Coordinate-free Negative Phase Velocity (NPV).

Important insight on dispersionless bianisotropic media.

A. Favaro*, M.W. McCall and P. Kinsler

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- Thesis, dispersion is the only way to get NPV.

What is Negative Phase Velocity (NPV)?

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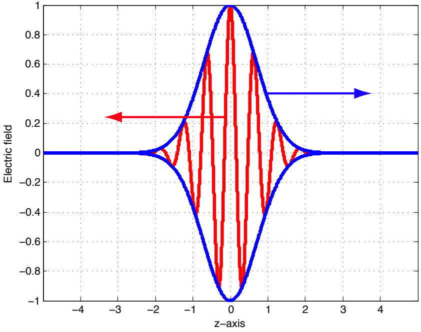
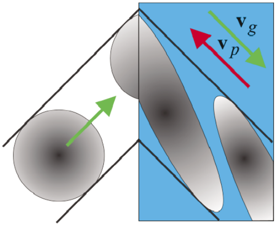
- Wave-fronts propagate opposite to the wave-packet.





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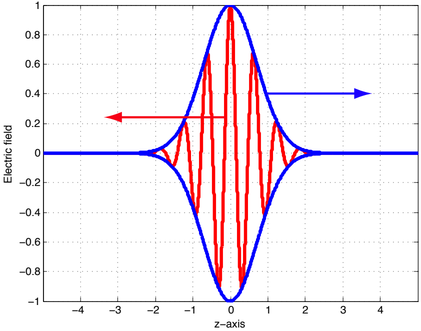

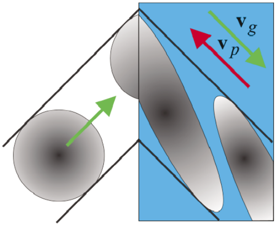

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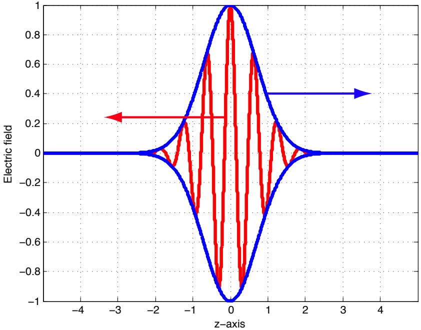

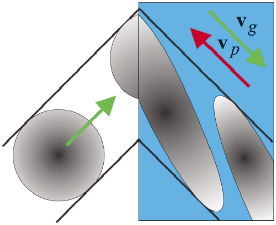

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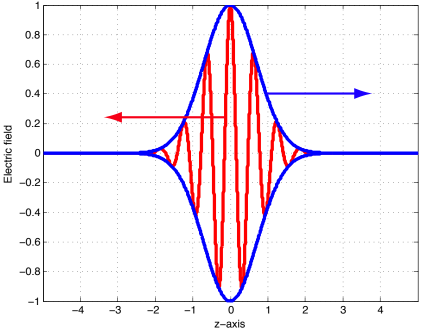

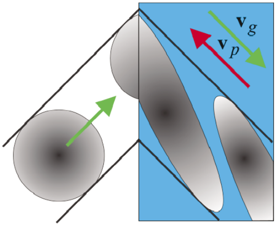

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...but not set in stone.

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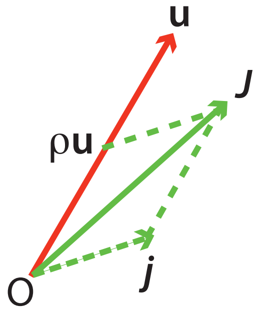
- E.g. electric 4-current $\mathcal{J} = (\rho, \mathbf{j}) \Rightarrow \mathcal{J} = \rho\mathbf{u} + \mathbf{j}$.



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Extracting $\vec{P}$, extracting $\vec{k}$ and forming $\vec{P}\cdot\vec{k}/\omega < 0$.

- $\vec{P}$: the time-space part of the energy-momentum tensor.

$$\boldsymbol{\mathcal{T}} = \left[\begin{array}{c|c} \text{time-time (scalar } U) & \text{time-space (vector } \mathbf{P}) \\ \hline \text{space-time (covec } -\mathbf{p}) & \text{space-space (matrix } -\mathbf{S}) \end{array}\right]$$

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where $\omega = -\mathbf{u}|\mathbf{K}$. Final result uses covariant quantities only + is pre-metric + useful in gas flows.

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  - Demonstrate that this $U < 0$ regime cannot be used to obtain negative refraction.

Various ways to get $U < 0$ in materials and curved vacuum.

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$\vec{P} \cdot \vec{k}/\omega = U < 0$ in materials.

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- Observer outside rotating black hole:

  ...similar to moving medium example?

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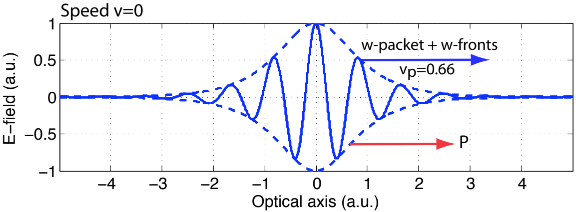
- E.g. propagate with $v_p = 0.66c$ in material's frame:



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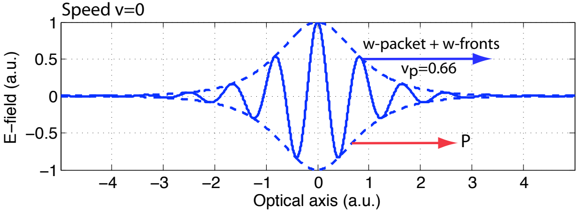

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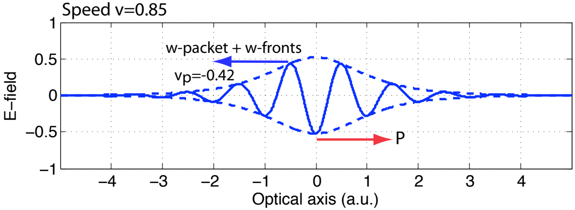

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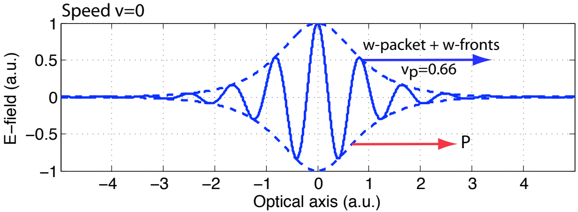

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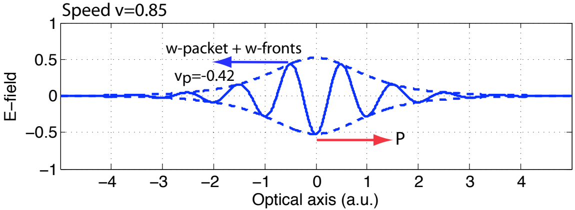

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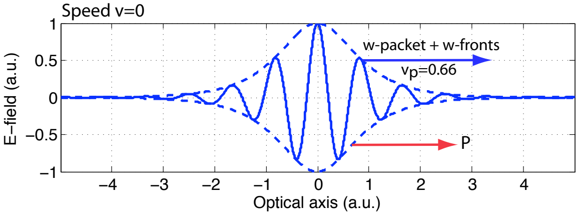

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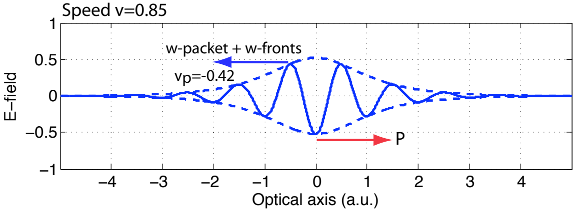

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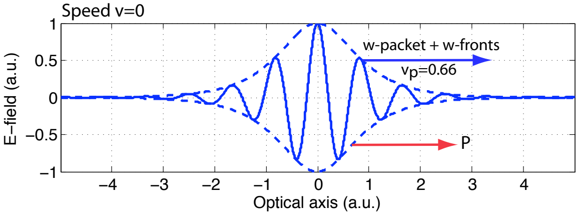

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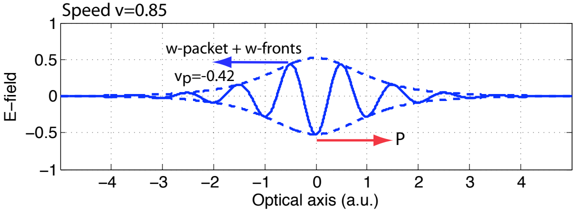

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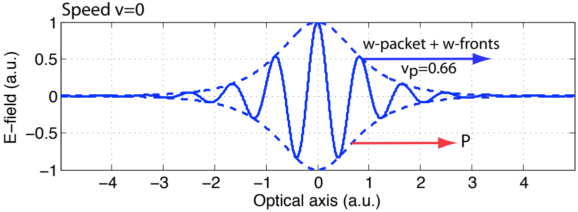

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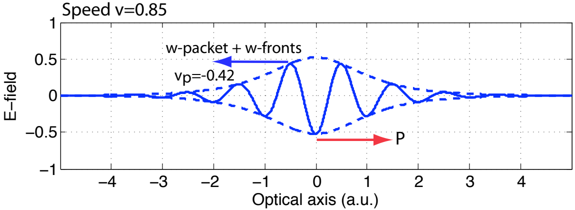

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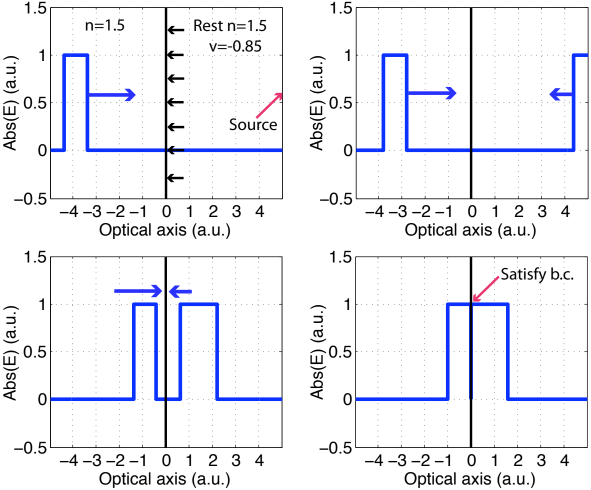

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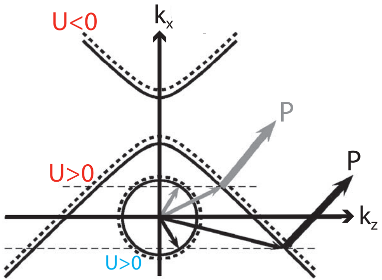

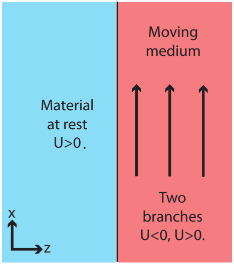

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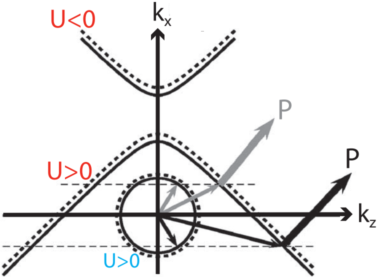

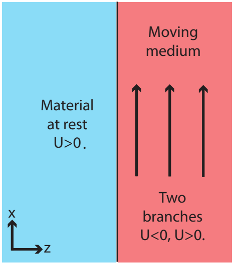

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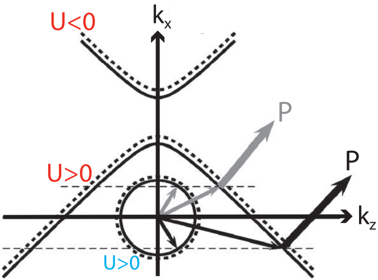

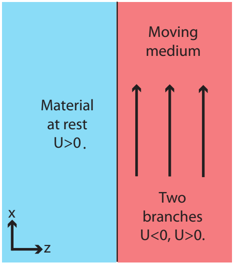

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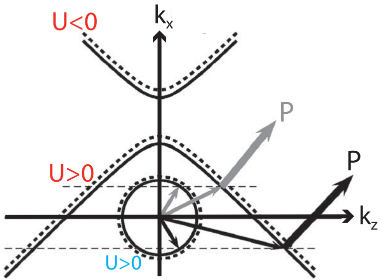

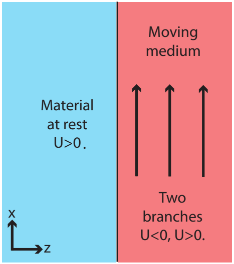

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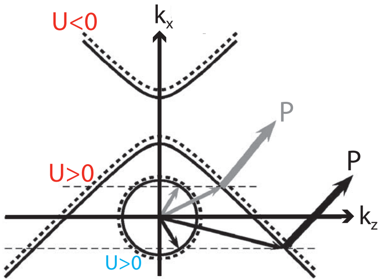

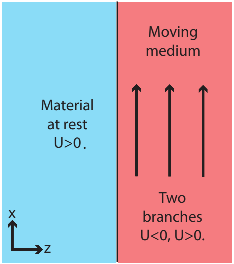

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Conclusion

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- Thesis: NPV always needs dispersion!

Thank-you!

Acknowledgements

- EPSRC, Grant No. EP/E031463/1.
- IoP, Quantum Electronics and Photonics Group, Research Student Conference Fund.