

6th exercise sheet on Relativity and Cosmology I

Winter term 2017/18

Deadline for delivery: Thursday, 30th November 2017 during the exercise class.

Exercise 16: *Curvature II*

Consider a family of Gaussian curves $z = \exp(-a^2 r^2)$ with $r^2 = x^2 + y^2$, embedded into a flat 3-dimensional space. Determine the metric on the surface formed by these Gaussian curves using polar coordinates (r, φ) and calculate the curvature at the apex using three different methods:

- 16.1 Use the two formulae given in the lecture: a) comparison of circumference and b) comparison of area.
- 16.2 Find the spherical shell with radius R that approximates the given surface best around the apex and use the known curvature of a sphere with radius R .

Exercise 17: *Christoffel symbols*

Derive the transformation properties of the Christoffel symbols

$$\Gamma_{\mu\nu\lambda} = \frac{1}{2} (g_{\mu\nu,\lambda} + g_{\lambda\mu,\nu} - g_{\nu\lambda,\mu})$$

under a coordinate transformation $x^\mu \rightarrow x'^\mu(x^\alpha)$.

Is this object a tensor?

Exercise 18: *Metricity*

It was stated in the lecture that the metric is *covariantly constant* for Riemannian spaces, meaning that its covariant derivative vanishes,

$$\nabla_\alpha g_{\mu\nu} = 0 \quad \text{and} \quad \nabla_\alpha g^{\mu\nu} = 0.$$

- 18.1 Prove the above two statements.
- 18.2 How does $\nabla_\alpha g_{\mu\nu} = 0$ transform under a coordinate transformation $x^\mu \rightarrow x'^\mu(x^\alpha)$?