University of Cologne www.thp.uni-koeln.de/gravitation/courses/rci1718.html
Institute for Theoretical Physics
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10th exercise sheet on Relativity and Cosmology I

Winter term 2017/18

Deadline for delivery: Thursday, 11th January 2017 during the exercise class.

Exercise 26: *Dust*

26.1 Derive the continuity equation and the Euler equation for dust given in the lecture within the framework of special relativity.

26.2 Show that in an arbitrary reference frame it follows from the conservation of the energy–momentum tensor of dust that dust particles move on geodesics.

Exercise 27: *Ideal fluid*

The energy–momentum tensor of an ideal fluid is given by

$$T^{\mu\nu} = \rho\, u^\mu u^\nu + P\left(u^\mu u^\nu + g^{\mu\nu}\right),$$

where u^μ is the four-velocity, ρ is the density and P is the pressure of the fluid.

27.1 Use the fact that the energy–momentum tensor of an ideal fluid is divergence-free to derive the continuity equation and the Euler equation.

27.2 Write out the continuity equation for the metric

$$g_{\mu\nu} = \mathrm{diag}\left[-1,\, a(t)^2,\, a(t)^2,\, a(t)^2\right].$$

Exercise 28: *Maxwell theory*

Consider the action of electromagnetic field in vacuum

$$S = \int \mathrm{d}^4x \left\{ -\frac{\sqrt{-g}}{16\pi} F_{\mu\nu} F^{\mu\nu} \right\}, \quad \text{where} \quad F_{\mu\nu} := \partial_\mu A_\nu - \partial_\nu A_\mu .$$

28.1 Derive the field equations by the action principle.

28.2 Calculate the Hilbert energy–momentum tensor $T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}}$.

28.3 Show by direct calculation that the divergence of the energy–momentum tensor $\nabla_\mu T^{\mu\nu}$ vanishes.