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www.thp.uni-koeln.de/gravitation/courses/rcii16.html

7th exercise sheet on Relativity and Cosmology II

Summer term 2016

Deadline for delivery: Thursday, 9th June 2016 during the exercise class.

Exercise 13 (4 credit points): *Accretion disks*

Give an estimate for the characteristic energy that is emitted by an accretion disk with radius R around a compact spherically symmetric object. For simplicity (even though this is not totally realistic), assume that the luminosity is that of a black body of radius R and temperature T and that it amounts to a given fraction ε of the Eddington luminosity. (At the end, use $\varepsilon \approx 0.5$.)

Exercise 14 (4 credit points): *Redshift in case of a gravitational collapse*

Consider an observer on the surface of a collapsing spherical star who emits radial light signals in short proper time intervals Δs, i. e. with a constant frequency $\omega_* = 2\pi/\Delta s$. These signals are received by a stationary observer at large distance $r = r_\mathrm{R}$, i.e. with a frequency $\omega_\mathrm{R} = 2\pi/\Delta t_\mathrm{R}$, where Δt_R refers to the Schwarzschild time.

Calculate the dependence of the frequency ratio $\omega_\mathrm{R}/\omega_*$ on t_R. Indicate the time scale of the redshift in terms of seconds if you measure M in solar masses.

Hint: Use Eddington–Finkelstein coordinates (as discussed in the lecture) and assume that the emitting observer is already located near the Schwarzschild radius.

Exercise 15 (12 credit points): *Newtonian cosmology*

15.1 In the Newtonian theory of gravity, how large is the total gravitational force of an infinitely extended mass distribution of mass density $\rho(\vec{x})$ on a mass m that is located at the origin? What kind of problem occurs with an infinitely large and homogeneous mass distribution?

15.2 Consider a "universe" with a homogeneous mass density $\rho(t)$. Write out the Newtonian equation of motion for an arbitrary galaxy of mass m whose radius vector with respect to the Earth be denoted by $\vec{x}(t)$. In order to calculate the gravitational force, use Newton's spherical shell theorem with the Earth as the origin. What kind of problem does this lead to?

According to the Hubble law, we have

$$\vec{x}(t) = a(t)\,\vec{x}_0 \qquad \text{with} \qquad \frac{\dot{a}}{a} =: H(t)\,.$$

($\vec{x}_0$ denotes the location for an arbitrarily given time.)
Formulate the equation of motion for $a(t)$. Use the continuity equation to eliminate $\rho(t)$. Is a static universe ($a = \text{const.}$) possible? Show that integration leads to the "energy theorem"

$$\dot{a}^2 - \frac{C}{a} + k = 0\,,$$

where $C > 0$ and k are constants. How can one interpret k? Sketch $a(t)$ roughly for the three cases $k < 0$, $k = 0$ and $k > 0$.

15.3 Add ad hoc a repulsive force $m\,\vec{x}\,\Lambda/3$ with "cosmological constant" $\Lambda > 0$ to the Newtonian gravitational force. Formulate the modified equation of motion for $a(t)$ as well as the modified "energy theorem". Is it now possible to have a static universe?