EMERGENT GRAVITY AND COSMOLOGY: THERMODYNAMIC PERSPECTIVE

Master Colloquium

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Work done at:
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Motivation:

Gravity and Quantum Theory

Points of contact and conflict

- Black hole singularity
- Big Bang singularity
- Cosmological constant problem

OUTLINE:

Notion of emergence in Gravity: AdS/CFT and Verlinde's Entropic gravity

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Notion of emergence in Gravity: AdS/CFT and Verlinde's Entropic gravity (Refer thesis)

1. Gravity as emergent phenomenon: Sakharov Paradigm

2. Temperature and Law of Equipartition in Gravity

3. BH thermodynamics → Horizon thermodynamics

4. Action Functional : Hint of alternative description

5. Holographically conjugated variables ↔ thermodynamic conjugacy

6. Holographic equipartition

7. Emergent Cosmology

8. Further Investigations and my work

1. Gravity as an emergent phenomenon

Sakharov Paradigm

Solids

Spacetime

Mechanics, Elasticity $(\rho, v \ldots)$

Einstein's Theory $(g_{ab} \ldots)$

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Statistical mechanics of atoms/molecules (well-known)

Spacetime

Einstein's Theory $(g_{ab} \ldots)$

Emergence : Different dynamical variables → different descriptions.

1. Gravity as an emergent phenomenon

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Mechanics, Elasticity $(\rho, v \ldots)$

↑

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Einstein's Theory $(g_{ab} \ldots)$

↑ $\langle g_{ab}(q^i, \lambda_A) \rangle$

Statistical mechanics of ``atoms of spacetime'' λ_A (?)

Emergence : Different dynamical variables → different descriptions.

2. Temperature and Law of Equipartition:

Boltzmann's postulate: Anything that can be heated has `atomic' structure!

Equipartition Law: $$E_1 = E_2 = ... = E_n \equiv \varepsilon = \frac{1}{2} k_B T$$

Equipartition of energy connects thermodynamics to microscopic d.o.f.

$$\boxed{\Delta n = \frac{\Delta E}{\left(\frac{1}{2}\right) k_B T}}$$

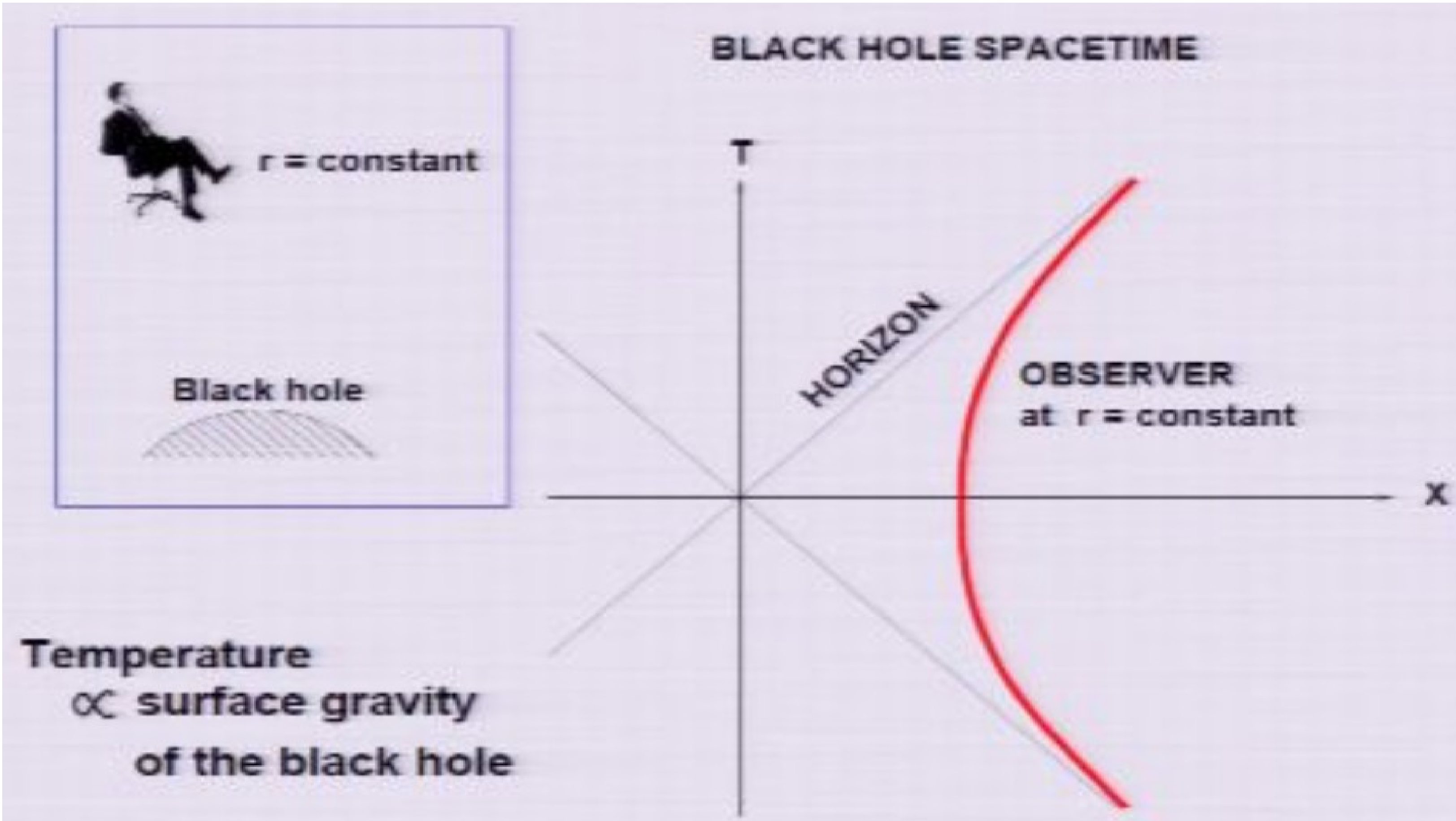
Temperature of matter told us that it has `atomic' structure

$$\boxed{E = n\varepsilon \to \int \mathrm{d}V \frac{\mathrm{d}n}{\mathrm{d}V} \frac{1}{2} k_B T = \frac{1}{2} k_B \int \mathrm{d}n T}$$

demands granularity with finite *n*; degrees of freedom scale as volume.

Spacetimes can be Hot!!

Static observer in Schwarzschild spacetime



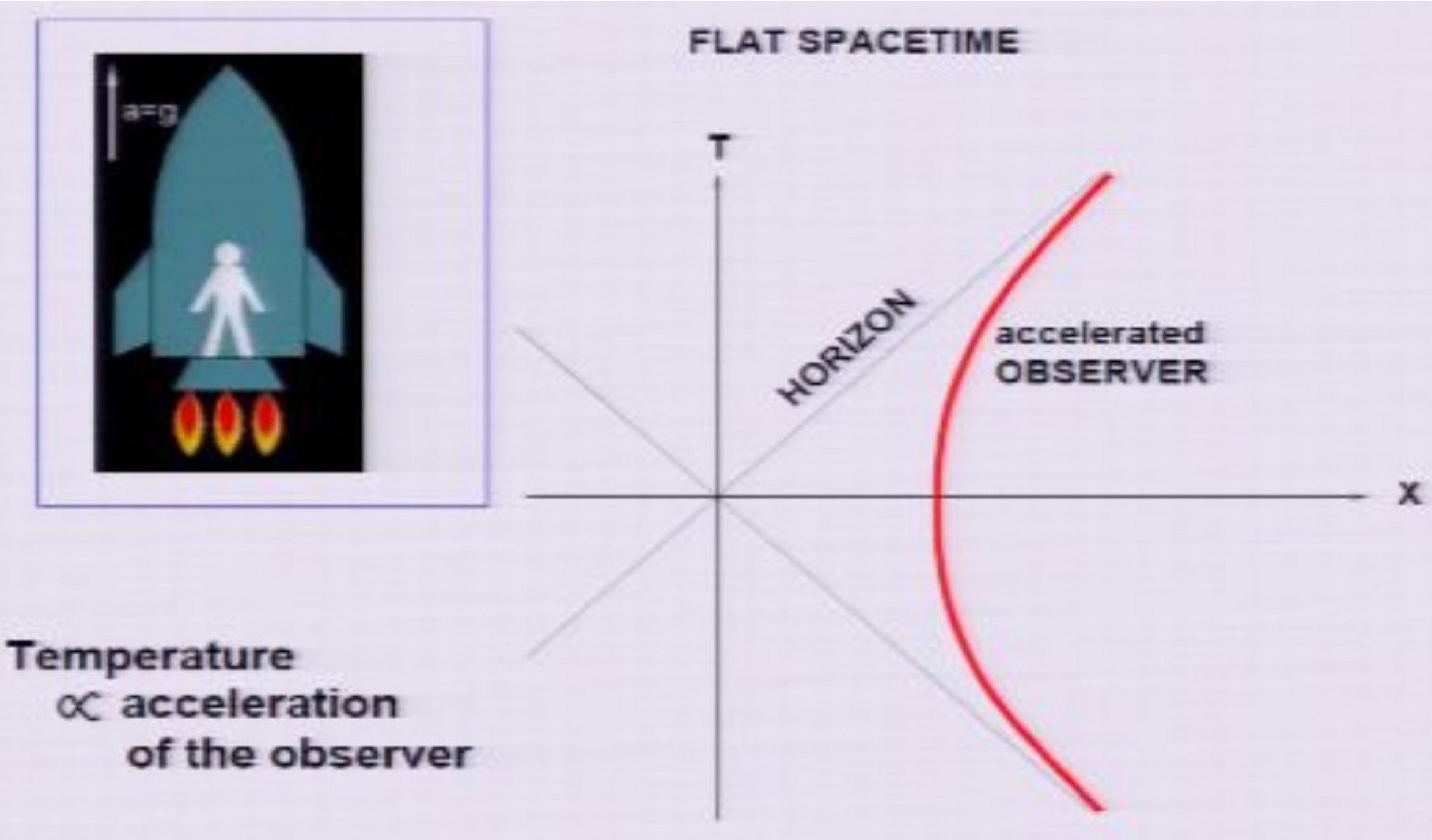
$$k_B T = \frac{\hbar}{c}\left(\frac{g}{2\pi}\right) = \frac{\hbar}{c}\frac{GM}{2\pi r^2}$$

Hawking temperature

[1975]

Spacetimes can be Hot!!

Rindler observer in flat spacetime



$$k_B T = \frac{\hbar}{c}\left(\frac{a}{2\pi}\right)$$

Davies-Unruh temperature [1976]

Indistinguishability of thermal and quantum fluctuations

Curved spacetime, $a = 0$

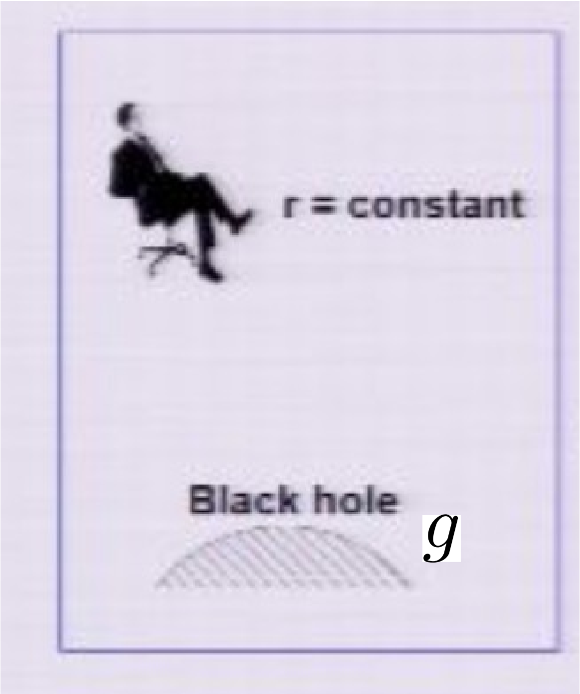
Flat spacetime, $a = g$

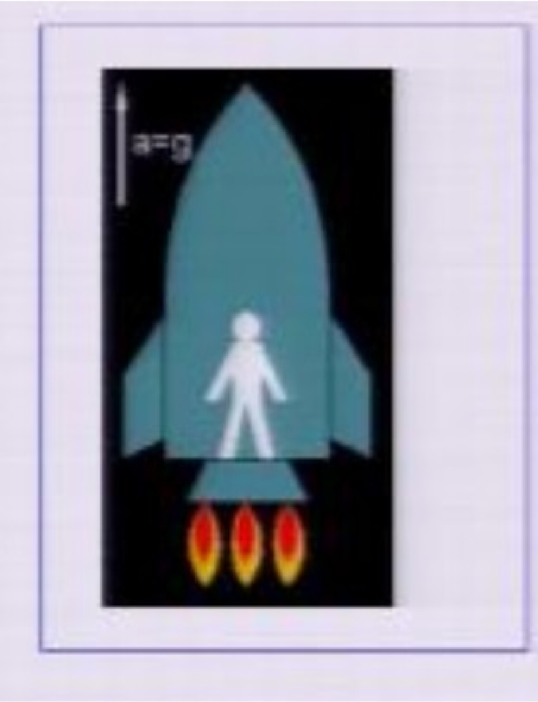
Hawking effect

$$\rho(T, 0) = \rho(0, T)$$

Unruh effect

Indistinguishability of thermal and quantum fluctuations





Curved spacetime, $a = 0$

Flat spacetime, $a = g$

Hawking effect

$$\rho(T, 0) = \rho(0, T)$$

Unruh effect

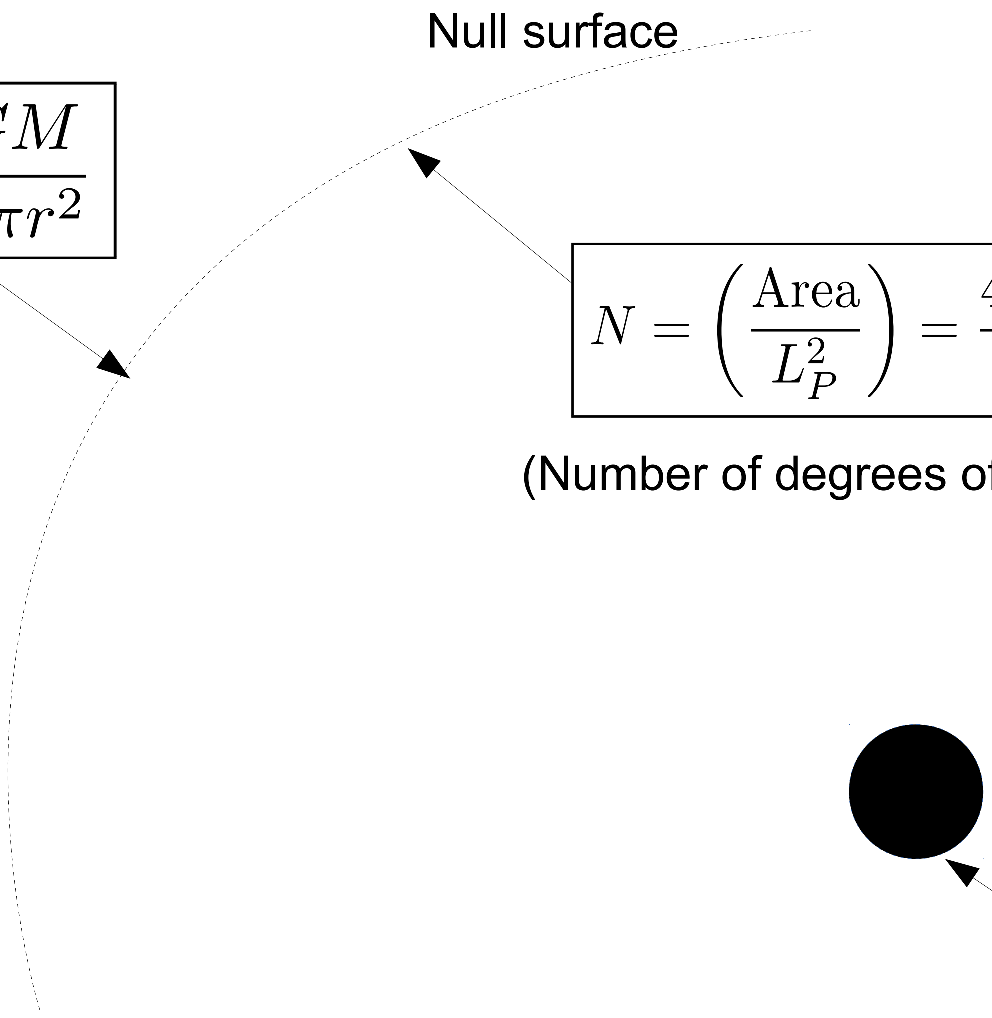
Generalisation to arbitrary spacetime:

$$\rho(T, T') = \rho(T', T)$$

Kolekar, T.P. [gr-qc/1308.6289v2]

Equipartition of `Area Bits' at null surface

TP(2004) [gr-qc/0308070]



$$k_B T = \frac{\hbar}{c}\left(\frac{g}{2\pi}\right) = \frac{\hbar}{c}\frac{GM}{2\pi r^2}$$

(Hawking Temperature)

$$N = \left(\frac{\text{Area}}{L_P^2}\right) = \frac{4\pi r^2 c^3}{G\hbar}$$

(Number of degrees of freedom)

$$R = \frac{2GM}{c^2}$$

Equipartition Law:

$$E = N \cdot \left(\tfrac{1}{2}\right) k_B T = \frac{\hbar}{c}\frac{GM}{4\pi r^2}\frac{4\pi r^2 c^3}{G\hbar} = Mc^2$$

3. Black hole thermodynamics → Horizon thermodynamics

BH Thermodynamics:

$$\delta M = \frac{g}{8\pi G}\delta A + \Omega_H \delta J$$

$$S = \frac{1}{4}\frac{A_H}{L_P^2}$$

$$\boxed{\mathrm{d}E = T\mathrm{d}S + P\mathrm{d}V}$$

3. Black hole thermodynamics → Horizon thermodynamics

Any static, spherically symmetric spacetime with horizon:

$$ds^2 = -f(r)c^2dt^2 + f^{-1}(r)dr^2 + r^2 d\Omega_2^2$$

Horizon: $r = a,\ f(a) = 0$

Temperature of horizon

$$k_B T = \frac{\hbar c f'(a)}{4\pi}$$

Einstein's equation evaluated at horizon

$$\frac{c^4}{G}\left[\frac{1}{2}f'(a)a - \frac{1}{2}\right] = 4\pi P a^2$$

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Horizons at $r = a$ and $r = a + \mathrm{d}a$:

$$\underbrace{\frac{\hbar c f'(a)}{4\pi}}_{T}\underbrace{\frac{c^3}{G\hbar}\mathrm{d}\left(\frac{1}{4}4\pi a^2\right)}_{\mathrm{d}S} - \underbrace{\frac{1}{2}\frac{c^4\mathrm{d}a}{G}}_{\mathrm{d}E} = \underbrace{P\mathrm{d}\left(\frac{4\pi}{3}a^3\right)}_{P\mathrm{d}V}$$

$$S = \frac{1}{4L_P^2}(4\pi a^2) = \frac{1}{4}\frac{A_H}{L_P^2}$$

$$\mathrm{L}_P^2 = \frac{G\hbar}{c^3}$$

BH Thermodynamics:

$$\delta M = \frac{\kappa}{8\pi G}\delta A + \Omega_H \delta J$$

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Horizon Thermodynamics:

$$\boxed{\underbrace{\frac{\hbar c f'(a)}{4\pi}}_{T}\underbrace{\frac{c^3}{G\hbar}\mathrm{d}\left(\frac{1}{4}4\pi a^2\right)}_{\mathrm{d}S} - \underbrace{\frac{1}{2}\frac{c^4\mathrm{d}a}{G}}_{\mathrm{d}E} = \underbrace{P\mathrm{d}\left(\frac{4\pi}{3}a^3\right)}_{P\mathrm{d}V}}$$

4. Action Functional: Hint of alternative description

$$A_{EH} = \int \mathrm{d}^4x \sqrt{-g} R = \int \mathrm{d}^4x \left(\mathcal{L}_{bulk} + \mathcal{L}_{sur} \right)$$

Important!

$A_{sur} \rightarrow$ entropy of horizon $\rightarrow$ *field equations*:

$$(\mathrm{G}_{ab} - 8\pi T_{ab}) u^a u^b = 0$$

TP(2009)[gr-qc/0912.3165]

4. Action Functional: Hint of alternative description

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Important!

Why this happens??

$$\mathcal{L}_{sur} = -\left[\partial_c \left(g_{ab} \frac{\partial \mathcal{L}_{bulk}}{\partial(\partial_c g_{ab})}\right)\right]$$

HOLOGRAPHIC REDUNDANCY!!

TP(2005)[gr-qc/0412068]

$A_{sur} \rightarrow$ entropy of horizon $\rightarrow$ *field equations*:

$$(\mathrm{G}_{ab} - 8\pi T_{ab}) u^a u^b = 0$$

TP(2009)[gr-qc/0912.3165]

5. Holographically conjugated variables → thermodynamic conjugacy

Majhi,Parattu,TP,(2013)

Canonical General Relativity via conjugated variables:

$$f^{ab} = \sqrt{-g}\, g^{ab} \qquad N^c_{ab} = \frac{\partial \mathcal{L}_{bulk}}{\partial(\partial_c f^{ab})} = -\Gamma^c_{ab} + \frac{1}{2}\left(\Gamma^d_{ad}\delta^c_b + \Gamma^d_{ad}\delta^c_b\right)$$

$$\mathcal{H}_g = f^{ab}\left(N^c_{ad}N^d_{bc} - \frac{1}{3}N^c_{ac}N^d_{bd}\right)$$

$$\boxed{\partial_c f^{ab} = \frac{\partial \mathcal{H}_g}{\partial N^c_{ab}}} \qquad \boxed{\partial_c N^c_{ab} = -\frac{\partial \mathcal{H}_g}{\partial f^{ab}} + 8\pi\left[T_{ab} - \frac{1}{2}g_{ab}T\right]}$$

$$(\nabla_c g^{ab}) = 0$$

Thermodynamic Conjugacy:

$$\boxed{\frac{1}{16\pi}\int d^3\Sigma_c\left(N^c_{ab}\delta f^{ab}\right) = T\mathrm{d}S}$$

$$\boxed{\frac{1}{16\pi}\int d^3\Sigma_c\left(f^{ab}\delta N^c_{ab}\right) = S\mathrm{d}T}$$

6. Holographic Equipartition

Surface DoF:

$$N_{sur} \equiv \frac{A}{L_P^2} = \int_{\partial V} \frac{\sqrt{\sigma}\mathrm{d}^2 x}{L_P^2}$$

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Evolution of geometry

$$\frac{1}{8\pi L_P^2} \int_V \mathrm{d}^3 x \sqrt{h} u_a g^{ij} \pounds_\xi N^a_{ij} =$$

$$= \underbrace{\int_{\partial V} \frac{\mathrm{d}^2 x \sqrt{\sigma}}{L_P^2} \epsilon \left(\frac{1}{2} k_{\mathrm{B}} T_{\mathrm{loc}} \right)}_{N_{sur}} - \underbrace{\int_{\mathrm{V}} \mathrm{d}^3 x \sqrt{h} \rho_{\mathrm{Komar}}}_{N_{bulk}}$$

Static geometry: no evolution

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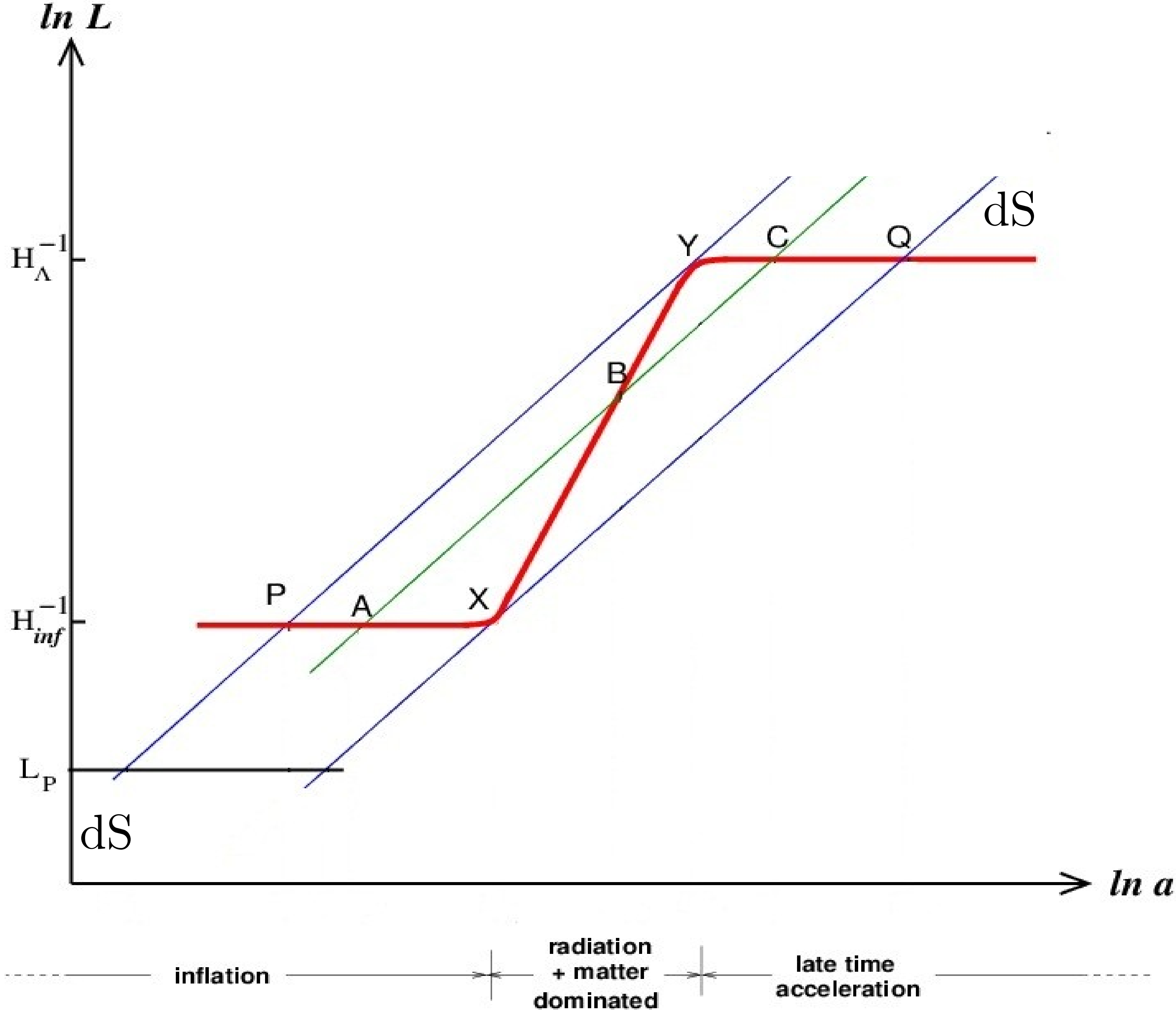
Static geometry: no evolution ⟶ Holographic equipartition! $N_{sur} - N_{bulk} = 0$

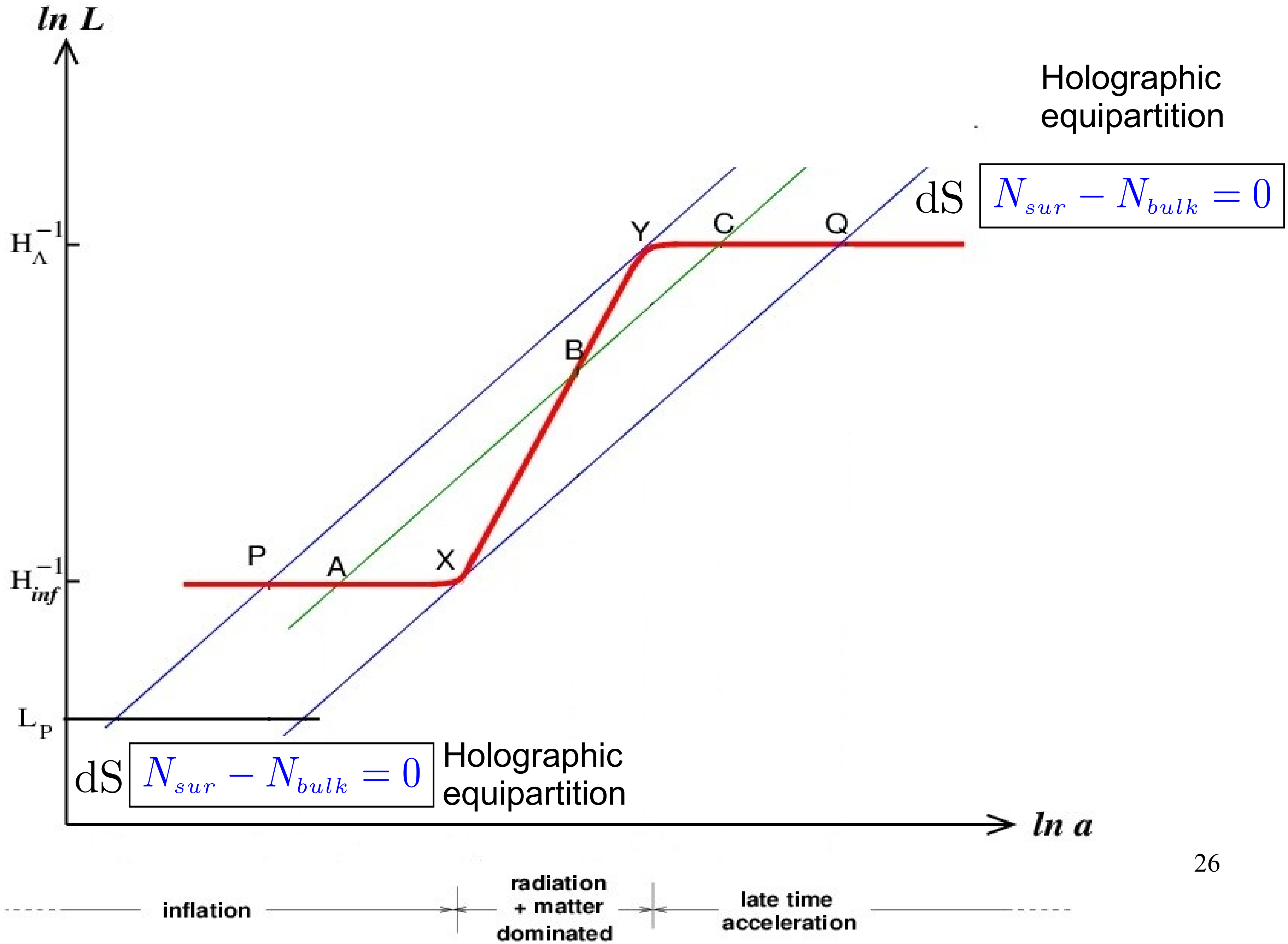
7. Emergent Cosmology

Bjorken(2006),TP(2013)



TP(2014)

7. Emergent Cosmology



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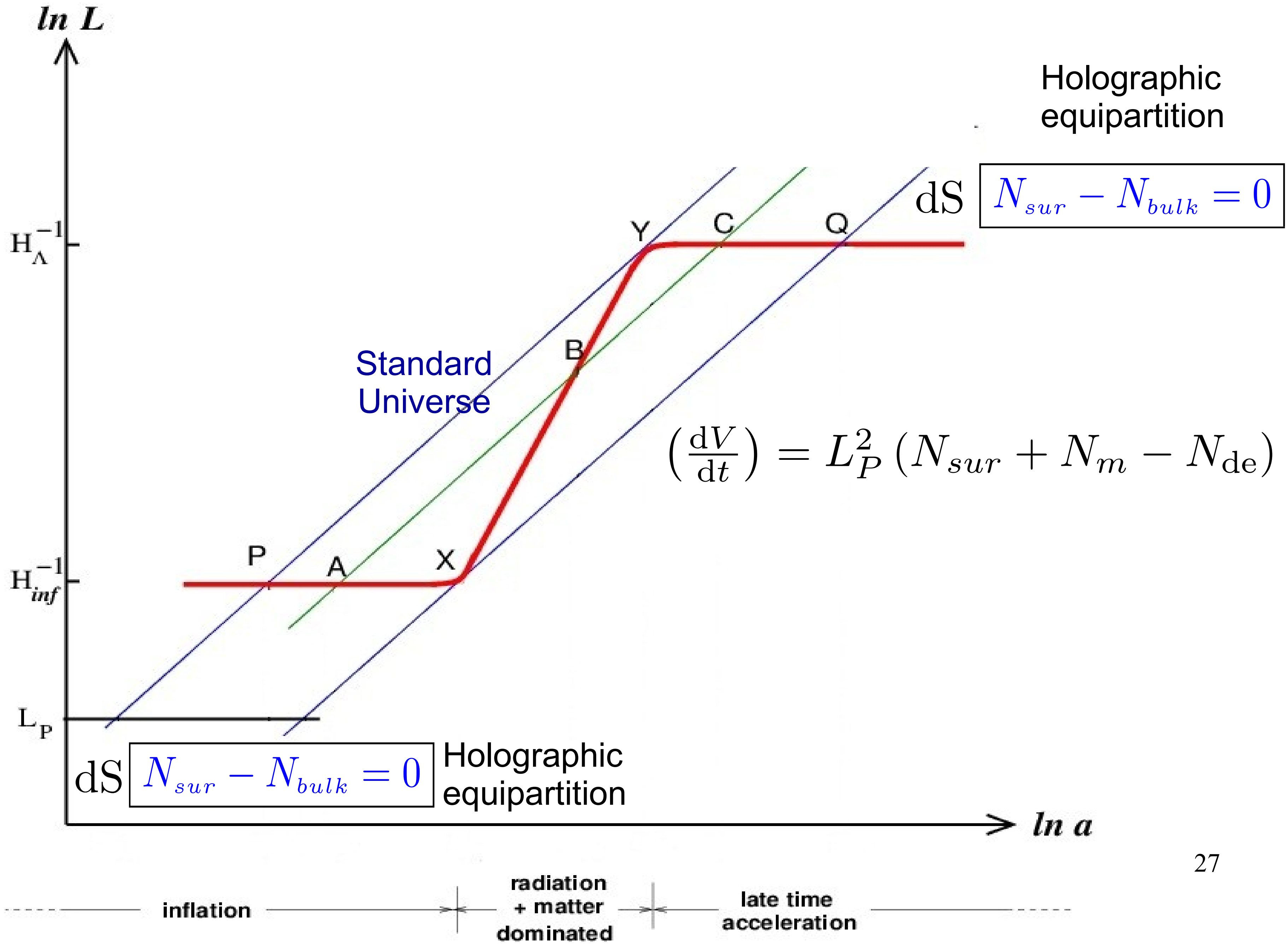
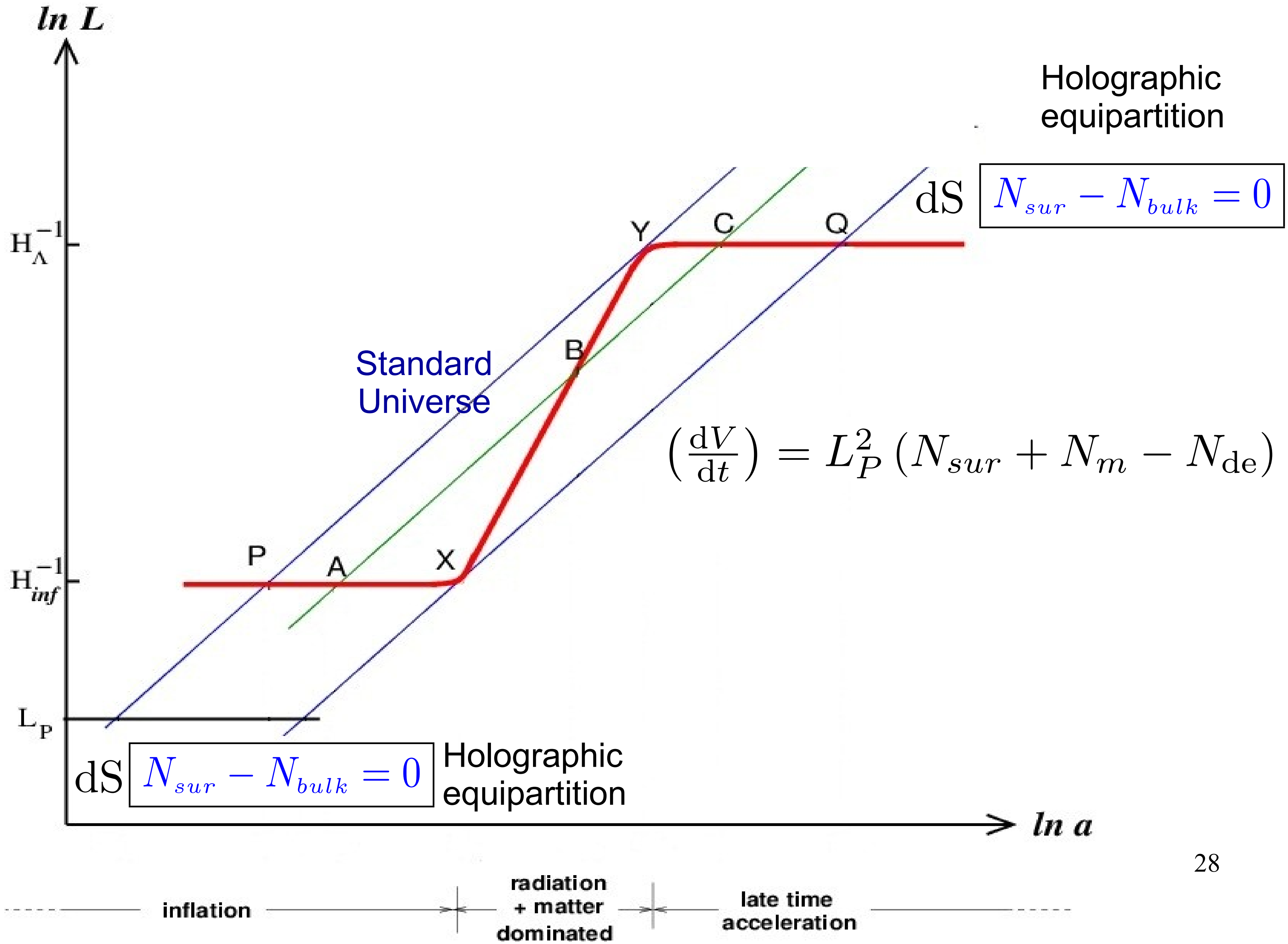


7. Emergent Cosmology

"Chicken is egg's way of making another egg!"

TP(2014)



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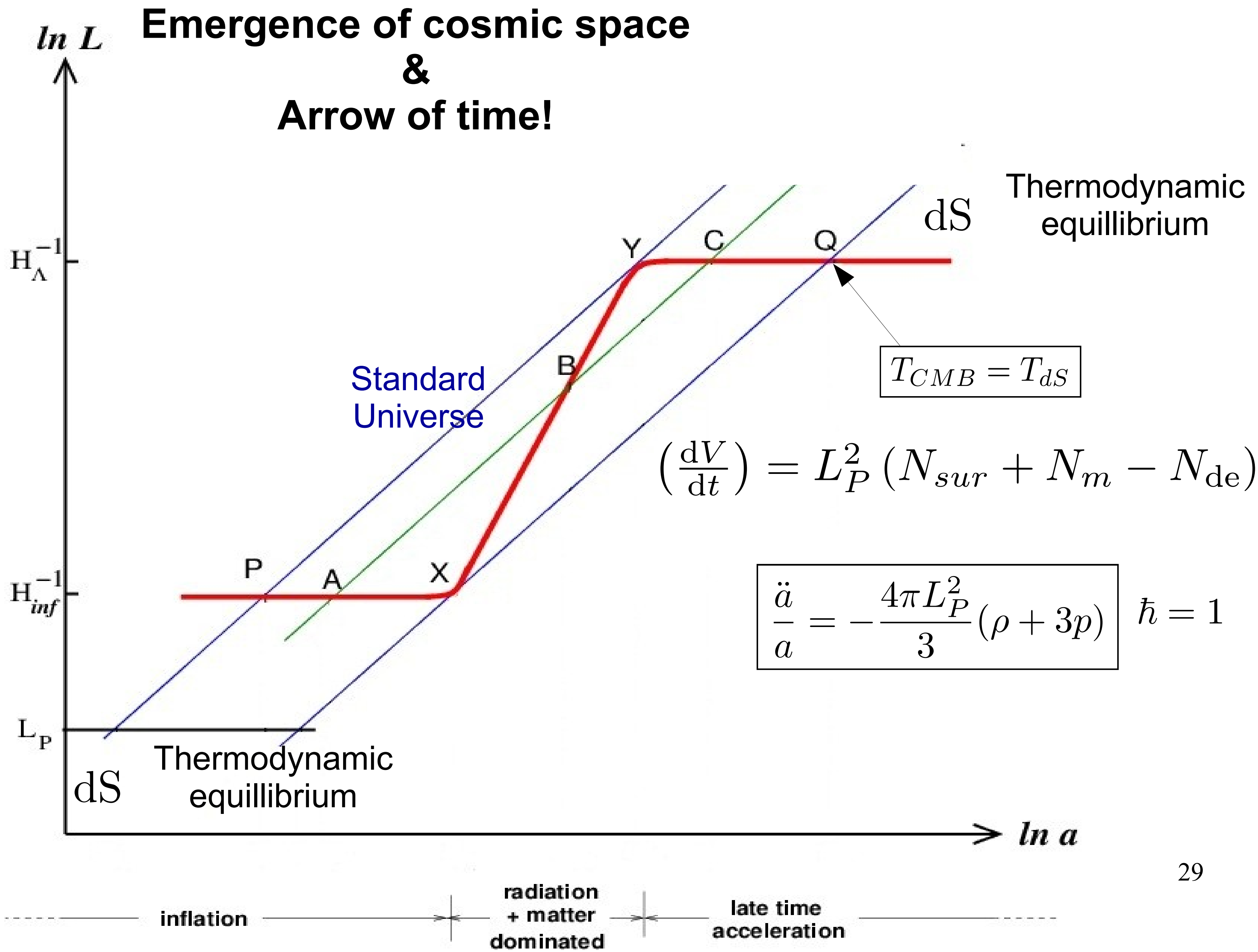
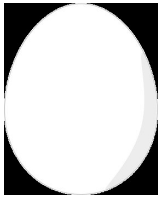


8. Further Investigations and my work

$$\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right) = L_P^2 \left(N_{sur} + N_m - N_{\mathrm{de}}\right)$$



dS

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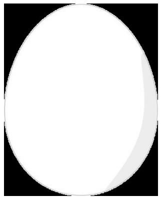

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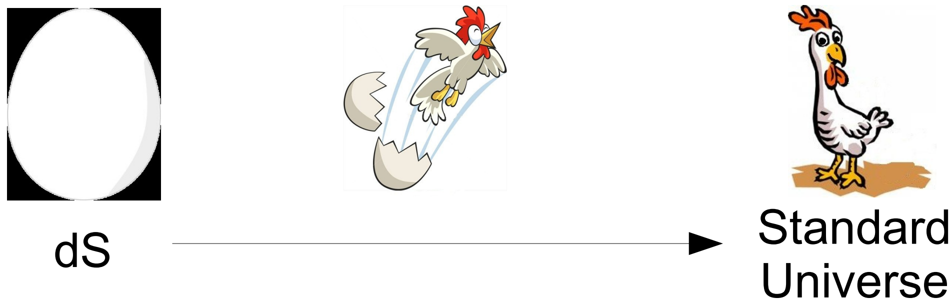

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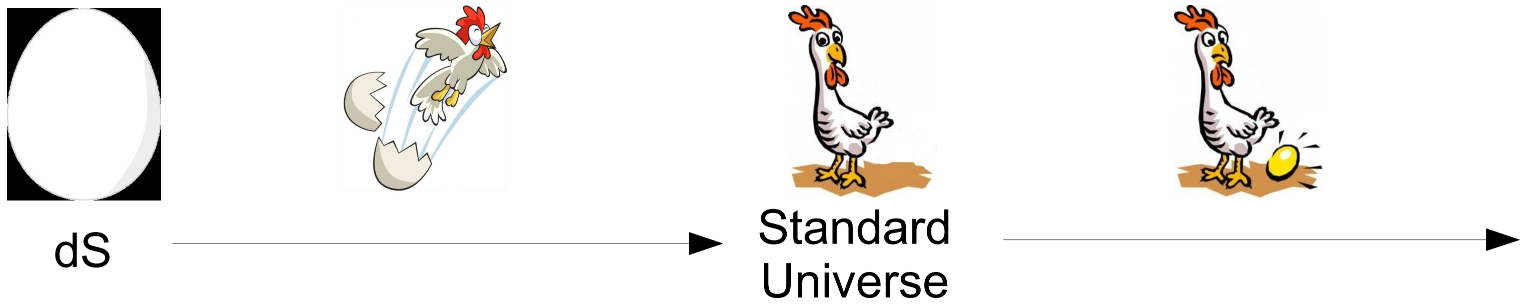

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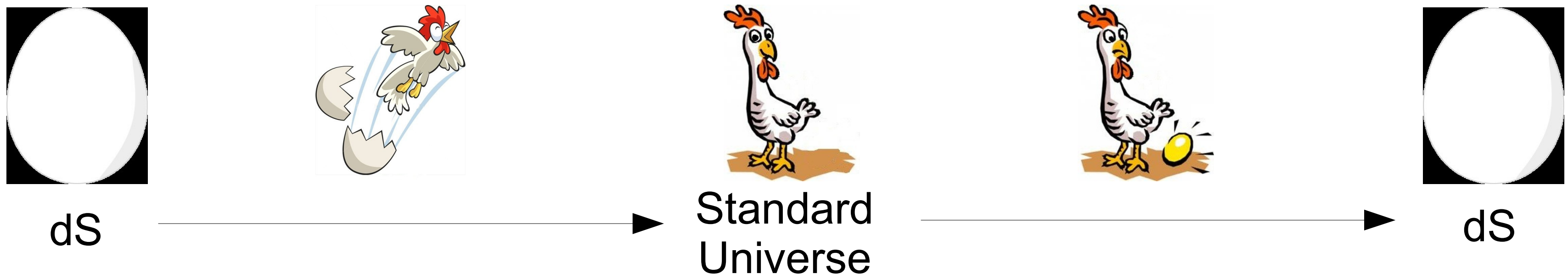

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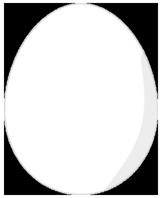

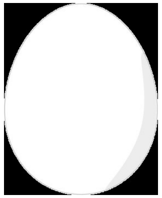

dS → Standard Universe → dS

Holographic discrepancy; Classical Universe; Holographic equipartition

$\frac{\mathrm{d}V}{\mathrm{d}t} = 0$ → $\frac{\mathrm{d}V}{\mathrm{d}t} \neq 0$ → $\frac{\mathrm{d}V}{\mathrm{d}t} = 0$

8. Further Investigations and my work

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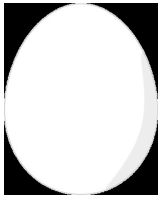



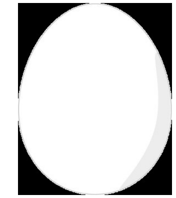

dS → Standard Universe → dS

Holographic discrepancy | Classical Universe | Holographic equipartition

$\frac{\mathrm{d}V}{\mathrm{d}t} = 0$ → $\frac{\mathrm{d}V}{\mathrm{d}t} \neq 0$ → $\frac{\mathrm{d}V}{\mathrm{d}t} = 0$

Conformal symmetry —Breaking of Conformal symmetry→ No Conformal symmetry —restoring Conformal symmetry→ Conformal symmetry

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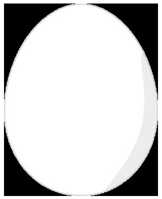
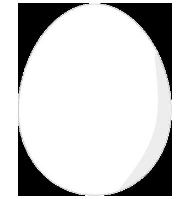
$$\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right) = L_P^2\left(N_{sur} + N_m - N_{\mathrm{de}}\right)$$

dS → Standard Universe → dS

Holographic discrepancy: $\frac{\mathrm{d}V}{\mathrm{d}t} = 0$ → Classical Universe $\frac{\mathrm{d}V}{\mathrm{d}t} \neq 0$ → Holographic equipartition: $\frac{\mathrm{d}V}{\mathrm{d}t} = 0$

(?)

Breaking of Conformal symmetry: Conformal symmetry → No Conformal symmetry → restoring Conformal symmetry: Conformal symmetry

(?)

Conformal symmetry ⟷ Holographic equipartition

Mechanism for symmetry breaking?

Needs more exploration!

8. Further Investigations and my work

1. Define conformal variables

2. Evaluate surface Hamiltonian H_{sur} in terms of *conformal variables*.

3. Check $\delta\overline{f}^{ab}$ and $\delta\overline{N}^{c}_{ab}$ for thermodynamic conjugacy relation.

Interesting: $\delta N^c_{ab} \propto \delta T$ — Conformal contributions / Non-conformal contributions

Formulate holographic equipartition and discrepancy
in terms of conformal variables.

Observation

CMB spectrum: nearly scale invariant → Look for breaking of conformal symmetry at high energy Within this framework

8. Further Investigations and my work

1. Conformal Variables

$$\boxed{f^{ab} = \Omega^2 \overline{f}^{ab}}$$

$$N^c_{ab} \quad = \quad \overline{N}^c_{ab} - \delta^c_{(a} k_{b)} - \overline{g}_{ab} k^c$$

$$k \equiv \ln \Omega \qquad\qquad k_a \equiv \frac{\partial_a \Omega}{\Omega} = \partial_a k \qquad\qquad \overline{k}^c \equiv \overline{g}^{cb} k_b$$

$$\mathcal{H}_g = \tfrac{1}{2} \left(\overline{N}^c_{ab} - \delta^c_{(a} k_{b)} - \overline{g}_{ab} k^c \right) \left[\Omega^2 \left(2 k_c \overline{f}^{ab} + \partial_c \overline{f}^{ab} \right) \right]$$

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2. … Work in progress!

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2. … Work in progress!

Thank you for your attention!

Holographic equipartition for de Sitter space

The dS maintain time translation invariance; natural choice for equillibrium.

For dS with Hubble radius H^{-1}

$$\mathrm{N}_{sur} = \frac{4\pi H^{-2}}{L_P^2} \text{ and } N_{bulk} = \frac{|E|}{\frac{1}{2}k_B T} = -\frac{2(\rho+3p)V}{k_B T}$$

For pure de Sitter universe, P=-ρ we get $$\mathrm{H}^2 = 8\pi L_P^2 \frac{\rho}{3}$$

Pure dS universe maintain holographic equipartition with constant V!

Comic expansion: Quest for holographic equipartition

Postulate: $\left(\frac{dV}{dt}\right) = L_P^2\left(N_{sur} - \epsilon N_{bulk}\right)$ $\epsilon = \pm 1$

using

$$\mathrm{V}{=}\left(\frac{4\pi}{3H^3}\right),\ T = \frac{H}{2\pi},\ N_{sur} = \frac{4\pi H^{-2}}{L_P^2},\ N_{bulk} = \frac{|E|}{\frac{1}{2}k_B T} = -\frac{2(\rho+3p)V}{k_B T}$$

We get standard FRW dynamics $\left(\frac{\ddot{a}}{a}\right) = -\frac{4\pi L_P^2}{3}(\rho + 3p)$

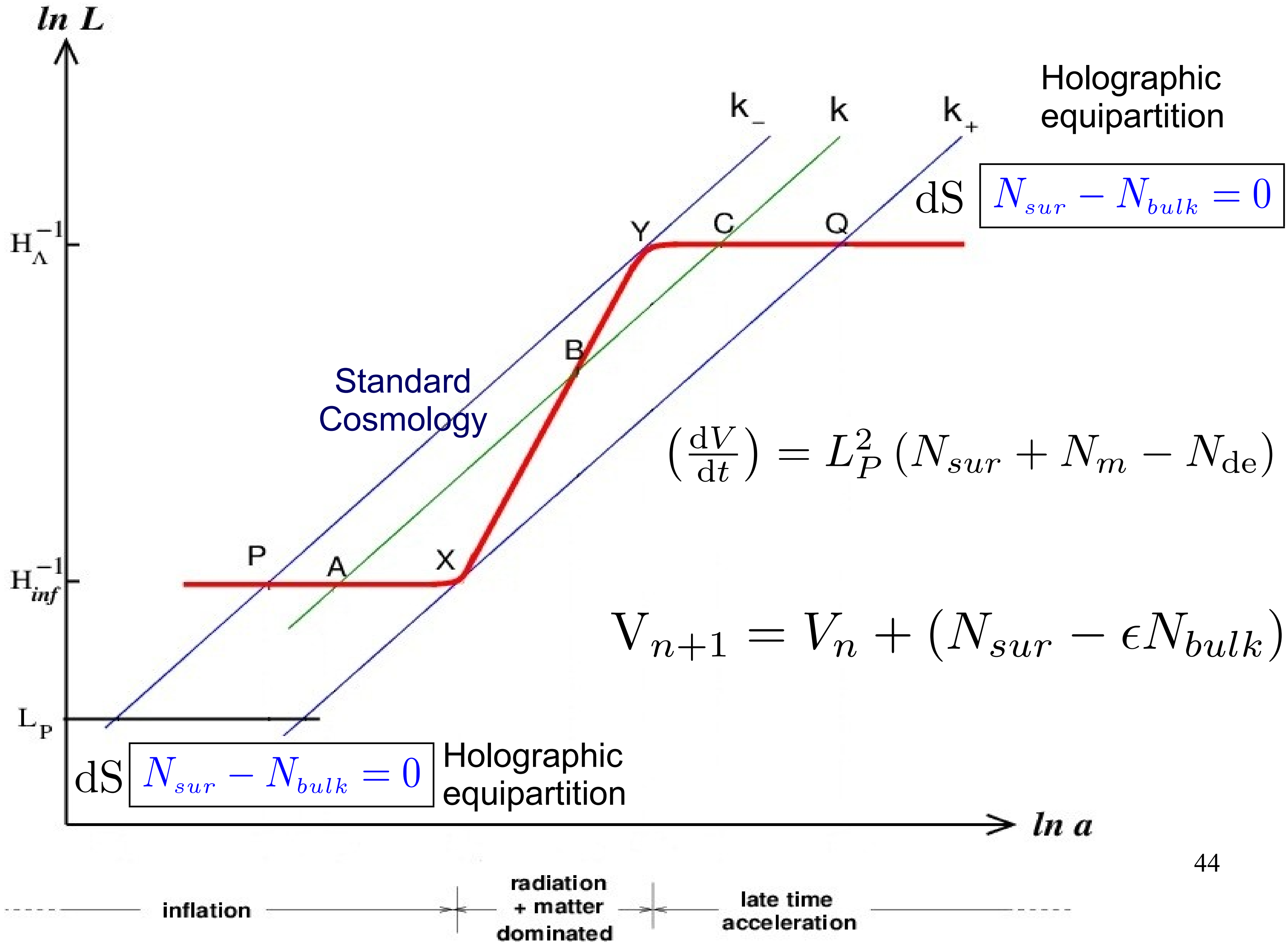
In Planck units, this has discrete version: $\mathrm{V}_{n+1} = V_n + \left(N_{sur} - \epsilon N_{bulk}\right)$

Alternate way of studying Quantum Cosmology!

7. Quantum Cosmology

"Chicken is egg's way of making another egg!"

TP(2014)



Attractive features

Beginning and the end of the universe: non-zero, finite volume

Arrow of cosmic time → Thermodynamic arrow $T_{\rm CMB} \to T_{\rm dS}$

Postulate $N_c \approx 4\pi$

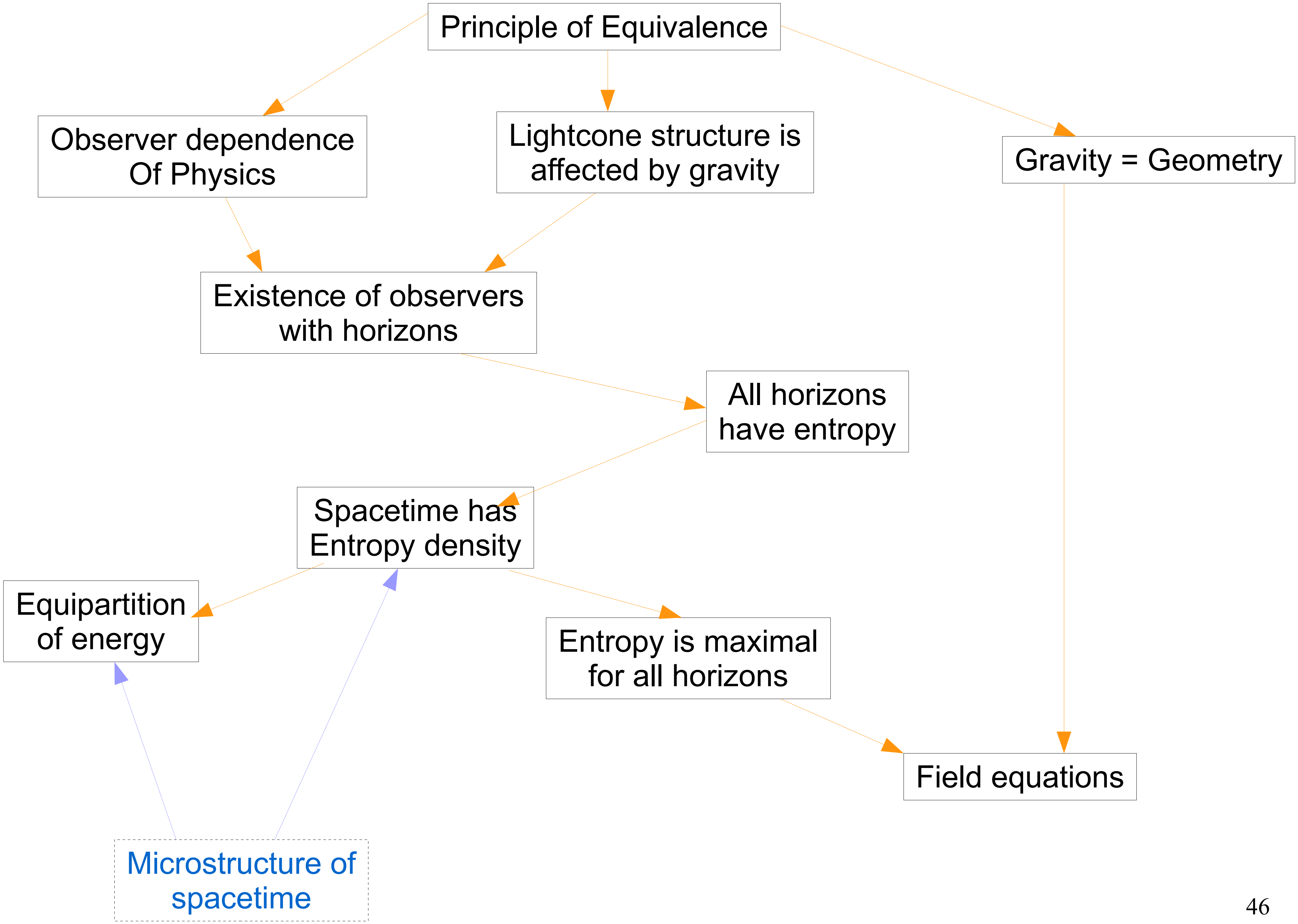
Reduction of Cosmological constant problem to CosMIn

$$\rho_\Lambda = \frac{4}{27}\left(\frac{\rho_{\rm inf}^{3/2}}{\rho_{\rm eq}^{1/2}}\right)\exp(-9\pi N_c)$$

will be determined by high energy physics

Recovery: Friedmann equations for $N_{sur} \neq N_{bulk}$

$$\frac{\ddot{a}}{a} = -\frac{4\pi L_P^2}{3}(\rho + 3p)$$


Principle of Equivalence
Observer dependence Of Physics
Lightcone structure is affected by gravity
Gravity = Geometry
Existence of observers with horizons
All horizons have entropy
Spacetime has Entropy density
Equipartition of energy
Entropy is maximal for all horizons
Field equations
Microstructure of spacetime