Electromagnetic media with no Fresnel (dispersion) equation and novel jump (boundary) conditions

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Outline

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Part 1. The local and linear electromagnetic response

- How to represent local linear media in 3D and in 4D.
- How to decompose electromagnetic response via index symmetries: principal part, skewon part and axion part.
- Geometrical optics as described by Fresnel (dispersion) equation. What is a medium with no Fresnel equation?

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Part 3. All local linear media with no Fresnel equation?

- We present strong evidence that there exist only three types of materials that give rise to no Fresnel equation.

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$$D^a = \varepsilon_0 \varepsilon^{ab} E_b + Z_0^{-1} \alpha^a{}_b B^b,$$
$$H_a = Z_0^{-1} \beta_a{}^b E_b + \mu_0^{-1} \mu_{ab}^{-1} B^b,$$

ε^{ab} called **permittivity**, μ_{ab}^{-1} called **impermeability**, $\alpha^a{}_b$ and $\beta_a{}^b$ called **magneto-electric** terms. But note, we make use of relative quantities. Also, $Z_0 = (\mu_0/\varepsilon_0)^{\frac{1}{2}}$.

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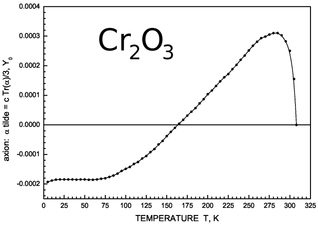
- Covariant (4D): $\mathfrak{G}^{\alpha\beta} = \frac{1}{2}\chi^{\alpha\beta\mu\nu} F_{\mu\nu}$. Now, decompose...

Principal-skewon-axion irreducible decomposition



- Split medium: **principal** part, **skewon** part, **axion** part.

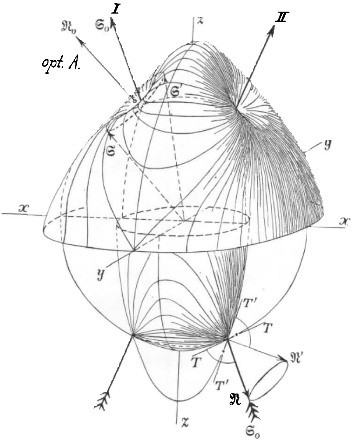
$$\chi^{\alpha\beta\mu\nu} = {}^{(1)}\chi^{\alpha\beta\mu\nu} + {}^{(2)}\chi^{\alpha\beta\mu\nu} + {}^{(3)}\chi^{\alpha\beta\mu\nu}.$$
$$36 = 20 \oplus 15 \oplus 1\,.$$

${}^{(1)}\chi^{\alpha\beta\mu\nu}$ is symmetric under $[\alpha\beta] \leftrightarrow [\mu\nu]$ and traceless.
${}^{(2)}\chi^{\alpha\beta\mu\nu}$ is antisymmetric under $[\alpha\beta] \leftrightarrow [\mu\nu]$. Moreover,
${}^{(3)}\chi^{\alpha\beta\mu\nu}$ is the trace w.r.t. the Levi-Civita symbol $\hat{\epsilon}_{\alpha\beta\mu\nu}$.

- **Finite axion** part observed in nature (Hehl et al. 2008). **Finite skewon** not yet, but magnetic groups identified (Dmitriev 1998). Know one route to find a **violation** of

$$\varepsilon^{ab} = \varepsilon^{ba}, \qquad \mu^{-1}_{ab} = \mu^{-1}_{ba}, \qquad \alpha^{a}{}_{b} = -\beta_{b}{}^{a}.$$

Electromag. waves: Fresnel (dispersion) equation



Medium: $\varepsilon^{ab} = \mathrm{diag}(\varepsilon_1, \varepsilon_2, \varepsilon_3)$, $\mu^{-1}_{ab} = \delta_{ab}$ and $\alpha^a{}_b = \beta_b{}^a = 0$.
Taken from: Schaefer (1932).

Electromag. waves: Fresnel (dispersion) equation

- $q_\alpha = (-\omega, k_i)$ is 4-dimensional **wave-covector**. Given propagation direction, what is inverse phase velocity k_i/ω of electromagnetic waves? Solve **Fresnel** equation:

$$\mathcal{G}(q) = \hat{\epsilon}_{\alpha\alpha_1\alpha_2\alpha_3}\hat{\epsilon}_{\beta\beta_1\beta_2\beta_3}\chi^{\alpha\alpha_1\beta\beta_1}\chi^{\alpha_2\rho\beta_2\sigma}\chi^{\alpha_3\tau\beta_3\upsilon}q_\rho q_\sigma q_\tau q_\upsilon = 0.$$

- $\mathcal{G}(q)$ **quartic** in q_α. Find **all** response tensors $\chi^{\alpha\beta\mu\nu}$ s.t.
  - $\mathcal{G}(q)$ = two coinciding quadratics = **no-birefringence**. Solved for zero skewon: Favaro+Bergamin (2011), Dahl (2012). Using: Lämmerzahl+Hehl (2004), Itin (2005).
  - $\mathcal{G}(q)$ = two distinct quadratics = **uniaxial generalised**. Solved for zero skewon, Lorentz signature: Dahl (2013). Using: Lindell+Wallén (2004), Schuller et al. (2010).
  - $\mathcal{G}(q)$ = zero for all possible q_α = **no Fresnel equation**. This talk and "PIER". Earlier: Hehl+Obukhov (2003).
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$[\,\cdot\,]$=value in one region minus value in other region as interface approached. n_a=surface normal (surf. 1-form).

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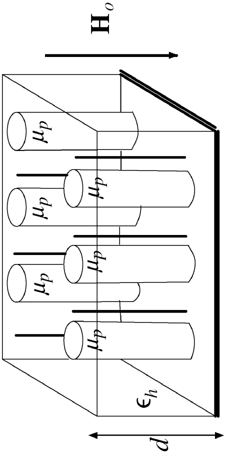
- Examples: perfect electromagnetic conductor (**PEMC**) and "**DB**" jump conditions. Have useful applications...

Perfect electromagnetic conductor (PEMC) jump condition



See: Lindell & Sihvola (2005). **Further details**: ask during Q&A.

- At interface with **vacuum**, given by

$$\epsilon^{abc} n_b (H_c + \alpha E_c) = 0,$$

$$\textbf{i.e.} \quad \vec{n} \times (\vec{H} + \alpha \vec{E}) = 0.$$

The jump operator $[\,\cdot\,]$ is not used.

- Obtain such PEMC jump condition with a **pure-axion** medium. In 4D,

$$\chi^{\alpha\beta\mu\nu} = \alpha \epsilon^{\alpha\beta\mu\nu}.$$

$\epsilon^{\alpha\beta\mu\nu}$ is Levi-Civita symbol. In 3D,

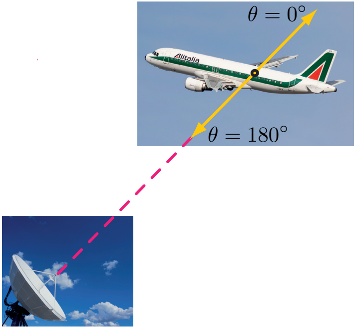
$$D^a = \alpha B^a, \qquad H_a = -\alpha E_a.$$

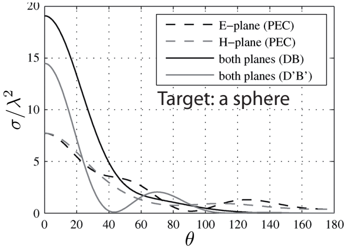
- Reflected light fully cross-polarised. Can build device: **twist polariser**.
- Can realise PEMC condition with **metamaterial layer** at interface.

Perfect electromagnetic conductor (PEMC) jump condition

A pure-axion medium has no Fresnel equation.

The "DB" jump conditions and radar invisibility





- The **DB** jump conditions at interface with vacuum are:

$$n_a D^a = 0, \qquad n_a B^a = 0.$$

- DB conditions at surface of highly symmetric object give **invisibility** to the monostatic radar (Lindell et al. 2009).

Skewon-axion media and DB jump conditions

- All skewon-axion media, ${}^{(1)}\chi^{\alpha\beta\mu\nu}=0$, can be written as:

$$\chi^{\alpha\beta\mu\nu} = \epsilon^{\alpha\beta\rho[\mu} \$_{\rho}{}^{\nu]} - \epsilon^{\mu\nu\rho[\alpha} \$_{\rho}{}^{\beta]} + \alpha\epsilon^{\alpha\beta\mu\nu} = 15 \oplus 1,$$

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- Nieves and Pal (1989): isotropic skewon-axion medium,

$$D^a = (-s + \alpha)B^a, \qquad H_a = (-s - \alpha)E_a.$$

  Link to general 4D law via $\$_{\alpha}{}^{\beta} = (s/2)\,\mathrm{diag}(-3, 1, 1, 1)$. See Hehl and Obukhov (2003) for a detailed treatment.

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Skewon-axion media have no Fresnel equation.

P-media and DB jump conditions

- P-**medium** constructed by straightforward P-roduct, as:

$$\chi^{\alpha\beta\mu\nu} = Y\epsilon^{\alpha\beta\rho\sigma}P_\rho{}^\mu P_\sigma{}^\nu.$$

- As a specific example, consider uniaxial P-medium. Set:

$$P_\alpha{}^\beta = (\psi - P_\perp)u_\alpha u^\beta + p_\| u_\alpha n^\beta + \pi_\| n_\alpha u^\beta + (P_\| - P_\perp)n_\alpha n^\beta + P_\perp \delta_\alpha^\beta.$$

  u^α is the medium 4-velocity and n^α is the preferred axis:

$$u_\alpha u^\alpha = -1, \qquad u_\alpha n^\alpha = 0, \qquad n_\alpha n^\alpha = +1.$$

  (To make example "**premetric**", need extra quantities.)

- Uniaxial P-medium as formulated in its rest frame, 3D:

$$D^a = \varepsilon_\perp \epsilon^{abc} n_b E_c + \alpha_\| n^a n_b B^b + \alpha_\perp(\delta_b^a - n^a n_b)B^b,$$

$$H_a = \mu_\perp^{-1}\hat{\epsilon}_{abc} n^b B^c + \beta_\| n_a n^b E_b + \beta_\perp(\delta_a^b - n_a n^b)E_b.$$

P-media and DB jump conditions (continued)

- Skewon: permittivity & permeability "solenoidal". Aside,

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- Above, seen that technologically useful PEMC and DB jump conditions are closely related to media with no Fresnel eq. **Identify all materials with this property!**

All local linear media with no Fresnel equation?

We found strong evidence that there exist **three types** of local linear materials whose Fresnel equation is satisfied for every wave-covector, $\mathcal{G}(q)=0$ for all $q_\alpha=(-\omega,k_i)$. Namely,

1. bivector $\otimes$ bivector + bivector $\otimes$ bivector + axion part,

$$\chi^{\alpha\beta\mu\nu} = A^{\alpha\beta}B^{\mu\nu} + C^{\alpha\beta}D^{\mu\nu} + \alpha\epsilon^{\alpha\beta\mu\nu};$$

2. skewon part + axion part [a.k.a. skewon-axion media],

$$\chi^{\alpha\beta\mu\nu} = \left(\epsilon^{\alpha\beta\rho[\mu}\$_\rho{}^{\nu]} - \epsilon^{\mu\nu\rho[\alpha}\$_\rho{}^{\beta]}\right) + \alpha\epsilon^{\alpha\beta\mu\nu};$$

3. every P-medium + axion part [a.k.a. P-axion media],

$$\chi^{\alpha\beta\mu\nu} = \left(\epsilon^{\alpha\beta\rho\sigma}P_\rho{}^\mu P_\sigma{}^\nu\right) + \alpha\epsilon^{\alpha\beta\mu\nu}.$$

Our derivation has some weak points [Lindell and Favaro, Prog. Electromagn. Res. B, vol. 51, pp. 269–289, 2013].

All media with no Fresnel? It is likely, because...

- Argument below suggests we **have** found all local linear materials whose Fresnel equation is satisfied identically.

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- Represent the medium tensor in an alternative way, as ${\kappa_{\alpha\beta}}^{\mu\nu} = \frac{1}{2}\hat{\epsilon}_{\alpha\beta\rho\sigma}\chi^{\rho\sigma\mu\nu}$. Inverse of this linear map is κ^{-1}.

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- Hence, if a medium tensor κ of full-rank has no Fresnel equation, its inverse κ^{-1} has no Fresnel equation too.
- But, taking inverse yields **no new classes** of materials:

material κ	inverse κ^{-1}
bivector pairs + axion	bivector pairs + axion
skewon-axion	specific P-axion
specific P-axion	skewon-axion
general P-axion	general P-axion

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- Represent the medium tensor in an alternative way, as ${\kappa_{\alpha\beta}}^{\mu\nu} = \frac{1}{2}\hat{\epsilon}_{\alpha\beta\rho\sigma}\chi^{\rho\sigma\mu\nu}$. Inverse of this linear map is κ^{-1}.
- A response tensor κ of full-rank and its inverse κ^{-1} have the **same** Fresnel equation, see Dahl (2012).
- Hence, if a medium tensor κ of full-rank has no Fresnel equation, its inverse κ^{-1} has no Fresnel equation too.
- But, taking inverse yields **no new classes** of materials:

material κ	inverse κ^{-1}
bivector pairs + axion	bivector pairs + axion
skewon-axion	specific P-axion
specific P-axion	skewon-axion
general P-axion	general P-axion

Conclusions

a) At an interface with vacuum. . . **PEMC** jump condition: build a twist polariser. **DB**: achieve invisibility to radar.

b) PEMC jump condition: from **pure-axion** medium. DB: from e.g. isotropic **skewon-axion** or uniaxial P-**medium**

c) These are all media with **no Fresnel equation**. Study local linear materials s.t. $\mathcal{G}(q) = 0$ is satisfied for all q_α.

d) Likely that all materials with no Fresnel are of 3 types: **bivector pairs** plus axion, **skewon-axion** and **P-axion**.

e) In support of this finding, taking the inverse does not produce new classes of local linear media $\sim$ completeness.

Thank-you!