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www.thp.uni-koeln.de/gravitation/courses/rcii16.html

10th exercise sheet on Relativity and Cosmology II

Summer term 2016

Deadline for delivery: Thursday, 30th June 2016 during the exercise class.

Exercise 20 (18 credit points): *Ideal fluids in Cosmology*

Consider an ideal fluid in a Friedmann–Lemaître model.

20.1 Show that comoving observers move on geodesics.

20.2 Evaluate the covariant conservation of the energy–momentum tensor of the ideal fluid for this kind of observers and show that this yields only *one* non-trivial equation, which on the other hand can also be deduced directly from the Friedmann equations.

20.3 Consider an equation of state of the form $p = w\rho$ with $w = \text{const.}$

Calculate the function $\rho(a)$. For which values of w does $\ddot{a} > 0$ hold? In which cases is the strong energy condition fulfilled?
Calculate $\rho(a)$ for a so-called "Chaplygin gas" whose equation of state is $p = -A/\rho$ ($A = \text{const.} > 0$) and discuss the extremal cases $a \to 0$ and $a \to \infty$.

20.4 Consider a flat Friedmann universe that satisfies $\Omega_\text{m} + \Omega_\text{x} = 1$, where Ω_m refers to pressureless matter and $\Omega_\text{x} = \rho_\text{x}/\rho_\text{c}$ denotes a hypothetical form of energy with density ρ_x and equation of state

$$p_\text{x} = w_\text{x}\rho_\text{x}\,.$$

Which condition has w_x depending on Ω_m to fulfill such that there is an accelerated expansion?
Calculate the Hubble parameter as a function of redshift, $H(z)$.

Exercise 21 (2 credit points): *Redshift of matter and vacuum energy density equality*

Current observations by the Planck satellite indicate that in the present universe $\Omega_{\text{m},0} \approx 0.31$ and $\Omega_\text{v} \approx 0.69$.

Calculate the redshift at which the energy density of matter was equal to that of the vacuum.