Quantum gravity and aspects of relativity

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WHO ARE WE???

Gravitation and Relativity research group
Prof. Dr. Claus Kiefer, Prof. Dr. Friedrich Hehl

- Institute of Theoretical Physics, Cologne

- have a look at the webpage of our group!

www.thp.uni-koeln.de/gravitation/

- join us at our seminars: Tuesdays 12h, Seminar Room 215

- email us if you have any questions, don’t be afraid! ☺

WHAT ARE WE GOING TO DO TODAY?

- tell you about what kind of interesting things we are doing in our group

- hang out with you during the coffee breaks and lunch

This talk:

A. Crash course in General Relativity

B. Crash course in Canonical Quantum Gravity

C. Overview of today's talks

A. Crash course in General Relativity

- Einstein, ~1916

geometry of non-empty (non-vacuum) 4D spacetime is not flat, but ***curved***!

Minkowski metric $\eta_{\mu\nu} = \mathrm{diag}(-1,+1,+1,+1)$ → general metric $g_{\mu\nu}(\vec{x},t)$
(encodes gravitational field)

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

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- **Einstein's Equations:**

geometry of 4D spacetime <~> matter

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

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curvature ($(\partial g_{\mu\nu})^2 , \partial^2 g_{\mu\nu}$)

cosmological constant

energy-momentum tensor

<u>to solve EE means:</u>

given matter distribution/symmetry

↓

find the metric $g_{\mu\nu}(\vec{x}, t)$

A. Crash course in General Relativity

- for example:

  - Friedman-Lemaitre-Robertson-Walker metric (models a homogeneous and isotropic universe)

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right)$$

scale factor

Friedmann equations:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$

$$\dot{H} + H^2 = \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)$$

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  - Schwarzschild solution (non-rotating Black Hole!)

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***2. Singularities in Generalized Chaplygin Gas model* (Arezu)**

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SINGULARITIES!

***3. Black Holes and Naked Singularities* (Alessandro)**

B. Crash course in Canonical Quantum Gravity

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  - three interactions of Nature can be unified in a common framework; include gravity ("theory of everything") ? → gravitational field must also be quantized!

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  - the problem of time:

    4D spacetime (described by metric tensor) is ***fixed*** in QFT

    VS

    in GR, spacetime is ***dynamical***

    so how do you describe a quantum field propagating on a cruved background?

B. Crash course in Canonical Quantum Gravity

→ semiclassical Einstein equations:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} \langle \psi | \hat{T}_{\mu\nu} | \psi \rangle$$

classical metric
(classical geometry)

quantum matter

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but Psi depends on the metric, which we cannot find without solving EE!?!
very, very, very non-linear problem!

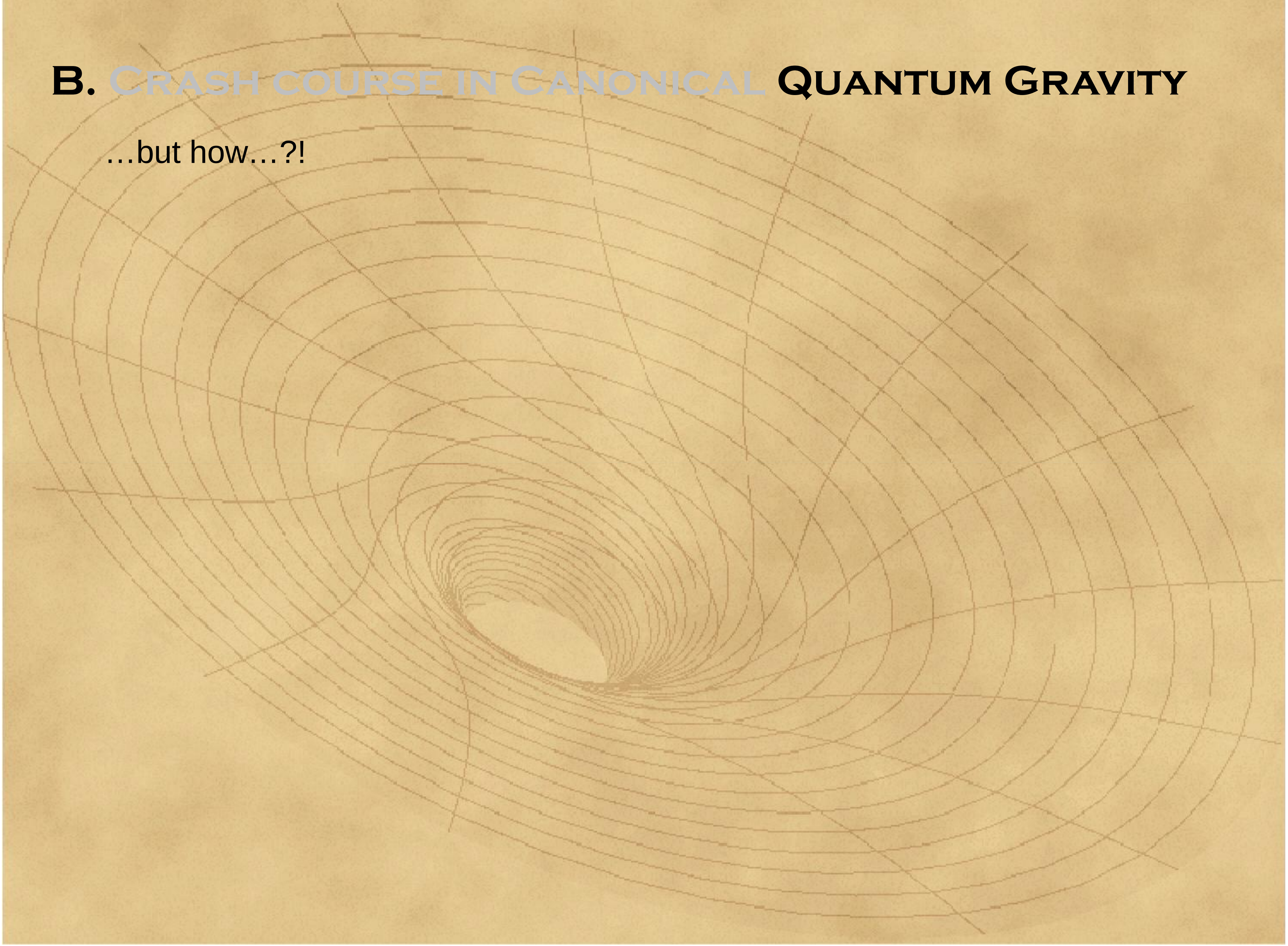
→ approximation to a more fundamental theory, which includes quantized metric, too

↑

a quantum theory of gravity

B. Crash course in Canonical Quantum Gravity

…but how…?!



B. Crash course in Canonical Quantum Gravity

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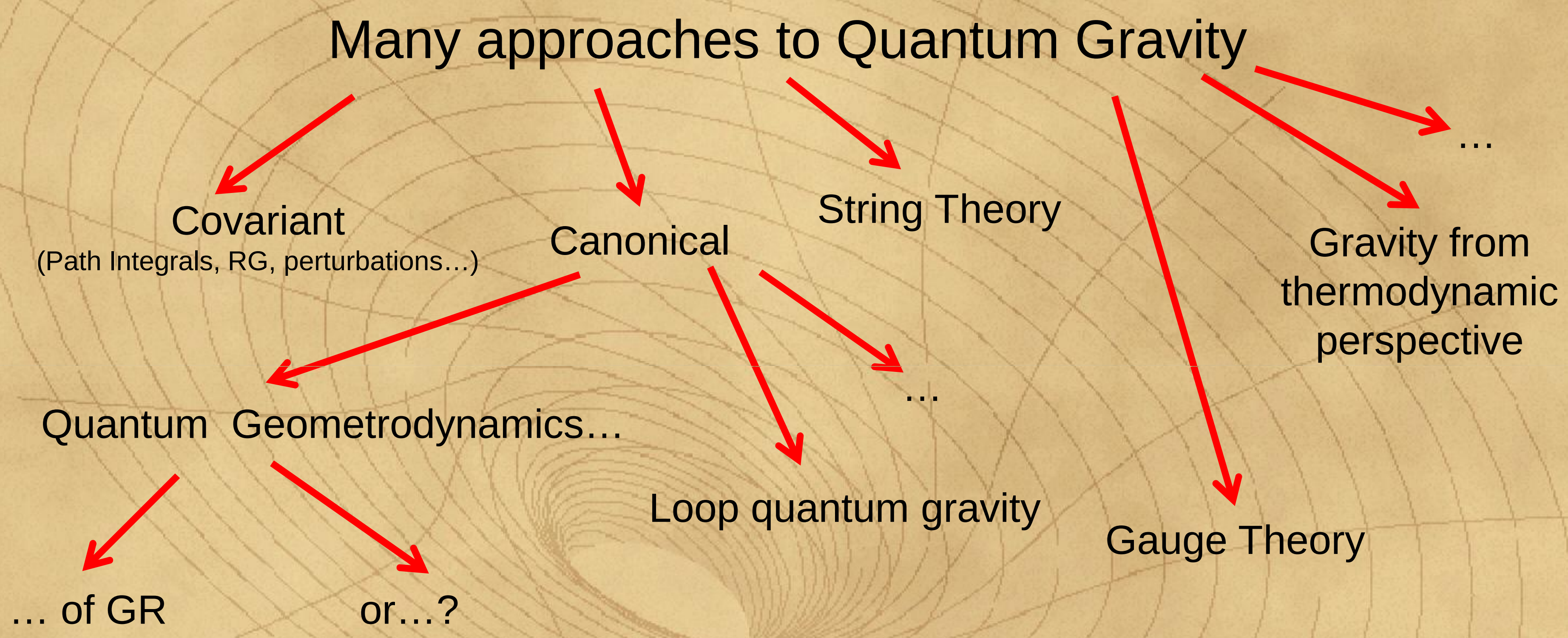

B. Crash course in Canonical Quantum Gravity

…this is how we try…

Many approaches to Quantum Gravity

Canonical

Gravity from thermodynamic perspective

Quantum Geometrodynamics…

Loop quantum gravity

Gauge Theory

… of GR

or…?

B. Crash course in Canonical Quantum Gravity

-you can write down the Hamiltonian for a particle in a potential
(using Hamiltonian formulation) - write

H = H (gen. coord., conj. mom.)

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- you can even write down the Hamiltonian of General Relativity! …how?

  - separate spacetime into 3D space + time, and look at geometry of 3D space only
  (write, hypersurfaces)

$$g_{\mu\nu} = \begin{pmatrix} -N^2 + N^j N_j & N_i \\ N_i & h_{ij} \end{pmatrix}$$

3-metric
“coordinate”

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conjugate momentum

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3-metric "coordinate" conjugate momentum

- Hamiltonian of GR (*ADM formalism*):

$$\mathcal{H}_0 = \frac{16\pi G}{\sqrt{h}} \mathcal{G}_{ijkl} p^{ij}_{ADM} p^{kl}_{ADM} - \frac{\sqrt{h}}{16\pi G}\left({}^{(3)}R - 2\Lambda\right) \approx 0$$

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- as in ordinary quantum mechanics, Dirac quantization procedure:

$$\hat{h}_{ij}(x)\Psi = h_{ij}(x)\Psi \qquad \hat{p}^{ij}(x)\Psi = -i\hbar\frac{\delta}{\delta h_{ij}(x)}\Psi$$

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…with which you can do almost no physics… ☺

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- what is Psi and defined on what kind of space? wave functional, lives in Superspace

- Hilbert space and unitarity?! no space, no time

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B. Crash course in Canonical Quantum Gravity

…that is why you have to pick a model… (minisuperspace model)

- gravity + scalar field → **wave function of the Universe:**

$$\mathcal{H}_0 \Psi_0(\alpha, \phi) \equiv$$
$$\frac{e^{-3\alpha}}{2}\left[\frac{1}{m_P^2}\frac{\partial^2}{\partial\alpha^2} - \frac{\partial^2}{\partial\phi^2} + e^{6\alpha}m^2\phi^2\right]\Psi_0(\alpha, \phi) = 0$$

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- semiclassical approximation: expansion in terms of Planck mass:

$$\Psi_k(\alpha, f_k) = e^{i\,S(\alpha, f_k)}$$

$$S(\alpha, f_k) = m_P^2\,S_0 + m_P^0\,S_1 + m_P^{-2}\,S_2 + \ldots.$$

- let's see what happens order by order…

B. Crash course in Canonical Quantum Gravity

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zeroth order: $$\left[\frac{\partial S_0}{\partial \alpha}\right]^2 - V(\alpha) = 0\,, \quad V(\alpha) := \mathrm{e}^{6\alpha} H^2$$

1st order: $$\mathrm{i}\,\frac{\partial}{\partial t}\,\psi_k^{(0)} = \mathcal{H}_k \psi_k^{(0)}$$

Schroedinger equation $$\frac{\partial}{\partial t} := -\,\mathrm{e}^{-3\alpha}\frac{\partial S_0}{\partial \alpha}\,\frac{\partial}{\partial \alpha}$$

2nd order: $$\mathrm{i}\,\frac{\partial}{\partial t}\,\psi_k^{(1)} = \mathcal{H}_k \psi_k^{(1)} - \frac{\mathrm{e}^{3\alpha}}{2m_{\mathrm{P}}^2 \psi_k^{(0)}}\left[\frac{(\mathcal{H}_k)^2}{V}\psi_k^{(0)} + \mathrm{i}\,\frac{\partial}{\partial t}\left(\frac{\mathcal{H}_k}{V}\right)\psi_k^{(0)}\right]\psi_k^{(1)}$$

quantum gravitational corrections *to Schroedinger equation*

B. Crash course in Canonical Quantum Gravity

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-- apply to the Cosmic Microwave Background power spectrum:

$$\Delta^2_{(1)}(k) \simeq \Delta^2_{(0)}(k) \times \left[1 - \frac{123.83}{k^3}\frac{H^2}{m_{\mathrm{P}}^2} + \frac{1}{k^6}\mathcal{O}\left(\frac{H^4}{m_{\mathrm{P}}^4}\right)\right]^2$$

$$\Delta^2_{(1)}(k) \simeq \Delta^2_{(0)}(k)\left[1 - 1.76 \times 10^{-9}\,\frac{1}{k^3} + \frac{\mathcal{O}(10^{-15})}{k^6}\right]^2$$

C. Overview of today's talks

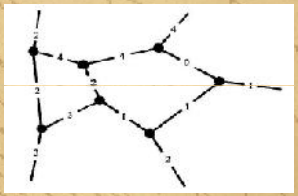
Canonical

Gravity from thermodynamic perspective

6. Pranjal

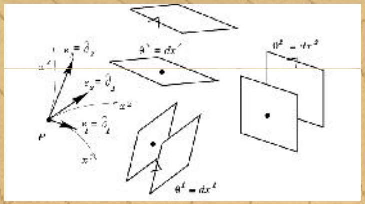
Quantum Geometrodynamics…

Loop quantum gravity



5. Patrick

Gauge Theory



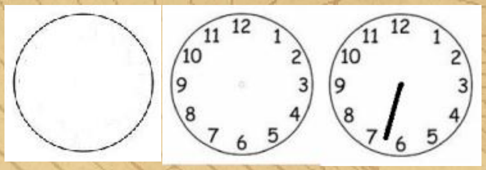
4. Jens

… of GR

2. Arezu

3. Alessandro

…of Conformal Gravity



7. Branislav