Loop Quantum Gravity

The Geometry of Quantized Space

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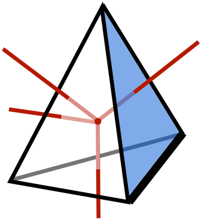
BCGS Weekend Seminar
Bad Honnef
19 April, 2015

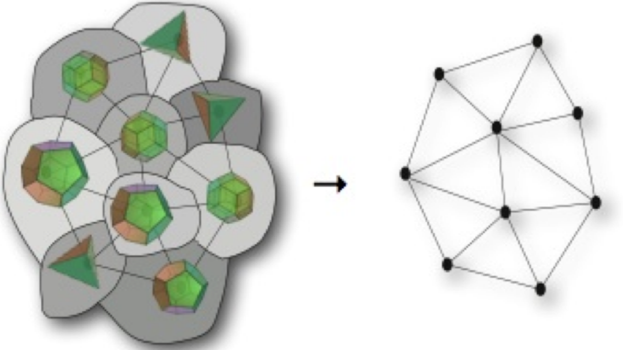
Overview

Elements of Loop Quantum Gravity

- Takes the Einsteinian interpretation of gravity literally
  - gravity $\Leftrightarrow$ the geometry of spacetime
  - "gravity does not exist"
- Gauge-like approach to gravity
  - $SU(2)$ symmetry
  - analogous to quantum angular momentum
- Build up quantum description of space

Discrete Quantum Spacetime





Quantum State Structure

Build formalism based on lattice gauge theory

- Variables: $(A^i{}_a, E_i{}^a)$
  - Like vector potential $\vec{A}$ and electric field $\vec{E}$ in EM
- Use holonomy

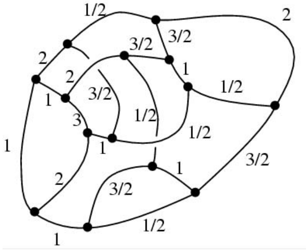
$$U_\gamma[A] = \exp\left\{ \oint_\gamma A(x)^i{}_\mu \tau^i \mathrm{d}x^\mu \right\}$$

- Lattice gauge theory: $U_\gamma[A]$ lives on edges
- Gravity: lattice $\Rightarrow$ Spin Network

Spin Networks

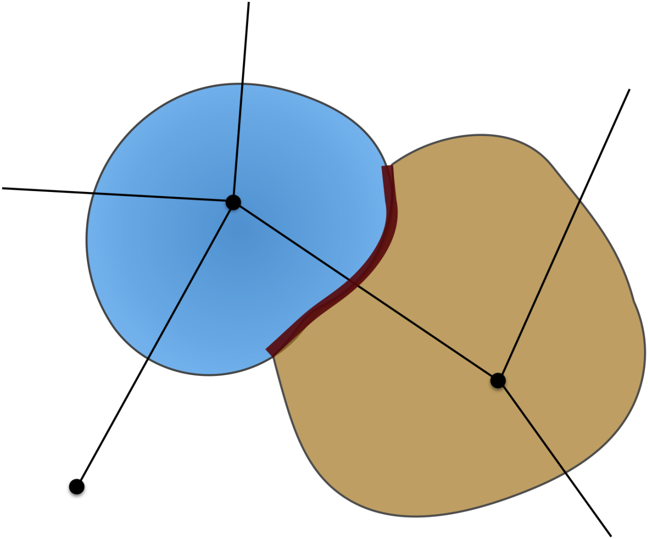
$$U_\gamma[A] = \exp\left\{ \oint_\gamma A(x)^i{}_\mu \tau^i \mathrm{d}x^\mu \right\}$$

U live on links, each with $SU(2)$ representation j



Spin networks are not states *on* spacetime,
they're states *of* spacetime!

Discrete Quantum Spacetime



Remarks on the variables

Hamiltonian phase space: $(A^i{}_a, E_i{}^a) \sim (q, p)$
What are they?

- Stem from frame-connection representation of gravity
- $E_i{}^a \sim$ frame field
- $A^i{}_a$ is a combination of two potentials:

$$A^i{}_a = \Gamma^i{}_a + \beta K^i{}_a$$

- For $(A^i{}_a, E_i{}^a)$ canonical, actually need $(A^i{}_a, E_i{}^a/\beta)$
  - Comes from $p = \frac{\delta S}{\delta q}$
- Symplectic structure defined by

$$\left\{A^i{}_a(x), E_j{}^b(y)\right\} = 8\pi G\beta\ \delta^b_a \delta^i_j \delta^3(x-y)$$

Quantum of Area

Classical:

$$\mathcal{A} = \int_\Sigma \mathrm{d}\sigma \sqrt{\det g^{(2)}} = \int_\Sigma \mathrm{d}\sigma \sqrt{E \cdot E}$$

Quantum:

$$\hat{\mathcal{A}} = \int_\Sigma \mathrm{d}\sigma \sqrt{\hat{E}^2}$$

$\hat{E}$ is $SU(2)$ like $\hat{L}$ in QM - eigenvalues of $\hat{L}^2$ are $j(j+1)$

$$\hat{\mathcal{A}}|\Psi\rangle = 8\pi\beta\ell_p^2\sqrt{j(j+1)}|\Psi\rangle \qquad (1)$$

Planck length $\ell_P = \sqrt{\hbar G}$

Discrete Quantum Spacetime

Length Scales for Space

Utah Salt Flats

$\sim$ Universe without matter

Space without matter

2× Zoom

4× Zoom

Invariance under scaling dilation

Space with matter

Scaling is fixed by a reference object!

Length scales

Require matter for well-defined length scale

$$\hat{\mathcal{A}}|\Psi\rangle = 8\pi\beta\ell_p^2\sqrt{j(j+1)}|\Psi\rangle$$
$$= 8\pi\ell_\star^2\sqrt{j(j+1)}|\Psi\rangle$$

To properly define $\ell_\star$, need to define β
Possibilies:

- β is itself a field: $\beta \to \beta(x)$
- β couples gravity to fermions, fermions provide length scale

Thank you for your attention!