

## 9<sup>th</sup> exercise sheet on Relativity and Cosmology I

Winter term 2017/18

**Deadline for delivery:** Thursday, 21<sup>st</sup> December 2017 during the exercise class.

### **Exercise 24:** *Contracted Bianchi identity from the action principle*

In the lecture we have derived the contracted Bianchi identity

$$\left( R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \right)_{;\mu} = 0.$$

Derive this identity this time from the Einstein–Hilbert action by demanding the action to be invariant under the infinitesimal coordinate transformations.

### **Exercise 25:** *Energy-momentum tensor of a scalar field*

One of the most important fields in physics is the scalar field  $\phi(x)$ . In General Relativity it is usually used to model various types of matter and so it is important to know its energy-momentum tensor  $T_{\mu\nu}$ .

**25.1** Derive the energy-momentum tensor from the action of a generic massive scalar field given by

$$S = \int d^4x \sqrt{-g} \left( -\frac{1}{2} g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} - \frac{m^2}{2} \phi^2 - V(\phi) \right),$$

where  $m$  is the mass parameter and  $V(\phi)$  is an arbitrary potential.

**25.2** Calculate the trace of this energy-momentum tensor.