Varying constants universes, singularities, and quantum cosmology

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Plan:

- 1. Introduction.
- 2. Varying constants theories.
- 3. Exotic singularities in Friedmann cosmology.
- 4. Dynamical scalar field framework and varying α.
- 5. Varying constants removing or changing singularities.
- 6. Varying constants quantum cosmology.
- 7. Conclusions.

1. Introduction.

- Pretty long story of varying constants theories:
- Hermann Weyl (1919): electron radius/its gravitational radius $\sim 10^{40}$
- Arthur Eddington (1935) discussed:

  1) proton-to-electron mass $1/\beta = m_p/m_e \sim 1840$

  2) an inverse of fine structure constant $1/\alpha = (hc)/(2\pi e^2) \sim 137$

  3) electromagnetic to gravitational force between a proton and an electron $e^2/(4\pi\epsilon_0 G m_e m_p) \sim 10^{40}$

  4) introduced “Eddington number” $N_{edd} \sim 10^{80}$
- P.A.M. Dirac (1937) interesting remarks about the relations between atomic and cosmological quantities: If $G \propto H(t) = (da/dt)/a$, then $a(t) \propto t^{1/3}$ and $G(t) \propto 1/t$ - fundamental constants must evolve in time.
- Nice conclusion: electromagnetic force is strong compared to gravitational since the universe is “old” i.e. $F_e/F_p \propto (e^2/m_e m_p)t \propto t$!!!

2. Varying constants theories.

First fully quantitative framework: Brans-Dicke scalar-tensor gravity (1961)

The gravitational constant G is associated with an average gravitational potential (scalar field) ϕ surrounding a given particle:

$<\phi>= GM/(c/H_0) \propto 1/G = 1.35 \times 10^{28} g/cm$. The scalar field gives the strength of gravity

$$G = \frac{1}{16\pi\Phi} \tag{1}$$

With the action

$$S = \int d^4x\sqrt{-g}\left(\Phi R - \frac{\omega}{\Phi}\partial_\mu\Phi\partial^\mu\Phi + \Lambda + L_m\right) \tag{2}$$

it relates to low-energy-effective superstring theory for $\omega = -1$

String coupling constant (running) $g_s = \exp\left(\phi/2\right)$ changes in time with ϕ - the dilaton and $\Phi = \exp\left(-\phi\right)$.

varying constants theories

Varying speed of light theories (VSL): Albrecht & Magueijo model (AM model) (1999)(Barrow 1999; Magueijo 2003):

$$c^4 = \psi(x^\mu) \tag{3}$$

and so the action is

$$S = \int d^4x \sqrt{-g} \left[\frac{\psi(R + 2\Lambda)}{16\pi G} + L_m + L_\psi \right] \tag{4}$$

AM model breaks Lorentz invariance (relativity principle and light principle) - preferred frame (cosmological or CMB) in which the field is minimally coupled to gravity.

Solves basic problems of standard cosmology: horizon problem and flatness problem.

Ansatz: Friedmann with $\rho = \rho_0 a^{-3\gamma}$, $c(t) = c_0 a^n$ - solution if $n \leq (1/2)(2 - 3\gamma)$.

varying constants theories

Magueijo covariant (conformally) and locally invariant model (2000, 2001):

$$\psi = \ln\left(\frac{c}{c_0}\right) \quad \text{or} \quad c = c_0 e^{\psi} \ , \tag{5}$$

with the action

$$S = \int d^4x\sqrt{-g}\left[\frac{c_0^4 e^{\alpha\psi}(R + 2\Lambda + L_\psi)}{16\pi G} + e^{\beta\psi} L_m\right] \ , \tag{6}$$

with

$$L_\psi = \kappa(\psi)\nabla_\mu\psi\nabla^\mu\psi \ . \tag{7}$$

Further assumption: $\alpha - \beta = 4$.

Interesting subcases:

$\alpha = 4; \beta = 0$ - Brans-Dicke with $\phi_{BD} = e^{4\psi}/G$ and $\kappa(\psi) = 16\omega_{BD}(\phi_{BD})$.

$\alpha = 0; \beta = -4$ - minimal VSL theory.

varying constants theories

Varying fine structure constant α (or charge $e = e_0\epsilon(x^\mu)$ theories (Webb et al. 1999, Sandvik 2002)

$$S = \int d^4x\sqrt{-g}\left(\psi R - \frac{\omega}{2}\partial_\mu\psi\partial^\mu\psi - \frac{1}{4}f_{\mu\nu}f^{\mu\nu}e^{-2\psi} + L_m\right) \tag{8}$$

with $\psi = \ln\epsilon$ and $f_{\mu\nu} = \epsilon F_{\mu\nu}$.
Assume linear expansion $e^\psi = 1 - 8\pi G\zeta(\psi - \psi_0) = 1 - \Delta\alpha/\alpha$ with the constraint on the local equivalence principle violence $\mid \zeta \mid \leq 10^{-3}$. The relation to dark energy is:

$$\gamma = w + 1 = \frac{(8\pi G\frac{d\psi}{d\ln a})^2}{\Omega_\psi} \tag{9}$$

(e.g. Vielzeuf and Martins 2012 - see further)

Observational constraints:

- $|(dG/dt)/G| < 9 \cdot 10^{-13}/year$ - from primordial nucleosynthesis (Accetta et al. 1990);
- $|(dG/dt)/G| < 1.6 \cdot 10^{-12}/year$ - from helioseismology (Guenther et al. 1998);
- $|(dG/dt)/G| < (4 \pm 9) \cdot 10^{-13}/year$ - from lunar laser ranging (LLR) (Williams et al. 1996);
- $\Delta\alpha/\alpha = (3.85 \pm 5.65) \cdot 10^{-8}$ - from Oklo phenomenon (Shlyakhter 1976, Petrov et al. 2006);
- $\Delta\alpha/\alpha = (-8 \pm 16) \cdot 10^{-7}$ - from meteorite dating (long-lived beta decays) (Olive et al. 2003);
- $\Delta\alpha/\alpha = (-0.5 \pm 1.3) \cdot 10^{-5}$ - from quasar absorption spectra with redshifts $2.33 < z < 3.08$ (Murphy et al. 2001);
- $\Delta\beta/\beta = (5.7 \pm 3.8) \cdot 10^{-5}$ ($\beta = m_e/m_p$) - from quasar absorption spectra (Ivanchik et al. 2005).

2. Exotic singularities in Friedmann cosmology

Standard Einstein-Friedmann equations are two equations for three unknown functions of time $a(t), p(t), \varrho(t)$

$$\varrho = \frac{3}{8\pi G}\left(\frac{\dot{a}^2}{a^2} + \frac{Kc^2}{a^2}\right) , \tag{10}$$

$$p = -\frac{c^2}{8\pi G}\left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{Kc^2}{a^2}\right) , \tag{11}$$

are usually solved by adding an equation of state (OES), e.g., of a barotropic type:

$$p(t) = w\varrho(t) \qquad \rightarrow a(t) \propto t^{\frac{2}{3(w+1)}} , \tag{12}$$

and the conservation equation is fulfilled. However, manipulating the equation of state (or just dropping it) allows to **enrich the possible ways for the Friedmann universe to evolve – non-standard (non-Big-Bang) singularities appear** which may violate: N(ull) E(nergy) C(ondition) $\varrho + p \geq 0$, W(eak) E(nergy) C(ondition) $\varrho + p \geq 0$, $\rho \geq 0$, D(ominant) E(nergy) C(ondition) $\mid p \mid \leq \varrho$, $\varrho \geq 0$.

“Exotic” singularities

An example is a Big-Rip (BR - type I): $\varrho, p \to \infty$ for $a \to \infty$ due to phantom $w < -1$ matter (which does not obey cosmic no-hair theorem)

$$\mid w+1 \mid = -(w+1) > 0 \; , \tag{13}$$

so $a(t) = t^{-2/3|w+1|}$ and $\varrho \propto a^{3|w+1|}$. **(It took R. Caldwell 3 years to publish the paper in PLB originally submitted to PRL.)**

Another example is a Sudden Future Singularity (SFS - type II) (Barrow 2004, Nojiri et al. 2005) which assumes an ansatz for the scale factor instead of EOS:

$$a(t) = a_s \left[\delta + (1-\delta) \left(\frac{t}{t_s} \right)^m - \delta \left(1 - \frac{t}{t_s} \right)^n \right] \; . \tag{14}$$

where $a_s \equiv a(t_s) =$ const. and $\delta, m, n =$ const. If $1 < n < 2$, then one has an acceleration $\ddot{a} \to -\infty$ **(“car-drag races”)** and so the pressure singularity $p \to \infty$ at $t = t_s$ and the DEC is violated. SFSs commonly appear in LQC (Cailletau et al., 2008).

singularities

Another example is a Finite Scale Factor - FSFS singularity (type III) characterized by (Nojiri, Odintsov, Tsujikawa 2005):

$$a = a_s = \text{const.}, \ \varrho, \dot{a}_s \to \infty, \ |p|, \ddot{a}_s \to \infty$$

which can be obtained by applying the scale factor as given previously for SFS, but with the range of parameter n changed from $1 < n < 2$ onto

$$0 < n < 1$$

Type IV singularity is when (Nojiri, Odintsov, Tsujikawa 2005):

$$a = a_s = \text{const.}, \ \varrho \to 0, \ p \to 0, \ \dot{p}, \dddot{a}, \ddot{H} \to \infty \ \text{etc.}$$

and so it has the divergence of the barotropic index $w(t) \to \infty$ $(p(t) = w(t)\varrho(t))$.

And what is more – they are **not necessarily "singularities"** in the sense of geodesic incompleteness.

Nature of singularities.

SFS and FSFS (and some other) do not exhibit geodesic incompleteness - simply geodesics do not feel them since geodesic equations are not singular for $a_s = a(t_s) =$ const. (Fernandez-Jambrina, Lazkoz PRD 74, 064030 (2006))

$$\begin{aligned} \left(\frac{dt}{d\tau}\right)^2 &= A + \frac{P^2 + KL^2}{a^2(t)} \ , && (15) \\ \frac{dr}{d\tau} &= \frac{P_1 cos\phi + P_2 \sin\phi}{a^2(t)} \sqrt{1 - Kr^2} \ , && (16) \\ \frac{d\phi}{d\tau} &= \frac{L}{a^2(t) r^2} \ . && (17) \end{aligned}$$

Geodesic deviation equation (tidal forces)

$$\frac{D^2 n^\alpha}{d\lambda^2} + R^\alpha_{\beta\gamma\delta} u^\beta n^\gamma u^\delta = 0 \ , \qquad (18)$$

do feel SFS since at $t = t_s$ we have the Riemann tensor $R^\alpha_{\beta\gamma\delta} \to \infty$.

Strength of singularities.

- Tipler's (Phys. Lett. A64, 8 (1977)) definition (of a strong singularity):
  $I^i_j(\tau) = \int_0^\tau d\tau' \int_0^{\tau'} d\tau'' |R^i_{ajb} u^a u^b|$
  diverges on the approach to a singularity at $\tau = \tau_s$
- i.e. an extended object is crushed to zero volume (represented by three linearly independent, vorticity-free geodesic deviation vectors at p parallely transported along causal geodesic l) at the singularity by infinite tidal forces
- Królak's (CQG 3, 267 (1988)) definition (of a strong singularity):
  $I^i_j(\tau) = \int_0^\tau d\tau' |R^i_{ajb} u^a u^b|$
  diverges on the approach to a singularity at $\tau = \tau_s$
- i.e. the expansion of every future-directed congruence of null (timelike) geodesics emanating from point p and containing l becomes negative somewhere on l
- For null geodesics one replaces Riemann by the Ricci tensor components.

Strength - extended objects through singularities.

Fernandez-Jambrina (PRD 82, 124004 (2010)) used Puiseux series expansion

$$a(t) = c_0 + (t_s - t)^{\eta_0} + c_1 (t_s - t)^{\eta_1} + c_2 (t_s - t)^{\eta_2} + \ldots \qquad \eta_0 < \eta_1 < \ldots \quad c_0 > 0 \tag{19}$$

to check the strength of exotic singularities (T - Tipler; K - Królak)

Balcerzak and MPD (2006) considered classical Polyakov strings

$$S = -\frac{T}{2} \int d\tau d\sigma \eta^{ab} g_{\mu\nu} \partial_a X^\mu \partial_b X^\nu \tag{20}$$

with an invariant size $S(\tau) = 2\pi a(\eta(\tau)) R(\tau)$ (circular ansatz with radius R) falling into exotic singularities to show that they are: infinitely stretched $S \to \infty$ at Big-Rip while for SFS and FSFS the scale factor is finite at η-time so that the invariant string size is also finite.

This means strings are **not destroyed** at these weak singularities.

The evolution through a singularity - averaging approach.

A.K. Raychaudhuri (PRL 80, 654 (1998)) proposed that one may average physical and kinematical scalars over the whole open spacetime (provided they vanish rapidly at spatial and temporal infinity) as follows

$$< \chi >= \lim_{x^a \to \infty} \frac{\int\int\int\int_{-x^a}^{x^a} \chi\sqrt{-g} d^4x}{\int\int\int\int_{-x^a}^{x^a} \sqrt{-g} d^4x} \tag{21}$$

By an open model it is meant that the ratio of the 3-volume hypersurfaces to a 4-volume of spacetime vanishes, i.e.,

$$\frac{\int\int\int \sqrt{|^3 g|} d^3x}{\int\int\int\int \sqrt{-g} d^4x} = 0. \tag{22}$$

His idea was to tight the vanishing of the average $< \chi >$ with the singularity avoidance in cosmology.

Spacetime averaging - density and pressure.

For the pressure, the energy density, and the average acceleration we have (MPD 2011)

$$< p >= - \lim_{\substack{t_0 \to 0 \\ t_1 \to \infty}} \frac{\int_{t_0}^{t_1} a^3 \left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} \right) dt}{\int_{t_0}^{t_1} a^3 dt} \tag{23}$$

and

$$< \varrho >= \lim_{\substack{t_0 \to 0 \\ t_1 \to \infty}} \frac{3 \int_{t_0}^{t_1} a^3 \left(\frac{\dot{a}^2}{a^2} \right) dt}{\int_{t_0}^{t_1} a^3 dt}. \tag{24}$$

$$< \dot{\theta} >= \lim_{\substack{t_0 \to 0 \\ t_1 \to \infty}} \frac{3 \int_{t_0}^{t_1} a^3 \left(\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} \right) dt}{\int_{t_0}^{t_1} a^3 dt}. \tag{25}$$

SFS universe through an exotic singularity.

One is able to construct a hybrid model which allows Big-Bang, SFS, and finally Big-Crunch given by:

$$a_L(t) = a_s \left[\delta + \left(1 + \frac{t}{t_B} \right)^m (1 - \delta) - \delta \left(- \frac{t}{t_B} \right)^n \right] \tag{26}$$

with $t_B < 0$ - the Big-Bang time, and $t = 0$ and SFS time;

$$a_R(t) = a_s \left[\delta + \left(1 - \frac{t}{t_C} \right)^m (1 - \delta) - \delta \left(\frac{t}{t_C} \right)^n \right] \tag{27}$$

with $t_C > 0$ - the Big-Crunch time. In the high pressure regime $t \rightarrow 0$ these are approximated by

$$a_L \approx a_s \left[1 + \frac{m}{t_B} (1 - \delta) t \right], \tag{28}$$

$$a_R \approx a_s \left[1 - \frac{m}{t_C} (1 - \delta) t \right]. \tag{29}$$

Subtle differences between singularities.

- BB singularities - all the energy conditions fulfilled, averages vanish (despite original claim of Raychaudhuri)
- BR singularities - no EC fulfilled, averages blow up
- SFS - only dominant energy violated, averages finite
- It seems that BR is stronger singularity that BB, BC on the ground of averaging.
- SFS is weaker, but FSF does not seem so.

This seems to be another measure for the strength of singularities.

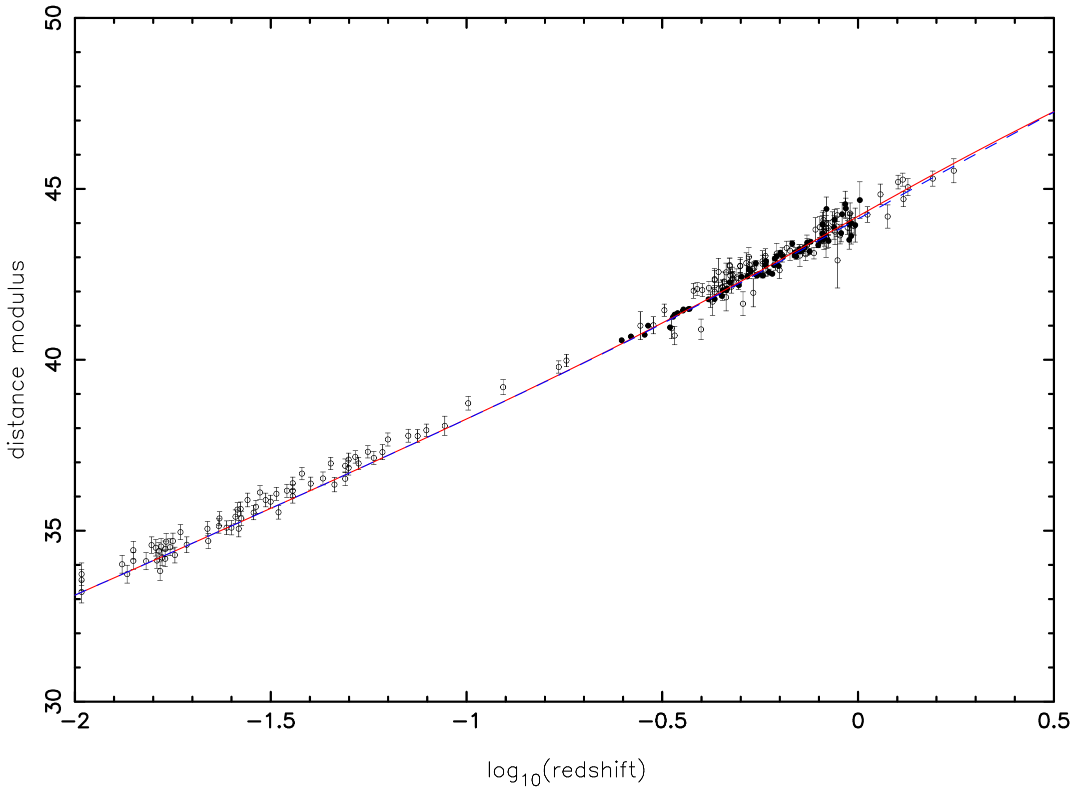
List of singularities (still not yet full ...).

- Type 0 - Big-Bang (Big-Crunch) $a \to 0$, $p \to \infty$, $\varrho \to \infty$
- Type I - Big-Rip $a(t_s) \to \infty$ ($t_s < \infty$), $p \to \infty$, $\varrho \to \infty$ (Caldwell 2002)
- Type II - Sudden Future (includes Big Boost and Big-Brake) $a(t_s) =$ const., $\varrho =$ const., $p \to \infty$ (Barrow 2004)
- Type IIg - Generalized Sudden Future $a(t_s)$= const., $\varrho =$ const., $p =$const., $\dddot{a} \to \infty$ etc., $w < \infty$ (Barrow 2004)
- Type III - Finite Scale Factor (also Big-Freeze) $a(t_s)$ = const., $\varrho \to \infty$, $p \to \infty$ (NOT 2005, Denkiewicz 2012)
- Type IV - Big Separation: $a(t_s)$= const., $p = \varrho = 0$, $w \to \infty$, $\dddot{a} \to \infty$ etc. (NOT 2005) (and generalizations $p = \varrho =$const. Yurov 2010)
- Type V - w-singularity $a(t_s)$= const., $p = \varrho = 0$, $w \to \infty$ (MPD, Denkiewicz 2009) (and generalizations $p =$const. Yurov 2010)
- More subtleties: Little-Rip $a(t_s) \to \infty$, $\varrho(t_s) \to \infty$ ($t_s \to \infty$) and Pseudo-Rip $\varrho(t_s) < \infty$ ($t_s \to \infty$) (Frampton et al. 2011, 2012)

Classification of singularities in Friedmann cosmology.

Type	Name	t sing.	a(t_s)	$\varrho(t_s)$	p(t_s)	$\dot{p}(t_s)$ etc.	w(t_s)	T	K
0	Big-Bang (BB)	0	0	∞	∞	∞	finite	strong	strong
I	Big-Rip (BR)	t_s	∞	∞	∞	∞	finite	strong	strong
I_l	Little-Rip (LR)	∞	∞	∞	∞	∞	finite	strong	strong
I_p	Pseudo-Rip (PR)	∞	∞	finite	finite	finite	finite	weak	weak
II	Sudden Future (SFS)	t_s	a_s	ϱ_s	∞	∞	finite	weak	weak
II_g	Gen. Sudden Future (GSFS)	t_s	a_s	ϱ_s	p_s	∞	finite	weak	weak
III	Finite Scale Factor (FSFS)	t_s	a_s	∞	∞	∞	finite	weak	strong
IV	Big-Separation (BS)	t_s	a_s	0	0	∞	∞	weak	weak
V	w-singularity (w)	t_s	a_s	0	0	0	∞	weak	weak

Non-standard singularities relation to dark energy.



SFS - supernovae only (MPD et al. 2007): distance modulus $\mu_L = m - M$ for the CC model ($H_0 = 72$kms^{-1}Mpc^{-1}, $\Omega_{m0} = 0.26$, $\Omega_{\Lambda 0} = 0.74$) (dashed curve) and SFS model ($m = 2/3 = 0.6666, n = 1.9999, \delta = -0.471, y_0 = 0.99936$ - **SFS in 8.7 mln years**) (solid curve). Open circles - 'Gold' data; filled circles - SNLS data

CMB shift parameter.

CMB shift parameter is:

$$\mathcal{R} = \frac{l_1'^{TT}}{l_1^{TT}} \tag{30}$$

where

l_1^{TT} – the temperature perturbation CMB spectrum multipole of the first acoustic peak in SFS model

$l_1'^{TT}$ – the multipole of a reference flat standard Cold Dark Matter model.

One usually uses a rescaled shift parameter ($y = t/t_s$):

$$\mathcal{R} = \frac{H_0 a_0}{c}\sqrt{\Omega_{m0}} r_{dec} = \sqrt{\Omega_{m0}} a'(y) \int_{y_{dec}}^{y_0} \frac{dy}{a(y)} = \sqrt{\Omega_{m0}} \int_0^{z_{dec}} \frac{dz}{E(z)}, \tag{31}$$

and WMAP data gives $\mathcal{R} = 1.70 \pm 0.03$ (Wang et al. 2006).

Baryon acoustic oscillations.

This can be done by measuring the transverse extend of an object (using the angular diameter distance $d_A = l/\Delta\theta$, where l is the linear size of an object) and the line-of-sight extend (using the redshift distance $\Delta x = c\Delta t/a(t) = ct_s\Delta y/a(y)$) (see e.g. Nesseris et al. 2006). As a result one defines the volume distance as

$$D_V^3 = d_A^2 \Delta x \quad , \tag{32}$$

so that one has

$$D_V = \left[\left(\int_{y_1}^{y_0} \frac{ct_s dy}{a(y)} \right)^2 \left(\frac{ct_s \Delta y}{a(y)} \right) \right]^{\frac{1}{3}} = \left[\left(\frac{c}{a_0 H_0} \int_0^z \frac{dz}{E(z)} \right)^2 \left(\frac{c}{a_0 H_0} \frac{\Delta z}{E(z)} \right) \right]^{\frac{1}{3}} \quad . \tag{33}$$

Eisenstein et al. (2005) gave $D_V(\Delta z = z_{BAO} = 0.35) = 1370 \pm 64$ Mpc (an acoustic peak for 46748 luminous red galaxies (LRG) selected from the SDSS.

Baryon acoustic oscillations - dimensionless parameter A.

For SFS models it is more convenient to use a dimensionless quantity $\mathcal{A}$ which is obtained multiplying D_V by $\sqrt{\Omega_{m0}}/(ct_s z_{BAO})$ or by $\sqrt{\Omega_{m0}}(a_0 H_0)/(cz_{BAO})$ to get

$$\mathcal{A} = \sqrt{\Omega_{m0}} a'(y_0) \left[\frac{a(y_{BAO})}{a'(y_{BAO}) a(y_0)} \right]^{\frac{1}{3}} \left[\frac{1}{z_{BAO}} \int_{y_{BAO}}^{y_0} \frac{dy}{a(y)} \right]^{\frac{2}{3}} \tag{34}$$

or

$$\mathcal{A} = \sqrt{\Omega_{m0}} E(z_{BAO})^{-1/3} \left[\frac{1}{z_{BAO}} \int_0^{z_1} \frac{dz}{E(z)} \right]^{2/3} \tag{35}$$

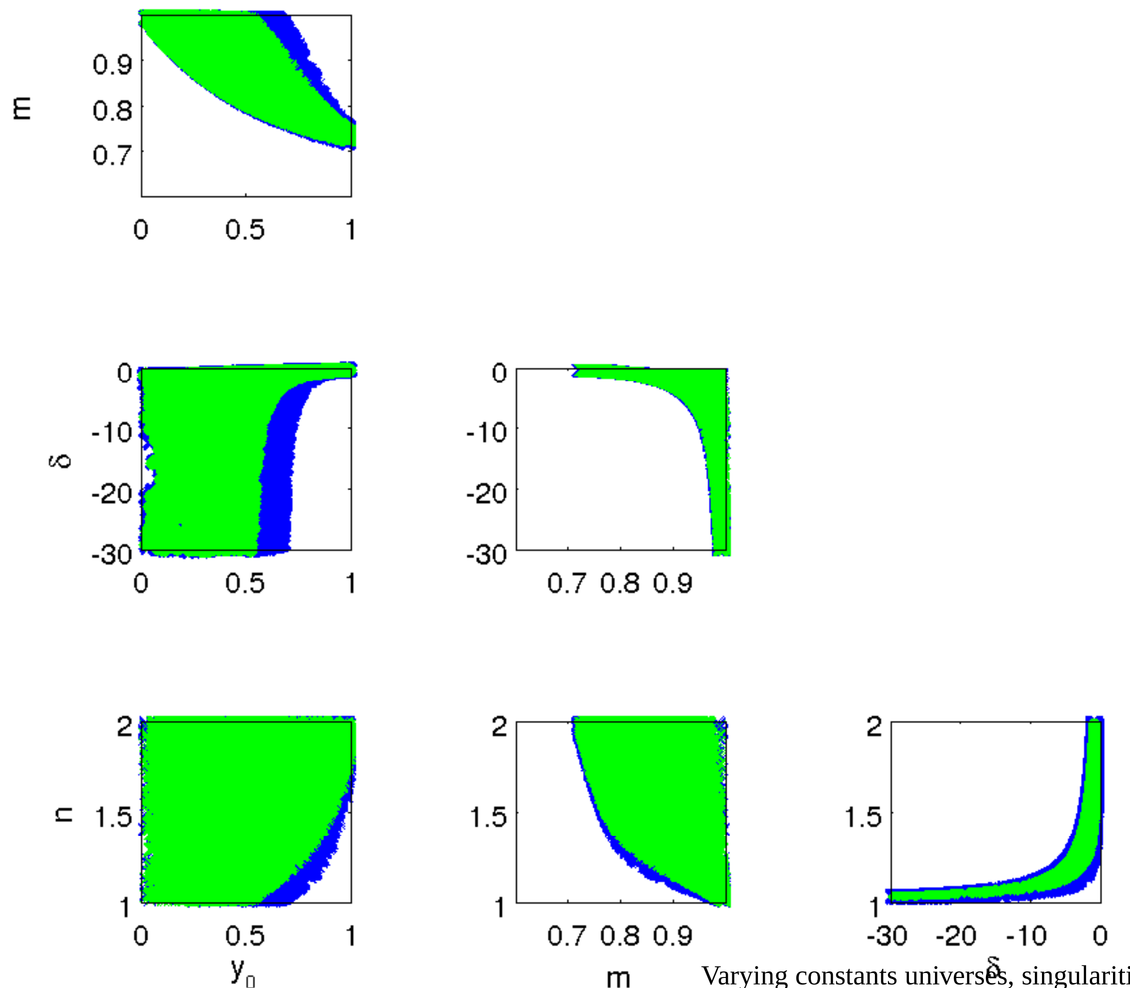
It should have the value (Eisenstein et al. 2005)

$$\mathcal{A} = 0.469 \left(\frac{n}{0.98} \right)^{-0.35} \pm 0.017 \ , \tag{36}$$

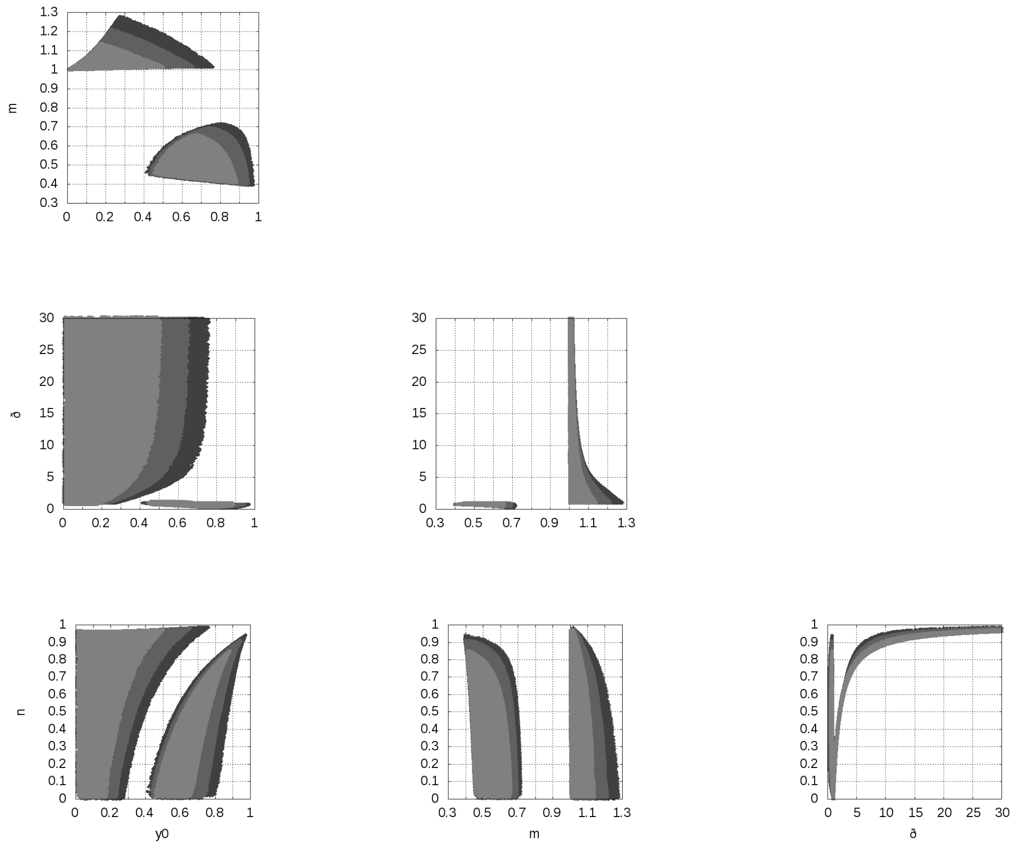
where n is the spectral index (now taken about ~ 0.96).

SFS: supernovae, CMB, BAO (Denkiewicz et al. 2012)

Fits if $m \approx 0.72$, $w = -0.082$ (slightly negative pressure); possibly $\approx$ tens of mln years in future



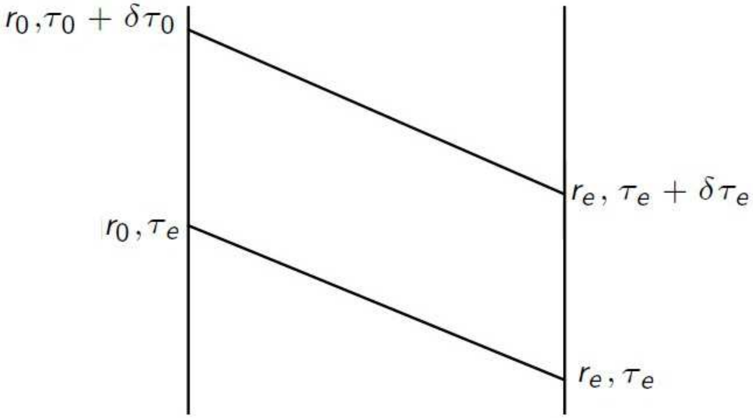
FSFS: supernovae, CMB, BAO (Denkiewicz 2012)



$m = 2/3$ (dust) matter allowed in the past; FSFS may happen in $2 \cdot 10^9$ years in future (stronger, and closer to big-bang since $a =$const, and big-bang has $a = 0$). Keresztes, et al. (2009, 2010) found similar value for the Big-Brake model which is a subcase of SFS model ($p = A/\rho$)

SFS, FSFS - redshift drift test of dark energy models.

Redshift drift (Sandage 1962) - the idea is to collect data from two light cones separated by 10-20 years to look for a change in redshift of a source as a function of time.



There is a relation between the times of emission of light by the source τ_e and $\tau_e + \Delta\tau_e$ and times of their observation at τ_o and $\tau_o + \Delta\tau_o$:

$$\int_{\tau_e}^{\tau_o} \frac{d\tau}{a(\tau)} = \int_{\tau_e+\Delta\tau_e}^{\tau_o+\Delta\tau_o} \frac{d\tau}{a(\tau)} , \tag{37}$$

which for small $\Delta\tau_e$ and $\Delta\tau_o$ reads as $\frac{\Delta\tau_e}{a(\tau_e)} = \frac{\Delta\tau_o}{a(\tau_o)}$.

Redshift drift in exotic singularity models.

The redshift drift is defined as

$$\Delta z = z_e - z_0 = \frac{a(t_0 + \Delta t_0)}{a(t_e + \Delta t_e)} - \frac{a(t_0)}{a(t_e)} \ , \tag{38}$$

which can be expanded in series and to first order in Δt as

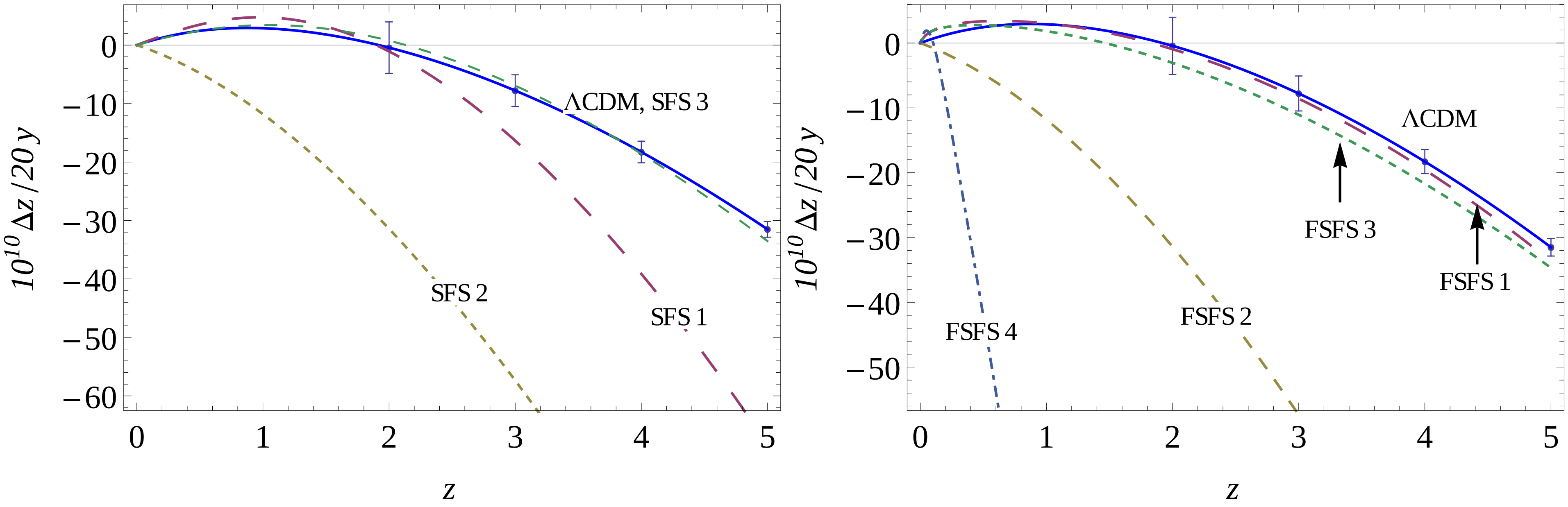
$$\begin{aligned} \Delta z &= \frac{a(t_0) + \dot{a}(t_0)\Delta t_0}{a(t_e) + \dot{a}(t_e)\Delta t_e} - \frac{a(t_0)}{a(t_e)} \\ &\approx \frac{a(t_0)}{a(t_e)} \left[\frac{\dot{a}(t_0)}{a(t_0)}\Delta t_0 - \frac{\dot{a}(t_e)}{a(t_e)}\Delta t_e \right] \ . \end{aligned} \tag{39}$$

Using above relations we have

$$\Delta z = \Delta t_0 \left[H_0(1+z) - H(t(z)) \right] = (1+z)\frac{\Delta v}{c} \ , \tag{40}$$

where Δv is the velocity shift and $H(t(z))$ is given in a standard way.

Redshift drift for SFS, FSF (Denkiewicz, MPD, Martins, Vielzeuf, 2014).



SFS3, FSFS1, FSFS3 can mimic ΛCDM

SFS1, FSFS4 differ from ΛCDM significantly

SFS2, FSFS2 - dust Friedmann model

$H_0 = 67.3\ km/s/Mpc$ and $\Omega_{m0} = 0.315$ (Planck 2013).

redshift drift - parameters

	m	δ	n	t_0/t_s
SFS 1	$2/3$	-0.43	1.9999	0.99
SFS 2	$2/3$	0.0	1.9999	0.99
SFS 3	0.749	-0.45	1.99	0.77
FSFS 1	0.56	0.42	0.8	0.96
FSFS 2	$2/3$	0.0	0.7	0.79
FSFS 3	$2/3$	0.24	0.7	0.96
FSFS 4	1.15	7.5	0.81	0.51

RD planned to be measured by ELT-HIRES high-resolution ultra-stable spectrograph for the E-ELT (European Extermely Large Telescope) - Lyman-α forest. Also SKA (Square Kilometre Array), CHIME (The Canadian Hydrogen Intensity Mapping Experiment). Plus DECIGO/BBO - grav. wave related measurements.

4. Dynamical scalar field framework and varying α.

We assume that the emergence of exotic singularities SFS and FSFS encoded in the behaviour of the scale factor (5) is due to a dynamical field Φ which at the same time is responsible for the variation of the fine structure constant α (Webb et al. 1999, Sandvik 2002) given by the Lagrangian (Nunes & Lidsey 2004)

$$S = \int d^4x\sqrt{-g}\left(-\frac{1}{2\kappa^2}R + \frac{\omega}{2}\partial_\mu\Phi\partial^\mu\Phi - V(\Phi) - \frac{1}{4}B_F(\Phi)F_{\mu\nu}F^{\mu\nu}\right) \; , \qquad (41)$$

where $\kappa^2 = 8\pi G/c^4$ and $B_F(\Phi) = \alpha_0/\alpha(\Phi)$. To a good approximation (small redshift) we may assume a linearized gauge kinetic function

$$B_F(\Phi) = 1 - \xi\kappa(\Phi - \Phi_0) \; , \qquad (42)$$

where ξ parametrizes the coupling between the scalar field and the electromagnetic sector. The evolution of α can be written as

$$\frac{\Delta\alpha}{\alpha} \equiv \frac{\alpha - \alpha_0}{\alpha_0} = B_F^{-1}(\Phi) - 1 = \xi\kappa(\Phi - \Phi_0) \; . \qquad (43)$$

Dynamical scalar field ...

The energy density is split from dust matter and the scalar field density as follows

$$\rho = \rho_\Phi + \rho_m \ , \rho_m = \Omega_m \rho_0 \left(\frac{a_0}{a}\right)^3 \tag{44}$$

In terms of density parameters

$$\rho = \frac{3H_0^2}{8\pi G}\left[\Omega_m \left(\frac{a_0}{a}\right)^3 + \Omega_\Phi\right] \tag{45}$$

so that

$$\Omega_\Phi = 1 - \Omega_{m0}\frac{H_0^2}{H^2}\left(\frac{a_0}{a}\right)^3 = 1 - \Omega_m . \tag{46}$$

The barotropic index of the canonical scalar field Φ is defined as $w_\Phi = p_\Phi/\rho_\Phi$, where $p_\Phi = \pm(1/2)\dot{\Phi}^2 - V(\Phi)$ and $\rho_\Phi = \pm(1/2)\dot{\Phi}^2 + V(\Phi)$ ("-" sign for phantom). The effective barotropic index of the equation of state is $w_{eff} = p/\rho$, and $p = p_\Phi$.

Dynamical scalar field and α.

Using $\pm\dot{\Phi}^2 = p_\Phi + \rho_\Phi$ and changing the derivative with respect to time into the derivative with respect to logarithm of the scale factor i.e. that $(...)' \equiv d/d\ln a = H^{-1}d/dt$ we have

$$w_\Phi + 1 = \pm\frac{\dot{\Phi}^2}{\rho_\Phi} = \pm\frac{(\kappa\Phi')^2}{3\Omega_\Phi} \ , \tag{47}$$

where Ω_Φ if the fraction of the universe's energy in the scalar field component

$$\Omega_\Phi = \frac{\rho_\Phi}{\rho_\Phi + \rho_m} = \frac{\rho_\Phi a^3}{\rho_0\Omega_{m0} + \rho_\Phi a^3} \ . \tag{48}$$

The equation of the field Φ can be integrated with respect to the scale factor by changing variables ($dz/(1+z) = da/a$)

$$\frac{\Delta\alpha}{\alpha}(z) = \pm\xi\int_0^z \sqrt{3\Omega_\Phi(\hat{z}) \mid (1+w(\hat{z})) \mid}\frac{d\hat{z}}{(1+\hat{z})} \,. \tag{49}$$

Dynamical scalar field - redshift

Since we mimic the SFS/FSFS scale factor (14) by the scalar field, then we define the redshift as

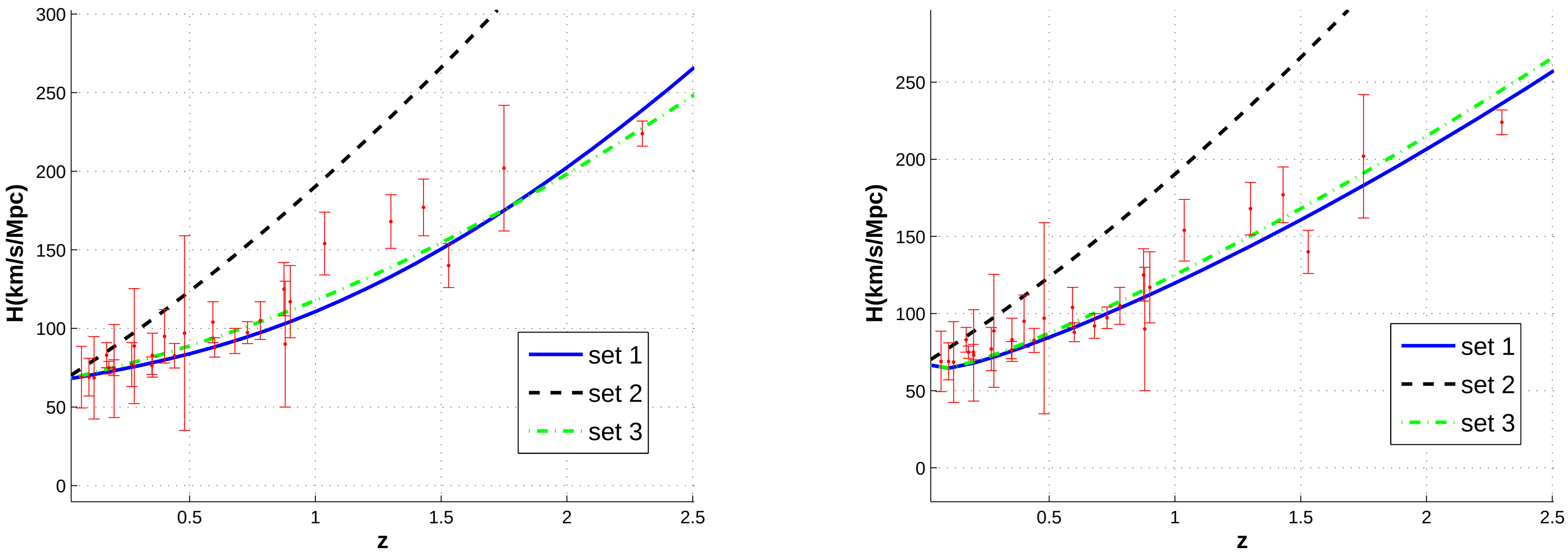
$$1 + z = \frac{a(t_0)}{a(t_1)} = \frac{\delta + (1-\delta)\left(\frac{t_0}{t_s}\right)^m - \delta\left(1 - \frac{t_0}{t_s}\right)^n}{\delta + (1-\delta)\left(\frac{t_1}{t_s}\right)^m - \delta\left(1 - \frac{t_1}{t_s}\right)^n} , \tag{50}$$

and the Hubble function as

$$H(t(z)) = \frac{1}{t_s} \frac{m(1-\delta)\left(\frac{t}{t_s}\right)^{m-1} + \delta n\left(1 - \frac{t}{t_s}\right)^{n-1}}{\delta + (1-\delta)\left(\frac{t}{t_s}\right)^m - \delta\left(1 - \frac{t}{t_s}\right)^n} , \tag{51}$$

Varying α as dark energy (MPD, Denkiewicz, Martins, Vielzeuf 2014).

Now we use the selected by the previous tests (SnIa, BAO, CMB shift parameter, redshift drift) dark energy models with SFS/FSFS singularities (see table above) to check if they can be mimicked by the dynamical scalar field varying α framework. First we check the evolution of the Hubble function $H(z)$ for SFS1,2,3 (left) and FSFS1,2,3 (right) models. SFS2 and FSFS2 (dust models) are ruled out but other models can mimic ΛCDM.



Measuring variation of α.

We use the data given by Webb et al. (PRL 107, 191101 (2011)) (Keck and VLT) as well as other specific measurements of α given in the table below (in parts per million):

Object	z	$\Delta\alpha/\alpha$	Spectrograph	Ref.
HE0515$-$4414	1.15	-0.1 ± 1.8	UVES	Molaro et al. (2008)
HE0515$-$4414	1.15	0.5 ± 2.4	HARPS/UVES	Chand et al. (2006)
HE0001$-$2340	1.58	-1.5 ± 2.6	UVES	Agafonowa et al. (2011)
HE2217$-$2818	1.69	1.3 ± 2.6	UVES–LP	Molaro et al. (2013)
Q1101$-$264	1.84	5.7 ± 2.7	UVES	Molaro et al. (2008)

UVES - Ultraviolet and Visual Echelle Telescope

HARPS - High Accuracy Radial velocity Planet Searcher

LP - Large Program measurement

Atomic clock Rosenband bound at $z = 0$

Rosenband (2008) measurement gives the following bound at $z = 0$

$$\left(\frac{\dot{\alpha}}{\alpha}\right)_0 = (-1.6 \pm 2.3) \times 10^{-17} \mathrm{yr}^{-1} \, . \tag{52}$$

which by using (49) can be transformed onto the bound for the scalar field coupling ξ:

$$\left|\frac{\dot{\alpha}}{\alpha}\right|_0 = |\xi| H_0 \sqrt{3\Omega_{\Phi 0} \mid 1 + w_{\Phi 0} \mid} \, , \tag{53}$$

which translates for $H_0 = (67.4 \pm 1.4) \, \mathrm{km.s}^{-1}\mathrm{Mpc}^{-1}$ Planck value) into the conservative (3σ) bound

$$|\xi| \sqrt{3\Omega_{\Phi 0} \mid 1 + w_{\Phi 0} \mid} < 10^{-6}, \tag{54}$$

Bounds on the coupling ξ from atomic clocks

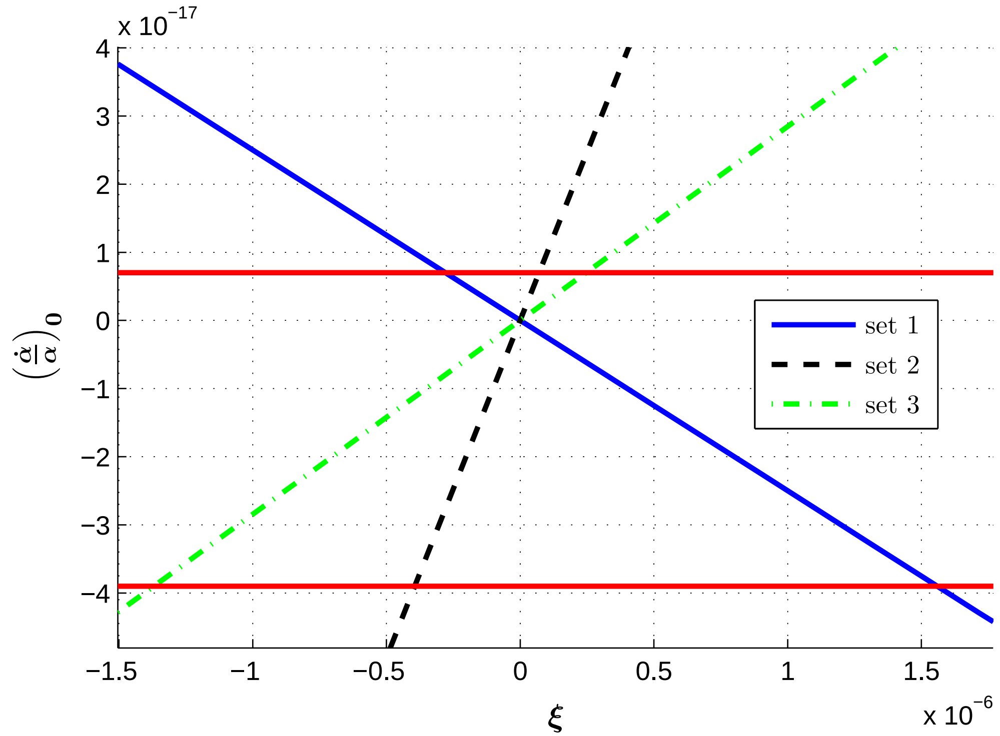
Maximum allowed variation of α in the redshift range $0 \leq z \leq 5$:

Model	$\Omega_{\Phi 0}$	$w_{\Phi 0}$	$\lvert\xi\rvert_{\max} \times 10^6$	$z\rvert_{\alpha_{max}}$	$\lvert\Delta\alpha/\alpha\rvert_{\max} \times 10^6$
SFS1	0.685	-1.06	2.76	1.4	1.47
SFS2	0.685	0.0	0.70	5.0	1.79
SFS3	0.685	-0.92	2.42	2.6	0.80
FSFS1	0.685	-3.49	0.44	0.2	0.08
FSFS2	0.685	0.0	0.70	5.0	1.79
FSFS3	0.685	-3.68	0.43	0.2	0.06

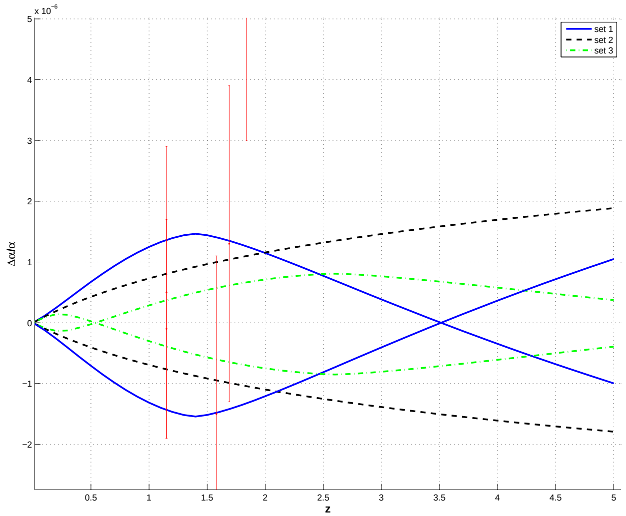
Important notice: large negative values of $w_{\Phi 0}$ lead to very tight bound on the coupling ξ and extremely small variations of α (100 times smaller than needed to explain Webb's et al. (2011) data and also difficult to reach by future generation telescopes).

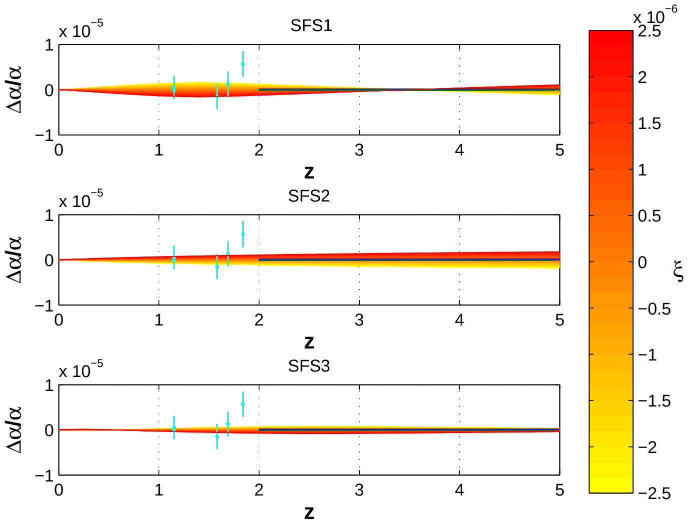
Atomic clock Rosenband (2008) bound at $z = 0$

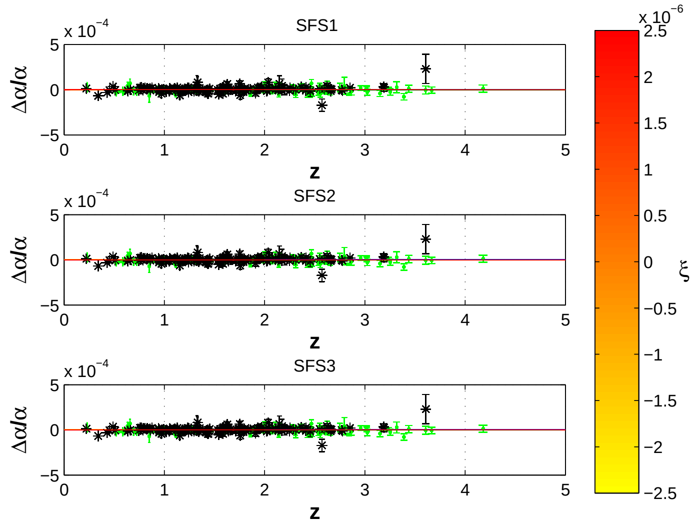
Then we have below the present-day drift rate of α as a function of the coupling ξ, for the SFS1,2,3 models under consideration, compared to the one-sigma Rosenband bound:



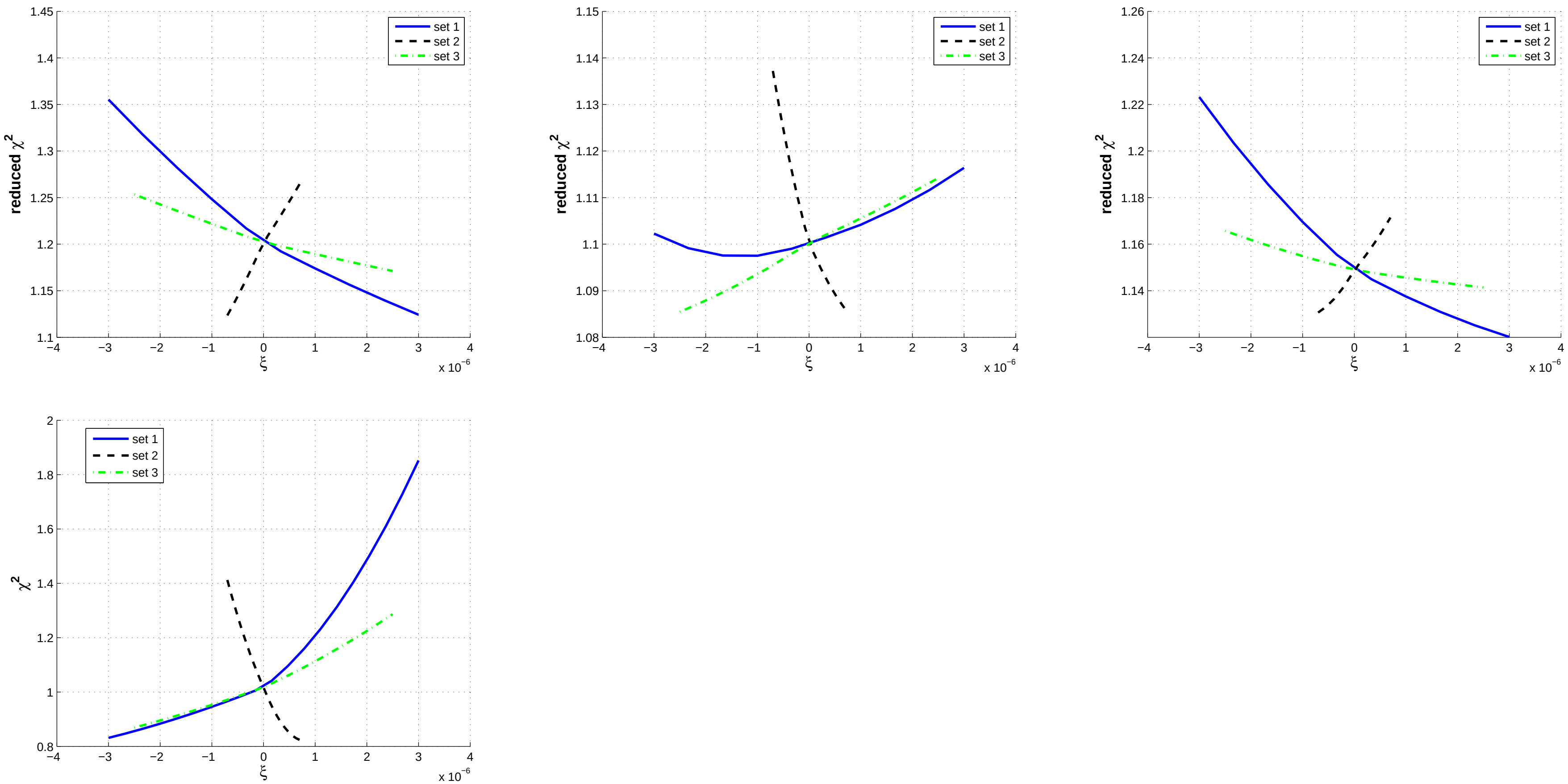
$\alpha(z)$ variation (Webb's data); black rectangle - sensitivity of E-ELT







χ^2 test of SFS models



Webb's Keck data (top left), Webb's VLT data (top middle), Webb's full dataset (top right) and the data from the Table (5 measurements)(bottom)

No minimum of χ^2 - coupling ξ incompatible with $z = 0$ atomic clock bound (3σ)

Redshift drift for VSL models.

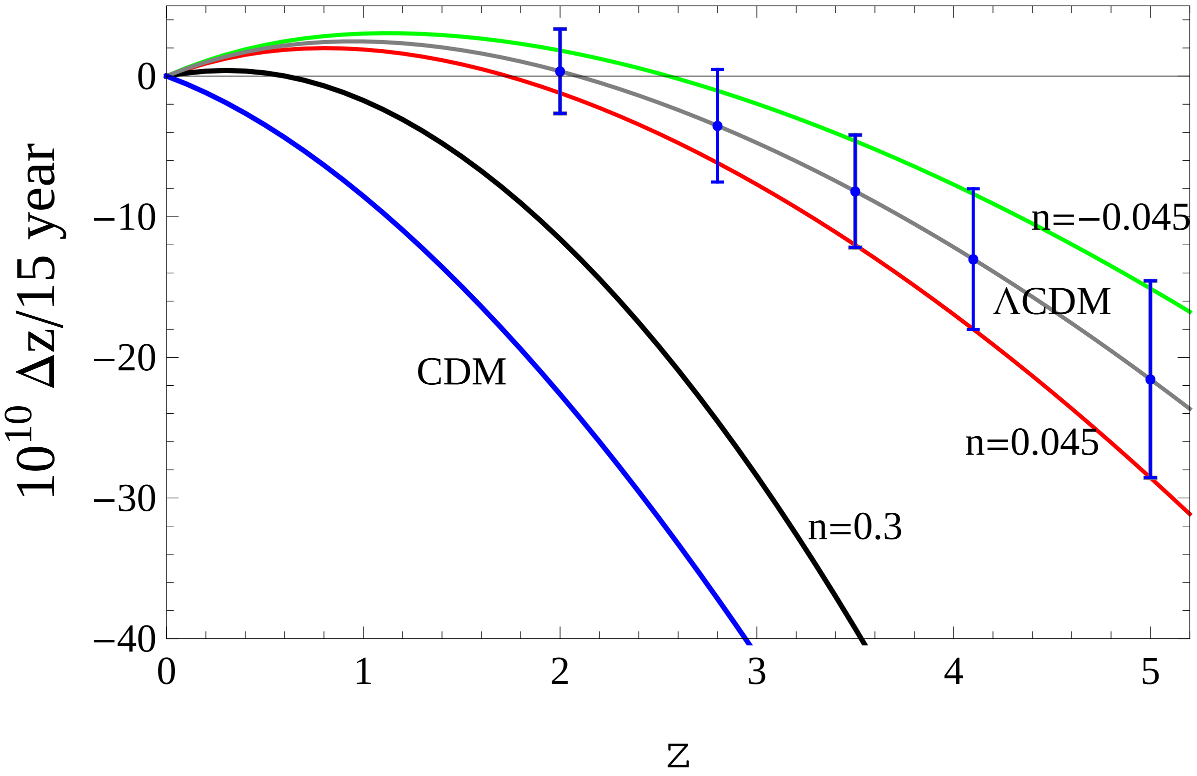
In varying speed of light theories one has (Balcerzak, MPD 2014)

$$\frac{\Delta z}{\Delta t_0} = \frac{\Delta z}{\Delta t_0}(z, n) = H_0(1+z) - H(z)(1+z)^n \quad , \tag{55}$$

or in terms of standard definitions of density parameters Ω (for $k = 0$) we have

$$\begin{aligned} \frac{\Delta z}{\Delta t_0} &= H_0 \left[1 + z - (1+z)^n \sqrt{\Omega_{m0}(1+z)^3 + \Omega_\Lambda}\right] \\ &= H_0 \left[1 + z - \sqrt{\Omega_{m0}(1+z)^{3+2n} + \Omega_\Lambda (1+z)^{2n}}\right] \end{aligned} \tag{56}$$

Redshift drift for VSL models.



Observationally, $n \sim -10^{-5} < 0$ (Murphy et al. 2007, King et al. 2012). Error bars due to Quercellini et al. 2012. For $|n| < 0.045$ one cannot distinguish between VSL and ΛCDM models.

5. Varying constants removing or changing singularities.

- It has been shown that quantum effects (e.g. Houndjo 1008.0664; Houndjo et al. 1203.6084) may change the strength of exotic singularities (e.g. SFS can be changed into either FSF or BR or BC).
- The application of quantum cosmology (cf. M. Bouhmadi-Lopez talk about type I-IV singularities) may remove classical singularities in the quantum sense.
- EiBl theory also removes the singularities (Bouhmadi-Lopez et al. 2014)
- Varying constants cosmologies have been applied to solve standard cosmology problems such as the horizon and flatness problem (e.g. Albrecht, Magueijo 1999; Barrow 1999).
- Here we can apply varying constants to **remove or change the nature of singularities** in cosmology.

varying constants versus cosmic singularities.

Einstein-Friedmann equations generalize in varying speed of light (VSL) theories and varying gravitational constant G theories to (ϱ - mass density; $\varepsilon = \varrho c^2(t)$ - energy density in $Jm^{-3} = Nm^{-2} = kgm^{-1}s^{-2}$)

$$\varrho(t) = \frac{3}{8\pi G(t)}\left(\frac{\dot{a}^2}{a^2} + \frac{kc^2(t)}{a^2}\right) , \tag{57}$$

$$p(t) = -\frac{c^2(t)}{8\pi G(t)}\left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{kc^2(t)}{a^2}\right) , \tag{58}$$

and the energy-momentum "conservation law" is

$$\dot{\varrho}(t) + 3\frac{\dot{a}}{a}\left(\varrho(t) + \frac{p(t)}{c^2(t)}\right) = -\varrho(t)\frac{\dot{G}(t)}{G(t)} + 3\frac{kc(t)\dot{c}(t)}{4\pi G a^2} . \tag{59}$$

General form of the scale factor.

We use a general form of the scale factor (MPD, K. Marosek, JCAP 02 (2013), 012), which **admits big-bang, big-rip, sudden future, finite scale factor and w-singularities** and reads as

$$a(t) = a_s \left(\frac{t}{t_s}\right)^m \exp\left(1 - \frac{t}{t_s}\right)^n , \tag{60}$$

with the constants t_s, a_s, m, n. For $k = 0$ we have

$$\varrho(t) = \frac{3}{8\pi G(t)} \left[\frac{m}{t} - \frac{n}{t_s}\left(1 - \frac{t}{t_s}\right)^{n-1}\right]^2 , \tag{61}$$

$$\begin{aligned} p(t) &= -\frac{c^2(t)}{8\pi G(t)} \left[\frac{m(3m-2)}{t^2} - 6\frac{mn}{tt_s}\left(1 - \frac{t}{t_s}\right)^{n-1}\right. \\ &+ \left. 3\frac{n^2}{t_s^2}\left(1 - \frac{t}{t_s}\right)^{2(n-1)} + 2\frac{n(n-1)}{t_s^2}\left(1 - \frac{t}{t_s}\right)^{n-2}\right] . \end{aligned} \tag{62}$$

The scale factor - parametrization.

For $m < 0$ we have **a big-rip singularity** - $a \to \infty$, $\varrho \to \infty$, $p \to \infty$ at $t = 0$;
For $1 < n < 2$ we have **a sudden future singularity** (SFS) which appears at $t = t_s$ ($a = a_s$, $\varrho =$ const., $p \to \infty$);
For $0 < n < 1$ we have **a (stronger) finite scale factor singularity** (FSF) at $t = t_s$ ($a = a_s$, $\varrho \to \infty$, $p \to \infty$).

In fact, for $1 < n < 2$ only the last term in the pressure of the type $(1 - t/t_s)^{n-2}$ blows-up, while for $0 < n < 1$ two more terms $(1 - t/t_s)^{n-1}$ and $(1 - t/t_s)^{2(n-1)}$ do.

Removing singularities by varying constants

One bears in mind the scale factor (60), the energy density (61) and pressure (62)

Regularizing a Big-Bang singularity by varying G:

If

$$G(t) \propto \frac{1}{t^2} \tag{63}$$

which is a faster decrease than in Dirac's LNH $G \propto 1/t$, but influences less the temperature of the Earth constraint (Teller 1948).

Both divergence in ϱ and p are removed, though at the expense of having the "singularity" of strong gravitational coupling $G \to \infty$ at $t \to 0$.

In the Dirac's case, only the ϱ singularity can be removed.

Regularizing an SFS singularity by varying c:

If

$$c(t) = c_0 \left(1 - \frac{t}{t_s}\right)^{\frac{p}{2}} , \tag{64}$$

then

$$p(t) = -\frac{c_0^2}{8\pi G}\left[\frac{m(3m-2)}{t^2}\left(1-\frac{t}{t_s}\right)^{p} - 6\frac{mn}{tt_s}\left(1-\frac{t}{t_s}\right)^{p+n-1} + 3\frac{n^2}{t_s^2}\left(1-\frac{t}{t_s}\right)^{p+2n-2} + 2\frac{n(n-1)}{t_s^2}\left(1-\frac{t}{t_s}\right)^{p+n-2}\right].$$

and the singularity of pressure is removed provided $p > 2 - n, (1 < n < 2)$.

removing singularities by varying constants: SFS.

Physical consequence: **light eventually stops** at the singularity. Same happens in loop quantum cosmology (LQC) where it is called the anti-newtonian limit $c = c_0\sqrt{1 - \varrho/\varrho_c} \to 0$ for $\varrho \to \varrho_c$ with ϱ_c being the critical density (Cailettau et al. 2012). The low-energy limit $\varrho \ll \varrho_0$ gives the standard limit $c \to c_0$.

It also appears naturally in Magueijo model ((Magueijo, PRD 63, 043502 (2001))) in which black holes are not reachable since the light stops at the horizon (despite they possess Schwarzschild singularity). An observer cannot reach this surface even in his finite proper time.

Strangely, both options $c = 0$ and $c = \infty$ are possible in Magueijo model.

Removing an SFS singularity by varying G:

If we assume that

$$G(t) = G_0 \left(1 - \frac{t}{t_s}\right)^{-r} , \tag{65}$$

($r =$ const., $G_0 =$ const.) which changes (61) and (62) to

$$\begin{aligned} \varrho(t) &= \frac{3}{8\pi G_0}\left[\frac{m^2}{t^2}\left(1-\frac{t}{t_s}\right)^{r} - \frac{2mn}{tt_s}\left(1-\frac{t}{t_s}\right)^{r+n-1}\right. \\ &+ \left.\frac{n^2}{t_s^2}\left(1-\frac{t}{t_s}\right)^{r+2n-2}\right] , \end{aligned} \tag{66}$$

$$\begin{aligned} p(t) &= -\frac{c^2}{8\pi G_0}\left[\frac{m(3m-2)}{t^2}\left(1-\frac{t}{t_s}\right)^{r} - 6\frac{mn}{tt_s}\left(1-\frac{t}{t_s}\right)^{r+n-1} + 3\frac{n^2}{t_s^2}\left(1-\frac{t}{t_s}\right)^{r+2n-2}\right. \\ &+ \left. 2\frac{n(n-1)}{t_s^2}\left(1-\frac{t}{t_s}\right)^{r+n-2}\right] . \end{aligned} \tag{67}$$

removing or changing singularities by varying constants: SFS

From (66) and (67) it follows that an SFS singularity $(1 < n < 2)$ is regularized by varying gravitational constant when

$$r > 2 - n \ , \tag{68}$$

and an FSF singularity $(0 < 1 < n)$ is regularized when

$$r > 1 - n \ . \tag{69}$$

On the other hand, assuming that we have an SFS singularity and that

$$-1 < r < 0 \, , \tag{70}$$

we get that varying G **may change an SFS singularity onto a stronger FSF singularity** when

$$0 < r + n < 1 \, . \tag{71}$$

Subtleties:

- In order to regularize an SFS or an FSF singularity by varying $c(t)$, the **light should slow and eventually stop propagating** at a singularity. Similar effects were found in loop quantum cosmology (LQC) as well as in VSL for Schwarzschild horizon (Magueijo 2001) - speed of light is either zero or infinity at $r = r_s$. An observer cannot reach this surface even in his finite proper time.
- To regularize an SFS, FSF by varying gravitational constant $G(t)$ - **the strength of gravity has to become infinite** at a singularity. On the one hand, it is quite reasonable because of the requirement to **overcome an infinite (anti-)tidal forces** at the singularity, but on the other hand, it makes another singularity - **a singularity of strong coupling** for a physical field such as $G \propto 1/\Phi$. Such problems were already dealt with in superstring and brane cosmology where both the curvature singularity and a strong coupling singularity appeared (choice of coupling, quantum corrections).

6. Varying constants quantum cosmology.

K. Leszczyńska, A. Balcerzak, MPD - in progress (cf. also R. Garattini and M. Sakellariadou 2014)
We consider quantum cosmology of VSL and VG theories with the ansätze:

$$c(t) = c_0 a^n(t) \ , \qquad G(t) = G_0 a^q(t) \ . \tag{72}$$

The integration of the conservation law and using the barotropic equation of state $p = (\gamma - 1)\rho$, $\gamma =$ const. gives

$$\varrho = \frac{M}{G_0 a^{3\gamma+q}} \tag{73}$$

$$+ \quad \frac{c_0^2}{4\pi G_0}\left[\frac{3kn}{2n+3\gamma-2}a^{2(n-1)-q} - \frac{\lambda(\frac{q}{2}-n)}{2n+3\gamma}a^{2n-q}\right] \ .$$

Substituting this into the Friedmann equation one has

$$\dot{a}^2 a^{3\gamma-2} - \frac{8\pi M}{3} + U(a) = 0 \ , \tag{74}$$

Varying constants quantum cosmology.

where

$$U(a) = -\left(\frac{k(2-3\gamma)c_0^2}{2n+3\gamma-2} + \frac{\Lambda(3\gamma+q)c_0^2a^2}{3(2n+3\gamma)}\right)a^{2n+3\gamma-2} \tag{75}$$

The potential (75) has the one zero at $a=0$ and another at

$$a_0 = \sqrt{\frac{3k(3\gamma-2)(2n+3\gamma)}{\Lambda(3\gamma+q)(2n+3\gamma-2)}} \tag{76}$$

provided it is real. In some cases it has the shape to allow quantum mechanical tunneling from $a=0$ to $a=a_0$. In the limit of constant G one recovers the result of Szydlowski and Krawiec (PRD, 2003) and A. Yurov and V. Yurov (0812.4738)

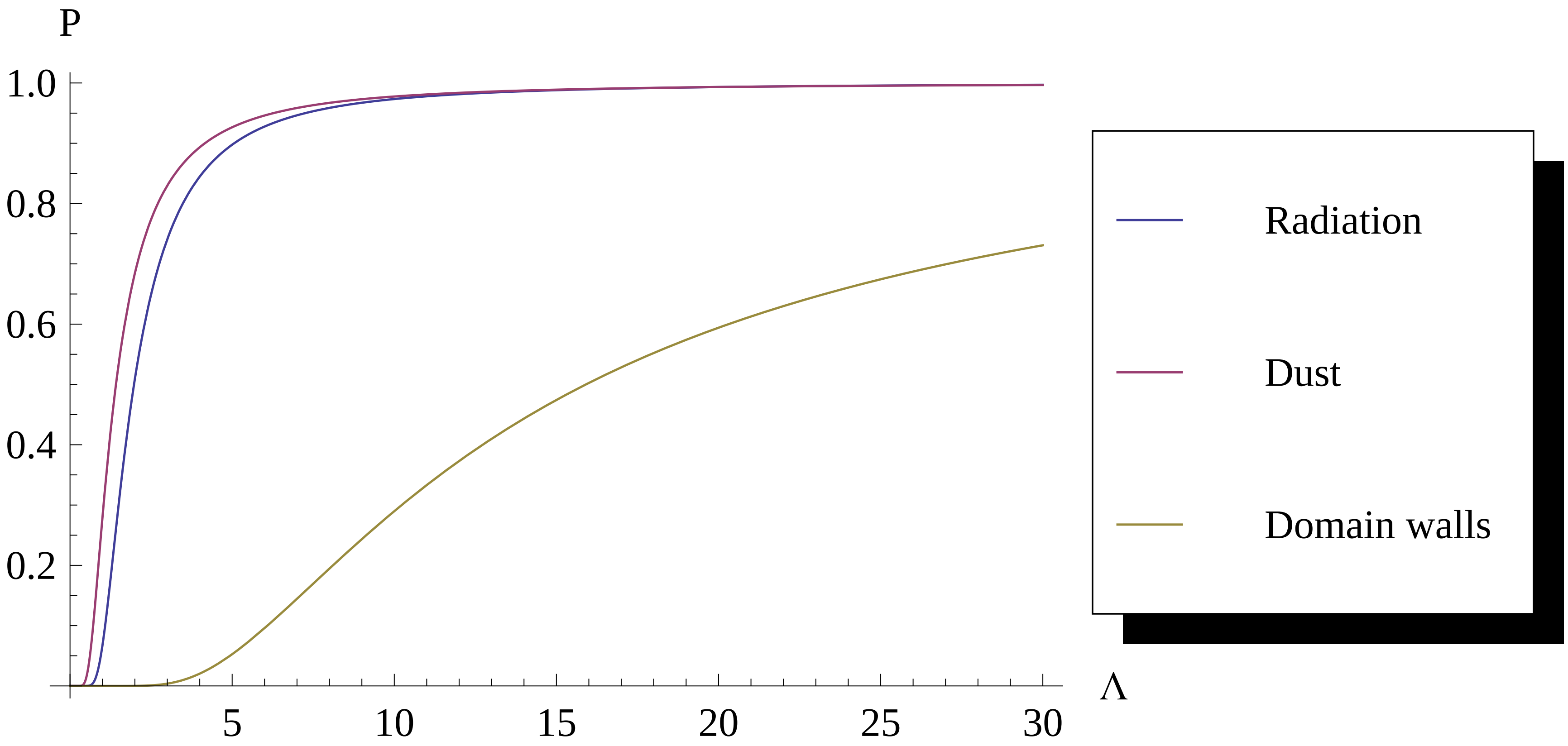
Varying constants quantum cosmology.

Two physically interesting cases of tunneling can be studied: first, when we have radiation matter ($\gamma = 4/3$) and dust matter ($\gamma = 1$), positive curvature $k = +1$ and positive cosmological constant $\Lambda > 0$. Second, when we have the network of red domain walls ($\gamma = 1/3$), negative curvature $k = -1$ and positive cosmological constant (similar case for $c =$const, $\Lambda < 0$ was considered by MPD and Larsen (PRD, 1985) and recently by Mithani and Vilenkin (JCAP, 2012) and Graham et al. 1109.0282)

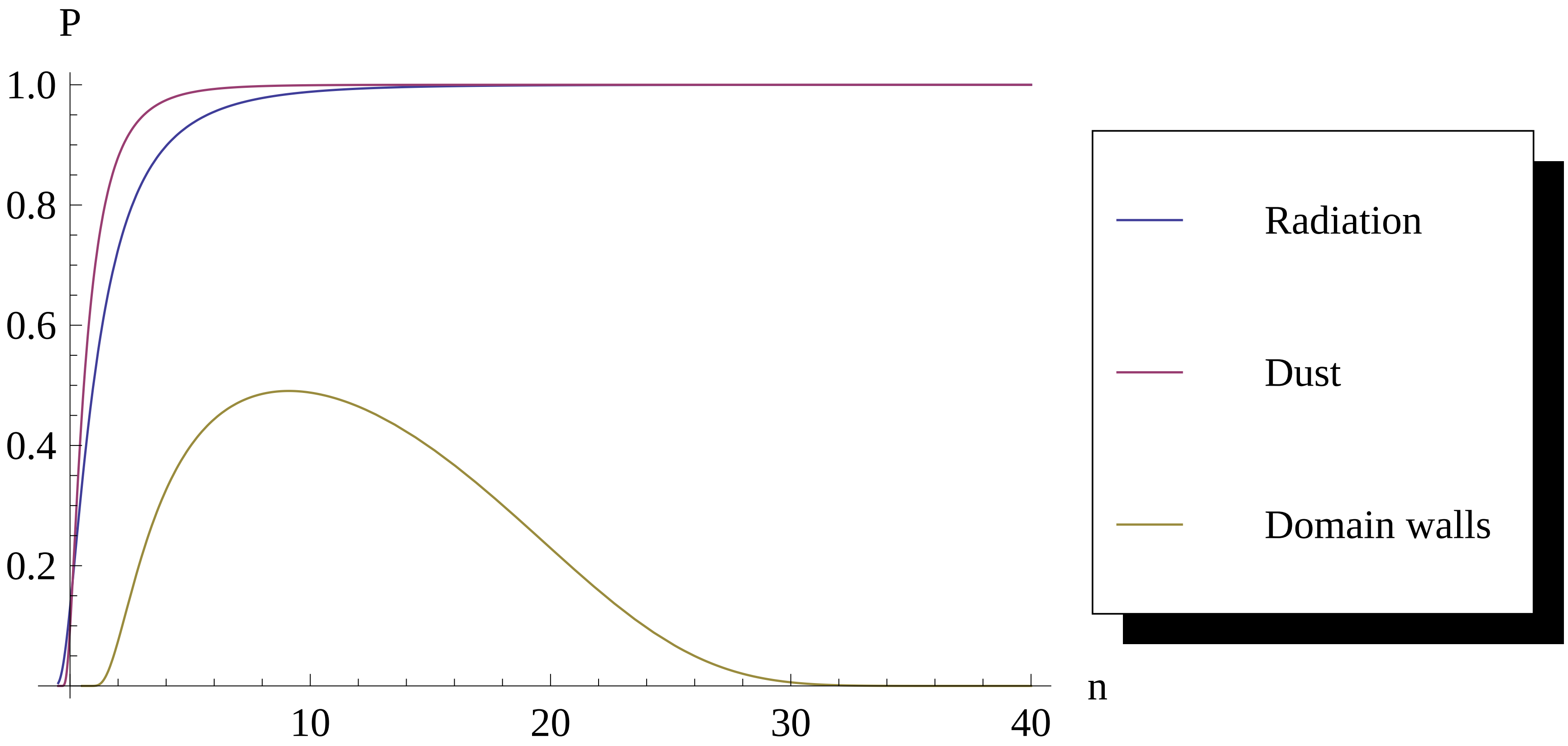
In WKB approximation the probability of tunneling the universe "from nothing" ($a = 0$) to a Friedmann geometry with $a = a_0$ (e.g. for the model with radiation) reads as

$$P \simeq \exp\left(- \frac{c_0}{\hbar} \sqrt{\frac{2\pi k}{n+1}} \, \frac{\Gamma(\frac{3+n}{2})}{\Gamma(3+\frac{n}{2})} \, a_0^{n+3} \right) \tag{77}$$

Varying constants quantum cosmology.



Varying constants quantum cosmology.



Varying constants quantum cosmology.

- The probability of tunneling is large for both the large values of the cosmological term and the large values of the speed of light ($n > 0$, large, $c = c_0 a^n$).
- The probability of tunneling for the universe with dust is smaller than with radiation when the speed of light diminishes $n < 0$ or slightly increases ($n > 0$, small) while it is larger if the speed of light increases strongly ($n > 0$, large) with some dividing value of n between both regimes which depends on Λ.

Varying constants quantum cosmology.

An interesting ansatz for the speed of light evolution (Buchalter astro-ph/0403202) is:

$$c(t) = \dot{a}(t) = H(t)a(t) \tag{78}$$

which replaces the Friedmann equation into:

$$\frac{\dot{a}^2}{a^2}(1+k) - \frac{\Lambda}{3}\dot{a}^2 = \frac{8\pi G(t)}{3}\varrho \tag{79}$$

There are interesting classical solutions (which are not standard) and also quantization is different due to a different form of the kinetic term.

7. Results and conclusions

- Currently one is able to **differentiate quite a number of "exotic" cosmological singularities** with completely different properties - despite many of them are geodesically complete, they still lead to a **blow-up of physical quantities** such as scale factor, energy density, pressure, physical fields etc.
- Some of these singularities **may serve as dark energy**. **SFS (type II) may even appear in near future (8.7 Myr)** while FSFS (type III) in more distant future (2 Gyr). They **can be fitted** to a combined SnIa, CMB, BAO data and can mimic ΛCDM model for redshift drift effect for specific choice of the parameters.
- **The "exotic" cosmological singularities can be influenced by varying constants**. It is possible to **remove or change** the type of these singularities with full physical consequences of this. However, we may face **new "singularity" in a physical field** responsible for the variability of constants but this is what happens in other physical theories (e.g. superstring) too.

Results and conclusions contd.

- Studies of the dynamical scalar field Φ coupling to electromagnetic field which is responsible for the **variation of the fine structure constant** α based on observational data show that FSFS (type III) dark energy models cannot be related to variation of α (they are too small, even to be detected)
- SFS (type III) dark energy models (SFS1, SFS3) allow larger variations of α, but the values of the coupling ξ to fit the data from α variation are still in more than three-sigma tensions with the local atomic clock bounds. However, non-monotonic redshift dependence of some SFS models may allow to find the range of parameters which will be tested by the new high-resolution spectrographs to give a definite answer if varying α models can serve dark energy.
- Quantum cosmology can be applied to discuss the influence of variability of physical constants onto the probability of creation of the universe. In particular, large values of the speed of light ($n > 0$) increase the probability of tunneling.

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