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http://www.thp.uni-koeln.de/gravitation/courses/rcii18.html

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11th exercise sheet on Relativity and Cosmology II

Summer term 2018

Deadline for delivery: Thursday, 12th July 2018 during the exercise class.

Exercise 50 (6 credit points): *Friedmann I*

Consider a Friedmann model with $k \neq 0$ and present density parameters $\Omega_{m,0}$, $\Omega_{r,0}$ and $\Omega_{v,0}$ as well as $\Omega := \Omega_{m,0} + \Omega_{r,0} + \Omega_{v,0}$. Furthermore let

$$\rho_c(a) = \frac{3\,\dot{a}^2}{8\pi G\, a^2}$$

be the critical density at a time when the scale parameter had the value a, and let $\Omega_m(a) = \rho_m(a)/\rho_c(a)$ etc. be the corresponding relative densities.

Determine the quantity $\Omega(a) - 1$ as a function of $\Omega_{m,0}$, $\Omega_{r,0}$, $\Omega_{v,0}$ and a. This quantity indicates how much the considered model "deviated" from a flat model at a certain time. What kind of problem with regard to the deviation from flatness at earlier times arises for a Friedmann model whose density parameter Ω differs only slightly from unity today?

Exercise 51 (8 credit points): *Friedmann II*

Solve the Friedmann equation for a universe that contains radiation as well as non-relativistic matter (dust). For this purpose, rewrite the Friedmann equation as a differential equation with respect to the conformal time η, solve this equation for the three possible values of k and write out the result in the form $(a(\eta), t(\eta))$.

Determine the t-dependence of $a(t)$ for early $(t \to 0)$ and late $(t \to \infty)$ times for all the possible values of k.

Exercise 52 (6 credit points): *Friedmann III*

Current observations indicate that we live in a flat $(k = 0)$ universe with positive cosmological constant, in which the contribution of radiation to the total energy density can be neglected, i. e. the only contributions come from non-relativistic matter (dust) and the cosmological constant.

Solve the Friedmann equation for this model. (*Hint:* The substitution $x^2 = (1/\Omega_{m,0} - 1)\, a^3$ could be helpful.) Determine the age of the universe as a function of H_0 and $\Omega_{m,0}$. How does $a(t)$ behave for large and small values of t?

According to well-established models for stellar evolution, several globular clusters in our galaxy are at least 12 billion years old. Draw a $(h - \Omega_m)$-diagram (h is the parameter in the definition of H_0) and sketch the contour lines for a constant age of the universe. Determine which parameter range is compatible with the above-mentioned observation. In doing so, only consider values $0.4 < h < 1$.