Poincaré gauge theory of gravity — An Introduction

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Motivation or “what if...”?

History:

- field equations of General Relativity, Einstein (1915)
- discovery of spin, Pauli (1924); Uhlenbeck & Goudsmit (1925)
- relativistic description of spin, Dirac (1928)
- gauge theories: Weyl (1918, 1929, 1950), Yang–Mills (1954)
- gravity as gauge theory: Utiyama (1956), Sciama (1960), Kibble (1961)

“Newton successfully wrote apple = moon, but you cannot write apple = neutron.”

– J. L. Synge

The Dirac equation, minimally coupled to gravity:

$$i\gamma^{\alpha} e^{j}{}_{\alpha} \left(\partial_j + \tfrac{i}{4}\Gamma_i\right)\Psi + m\Psi = 0$$

Problem: the frame field has to be put into General Relativity by hand.

What if spin had been discovered before General Relativity?

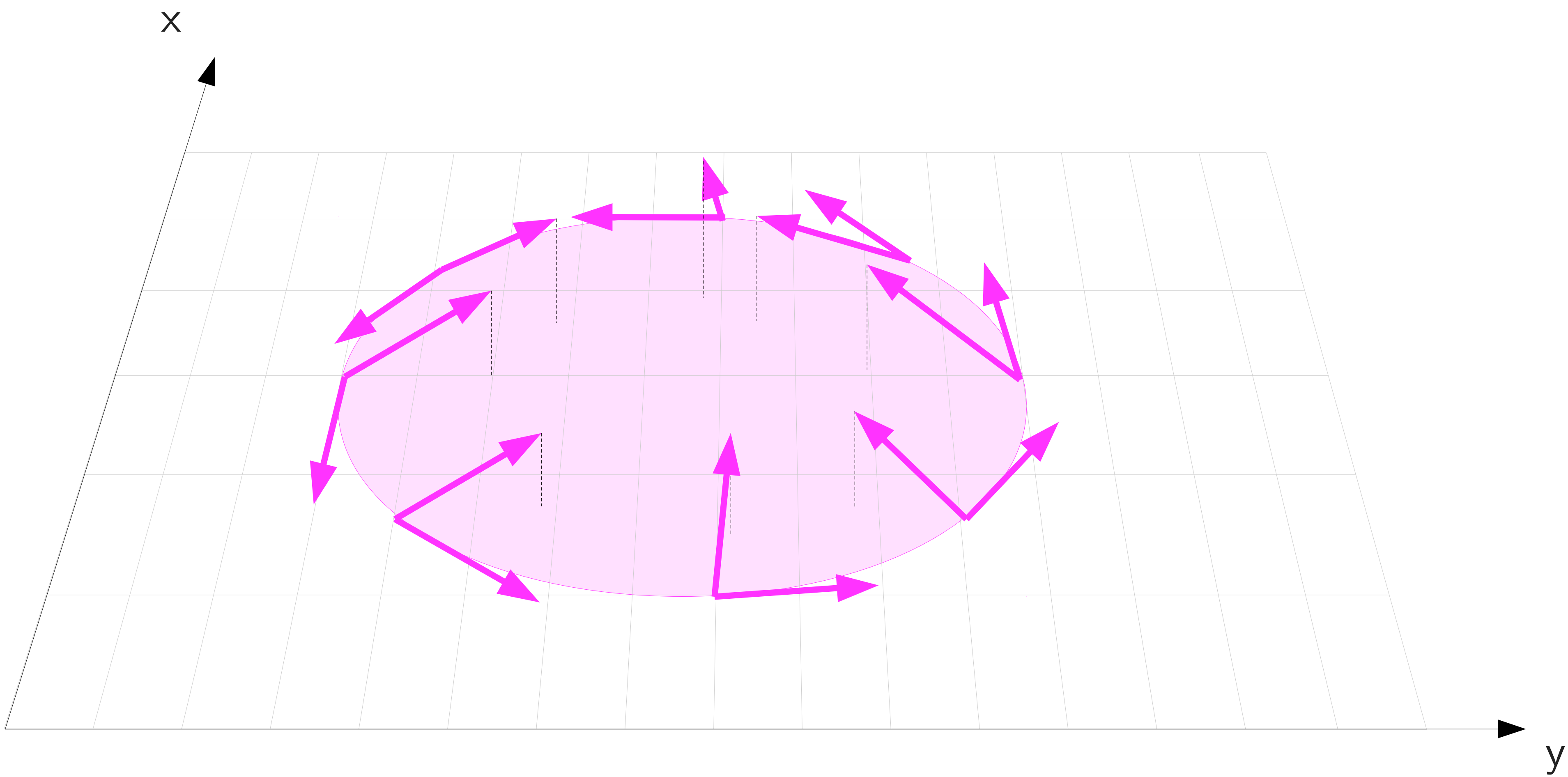
Would Einstein have applied the equivalence principle to a neutron instead?

Physical interpretation of the frame field

The frame field $e^j{}_\mu$ supplies us with orthonormal basis vectors on the curved space:
(necessary for spinor representation ⟶ it is a field of **fundamental** importance)

Physical interpretation of the frame field

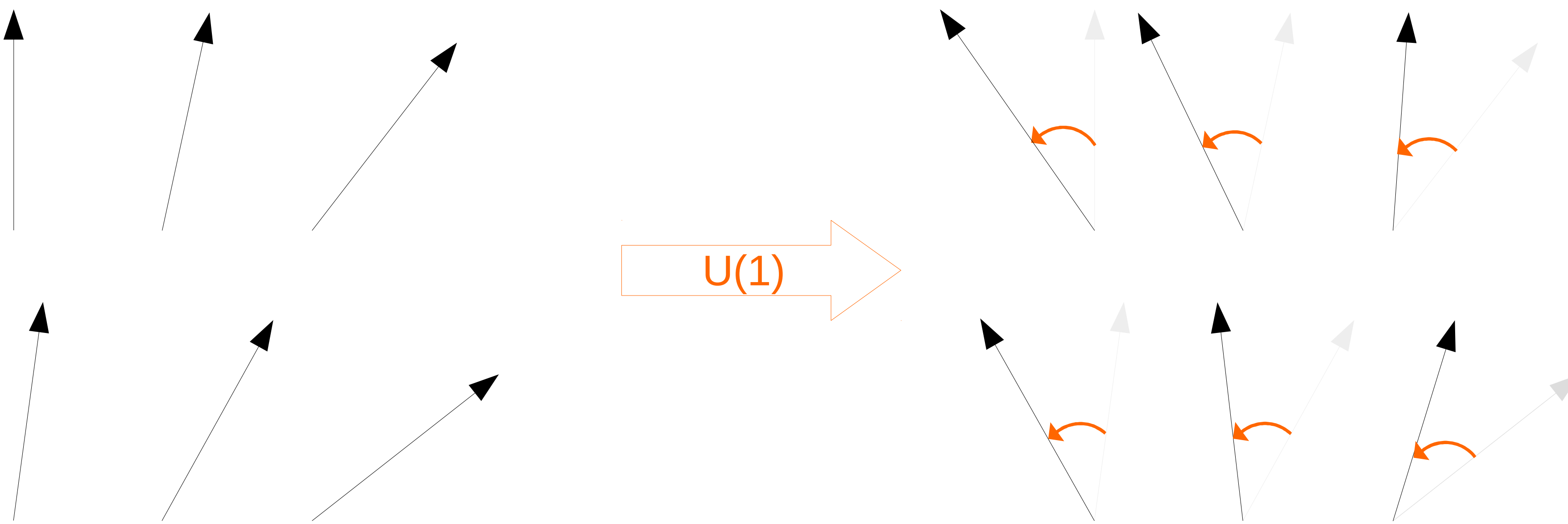
Think about the frame field displayed in a random coordinate space: it rotates!



Is there a gauge principle involved?

Example: a brief description of U(1) gauge theory

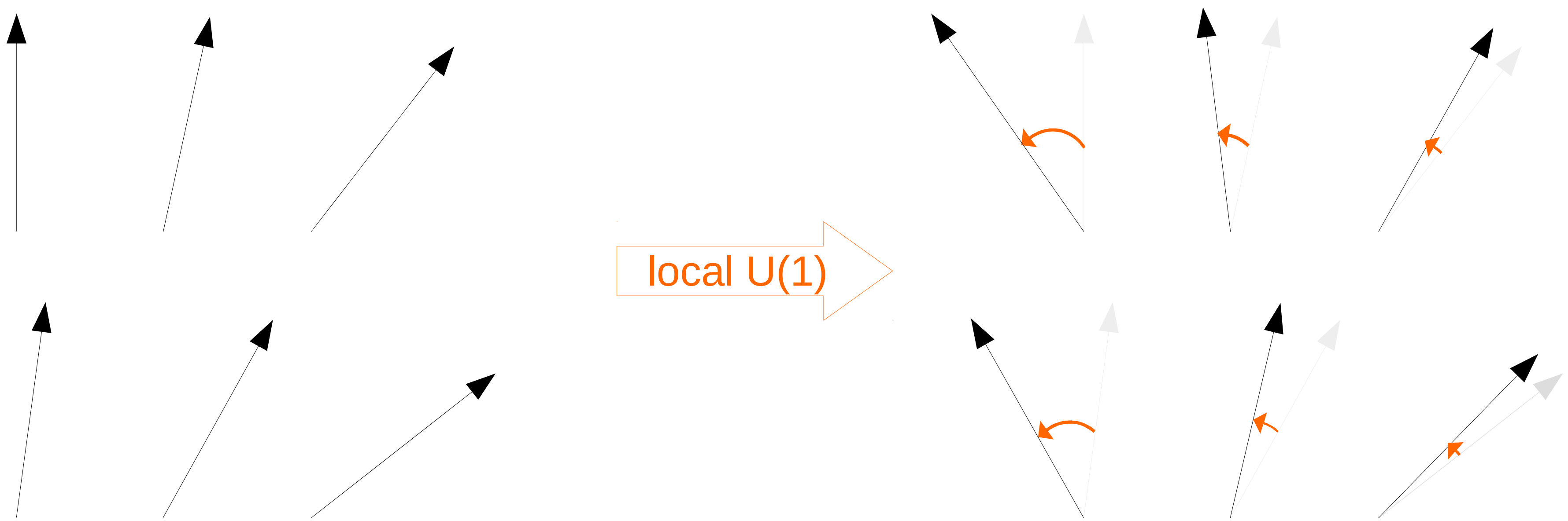
Consider a complex field ϕ under a global U(1) transformation $\phi \mapsto e^{i\alpha}\phi$, with $\alpha \in \mathbb{R}$:



If the theory is invariant under this transformation, we call U(1) a **rigid symmetry**.

Example: a brief description of U(1) gauge theory

Now carry out a local transformation $\phi \mapsto e^{i\alpha(x)}\phi$:

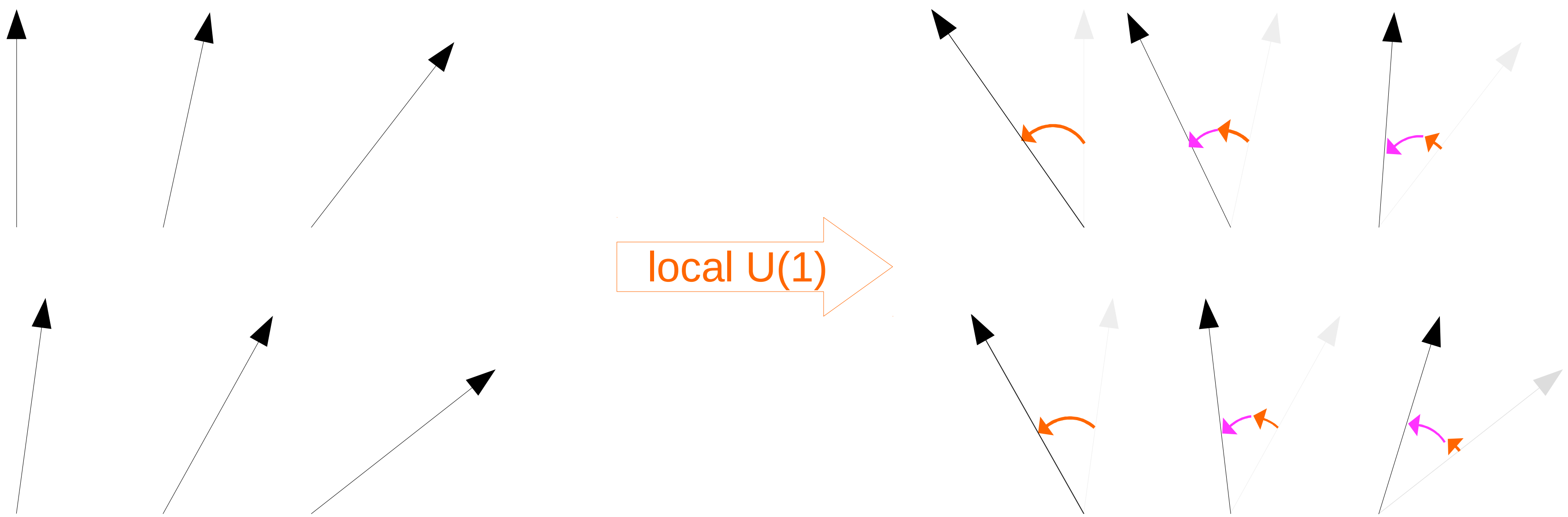


Due to x-dependence, any dynamical theory is **not invariant anymore**.

How do we rescue this? We need a gauge potential A!

Example: a brief description of U(1) gauge theory

The gauge potential restores gauge invariance by $\mathrm{d} \mapsto \mathrm{d} + ie A$:



What have we gained? We can construct a Lagrangian for A using $F := \mathrm{d}A$:

$\mathcal{L} := F \wedge \star F + j \wedge A$ yields **electrodynamics** with conserved current j

Towards a gauge theory of gravity

We saw: to describe electrodynamics as a gauge theory, we have to

1. **forget** about electrodynamics (!)
2. carry out a **gauge procedure** with a suitable group, here: U(1)
3. obtain electrodynamics **for free** from gauge curvature Lagrangian

To describe gravity as a gauge theory, we first **have to forget about gravity**.

What remains if we do that?

special relativity,
and fields propagating on flat Minkowski space

Note the difference: symmetries in **external** space, not in internal space.

Symmetries of Minkowski space

Translational invariance: four parameters
conserved energy momentum

Rotational invariance: six parameters
conserved spin-angular momentum

Total symmetry group: the **Poincaré group** $\mathrm{P}(1,3) = \mathrm{T}^{1,3} \rtimes \mathrm{SO}(1,3)$

Poincaré gauge theory of gravity

After applying the gauge procedure, there are **two** gauge fields:

- the coframe $\vartheta^\mu = \mathrm{e_i}^\mu \mathrm{dx^i}$, which is essentially the frame field
  translational invariance, four parameters
  field strength: **torsion**
  source for torsion: **spin-angular momentum**

- the Lorentz connection $\Gamma_{\mu\nu}$, an additional gauge potential
  rotational invariance, six parameters
  field strength: **curvature**
  source for curvature: **energy-momentum**

These gauge potentials can be used to define a viable theory of gravity (Einstein–Cartan theory in a spacetime with curvature and torsion).

We fall back to General Relativity for vanishing torsion.

Conclusions & Outlook

- Yes, it is possible to formulate gravity as a gauge theory.
- In Poincaré gauge theory, the frame field $e^j{}_\mu$ is the gauge potential of translations, and it is accompanied by the Lorentz connection $\Gamma_{\mu\nu}$ as the rotational potential
- Gauge approach helpful for quantization?
- see also: Loop Quantum Gravity (but vanishing torsion)

Literature:

M. Blagojevic and F. W. Hehl (eds.), "Gauge Theories of Gravitation – A reader with commentaries," (Imperial College Press, London, 2013)