery: Thursday, 3rd December 2015 during the exercise class.

Exercise 15 (5 credit points): *Curvature II*

Consider a family of Gaussian curves $z = \exp(-a^2 r^2)$ with $r^2 = x^2 + y^2$, embedded into a flat 3-dimensional space. Determine the metric on the surface formed by these Gaussian curves using polar coordinates (r, φ) and calculate the curvature at the apex using three different methods:

15.1 Use the two formulae given in the lecture: a) comparison of circumference and b) comparison of area.

15.2 Find the spherical shell with radius R that approximates the given surface best around the apex and use the known curvature of a sphere with radius R.

Exercise 16 (6 credit points): *Christoffel symbols*

Derive the transformation properties of the Christoffel symbols

$$\Gamma_{\mu\nu\lambda} = \frac{1}{2}\left(g_{\mu\nu,\lambda} + g_{\lambda\mu,\nu} - g_{\nu\lambda,\mu}\right)$$

under a coordinate transformation $x^\mu \to x'^\mu(x^\alpha)$.

(The result shows that the Christoffel symbols do not form a tensor.)

Exercise 17 (9 credit points): *Metricity*

It was stated in the lecture that the metric is *covariantly constant* for Riemannian spaces, meaning that its covariant derivative vanishes,

$$\nabla_\alpha g_{\mu\nu} = 0 \qquad \text{and} \qquad \nabla_\alpha g^{\mu\nu} = 0\,.$$

17.1 Prove the above two statements.

17.2 How does $\nabla_\alpha g_{\mu\nu} = 0$ transform under a coordinate transformation $x^\mu \to x'^\mu(x^\alpha)$?