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The Emergence of Classical Properties Through Interaction with the Environment

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The dependence of macroscopic systems upon their environment is studied under the assumption that quantum theory is universally valid. In particular scattering of photons and molecules turns out to be essential even in intergalactic space in restricting the observable properties by locally destroying the corresponding phase relations. The remaining coherence determines the 'classical' properties of the macroscopic systems. In this way local classical properties have their origin in the nonlocal character of quantum states.

The effect of the interaction depends essentially on whether it permanently 'measures' discrete or continuous quantities. For discrete variables (here exemplified by two-state systems) the classical properties are given by the measurement basis. The continuous case, studied for translational degrees of freedom, leads to a competition between destruction of coherence by the interaction and dispersion of the wave packet by the internal dynamics. A non-phenomenological Boltzmann-type master equation is derived for the density matrix of the center of mass. Its solutions show that the much-discussed dispersion hardly ever shows up even for small dust particles or large molecules. Instead the coherence length decreases towards the thermal de Broglie wave length of the object, whereas the incoherent spread increases. The Ehrenfest theorems are shown nevertheless to remain valid for recoil-free interactions. Some consequences of these investigations for the quantum theory of measurement are pointed out.

I. Introduction

The relation between classical and quantum mechanics is at the heart of the interpretation problem of quantum theory. Outcomes of measurements are usually expressed in classical terms at a certain level of description: the pointer position is assumed to be definite like the position of a classical point mass in space. On the other hand, the general applicability of quantum theory – that is, essentially, the superposition principle – is important for many phenomena of macroscopic objects, for example, in solid state physics. However, if applied rigorously, this principle would lead to possible states never observed in nature, like superpositions of macroscopic objects in very different positions or of other 'macroscopically different' states. One may also wonder why microscopic objects are usually found in energy eigenstates, whereas macroscopic objects occur in time-dependent states.

In addition to obviously macroscopic bodies, nearly all molecules except the very small ones seem to have a well-defined spatial structure. Especially molecules like sugar, alanine etc. appear only in right-handed or left-handed (chiral) configurations, although eigenstates of a parity-conserving Hamiltonian, in particular the ground state, are symmetric (or antisymmetric) under parity transformation and therefore cannot be chiral unless in case of degeneracy. This phenomenon has sometimes been called the 'paradox of optical isomers'. The smallness of the transition matrix element $\langle \varphi_L | H | \varphi_R \rangle$ between chiral states is of importance, but is not sufficient to explain the non-occurrence of eigenstates of the molecule. Therefore this superselection rule must have reasons which lie outside the molecule.

Another important consequence of the superposition principle is the kinematical 'quantum nonlocality'. It means that for two spatially separated systems there are in general 'quantum-correlated' states of type $\sum_{n,m} c_{n,m} |\varphi_n\rangle |\Phi_m\rangle \neq |\varphi\rangle |\Phi\rangle$. Such superpositions together with the interferences contained therein are 'real' in the sense that the latter may have observable dynamical effects – they are not due to incomplete information as for classical ensembles. In contrast, classical concepts are kinematically local, even those used for the description of extended objects like classical waves. Hence, locality would have to be *derived* together with classical concepts for the appropriate macroscopic situations, if quantum theory were assumed to be universally valid. This kinematical nonlocality is more specific than the general (perhaps dynamical) nonlocality, which has been demonstrated by Bell to be required for possible theories behind quantum theory (hidden variables theories).

As is well known, Schrödinger's original attempts [1] to derive classical concepts and equations of motion by means of wave packets obeying his equation have failed. More recently the importance of the environment for the microscopic dynamical behaviour of macroscopic quantum systems has been reckognized [2–6]. In principle, taking into account such interactions is by no means new. In particular the sensitivity of interference terms with respect to interactions with the outside world had in many situations to be carefully discussed. Nevertheless, their general role and the underlying dynamical mechanism both seem to have been misinterpreted. Otherwise it would not be understandable that Ehrenfest's theorems are still widely used to justify classical motion, or the von-Neumann equation (quantum Liouville equation) to 'derive' master equations. The destruction of interference terms is often considered as caused in a classical way by an 'uncontrollable influence' of the environment on the system of interest. In fact, this interpretation seems to date back to Heisenberg [7]. But the opposite is true: The system disturbs the environment, thereby dislocalizing the phases. If the system is originally in a superposition $\sum_n c_n |\varphi_n\rangle$, it may influence the environment $|\Phi_0\rangle$ as if being measured by it according to [8]

$$\sum_n c_n |\varphi_n\rangle |\Phi_0\rangle \rightarrow \sum_n c_n |\varphi_n\rangle |\Phi_n\rangle. \tag{1.1}$$

Only because of the kinematical nonlocality (quantum correlation) does this interaction process have an effect on later measurements performed at the system. The interference terms still exist, but they are not *there*! (The synonymy of everyday language contains a classical prejudice.) No internal (even ergodic) unitary evolution can explain the disappearance of local phases [9, 10], just as the interaction with the environment cannot describe a non-unitarity in the 'total' system.

What is 'there' (what can be observed locally at the considered system) is in quantum theory described by the system's density matrix. The use of the local density matrix allows at most only a partial derivation of classical concepts for two reasons: it already assumes a local description, and it presupposes the probabilistic interpretation leading to the collapse of the state vector at some stage of a measurement. This paper is concerned only with this partial derivation. The interaction will thereby be treated nonphenomenologically – in contrast to conventional treatments of open systems. The locality assumption may perhaps be justified by a fundamental (underivable) assumption about the local nature of the observer together with the usual locality of interactions [11]. The collapse could then be based on an assumption on how a nonlocal reality is experienced subjectively by a local observer [2, 12]. The difficulties in giving a *complete* derivation of classical concepts may as well signal the need for entirely novel concepts.

The density matrix of a subsystem φ is given by

$$\rho_\varphi = \mathrm{Tr}_\Phi \, \rho_{\mathrm{total}}, \tag{1.2}$$

where Tr_Φ is the trace over the degrees of freedom of the environment and ρ_{total} is the density matrix of the system plus environment – ideally the dyadic product $|\Psi\rangle\langle\Psi|$ of a pure state of the whole universe. Since ρ_φ will in general correspond to a 'mixed state' ('improper mixture'), certain phase relations are not defined locally. Observables that depend on such phases can not be measured (or measurements will give trivial information). If the dislocalization of certain phases by the natural environment is unavoidable, superselection rules arise: Quantities corresponding to those subspaces between which interference terms are permanently destroyed locally can be thought of as always existing – though not necessarily behaving deterministically. Within the limits of resolution of 'measurement' their paths may be assumed to exist, just as if they were classical quantities.

This 'coming into being of classical properties' appears related to what Heisenberg may have meant by his famous remark [7]: "Die 'Bahn' entsteht erst dadurch, daß wir sie beobachten." It clearly emphasizes the irreversible nature of observation. However, the Copenhagen interpretation considers classical conceptions a necessary and fundamental (non-derivable) prerequisite for the description of measurements, and therefore has to deny the possibility of analyzing them entirely within quantum mechanical terms. The point of view assumed here is rather related to Mott's analysis of α-particle tracks [13].

Quantities under the described conditions are called 'continuously measured'. In contrast to genuine measurements (as referred to by Heisenberg) the measurement-like interaction (1.1) need not be controllable. It is only required that the resulting environment states $|\Phi_n\rangle$ are (approximately) orthogonal – not that the information indicated by the letter n is in any way extractable or classical itself in the sense of the above. Measurements proper (including the reading of the results by an observer) are described by the collapse of the (total) wave function – leading to one of its components $|\varphi_n\rangle|\Phi_n\rangle$ –, whereas the (controllable or uncontrollable) measurement without reading retains the complete superposition (1.1) with its resulting density matrix (1.2). Continuous measurement (even if 'ideal' as in (1.1)) will also have certain dynamical consequences: it may either lead to a freezing of the motion [14, 15] (more generally to enforced motion [16]) – called 'Zeno's quantum paradox' or the 'watchdog effect' – or to the validity of probabilistic master equations like those expressed by Fermi's Golden Rule instead of a unitary evolution [6]. It is these instead of the Schrödinger equation which should therefore serve to derive classical equations of motion.

The investigation of spatial motion under continuous measurement forms the main part of this paper. This subject is discussed in Sect. III.2.2. In addition also the simpler situation of two-state systems (parity versus chirality eigenstates) is studied in detail (see Sects. II, III.2.1 and III.3). Section II presents some introductory remarks about stationary correlations, whereas the realistic situation of time-dependent states is considered in Sect. III. Subsection III.3 is devoted to a comparison of time-dependent and time-independent environmental states, in particular in the case of thermal equilibrium. Section IV gives a short summary of the main results and discusses some consequences of this work for the interpretations of measurement in quantum mechanics.

II. Time-Independent Correlations

The total Hamiltonian can be written as

$$H = H_\varphi + H_\Phi + W, \tag{2.1}$$

where φ refers to the system and Φ to the environment. The stationary states (eigenstates of H) may be approximated by product states ('local states') only if the interaction W can be considered as a perturbation. Since the density of states is very large for macroscopic objects, even extremely small interactions must not in general be considered as perturbations. As an example, the level density of a macroscopic rotator (neglecting all other degrees of freedom) has been compared with the interaction matrix elements between two elementary dipoles, one of them positioned on the rotator and the other at an astronomical distance [2]. The exact energy states are clearly non-perturbative in this case.

Hence, if the total system is described by an eigenstate of H, a macroscopic system cannot be in a state by itself at all, that is, it is usually strongly correlated with the environment. The sensitivity of time-dependent states with respect to interactions with the environment as well as that of interactions with a time-dependent environment will be discussed in Sect. III.

If the total system is described by a stationary density matrix of large entropy, as for example the canonical one, the subsystem's density matrix (1.2) may instead be insensitive to small interactions, as

$$\begin{aligned}\rho_{\mathrm{can}} &= Z^{-1}\exp[-(H_\varphi + H_\Phi + W)/kT]\\ &\simeq Z^{-1}\exp[-(H_\varphi + H_\Phi)/kT],\end{aligned} \tag{2.2}$$

and therefore

$$\begin{aligned}\rho_\varphi &= Z_\varphi^{-1}\,\mathrm{Tr}_\Phi \exp[-(H_\varphi + H_\Phi + W)/kT]\\ &\simeq Z_\varphi^{-1}\exp[-H_\varphi/kT].\end{aligned} \tag{2.3}$$

This insensitivity of the canonical density matrix is facilitated by the almost-degeneracy of the eigenvalues of the resulting ρ_φ. Although the eigenstates of H are still extremely sensitive to the existence of W, the interference terms arising from the individual eigenstates of H in the eigenbasis of ρ_φ cancel (compare also III.3.1). They would become important if the situation corresponded to a genuine measurement including the 'reading', which could single out the eigenstates. Of course, an environment in thermal equilibrium is inappropriate as a measurement device.

However, an equilibrated environment *is* appropriate for the dislocalization of phases. Consider for example the parity eigenstates $|\varphi_\pm\rangle$ of a chiral molecule in interaction with an unpolarized photon. The energy difference between these states is in most such cases extremely small, as the transition between chirality eigenstates would require a simultaneous tunneling of several atoms. In this sense chirality resembles a macroscopic property. If $|\chi_{L,\pm}\rangle$ are angular momentum and parity eigenstates of the scattered photon, the four scattering eigenstates for each value of L can be chosen to be total parity eigenstates

$$|\Psi^{s+}\rangle = c_1^{s+}|\varphi_+\rangle|\chi^s_{L+}\rangle + c_2^{s+}|\varphi_-\rangle|\chi^s_{L-}\rangle, \tag{2.4 a}$$

$$|\Psi^{s-}\rangle = c_1^{s-}|\varphi_+\rangle|\chi^s_{L-}\rangle + c_2^{s-}|\varphi_-\rangle|\chi^s_{L+}\rangle, \tag{2.4 b}$$

where $s=\pm$ distinguishes the two solutions for each total parity value $\pi=\pm$ assigned by the second binary upper index. In a spherical box of radius R these are also the energy eigenstates according to the boundary condition

$$\sin(kR - L\pi/2 + \delta) = 0 \rightarrow k^{s\pi}_{Ln} = (n\pi + L\pi/2 - \delta^{s\pi}_{Ln})/R. \tag{2.5}$$

This is an implicit definition, as the phase shifts δ depend (weakly) on k. In order to measure chirality in the time dependent scattering process (1.1), the states $|\varphi_n\rangle$ of that equation now have to be the chirality states $|\varphi_{R,L}\rangle = 2^{-1/2}(|\varphi_+\rangle \pm |\varphi_-\rangle)$. Taking the limit of an infinitesimal time in (1.1), one finds [17] that the total Hamiltonian (for each subspace of L) can be written as

$$\begin{aligned} H = \frac{-\Delta E}{2}&[|\varphi_+\rangle\langle\varphi_+| - |\varphi_-\rangle\langle\varphi_-|] \otimes 1_\chi + H_0^{\text{Photon}} \\ &+ \gamma\{|\varphi_R\rangle\langle\varphi_R| \otimes [|\chi_1^+\rangle\langle\chi_1^-| - |\chi_2^-\rangle\langle\chi_2^-|] \\ &- |\varphi_L\rangle\langle\varphi_L| \otimes [|\chi_1^-\rangle\langle\chi_1^-| - |\chi_2^+\rangle\langle\chi_2^+|]\}, \end{aligned} \tag{2.6}$$

where $\gamma = \gamma(n, L, R)$ is a coupling constant and $|\chi^s_{1,2}\rangle = 2^{-1/2}(|\chi^s_+\rangle \pm |\chi^s_-\rangle)$. These linear combinations are required for total parity conservation. $|\chi^\pm_{1,2}\rangle$ are assumed to contain the exact radial solutions (see also Sect. III.3). The interaction commutes [18] with the projection operators $|\varphi_R\rangle\langle\varphi_R| \otimes 1_\chi$ and $|\varphi_L\rangle\langle\varphi_L| \otimes 1_\chi$. For $\gamma/\Delta E \rightarrow 0$ one obtains the eigenstates $|\varphi_+\rangle\ |\chi_\mp\rangle$ or $|\varphi_-\rangle\ |\chi_\pm\rangle$ with factorizing parity and energy being independent of photon parity. For $\Delta E/\gamma \rightarrow 0$ the stationary states can be chosen either as the total parity eigenstates

$$|\Psi^{++}\rangle = 2^{-1/2}(|\varphi_+\rangle|\chi^+_+\rangle + |\varphi_-\rangle|\chi^+_-\rangle), \tag{2.7 a}$$

$$|\Psi^{--}\rangle = 2^{-1/2}(|\varphi_-\rangle|\chi^+_-\rangle + |\varphi_-\rangle|\chi^+_+\rangle), \tag{2.7 b}$$

and analogously for $s=-$, or as their superpositions

$$|\Psi_R^+\rangle = |\varphi_R\rangle|\chi_1^+\rangle, \quad |\Psi_L^-\rangle = |\varphi_L\rangle|\chi_2^+\rangle \tag{2.8 a}$$

and

$$|\Psi_R^-\rangle = |\varphi_R\rangle|\chi_2^-\rangle, \quad |\Psi_L^-\rangle = |\varphi_L\rangle|\chi_1^-\rangle. \tag{2.8 b}$$

Such parity-mixed states are possible as a consequence of a degeneracy of eigenstates with different parity and the same value of s. Hence, in the case of strong coupling corresponding to a 'continuous measurement' of chirality, the chirality states $|\varphi_{R,L}\rangle$ facilitate factorizing energy eigenstates. This degeneracy is the stationary version of the 'watchdog effect'.

Practically, the coupling to a single photon is very small, and the limit $\Delta E/\gamma \rightarrow 0$ is not obtained. For a non-negligible effect many photons are needed. Pfeifer [19] has in fact demonstrated that for sugar molecules virtual (bound) photons are sufficient to produce stationary states of type (2.8) – now with infinitely many photons instead of one. But virtual photons have to be considered as part of the 'dressed' sugar molecule – not as an environment. There now exist the two degenerate ground states (for $\gamma > 0$) of type (2.8 b) for the dressed sugar molecule: The renormalized energy difference is $\Delta E^{\text{new}} = 0$, whereas the excited states (2.8 a) will not be bound. Therefore superpositions of the two dressed states (2.8 b) are still possible. This means that the result by itself does not explain why chirality of macromolecules is a 'classical' property. Pfeifer's further argument is based on proposals by Jauch [20], Hepp [21], and Primas [22] according to which observables must be constructed in Fock space, whereas the renormalized states of type (2.8 b) essentially require a non-denumerable number of contributions. If true, this *postulated* superselection rule would mean that non-trivially dressed particles could never be observed. The solution of this apparent paradox is that the observables also have to be non-trivially dressed, that is, they have to be operators for 'physical' systems. However, the important result $\Delta E^{\text{new}} = 0$ (corresponding to a stationary watchdog effect) renders the stationary scattering states with *non-virtual* photons extremely sensitive to the coupling. Since $\Delta E^{\text{new}}/\gamma = 0$ one always has the situation described by (2.8), where $|\varphi\rangle$ are now the dressed states of the molecule, and $|\chi\rangle$ describes a real photon again: In a stationary situation the dressed sugar molecule would be strongly correlated with its environment of free photons.

The canonical density matrix (that means, in particular, an unpolarized photon) is

$$\begin{aligned} \rho_{\text{can}} = Z^{-1}\{&e^{-\gamma/kT}(|\Psi_R^+\rangle\langle\Psi_R^-| + |\Psi_L^-\rangle\langle\Psi_L^+|) \\ &+ e^{\gamma/kT}(|\Psi_R^-\rangle\langle\Psi_R^-| + |\Psi_L^-\rangle\langle\Psi_L^-|\}. \end{aligned} \tag{2.9}$$

Partical tracing gives

$$\rho_\varphi = \tfrac{1}{2}, \tag{2.10}$$

that is, no interference terms in any basis. In contrast, the density matrix of an ideal non-degenerate gas

$$\rho_{\mathrm{can}}(\mathbf{x},\mathbf{x}')=Z^{-1}\int d^3k\,\exp[-k^2/2mk_BT+i\mathbf{k}(\mathbf{x}-\mathbf{x}')]$$
$$\sim\exp[-(\mathbf{x}-\mathbf{x}')^2 mk_BT/2] \tag{2.11}$$

contains coherence effects over the range of the thermal de Broglie wavelength

$$\lambda_{\mathrm{th}}=(2\pi mk_BT)^{-1/2}. \tag{2.12}$$

The extreme result (2.10) is a consequence of complete equilibration in connection with the degeneracy leading to (2.8). The classical significance of the chirality states has to be investigated for a time-dependent situation (see Sect. III.2.1). It will turn out from quantitative considerations that the chirality-sensitive scattering of photons by sugar is too weak to lead to noticeable evolution towards equilibrium within reasonable times. Other scattering processes of the same (chirality-measuring) type are responsible for the chirality superselection rule.

Stationary correlations analogous to those with virtual photons have also been considered for superpositions of states with different charge (Sect. 4 of Ref. 23). Different charges lead to different polarization of the surrounding matter by the long range electromagnetic field. But this argument in favor of charge superselection rules is subject to a similar critisism as before: Static polarization describes merely a renormalization of the charged particle. One may conjecture that only the radiation part of a particle's field has to be used for calculating correlations with the environment, whereas the generalized Coulomb part [24] contributes a matter-induced renormalization for building up a 'quasi-particle'. The dislocalization of phases is then obviously a time-dependent – in fact irreversible – process, as the Sommerfeld radiation condition has to be employed. When generalized this means that macroscopic properties can only occur in an irreversible situation [25].

III. Time-Dependent Correlations

III.1. Short-Time Behaviour

In classical mechanics, where a system and its environment both possess their own states, an interaction W will disturb the independent motion. This kind of dynamical influence may as well occur for time-dependent states in quantum mechanics, but in addition the interaction will here tend to correlate two initially factorizing states. Both effects can be investigated and distinguished by representing the state $|\Psi\rangle$ of the whole system by the Schmidt-canonical form [23], that is, as a single sum,

$$|\Psi(t)\rangle=\sum_i p_i^{1/2}(t)\,|\bar\varphi_i(t)\rangle\,|\bar\Phi_i(t)\rangle, \tag{3.1}$$

where $\{\bar\varphi_i\}$ and $\{\bar\Phi_i\}$ are certain time-dependent orthonormal systems in the Hilbert spaces of system and environment, and $\sum_i p_i=1$. For an initially separating state

$$|\Psi(0)\rangle=|\bar\varphi_0(0)\rangle|\bar\Phi_0(0)\rangle, \tag{3.2}$$

that is, $p_0(0)=1$, one finds in second order of time

$$p_0(t)=1-At^2, \tag{3.3}$$

where (now dropping the argument $t=0$)

$$A:=\sum_{j\neq 0,\,m\neq 0}|\langle\bar\varphi_j\bar\Phi_m|\,H\,|\bar\varphi_0\bar\Phi_0\rangle|^2$$
$$=\sum_{j\neq 0,\,m\neq 0}|\langle\bar\varphi_j\bar\Phi_m|\,W\,|\bar\varphi_0\bar\Phi_0\rangle|^2. \tag{3.4}$$

This 'rate of de-separation' [23], which measures the amount, at which the two systems become correlated, has to be distinguished from the total rate of change of the initial state, which is given by

$$B:=\sum_{j,m\neq 0,0}|\langle\bar\varphi_j\bar\Phi_m|\,H\,|\bar\varphi_0\bar\Phi_0\rangle|^2. \tag{3.5}$$

The difference between the two quantities

$$B-A=\sum_{j\neq 0}|\bar\varphi_j\bar\Phi_0|\,H\,|\bar\varphi_0\bar\Phi_0\rangle|^2$$
$$+\sum_{m\neq 0}|\langle\bar\varphi_0\bar\Phi_m|\,H\,|\bar\varphi_0\bar\Phi_0\rangle|^2 \tag{3.6}$$

is the 'classical rate of change' and therefore the sum of the rates for both systems.

For example, the von-Neumann-type interaction (1.1) may be described by the Hamiltonian

$$H=\gamma\sum_n\alpha_n|\varphi_n\rangle\langle\varphi_n|\,\hat p, \tag{3.7}$$

where γ is a coupling constant, α_n a 'pointer scale' and $\hat p$ the infinitesimal translation operator (momentum operator) of the pointer. The wave function of the pointer (environment) after time t is then

$$\Phi_n(x,t)=\bar\Phi_0(x-\gamma\alpha_n t). \tag{3.8}$$

A measurement is 'complete' when the wave packets Φ_n for different n are approximately orthogonal (and therefore equal to the Schmidt states $\bar\Phi_n(t)$). The de-separation rate (3.4) can conveniently be written as

$$A=\langle\bar\varphi_0\bar\Phi_0|\,H(1-|\bar\varphi_0\rangle\langle\bar\varphi_0|)(1-|\bar\Phi_0\rangle\langle\bar\Phi_0|)\,H\,|\bar\varphi_0\bar\Phi_0\rangle \tag{3.9}$$

and be calculated for the case (3.7) with $\bar\varphi_0=\sum_n c_n\varphi_n$

to give

$$A=\gamma^2\langle\bar{\Phi}_0|\,\hat{p}^2\,|\bar{\Phi}_0\rangle\,\{\sum_n\alpha_n^2\,|c_n|^2-\sum_{n,m}\alpha_n\,\alpha_m|c_n|^2\,|c_m|^2\}$$

$$=\gamma^2\langle\bar{\Phi}_0|\,\hat{p}^2\,|\bar{\Phi}_0\rangle\,\overline{(\alpha_n-\overline{\alpha_n})^2} \qquad (3.10)$$

if $\langle\bar{\Phi}_0|\,\hat{p}|\bar{\Phi}_0\rangle=0$. From the uncertainty relation (Fourier theorem) one obtains

$$A\geqq\gamma^2\,\overline{(\alpha_n-\overline{\alpha_n})^2}/4b^2, \qquad (3.11)$$

where $b^2=\overline{(x-\bar{x})^2}$ defines the width b of the wave packet $\Phi_0(x)$. γ/b describes the 'effectivity of measurement' [6]. The total rate of change

$$B=\gamma^2\langle\bar{\Phi}_0|\,\hat{p}^2\,|\bar{\Phi}_0\rangle\,\overline{\alpha_n^2}\geqq\gamma^2\,\overline{\alpha_n^2}/4b^2 \qquad (3.12)$$

depends on $\overline{\alpha_n^2}$, whereas A is determined solely by the spread of the α_n.

For coupled harmonic oscillator Hamiltonians the de-separation rate is proportional to the product nN of quantum numbers, if the initial states $|\bar{\varphi}_0\rangle$ and $|\bar{\Phi}_0\rangle$ are assumed to be eigenstates $|\varphi_n\rangle$ and $|\Phi_N\rangle$ of H_φ and H_Φ respectively (see Sect. 4 of Ref. 23). This means that the sensitivity increases with level density – in accord with stationary considerations (Sect. II). However, if the initial states are coherent states (eigenstates of the annihilation operator), the rate is independent of the eigenvalues and remains small with small interaction. Therefore coherent states can remain approximately uncorrelated even in the macroscopic limit. This argument may explain the dominance of the field aspect over the particle aspect for boson fields.

Another example appropriate for investigation by this method is an interaction by long range forces. If in an initially factorizing two-particle state

$$\Psi(\mathbf{r}_1,\mathbf{r}_2)=\varphi(\mathbf{r}_1)\,\Phi(\mathbf{r}_2) \qquad (3.13)$$

the two wave packets have width b_i and are localized at $\mathbf{r}_i^0$ with $b_i\ll|\mathbf{r}_1^0-\mathbf{r}_2^0|$ the interaction part V of the Hamiltonian

$$H=\frac{p_1^2}{2m_1}+\frac{p_2^2}{2m_2}+V(\mathbf{r}_1-\mathbf{r}_2) \qquad (3.14)$$

may be expanded at $\mathbf{r}_1^0-\mathbf{r}_2^0$

$$V(\mathbf{r}_1-\mathbf{r}_2)\simeq V(\mathbf{r}_1^0-\mathbf{r}_2^0)+\sum_k(r_1-r_2-r_1^0+r_2^0)_k\,\partial_k\,V(\mathbf{r}_1^0-\mathbf{r}_2^0)$$
$$+\tfrac{1}{2}\sum_{k,l}(r_1-r_2-r_1^0+r_2^0)_k(r_1-r_2-r_1^0+r_2^0)_l$$
$$\cdot\ \partial_k\partial_l\,V(\mathbf{r}_1^0-\mathbf{r}_2^0). \qquad (3.15)$$

For Gaussian wave packets the rate of de-separation is in this approximation

$$A=b_1^2 b_2^2\sum_{k,l}V_{kl}^2, \qquad (3.16)$$

where

$$V_{kl}:=\partial_k\,\partial_l\,V(\mathbf{r}_1^0-\mathbf{r}_2^0). \qquad (3.17)$$

Equation (3.16) is of course invariant under exchange of the particles and independent of particle masses. (The formation of correlations is independent of the internal dynamics of each system in second order of time, see (3.4).) The interaction can be interpreted as a measurement of the position of particle 1 by particle 2 and vice versa – unlike von Neumann's interaction (3.7), which neglects momentum conservation. The 'resolution' of the 'pointer' i is $1/b_i$, that is the width of the momentum wave function $\tilde{\varphi}(\mathbf{p}_1)$ or $\tilde{\Phi}(\mathbf{p}_2)$, since up to order t^2 an interaction potential transfers only momentum (without shifting the wave packet spatially).

The significance of the quantity A can for this situation be demonstrated by a simple calculation beyond the short-time expansion. For macroscopic objects (with large masses) the effect of the spreading of the wave packet can be neglected. Hence, in a first approximation only the interaction $V(\mathbf{r}_1-\mathbf{r}_2)$ in the Hamiltonian may be retained. The density matrix of the whole system then evolves according to

$$\rho(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}'_1,\mathbf{r}'_2,t)$$
$$=\rho(t=0)\exp-it[V(\mathbf{r}_1-\mathbf{r}_2)-V(\mathbf{r}'_1-\mathbf{r}'_2)]. \qquad (3.18)$$

The initial state will again be assumed to factorize,

$$\rho(t=0)=\varphi(\mathbf{r}_1)\,\Phi(\mathbf{r}_2)\,\varphi^*(\mathbf{r}'_1)\,\Phi^*(\mathbf{r}'_2). \qquad (3.19)$$

The density matrix for, say, particle 1 after time t is

$$\rho_\varphi(\mathbf{r}_1,\mathbf{r}'_1,t)=\mathrm{Tr}_\Phi\,\rho$$
$$=\rho_\varphi(\mathbf{r}_1,\mathbf{r}'_1,0)\int d^3r|\Phi(\mathbf{r})|^2$$
$$\cdot\ \exp\{-it[V(\mathbf{r}_1-\mathbf{r})-V(\mathbf{r}'_1-\mathbf{r})]\}. \qquad (3.20)$$

Hence, the initial density matrix of particle 1 is multiplied by a factor, originating from the formation of correlations between the two particles and expected to yield a suppression of interference terms between different locations $\mathbf{r}_1$ and $\mathbf{r}'_1$. Expanding $V(\mathbf{r}_1-\mathbf{r}_2)$ again as in (3.15) gives

$$\rho_\varphi(\mathbf{r}_1,\mathbf{r}'_1,t)=\rho_\varphi(\mathbf{r}_1,\mathbf{r}'_1,0)$$
$$\cdot\ \exp\{-it[\sum_k(r_1-r'_1)_k\,\partial_k\,V+\sum_{k,l}V_{kl}$$
$$\cdot\ \{(r_{1k}\,r_{1l}-r'_{1k}\,r'_{1l})/2+(r_1-r'_1)_k(r_2^0-r_1^0)_l\}]\}$$
$$\cdot\ \int d^3r\,|\Phi(\mathbf{r})|^2\exp\{+it\sum_{k,l}(r_1-r'_1)_k\,r_l\,V_{kl}\}. \qquad (3.21)$$

Obviously only the last factor describes a damping of interference terms. It may be discussed by inserting a Gaussian wave function of width b for particle 2. Up to a phase factor the integral I in (3.21) then becomes

$$I = \exp\left[-t^2 b^2 \sum_l \{\sum_k V_{kl}(r_1 - r_1')_k\}^2\right]. \tag{3.22}$$

For small arguments the damping factor reads

$$I \simeq 1 - t^2 b^2 \sum_l \{\sum_k V_{kl}(r_1 - r_1')_k\}^2. \tag{3.23}$$

The similarity of the second term with the de-separation parameter A in (3.16) is obvious. For a Coulomb-like potential with coupling constant g

$$V(\mathbf{r}) = \frac{g}{|\mathbf{r}|}, \tag{3.24}$$

one has

$$V_{kl} = g\,\frac{3r_k r_l - r^2 \delta_{kl}}{r^5}. \tag{3.25}$$

The order of magnitude of suppression of interference terms over a distance $l := |\mathbf{r}_1 - \mathbf{r}_1'|$ can now be estimated by combining (3.23) and (3.25) and considering the special case, where $\mathbf{r}_1 - \mathbf{r}_1'$ is parallel to $\mathbf{r}_2^0 - \mathbf{r}_1^0$. Then

$$I \simeq 1 - \frac{4l^2 b^2 g^2}{a^6}\,t^2, \tag{3.26}$$

where $a := |\mathbf{r}_2^0 - \mathbf{r}_1^0|$ is the distance between the two particles. In the range where the above approximations are valid, (3.26) can be used to estimate the coherence range l under the influence of the second particle with wave packet width b. Its influence is appreciable if the second term in (3.26) is non-negligible, roughly if

$$\frac{l^2 b^2 g^2 t^2}{a^6} \simeq 1. \tag{3.27}$$

For simplicity, the symmetrical situation $b = l$ is considered. Table 1 illustrates the case of electrostatic interaction $g = q_1 q_2 / 4\pi\varepsilon_0$ between two particles each carrying one elementary charge.

For gravitational interaction the small value of the coupling constant can be compensated by sufficiently large masses. For example, two dust particles with $m = 10^{-6}$ g at a distance of $a = 1$ cm will loose interference properties over a range $l = 10^{-4}$ cm after $t = 1$ s. Two earthlike bodies at earth-moon distance would reduce their coherence lengths to 10^{-15} cm within 10^{-10} s with respect to an external observer. Hence the center of mass states of macroscopic bodies may always be assumed to be well localized.

Table 1. Coherence range for two elementary charges after various times t at various distances a, as resulting from their Coulomb interaction and calculated in short-time approximation

	$t = 10^{-5}$ s	$t = 1$ s	$t = 1$ year
$a = 1$ km	10^6 cm	10^3 cm	10^{-1} cm
$a = 1$ m	10^1 cm	10^{-1} cm	10^{-5} cm
$a = 1$ cm	10^{-2} cm	10^{-4} cm	10^{-8} cm

III.2. Irreversible Production of Correlations

If a system is originally in a superposition $\sum_n c_n |\varphi_n\rangle$ with respect to the measurement basis, its density matrix in this basis is $\rho_{nn'} = c_n c_{n'}^*$. A measurement of type (1.1) will be called 'incomplete', if the apparatus or environment states $|\Phi_n\rangle$ are not (approximately) orthogonal after measurement. The matrix elements $\rho_{nn'}$ are thereby multiplied by factors $\langle \Phi_n | \Phi_{n'} \rangle$ with $|\langle \Phi_n | \Phi_{n'} \rangle| < 1$ for $n \neq n'$, that is, the nondiagonal elements will be suppressed. In collision-type interactions these processes will last only for a short time, but may occur repeatedly. Many individually ineffective collisions ($\langle \Phi_n | \Phi_{n'} \rangle \simeq 1$) may in this way become important. Multiplying the density matrix elements repeatedly with interference-reducing factors $\langle \Phi_n | \Phi_{n'} \rangle$ corresponds to the integration of an irreversible master equation analogous to Boltzmann's Stoßzahlansatz.

If, in addition, the duration of a scattering process is short compared to the typical time scales of evolution of the object by itself, the total evolution can approximately be described by an alternating dynamics: The change of ρ is equal to the change given by the internal dynamics interrupted by complete scattering processes, or formally

$$i\frac{\partial \rho}{\partial t} = [H_{\text{internal}}, \rho] + i\left.\frac{\partial \rho}{\partial t}\right|_{\text{scatt.}}, \tag{3.28}$$

where $\left.\frac{\partial \rho}{\partial t}\right|_{\text{scatt.}}$ is defined by means of the S-matrix.

To work out such a Boltzmann-equation one has only to solve the equations of motion for the internal dynamics and the scattering process *separately* (instead of a Schrödinger equation with the full Hamiltonian and complicated initial conditions). To begin with, only the scattering term in (3.28) will be considered, neglecting the internal dynamics of the system.

The most obvious measuring process for macroscopic bodies is scattering of light. To justify this statement one just has to open one's eyes. It will turn out, that even the cosmic background radiation can act as an effective measuring device. As discussed in Sect. II for time-independent correlations, an equilibrated environment like thermal radiation is able to destroy (or dislocalize) interference terms – although it cannot really measure anything. This argument will now be extended to the case of time-dependent formation of correlations.

Consider an environment equally distributed over N possible values of the relevant degree of freedom ('pointer positions'),

$$\rho_\Phi = N^{-1} \sum_{n=1}^{N} |\Phi_n\rangle\langle\Phi_n|. \tag{3.29}$$

Using $\Phi_n(x) = \Phi_0(x - n\Delta x)$ and von Neumann's unitary interaction (3.7) with $\gamma\alpha_n T = n\Delta x$ for simplicity, one obtains for the density matrix of the whole system

$$\rho(0) = \sum_{m,m'} c_m c_{m'} |\varphi_m\rangle\langle\varphi_{m'}| \otimes N^{-1} \sum_n |\Phi_n\rangle\langle\Phi_n|$$
$$\to \rho(T) = N^{-1} \sum_{m,m',n} c_m c^*_{m'} |\varphi_m\rangle\langle\varphi_{m'}| \otimes |\Phi_{n+m}\rangle\langle\Phi_{n+m'}|. \tag{3.30}$$

If the width of the wave packet Φ_0 is much smaller than Δx (complete measurement after one or several collisions), one obtains

$$\rho_\varphi(T) = \sum_m |c_m|^2 |\varphi_m\rangle\langle\varphi_m|. \tag{3.31}$$

The phases have been dislocalized as before. Nevertheless, if the 'pointer variable' x is cyclic, $\Phi_0(x - N\Delta x) = \Phi_0(x)$, or if $N \to \infty$, 'reading' the result at the pointer would not give any information about φ, as

$$\begin{aligned}\rho_\Phi(T) &= \mathrm{Tr}_\varphi\, \rho(T)\\ &= N^{-1} \sum_{m,n} |c_m|^2 |\Phi_{n+m}\rangle\langle\Phi_{n+m}|\\ &= N^{-1} \sum_n |\Phi_n\rangle\langle\Phi_n| = \rho_\Phi(0).\end{aligned} \tag{3.32}$$

The environment is therefore inappropriate for a measurement (the interaction is 'uncontrollable') in spite of (3.31).

III.2.1. Chirality

Two-state systems provide the simplest examples for studying the effect of repeated scattering processes. As previous work [6] has shown, systems with few degrees of freedom are very sensitive to such continuous measurements. For example, chirality states may become dynamically stabilized by repeated chirality-dependent scattering processes. Important work concerning this question was done by Simonius [26], Pfeifer [19] and Harris and Stodolsky [27]. The latter considered the effect of irreversible scattering off a chiral molecule in great detail. Their method will be briefly described in the following paragraph.

If the state of molecule plus environment is written as

$$|\Psi\rangle = |\varphi_R\rangle|\Phi_R\rangle + |\varphi_L\rangle|\Phi_L\rangle, \tag{3.33}$$

the coherence between the chiral states $|\varphi_R\rangle$ and $|\varphi_L\rangle$ is given by the off-diagonal term ρ_{RL} in the density matrix of the considered molecule,

$$\rho_{RL} = \langle\Phi_L|\Phi_R\rangle. \tag{3.34}$$

In a chirality-dependent scattering process a one-particle state $|\chi\rangle$ leads to a change of ρ_{RL} according to

$$\rho_{RL} \to \rho_{RL}\langle\chi|S_L^\dagger S_R|\chi\rangle, \tag{3.35}$$

where $S_{R,L}$ are scattering matrices of the particles, assumed to depend on the chirality of the molecule. Then the second term in (3.28) reads

$$\left.\frac{\partial\rho_{RL}}{\partial t}\right|_{\mathrm{scatt.}} = -\lambda\rho_{RL}(t), \tag{3.36}$$

where

$$\lambda := (1 - \langle\chi|S_L^\dagger S_R|\chi\rangle) \cdot \text{number of collisions per s.} \tag{3.37}$$

The parameter λ may be expressed in terms of scattering phase shifts. However, Ref. [26] does not contain numerical estimates of λ.

Experimental evidence for chirality is best known from the phenomenon of optical activity. The observability of this effect demonstrates that chirality is measured by light. Thus rotating the photon polarization reduces the coherence between left and right handedness. An essential part of the natural surrounding of a molecule like sugar is thermal radiation of, say, room temperature or day light. Here again it is important to realize, that the incident light need not be polarized in order to destroy interferences.

The order of magnitude of this effect may be estimated as follows. The scattering of a plane wave $|\alpha\rangle$ with angle α of polarization with respect to some reference frame can be assumed to be dominated by the lowest multipole part (p-wave). Then the effect of optical activity may be described by

$$|\varphi_{R,L}\rangle|p,\alpha\rangle \to |\varphi_{R,L}\rangle(\cos\delta|p,\alpha\rangle \pm \sin\delta|p,\alpha+\pi/2\rangle), \tag{3.38}$$

where $|p, \alpha\rangle$ represents the p-wave part of a plane wave with polarization α. If, for example, the molecule were initially in its ground state,

$$|\varphi_+\rangle = 2^{-1/2}(|\varphi_R\rangle + |\varphi_L\rangle), \tag{3.39}$$

the density matrix of the total system with undetermined photon polarization would read

$$\begin{aligned}\rho(0) = {} & \tfrac{1}{4}(|\varphi_R\rangle + |\varphi_L\rangle)(\langle\varphi_R| + \langle\varphi_L|) \\ & \otimes(|\alpha\rangle\langle\alpha| + |\alpha+\pi/2\rangle\langle\alpha+\pi/2|),\end{aligned} \tag{3.40}$$

where $\rho_{RL}(0) = 1/2$. In a scattering process the two parts of the photon polarization contribute independently to (3.38). (The form of the photon density matrix is here independent of the choice of a special basis. For a discussion see Sect. III.3.) Therefore ρ_{RL} after scattering is $\rho_{RL} = \frac{1}{2}(\cos^2\delta\text{-}\sin^2\delta)$, whereas ρ_{phot} remains unchanged. In this 'uncontrollable measurement' no information is transferred to the environment.

For a single scattering process δ is obviously very small, hence

$$\rho_{RL} \rightarrow \rho_{RL} \cos(2\delta) \simeq \rho_{RL}(1 - 2\delta^2). \tag{3.41}$$

If Ω scattering events per second occur, one has an approximately exponential decrease of ρ_{RL},

$$\begin{aligned}\rho_{RL}(t) & = \rho_{RL}(0)\exp(-2\delta^2\Omega t) \\ & =: \rho_{RL}(0)\exp(-t/\tau).\end{aligned} \tag{3.42}$$

The characteristic time τ may be estimated from experimental data of optical activity (coherent forward scattering). The observed angle φ of rotation is obtained as the sum of contributions from all molecules [28]. The scattered waves interfere constructively only in forward direction. The angle φ turns out to depend on δ, the vessel length d, the density n of molecules and the wave number k of the photons according to $\varphi \sim dn\delta/k^2$. For sugar the value of δ derived from measurement of φ is about $5\cdot 10^{-16}$ radians (for visible light). For sunlight $\Omega \simeq 10^{11}\,\text{s}^{-1}$, leading to a characteristic time $\tau \simeq 10^{11}$ years.

One therefore has to conclude that optical activity is not sufficient to explain the absence of coherence between chiral states. However, quite general circumstances provide measurements of chirality together with *other* properties of spatial orientation. For example, scattering of air molecules at room temperature (with thermal de Broglie wavelength of $\lambda_{th} \simeq 10^{-9}$ cm) corresponds to a characteristic time $\tau \simeq 10^{-9}$ s, as the wave lengths are small enough for scattering to distinguish between chiral states with almost perfect efficiency.

III.2.2. Localization of Macroscopic Objects

As mentioned before, macroscopic objects cannot avoid scattering photons and other particles. The effect of these scattering processes on the scattered particles of course depends on the position of the scattering object. As a consequence, this position is measured. In this way certain interference terms in the density matrix of the macroscopic object will be destroyed.

The mechanism of position measurement by scattering can be described in analogy to that of chirality measurement. If a particle is scattered off a macroscopic object, its resulting state $|\chi_{\mathbf{x}}\rangle$ will depend on the location $\mathbf{x}$ of the scattering center,

$$|\mathbf{x}\rangle|\chi\rangle \rightarrow |\mathbf{x}\rangle|\chi_{\mathbf{x}}\rangle = |\mathbf{x}\rangle S_{\mathbf{x}}|\chi\rangle, \tag{3.43}$$

where $S_{\mathbf{x}}$ is the scattering matrix for an object with center of mass positioned at $\mathbf{x}$. The mass of the macroscopic object is considered as infinite in this treatment (recoil-free collisions). If the position of the center of mass is instead described by a wave function $\varphi(\mathbf{x})$, the state of the whole system evolves according to

$$\int d^3x\,\varphi(\mathbf{x})|\mathbf{x}\rangle|\chi\rangle \rightarrow \int d^3x\,\varphi(\mathbf{x})|\mathbf{x}\rangle S_{\mathbf{x}}|\chi\rangle. \tag{3.44}$$

The density matrix of the position of the scattering center after scattering reads

$$\rho(\mathbf{x}, \mathbf{x}') = \varphi(\mathbf{x})\,\varphi^*(\mathbf{x}')\langle\chi|S^\dagger_{\mathbf{x}'}S_{\mathbf{x}}|\chi\rangle. \tag{3.45}$$

Hence, the density matrix elements are multiplied by factors $\langle\chi|S^\dagger_{\mathbf{x}'}S_{\mathbf{x}}|\chi\rangle$, the matrix elements between the corresponding 'pointer positions'. For sufficiently large $\Delta x := |\mathbf{x} - \mathbf{x}'|$ one may expect $\langle\chi|S^\dagger_{\mathbf{x}'}S_{\mathbf{x}}|\chi\rangle \simeq 0$. For such distances one clearly has a strong damping of interference terms. On the other hand, for small values of Δx a single scattering process will merely have a small influence, that is $|\langle\chi|S^\dagger_{\mathbf{x}'}S_{\mathbf{x}}|\chi\rangle \simeq 1$. This situation will be assumed in the following, thus greatly underestimating the rate of damping for large distances, where the interference terms will very soon vanish anyhow.

If the scattering interaction is invariant under a translation in space, the scattering matrices for scattering at $\mathbf{x}$ and $\mathbf{x}'$ are in momentum representation related by a phase factor,

$$S_{\mathbf{x}}(\mathbf{k}, \mathbf{k}') = S(\mathbf{k}, \mathbf{k}')\,\mathrm{e}^{-i(\mathbf{k}-\mathbf{k}')\mathbf{x}}, \tag{3.46}$$

where $S(\mathbf{k}, \mathbf{k}')$ is the usual scattering matrix with the scattering center at the origin.

The expression to be calculated is

$$\begin{aligned}&\langle\chi|\,S^{\dagger}_{\mathbf{x}'}S_{\mathbf{x}}|\chi\rangle\\&=\int d^3k\,d^3k'\,d^3k''\,S_{\mathbf{x}}(\mathbf{k},\mathbf{k}')\,S^{*}_{\mathbf{x}'}(\mathbf{k},\mathbf{k}'')\,c(\mathbf{k}')\,c^{*}(\mathbf{k}'')\\&=\int d^3k\,d^3k'\,d^3k''\,S(\mathbf{k},\mathbf{k}')\,S^{*}(\mathbf{k},\mathbf{k}'')\\&\cdot\,\mathrm{e}^{-i\mathbf{k}(\mathbf{x}-\mathbf{x}')}\,\mathrm{e}^{i\mathbf{k}'\mathbf{x}-i\mathbf{k}''\mathbf{x}'}c(\mathbf{k}')\,c^{*}(\mathbf{k}''),\end{aligned}\tag{3.47}$$

where $c(\mathbf{k})$ is the momentum wave function of the scattered particle.
In the considered range of ineffective single scattering, the typical wavelength of the measuring particle is large compared to the distance $|\mathbf{x}-\mathbf{x}'|$ between the two positions of the scattering center, that is, the wave function may be replaced by a plane wave with appropriate normalization (one particle in a cube of length L). The momentum wave function may then be approximated by

$$c(\mathbf{k})\simeq L^{-3/2}\,\delta^3(\mathbf{k}-\mathbf{k}_0),\tag{3.48}$$

where $\mathbf{k}_0$ is the momentum of the incident particle and $k_0|\mathbf{x}-\mathbf{x}'|\ll 1$. The scattering matrix can be expressed in terms of the scattering amplitude as

$$S(\mathbf{k},\mathbf{k}')=\delta^3(\mathbf{k}-\mathbf{k}')+\frac{i}{2\pi k}f(\mathbf{k},\mathbf{k}')\,\delta(k-k').\tag{3.49}$$

Inserting (3.48) and (3.49) in (3.47) and expanding the exponential yields

$$\begin{aligned}&\langle\chi|\,S^{\dagger}_{\mathbf{x}'}S_{\mathbf{x}}|\chi\rangle=1+\frac{1}{\pi^2L^2}\int\frac{d^3k}{k^2}\,|f(\mathbf{k},\mathbf{k}_0)|^2\,\delta(k-k_0)\\&\cdot\{-i(\mathbf{k}-\mathbf{k}_0)(\mathbf{x}-\mathbf{x}')-[(\mathbf{k}-\mathbf{k}_0)(\mathbf{x}-\mathbf{x}')]^2\}.\end{aligned}\tag{3.50}$$

Integrations over angles can be performed elementarily. Let the angle between $\mathbf{x}-\mathbf{x}'$ and the direction of incidence $\mathbf{k}_0$ be α. Then

$$\begin{aligned}&\langle\chi|\,S^{\dagger}_{\mathbf{x}'}S_{\mathbf{x}}|\chi\rangle=1+\frac{i}{4\pi^2L^2}\,k_0|\mathbf{x}-\mathbf{x}'|\cos\alpha\\&\cdot\left\{\sigma_{\text{tot}}-2\pi\int d\cos\Theta\,|f(\cos\Theta)|^2\cos\Theta\right\}\\&-\frac{(k_0|\mathbf{x}-\mathbf{x}'|)^2}{8\pi^2L^2}\left\{\frac{\sigma_{\text{tot}}}{2}(1+\cos^2\alpha)\right.\\&+\pi(3\cos^2\alpha-1)\int d\cos\Theta\,|f(\cos\Theta)|^2\cos^2\Theta\\&\left.-4\pi\cos^2\alpha\int d\cos\Theta\,|f(\cos\Theta)|^2\cos^2\Theta\right\}.\end{aligned}\tag{3.51}$$

The above expression of course depends on α. If the scattered particles (e.g. photons) are randomly distributed, one can average over α and replace the result by

$$\begin{aligned}&1-\frac{(k_0|\mathbf{x}-\mathbf{x}'|)^2}{8\pi^2L^2}\\&\cdot\left\{3/4\,\sigma_{\text{tot}}+\pi/2\int d\cos\Theta\,|f(\cos\Theta)|^2\cos^2\Theta\right.\\&\left.-2\pi\int d\cos\Theta\,|f(\cos\Theta)|^2\cos\Theta\right\}.\end{aligned}\tag{3.52}$$

The imaginary part, containing information about the direction of incidence, has disappeared.
The effect of a single scattering process on the density matrix is therefore approximately given by

$$\begin{aligned}&\rho(\mathbf{x},\mathbf{x}')\to\rho(\mathbf{x},\mathbf{x}')\left(1-\frac{(k_0|\mathbf{x}-\mathbf{x}'|)^2}{8\pi^2L^2}\,\sigma_{\text{eff}}\right)\\&\simeq\rho(\mathbf{x},\mathbf{x}')\exp\left(-\frac{(k_0|\mathbf{x}-\mathbf{x}'|)^2}{8\pi^2L^2}\,\sigma_{\text{eff}}\right),\end{aligned}\tag{3.53}$$

where

$$\sigma_{\text{eff}}:=\frac{\pi}{2}\int d\cos\Theta\,|f(\cos\Theta)|^2[(2-\cos\Theta)^2-1].\tag{3.54}$$

The effect of n subsequent independent scattering processes is

$$\rho(\mathbf{x},\mathbf{x}')\to\rho(\mathbf{x},\mathbf{x}')\exp\left(-n\frac{(k_0|\mathbf{x}-\mathbf{x}'|)^2}{8\pi^2L^2}\,\sigma_{\text{eff}}\right).\tag{3.55}$$

where n is given by

$$\begin{aligned}n&=L^2\cdot\text{flux}\cdot t\\&=L^2\cdot\text{particle density}\cdot\text{mean velocity}\cdot t=L^2\,\frac{N}{V}\,vt.\end{aligned}\tag{3.56}$$

The density matrix thus changes according to

$$\rho(\mathbf{x},\mathbf{x}')\to\rho(\mathbf{x},\mathbf{x}')\exp\{-\Lambda t(\mathbf{x}-\mathbf{x}')^2\}.\tag{3.57}$$

where

$$\Lambda:=\frac{k_0^2\,\sigma_{\text{eff}}\,Nv}{8\pi^2V}.\tag{3.58}$$

The parameter Λ, which may be called 'localization rate', describes the overall effect of many individually ineffective measurements of position by scattering. Using (3.57), the contribution from scattering to the Boltzmann equation (3.28) can now be written as

$$\left.\frac{\partial\rho(\mathbf{x},\mathbf{x}')}{\partial t}\right|_{\text{scatt}}=-\Lambda(\mathbf{x}-\mathbf{x}')^2\rho(\mathbf{x},\mathbf{x}').\tag{3.59}$$

This equation describes an irreversible damping of interference terms in the density matrix for the position of the scattering center. A phenomenological rate equation of this type (resp. (3.28)) has been suggested by Wigner [4]. Equation (3.59) as well as (3.28) will in the following be discussed for the case of various objects scattering photons or other particles.

III.2.2.1. Various Position Measurements. Scattering of electromagnetic waves by small objects (i.e. with size much smaller than the wavelength) is described by Rayleigh's law. Scattering will then be dominated by the dipole contribution and the well-known frequency dependence $\sigma \sim \omega^4$ arises. The cross sections for simple situations can easily be calculated. Some examples will be employed in order to demonstrate the suppression of interference terms between different locations for small objects.

Let the scattering object be a small dielectic sphere with radius a and uniform isotropic dielectric constant $\varepsilon(\omega)$. Then the differential cross section for scattering radiation with wave vektor k is (after averaging over polarizations) [29]

$$\frac{d\sigma}{d\Omega} = k^4 a^6 \left(\frac{\varepsilon - 1}{\varepsilon + 2}\right)^2 (1 + \cos^2 \Theta)/2. \tag{3.60}$$

Hence (3.54) gives

$$\sigma_{\text{eff}} = \frac{34\pi}{15} k^4 a^6 \left(\frac{\varepsilon - 1}{\varepsilon + 2}\right)^2 \tag{3.61}$$

and

$$\Lambda_k = \frac{17}{4\pi} \left(\frac{\varepsilon - 1}{\varepsilon + 2}\right)^2 \frac{Nc}{V} k^6 a^6. \tag{3.62}$$

For black body radiation the wave-numbers k are occupied according to Planck's law

$$n(\mathbf{k}) = \frac{2}{\exp(ck/k_B T) - 1}, \tag{3.63}$$

giving the density

$$\frac{N}{V} = \int \frac{d^3k}{(2\pi)^3} n(\mathbf{k}) = \frac{2}{\pi^2} \zeta(3) \left(\frac{k_B T}{c}\right)^3. \tag{3.64}$$

Λ_k may be replaced by its average value

$$\begin{aligned} \Lambda &= \int \frac{d^3k}{(2\pi)^3} n(\mathbf{k}) \Lambda_k \Big/ \int \frac{d^3k}{(2\pi)^3} n(\mathbf{k}) \\ &= \frac{17}{4\pi^3} a^6 c \int dk \left(\frac{\varepsilon(k) - 1}{\varepsilon(k) + 2}\right)^2 \frac{k^8}{\exp(ck/k_B T) - 1}. \end{aligned} \tag{3.65}$$

If $\varepsilon(k)$ is approximately constant (for frequencies below the resonance region it can be replaced by its static value), the result is

$$\Lambda = \left(\frac{k_B T}{c}\right)^9 \left(\frac{\varepsilon - 1}{\varepsilon + 2}\right)^2 7! \frac{34}{\pi^3} a^6 c \zeta(9), \tag{3.66}$$

where $\zeta(x)$ is the Riemann ζ-function, with $\zeta(9) \simeq 1.002$.

The time dependence of the density matrix (3.57) can be written as

$$\rho(t) = \rho(0) \exp[-(\mathbf{x} - \mathbf{x}')^2 / 8 l^2(t)]. \tag{3.67}$$

This means that interferences between a characteristic distance

$$l(t) := (8 \Lambda t)^{-1/2} \tag{3.68}$$

are suppressed (l is defined such that $l^2 = \overline{(x - \bar{x})^2}$ for a Gaussian wave packet, comp. (3.98)). The order of magnitude of this effect can now easily be calculated by inserting some typical values in the above equations. The resulting coherence length may be illustrated for $t = 1$ s by the following examples. For a large dust particle ($a = 10^{-2}$ cm) background radiation leads to $l = 10^{-6}$ cm, whereas for a smaller one ($a = 10^{-5}$ cm) a value of only $l = 10^3$ cm results. For $T = 300$ K the latter example yields $l = 10^{-6}$ cm. Thermal radiation alone would therefore prevent observation of interference effects even for very small dust particles.

Even free electrons (or other charged particles) are affected by thermal radiation. Inserting the classical Thomson cross section

$$\frac{d\sigma_T}{d\Omega} = \left(\frac{e^2}{mc^2}\right)^2 (1 + \cos^2 \Theta)/2, \tag{3.69}$$

where e^2/mc^2 is the 'classical electron radius', gives

$$\Lambda_k = \frac{17}{\pi} \frac{Nc}{V} k^2 \left(\frac{e^2}{mc^2}\right)^2 \tag{3.70}$$

and as average value

$$\begin{aligned} \Lambda &= \frac{17}{4\pi^3} \left(\frac{e^2}{mc^2}\right)^2 c \int dk \frac{k^4}{\exp(ck/k_B T) - 1} \\ &= \frac{102}{\pi^2} \zeta(5) c \left(\frac{e^2}{mc^2}\right)^2 \left(\frac{k_B T}{c}\right)^5 \end{aligned} \tag{3.71}$$

with $\zeta(5) \simeq 1.03$. For $t = 1$ s a 'coherence length' of $l(t = 1 \text{ s}) = 10^4$ cm for $T = 3$, and $l(t = 1 \text{ s}) = 0.1$ cm for $T = 300$ results. All values of l depend on time as $t^{-1/2}$ (see (3.68)).

On the other hand, if the wavelength of the scattered particle is comparable with or smaller than the extension of the scattering object (e.g. a dust particle), the geometrical cross section becomes relevant. In particular the comparatively small de-Broglie wavelength (2.12) for particles of, say, room temperature, leads to a strong coherence damping for objects subjected to scattering by air molecules even in the best available vacuum. For the case of geometrical scattering one has in (3.58)

$$\sigma_{\text{eff}} \simeq \sigma_{\text{geom.}} \simeq \pi a^2, \tag{3.72}$$

where a is the extension of the scattering object. Averaging over wavelengths gives, e.g. for thermal

Table 2. Localization rate Λ in $\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$ for three sizes of 'dust particles' and various scattering processes

	$a=10^{-3}$ cm dust	$a=10^{-5}$ cm dust	$a=10^{-6}$ cm large molecule
Cosmic background radiation	10^{6}	10^{-6}	10^{-12}
Room temperature	10^{19}	10^{12}	10^{6}
Sunlight (on earth)	10^{21}	10^{17}	10^{13}
Air	10^{36}	10^{32}	10^{30}
Laboratory vacuum (10^{6} particles/cm^{3})	10^{23}	10^{19}	10^{17}

radiation,

$$\Lambda=\frac{24}{(2\pi)^3}\,a^2 c\left(\frac{k_B T}{c}\right)^5 \zeta(5) \tag{3.73}$$

(compare (3.66), where Λ depends in a quite different way on radius and temperature). Table 2 gives a listing of various scattering processes for three sizes a of 'dust particles'. The last value $a=10^{-6}$ cm corresponds to large molecules. The table contains rough estimates for the localization rate Λ for the different measuring agents listed in the first column. Λ is given in units of $\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$.

The table shows that in general scattering of air molecules is most important, mainly because of the small thermal de Broglie wavelength of the scattered particles.

III.2.2.2. Equation of Motion. In the preceding sections the influence of scattering processes on the density matrix was considered neglecting internal dynamics. For a complete treatment including also the unitary evolution of the system itself, the full Eq. (3.28) has to be discussed.

For a 'free' mass point one has

$$H_{\text{internal}}=\frac{p^2}{2m}. \tag{3.74}$$

The quantum mechanical evolution of the system described by this Hamiltonian leads to the well-known spreading of wave-packets, corresponding to an increasing coherence length. The opposite effect is generated by the scattering term. The equation of motion (3.28) now reads (for one space dimension)

$$i\frac{\partial\rho(x,x',t)}{\partial t}=\frac{1}{2m}\left(\frac{\partial^2}{\partial x'^2}-\frac{\partial^2}{\partial x^2}\right)\rho-i\Lambda(x-x')^2\rho. \tag{3.75}$$

It is convenient to introduce rotated variables

$$y:=x-x', \tag{3.76 a}$$

$$z:=x+x', \tag{3.76 b}$$

which transform the above equation into

$$i\frac{\partial\rho(y,z,t)}{\partial t}=-\frac{2}{m}\frac{\partial^2\rho}{\partial y\,\partial z}-i\Lambda y^2\rho. \tag{3.77}$$

One may construct special solutions of this equation by a Gaussian ansatz

$$\rho(y,z,t)=\exp-[A(t)y^2+iB(t)yz-C(t)z^2-D(t)], \tag{3.78}$$

where ρ is Hermitean if the time-dependent coefficients A, B, C are real. $D(t)$ normalizes trace ρ to unity. This ansatz appears sufficiently general to exhibit the essential properties of the solutions of (3.75). Obviously $A(t)$ describes the range of coherence contained in ρ, whereas $C(t)$ specifies the extension in space of the ensemble, explicitely

$$(\Delta x)^2=\frac{1}{8C}. \tag{3.79}$$

The spread in momentum is given by

$$(\Delta p)^2=2\left(A+\frac{B^2}{4C}\right), \tag{3.80}$$

hence the lefthand side of the uncertainty relation can be written as

$$(\Delta x)^2(\Delta p)^2=\frac{1}{4}\left(\frac{A}{C}+\frac{B^2}{4C^2}\right). \tag{3.81}$$

For $A=C$ (pure state) and $B=0$ one has the well-known case of a real Gaussian wave packet with minimum uncertainty. The ansatz (3.78) leads to the coupled ordinary differential equations

$$\frac{dA}{dt}=\frac{4}{m}AB+\Lambda, \tag{3.82 a}$$

$$\frac{dB}{dt}=\frac{2}{m}B^2-\frac{8}{m}AC, \tag{3.82 b}$$

$$\frac{dC}{dt}=\frac{4}{m}BC, \tag{3.82 c}$$

while $D(t)$ is given by

$$\mathrm{e}^{-D}=2(C/\pi)^{1/2}. \tag{3.83}$$

Rescaling the time t by introducing $\tau := t/m$ retains only the product Λm as a dynamical parameter,

$$\frac{dA}{d\tau} = 4AB + \Lambda m, \tag{3.84 a}$$

$$\frac{dB}{d\tau} = 2B^2 - 8AC, \tag{3.84 b}$$

$$\frac{dC}{d\tau} = 4BC. \tag{3.84 c}$$

The solution is characterized by a third-order polynomial $G(\tau)$ obeying

$$G''' = 32\Lambda m. \tag{3.85}$$

Then

$$A(\tau) = -\frac{G'^2}{64G} + \frac{G''}{32}, \tag{3.86 a}$$

$$B(\tau) = -\frac{G'}{4G}, \tag{3.86 b}$$

$$C(\tau) = \frac{1}{G} \tag{3.86 c}$$

identically fulfill (3.84). Writing

$$G(\tau) = \frac{16}{3}\Lambda m \tau^3 + c_2\tau^2 + c_1\tau + c_0, \tag{3.87}$$

the coefficients may be determined by initial conditions, e.g., at $\tau = 0$, as

$$c_0 = \frac{1}{C(0)}, \tag{3.88 a}$$

$$c_1 = -4\frac{B(0)}{C(0)}, \tag{3.88 b}$$

$$c_2 = 4\left(4A(0) + \frac{[B(0)]^2}{C(0)}\right). \tag{3.88 c}$$

They are obviously related to the physical quantities $(\Delta x)^2$ and $(\Delta p)^2$ (see (3.79) and (3.80)).

The Gaussian density matrix (3.78) has the further advantage that its diagonal representation

$$\rho = \sum_n p_n |\varphi_n\rangle\langle\varphi_n| \tag{3.89}$$

can be explicitely constructed by solving the integral equation

$$p_n\varphi_n(x) = \int dx'\rho(x,x')\,\varphi_n(x'). \tag{3.90}$$

Solutions [30] are the harmonic oscillator wave functions

$$\varphi_n(x) = NH_n[2(AC)^{1/4}x]\exp - x^2[2(AC)^{1/2} + iB] \tag{3.91}$$

with

$$N = \frac{(AC)^{1/4}}{2^{n-1}n!\,\pi^{1/2}} \tag{3.92}$$

(H_n are Hermite polynomials). The eigenvalues p_n are given by

$$p_n = p_0 q^n, \tag{3.93}$$

where

$$p_0 = \frac{2C^{1/2}}{A^{1/2} + C^{1/2}}, \qquad q = \frac{A^{1/2} - C^{1/2}}{A^{1/2} + C^{1/2}}. \tag{3.94}$$

The diagonal representation (3.89) also allows calculation of the entropy ($k_B = 1$),

$$\begin{aligned} S &= -\langle \ln \rho \rangle \\ &= -\mathrm{trace}(\rho \ln \rho) = -\sum_n p_n \ln p_n \\ &= -p_0^{-1}(p_0 \ln p_0 + q \ln q). \end{aligned} \tag{3.95}$$

It may be more convenient to use the 'linear entropy'

$$\begin{aligned} S_{\mathrm{lin}} &:= -\langle \rho \rangle \\ &= -\mathrm{trace}(\rho^2) = -\sum_n p_n^2 \\ &= -(C/A)^{1/2} \end{aligned} \tag{3.96}$$

with $-1 \leqq S_{\mathrm{lin}} < 0$. S_{lin} is obviously monotonuously connected with S in this case. As (3.75) is a master equation, the entropy is expected to increase. This is indeed so, as

$$\begin{aligned} \frac{dS_{\mathrm{lin}}}{dt} &= -2\,\mathrm{trace}\,\rho\dot{\rho} \\ &= -2\int dx'\rho(x,x')\,\dot{\rho}(x',x) \\ &= +2\Lambda\int dx'\rho(x,x')(x-x')^2\rho(x,x') \\ &= 2\Lambda\int dx'(x-x')^2|\rho(x,x')|^2 > 0. \end{aligned} \tag{3.97}$$

For $C > A$ the entropy S is negative (resp. $S_{\mathrm{lin}} < -1$) – corresponding to the presence of negative probabilities. This range has to be excluded. It would occur by integrating (3.75) backwards in time beyond a pure state, just as exponential decay would lead to normalizations greater than unity for negative times.

The following discussion will concentrate on the question, over what distances interferences can be observed at the mass point under consideration, described by a density matrix ρ of type (3.78). The crucial ‘coherence width’ l of the density matrix (3.78) is given by

$$l(t) := [8A(t)]^{-1/2}. \tag{3.98}$$

It describes the width of ρ in x, for which coherence is present. For a pure initial Gaussian state of width b,

$$\varphi(x,0) = (2\pi b^2)^{-1/4} \exp(-x^2/4b^2), \tag{3.99}$$

one has

$$A(0) = C(0) = \frac{1}{8b^2}, \qquad B(0) = 0. \tag{3.100}$$

The time dependence of $A(\tau)$ according to the master equation (3.75) as described by (3.86a) is then given by

$$A(\tau) = \frac{4\Lambda^2 m^2 b^2 \tau^4 + 2\Lambda m \tau^3 + 24\Lambda m b^4 \tau + 3b^2}{2(8\Lambda m b^2 \tau^3 + 3\tau^2 + 12b^4)}, \tag{3.101}$$

while

$$C(\tau) = \frac{3b^2}{2(8\Lambda m b^2 \tau^3 + 3\tau^2 + 12b^4)}. \tag{3.102}$$

$A(t)$ increases linearly with time for small t,

$$A(t) = \frac{1}{8b^2} + \Lambda t + O(t^2), \tag{3.103}$$

as well as for large t,

$$A(t) = \frac{\Lambda t}{4} + \frac{1}{32b^2} + O(1/t). \tag{3.104}$$

In the first case the term Λt arises from the scattering term in (3.75), whereas for large times 3/4 of it is compensated by the internal dynamics. In both cases the destruction of coherence dominates the dispersion of the wave packet.

As mentioned above, the linear time dependence (3.103) would lead to negative entropy and negative eigenvalues of the density matrix for negative times. In fact, the exact behavior for very small times is characterized by a quadratic time dependence (compare Sect. III.1). As the linear time dependence can be justified by averaging over many scattering events, the master equation (3.75) will hold for times t being large compared to the time interval Δt between two collisions: $t \gg \Delta t$. The initial condition (3.100) is therefore consistent with the master equation only if it is not essentially changed by a few measuring collisions. This holds true if the original width b is small compared to the effective wave length λ of the scattered particles: $b \ll \lambda$. Because of the very small thermal de Broglie wavelengths of the molecules, which dominate as a measuring agent in almost all situations, this consistency condition for pure initial states could be realized in nature only for very small initial widths b.

The large-time behavior (3.104) corresponds to a coherence length decreasing as $(\Lambda t)^{-1/2}$. This again holds only as an approximation, since the coherence length has to asymptotically approach the thermal de Broglie wavelength λ_{th} of the *object* as given by the temperature of the particle bath, that is $A(t) \ll \lambda_{\text{th}}^{-2}$. This limitation of the present model is due to the assumption of passive measurements (recoil-free collisions), dynamically equivalent to an infinite mass, and therefore vanishing de Broglie wavelength.

Investigation of the higher order terms of the expansions (3.103) and (3.104) shows that they are negligible if the sets of inequalities

$$\Delta t \ll t \ll \Lambda m^2 b^6 \ll \Lambda m^2 \lambda^6 \quad \text{and}$$
$$t^3 \ll \frac{b^2 m^2}{\Lambda} \ll \frac{\lambda^2 m^2}{\Lambda} \tag{3.105}$$

for small times, or

$$t \gg \frac{1}{b^2 \Lambda} \gg \frac{1}{\lambda^2 \Lambda} \quad \text{and} \quad t^3 \gg \frac{b^2 m^2}{\Lambda} \tag{3.106}$$

asymptotically, are simultaneously fulfilled. Here the consistency conditions discussed above have been added for completeness. Although, according to (3.104), $A(t)$ becomes asymptotically independent of the original width b, the range of validity of asymptotic behavior strongly depends on it. The complete shape of the solutions differing in the physical parameters b, m and Λ is characterized solely by the dimensionless quantity $\mu := \Lambda m b^4$. For example, from the expression (3.101) one may derive the result, that $A(t)$ can never drop below $A(0)$ if $\mu > 192^{-1/2}$.

In order to illustrate the different types of solutions for $A(t)$, the coherence length $l(t) = (8A(t))^{-1/2}$ is presented in Fig. 1 for the six sets of parameters that result from combinining $T = 0$, 3 K and 300 K with $b = 10^{-2}$ cm and 10^{-5} cm, and $m = 10^{-14}$ g (small dust particle). Measurement by thermal radiation only – although not being very realistic – is considered, as it allows to display the different types most distinctly. The resulting values of the dimensionless quantity μ are 0, 10^{-1}, 10^{17} for $T = 0$, 3 K, 300 K, respectively, for $b = 10^{-2}$ cm, and $\mu = 0$, 10^{-13}, 10^5 for $b = 10^{-5}$ cm. Only for the combination $b = 10^{-5}$ cm and $T = 300$ K the consistency condition for pure initial states is not fulfilled.

As can be seen from Table 2, scattering of air mole-

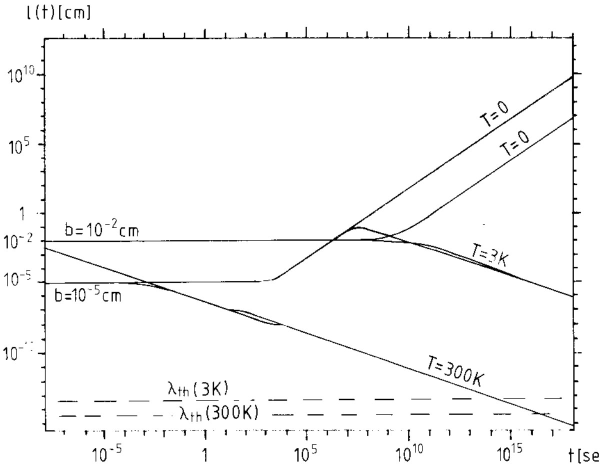

Fig. 1. Time dependence of the coherence length $l(t) := (8A(t))^{-1/2}$ for center of mass of a small dust particle with radius $a = 10^{-5}$ cm (mass $m = 10^{-14}$ g) under continuous measurement by thermal radiation only. Six typical situations differing in temperature T and coherence length $b = l(0)$ of the initial Gaussian wave packet are shown. All curves with $T \neq 0$ start with negative derivative (compare (3.104)) – reckognizable only for $T = 300$ K. Notice that increase of coherence length corresponding to quantum mechanical spreading of wave packets dominates only in a limited time range and if b is sufficiently small, or if $T = 0$ (no measurement). For large times l decreases in all cases $T \neq 0$ with $t^{-1/2}$. Decrease of l will cease finally at the thermal de Broglie wave length λ_{th} of the dust particle (not described by the model). In realistic situations scattering of molecules will reduce the coherence length even more effectively

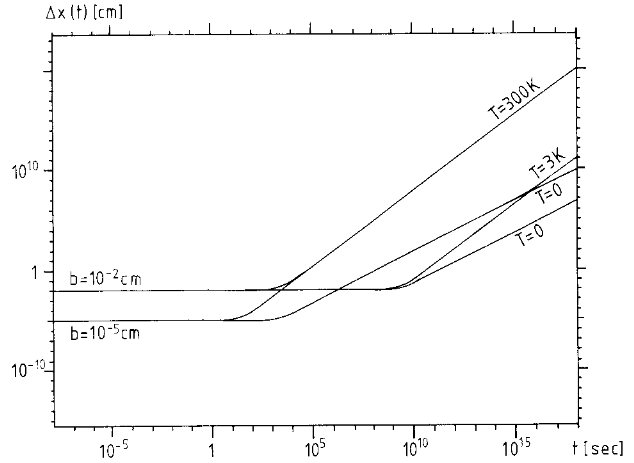

Fig. 2. Time dependence of the spatial extension $\Delta x(t) = (8C(t))^{-1/2}$ of the ensemble evolving from an initial Gaussian wave packet for the same situations as in Fig. 1. In all cases with $T > 0$ Δx increases faster than without measurement ($T = 0$)

cules dominates in effectivity under realistic conditions. mainly because of the small thermal de Broglie wavelength, e.g. $\lambda \simeq 10^{-9}$ cm for $T = 300$ K. For the same dust particle as in Fig. 1 with radius $a = 10^{-5}$ cm, now with initial wave function width $b = 10^{-10}$ cm, one finds $\mu = 10^5$ and $\Delta t \simeq 10^{-14}$ s. The asymptotic region will then begin already at $t \simeq 10^{-8}$ s. Hence, under usual circumstances all 'macroscopic' objects can be assumed to be localized within *their* thermal de Broglie wavelengths.

Another important physical quantity is the spatial extension of the ensemble described by the density matrix ρ. It is given by the coefficient $C(t)$, namely

$$(\Delta x)^2 = \frac{1}{8C(t)} = \frac{1}{12}\left(8\Lambda m\tau^3 + \frac{3}{b^2}\,\tau^2 + 12b^2\right) \tag{3.107}$$

(compare (3.79) and (3.102)). Again – as is the case for the coherence length – for large times the Λ-dependent term dominates. Hence the spreading of the ensemble is more rapid than had to be expected from a usual dispersion of the wave packet by itself. Figure 2 presents $\Delta x(t)$ for the same situations as in Fig. 1.

The strong influence of the environment can also be seen by considering the entropy S of the density matrix ρ. In the case of a dust particle measured by air molecules, the entropy, when expressed as the number $N = e^S$ of states contained in the orthogonalizing ensemble turns out, for example for $t = 10^{-6}$ s, to be about $N = 5 \cdot 10^6$ (see (3.95)).

III.2.2.3. Ehrenfest Theorems. The time dependence of the mean values of position and momentum of a free particle follows the 'classical' equations

$$\frac{d}{dt}\langle x\rangle = \frac{\langle p\rangle}{m}, \tag{3.108 a}$$

$$\frac{d}{dt}\langle p\rangle = 0. \tag{3.108 b}$$

These equations are special cases of what is known as 'Ehrenfest theorems', which formally connect the time dependence of mean values of canonically conjugate observables with the Hamilton equations of classical mechanics. They are often used to link quantum mechanics with classical mechanics. However, theorems on expectation values are by no means sufficient. Nevertheless, the time dependence of mean values is of importance for a *partial* derivation of classical motion as discussed in the introduction. Equations (3.108) have to be rederived for systems under continuous measurement, as the density matrix does then not obey a von Neumann equation. This could invalidate the Ehrenfest theorem. In addition, the so-called 'watchdog-effect' or 'Zeno paradox' might influence the motion of the particle. This effect is a consequence of repeated (even ideal) measurements and can lead to a freezing of the motion of the continuously observed object. Analysis of this mechanism has shown [6], that systems with dense spectra are not as sensitive to this freezing as, e.g., two-state systems. In the case of a 'free' particle one has a continuum of states, therefore ideal measurements are not expected to lead to a freezing of the motion.

The Ehrenfest theorem in the form of Eqs. (3.108) can be shown to remain valid by direct calculation. Writing the equation of motion (3.75) in operator notation as

$$i\frac{d\rho}{dt}=\left[\frac{p^2}{2m},\rho\right]+i\Sigma, \tag{3.109}$$

where

$$\langle x|\Sigma|x'\rangle=-\Lambda(x-x')^2\rho(x,x'), \tag{3.110}$$

the time derivative of the expectation value of position is

$$\begin{aligned}\frac{d}{dt}\langle x\rangle&=\mathrm{Tr}\left\{x\frac{d\rho}{dt}\right\}=-i\,\mathrm{Tr}\left\{x\left[\frac{p^2}{2m},\rho\right]\right\}+\mathrm{Tr}\{x\Sigma\}\\&=\frac{\langle p\rangle}{m}+\mathrm{Tr}\{x\Sigma\},\end{aligned} \tag{3.111}$$

while for momentum one has

$$\begin{aligned}\frac{d}{dt}\langle p\rangle&=\mathrm{Tr}\left\{p\frac{d\rho}{dt}\right\}=-i\,\mathrm{Tr}\left\{p\left[\frac{p^2}{2m},\rho\right]\right\}+\mathrm{Tr}\{p\Sigma\}\\&=\mathrm{Tr}\{p\Sigma\}.\end{aligned} \tag{3.112}$$

These equations are identical to (3.108) if the terms containing Σ vanish. This can be seen immediately, since

$$\mathrm{Tr}\{x\Sigma\}=\Lambda\int dx\,dx'\,\delta(x-x')\,x(x-x')^2\rho(x,x')=0, \tag{3.113}$$

$$\mathrm{Tr}\{p\Sigma\}=-i\Lambda\int dx\,dx'\,\delta'(x-x')(x-x')^2\rho(x,x')=0. \tag{3.114}$$

Hence, the connection between the expectation values of position and momentum, as given by the Ehrenfest theorem, is retained. A continuous measurement by (ideal) scattering processes, when described by (3.75), does not lead to a damping of the spatial motion.

III.3. Stationary Environment

The measurement of a certain property leading to the destruction of local phases requires that the environment changes in dependence on the value of the property to be measured, as described by von Neumann's interaction (1.1). As discussed previously, the measurement may well be 'uncontrollable' in the sense that the environment is in a homogenuous mixture of all 'pointer positions' before and therefore also after measurement. If the probabilities for all pointer positions are equal, the initial density matrix of the environment can be diagonalized as well by means of an orthogonal set of superpositions of pointer positions. Especially one may choose eigenstates of the interaction Hamiltonian (in the factor space of the environment), that is, states that can*not* vary in dependence on the measured property. This raises the question (to be discussed in III.3.1) of how the destruction of phases can be understood in this 'stationary representation'. Homogeneous mixtures occur in particular in canonical ensembles like black body radiation, for instance if chirality is measured by unpolarized photons.

In Sect. III.2 measurements were described by means of successive collisions. This procedure is equivalent to the application of a master equation, and to the assumption of a time-dependent environment (incoming wave packets). In the case of an equilibrated environment, eigenstates of the interaction-free Hamiltonian have to be considered as initial environment states. Since

$$\exp(-H_\varphi/kT)\otimes\exp(-H_\Phi/kT)$$

is only an approximation of $\exp[-(H_\varphi+H_\Phi+W)/kT]$, information may then still flow into correlations between the systems. The use of a master equation in the case of a stationary state of the environment will be justified in Sect. III.3.2.

III.3.1. Measurement by Eigenstates of the Interaction. Consider the situation of a two-state system φ measured by another two-state system χ, by means of the Hamiltonian

$$\begin{aligned}H=\gamma&[|\varphi_R\rangle\langle\varphi_R|-|\varphi_L\rangle\langle\varphi_L|]\\&\otimes[|\chi_+\rangle\langle\chi_-|+|\chi_-\rangle\langle\chi_+|].\end{aligned} \tag{3.115}$$

It corresponds to chirality being measured by polarization as in Sect. II (with $\Delta E=0$) except for the important difference of spatial degrees of freedom now being neglected. The polarization χ rotates in dependence on chirality like

$$|\varphi_{R,L}\rangle[\cos(\gamma t)|\chi_+\rangle\pm i\sin(\gamma t)|\chi_-\rangle]. \tag{3.116}$$

This time dependence is reversible, since the particle carrying the polarization cannot 'run away' in this model. An initial superposition $2^{-1/2}(|\varphi_R\rangle+|\varphi_L\rangle)$ leads to a density matrix of the subsystem φ given by

$$\rho_\varphi(t)=\tfrac{1}{2}\begin{pmatrix}1&\cos(2\gamma t)\\\cos(2\gamma t)&1\end{pmatrix} \tag{3.117}$$

in the chirality basis. There is no linear term in t corresponding to a master equation for a two-state system. The interference disappears reversibly at $2\gamma t=\pi/2, 3\pi/2$, etc. Since the same result is obtained for an initial polarization $|\chi_-\rangle$ instead of $|\chi_+\rangle$, it holds as well for a mixture of both.

A mixture corresponding to the unity matrix can also be written as a mixture of the superpositions $|\chi_{1,2}\rangle=2^{-1/2}(|\chi_+\rangle \pm |\chi_-\rangle)$, being eigenstates of the Hamiltonian. They are not able to measure chirality, as they obey the trivial time dependence

$$|\varphi_{R,L}\rangle|\chi_1\rangle \exp[\pm i\gamma t] \quad \text{and} \\ |\varphi_{R,L}\rangle|\chi_2\rangle \exp[\mp i\gamma t]. \tag{3.118}$$

In this case the superposition $2^{-1/2}(|\varphi_R\rangle+|\varphi_L\rangle)$ leads to the density matrix of a pure state for all times,

$$\rho_\varphi(t)=\tfrac{1}{2}\begin{pmatrix} 1 & \exp[\pm 2i\gamma t] \\ \exp[\mp 2i\gamma t] & 1 \end{pmatrix} \tag{3.119}$$

with the upper or lower sign corresponding to an initial state $|\chi_1\rangle$ or $|\chi_2\rangle$, respectively. All interference terms are locally preserved. Both systems remain passive according to (3.118), and neither measures the other one. Nevertheless, a mixture of $|\chi_1\rangle$ and $|\chi_2\rangle$ with equal weights leads to (3.117) again, and therefore surprisingly *is* able to 'measure' chirality (to destroy local phases).

This situation can be illuminated by considering the case of an initial mixture ρ_χ, which is not due to incomplete information, but instead due to a kinematical correlation with an environment Φ of the environment χ ('improper mixture'), $2^{-1/2}(|\chi_1\rangle|\Phi_1\rangle + |\chi_2\rangle|\Phi_2\rangle)$, without interaction between Φ and χ and $\langle\Phi_1|\Phi_2\rangle=0$. This corresponds to the same initial density matrix $\rho_\chi=1/2$ that leads to (3.117). The time dependence is now given by (compare 3.118)

$$\begin{aligned}&|\varphi_{R,L}\rangle\{|\chi_1\rangle|\Phi_1\rangle\exp[\pm i\gamma t]+|\chi_2\rangle|\Phi_2\rangle\exp[\mp i\gamma t]\}\\ &=|\varphi_{R,L}\rangle\{(|\chi_1\rangle|\Phi_1\rangle+|\chi_2\rangle|\Phi_2\rangle)\\ &\quad\cdot\cos(\gamma t)\pm i(|\chi_1\rangle|\Phi_1\rangle-|\chi_2\rangle|\Phi_2\rangle)\sin(\gamma t)\}.\end{aligned} \tag{3.120}$$

The combined states $2^{-1/2}(|\chi_1\rangle|\Phi_1\rangle \pm |\chi_2\rangle|\Phi_2\rangle)$ now lead to a time-dependence that does measure chirality, and thereby they assume the role of $|\chi_\pm\rangle$ in (3.116). As the 'Schmidt representation' (3.1) is degenerate in this case,

$$|\chi_1\rangle|\Phi_1\rangle+|\chi_2\rangle|\Phi_2\rangle=|\chi_+\rangle|\Phi_+\rangle+|\chi_-\rangle|\Phi_-\rangle, \tag{3.121}$$

Eq. (3.120) can also be written

$$\begin{aligned}&|\varphi_{R,L}\rangle\{[|\chi_-\rangle\cos(\gamma t)\pm i|\chi_-\rangle\sin(\gamma t)]|\Phi_+\rangle\\ &\qquad+[\pm i|\chi_+\rangle\sin(\gamma t)+|\chi_-\rangle\cos(\gamma t)]|\Phi_-\rangle\}\end{aligned} \tag{3.122}$$

wherein $|\Phi_\pm\rangle$ remain completely passive and $|\chi_\pm\rangle$ resume their original role as in (3.116). In the generic case the Schmidt representation of a correlated state is unique, and the phase relations defining the states which diagonalize the density matrix ρ_χ are given unambiguously.

The situation described by this simple model essentially holds in more complicated systems except that the recurrence time for local phases dramatically increases with the number of effective degrees of freedom of the environment.

III.3.2. Measurement by a Stationary State of the Environment. The preceding simple two-state model for the environment led to an oscillating time-dependence of the interference term ρ_{RL}. A more realistic environment $|\Phi(t)\rangle$ will in general cause a more complicated behaviour. The Boltzmann-type master equation (3.28) – corresponding to a linear time-dependence – is usually justified by averaging over collisions occurring at a constant rate. The exact time-dependence of ρ_{RL} would then be given in a complicated way by the shape of the incoming wave packets.

For stationary initial states of the environment constant rates may be derived in the usual manner from a Born approximation. In a one-dimensional model the interaction with the scattered particles may be assumed to be of type

$$\begin{aligned}W=&[|\varphi_R\rangle\langle\varphi_R|-|\varphi_L\rangle\langle\varphi_L|]\\ &\otimes[|\chi_1\rangle\langle\chi_2|+|\chi_2\rangle\langle\chi_1|]\,V(x).\end{aligned} \tag{3.123}$$

where $|\chi\rangle$ refers solely to the particle spin. If the initial state is assumed to be

$$|\Psi(0)\rangle=\tfrac{1}{2}(|\varphi_R\rangle+|\varphi_L\rangle)(|\chi_1\rangle+|\chi_2\rangle)|\Phi(0)\rangle \tag{3.124}$$

it will evolve according to

$$\begin{aligned}|\Psi(t)\rangle=&\tfrac{1}{2}(|\varphi_R\rangle|\chi_1\rangle+|\varphi_L\rangle|\chi_2\rangle)|\Phi_+(t)\rangle\\ &+\tfrac{1}{2}(|\varphi_R\rangle|\chi_2\rangle+|\varphi_L\rangle|\chi_1\rangle)|\Phi_-(t)\rangle,\end{aligned} \tag{3.125}$$

where $|\Phi_\pm(t)\rangle$ is the spatial part of the scattered particle state, corresponding to the potential $\pm V(x)$, respectively. The interference term

$$\begin{aligned}\rho_{RL}(t)&=\tfrac{1}{4}[\langle\Phi_+(t)|\Phi_-(t)\rangle+\langle\Phi_-(t)|\Phi_+(t)\rangle]\\ &=\tfrac{1}{2}\,\mathrm{Re}\,\langle\Phi_+(t)|\Phi_-(t)\rangle\end{aligned} \tag{3.126}$$

is most conveniently calculated from the interaction representation (in second order)

$$\begin{aligned}\rho_{RL}(t)=\tfrac{1}{2}\,\mathrm{Re}\Big\{&1-2\langle\Phi(0)|\int_0^t d\tau\int_0^\tau d\tau'\,V_W(\tau)\,V_W(\tau')|\Phi(0)\rangle\\ &-\langle\Phi(0)|\Big[\int_0^t d\tau\,V_W(\tau)\Big]^2|\Phi(0)\rangle\Big\}\end{aligned} \tag{3.127}$$

with $V_W(\tau)=\exp(iH_0\tau)\,V(x)\exp(-iH_0\tau)$. Inserting a complete system of plane waves $|k'\rangle$ normalized on an interval of length $2L$ and assuming $|\Phi(0)\rangle$ also to be an eigenstate of H_0 with eigenvalue $E(k)$, one obtains the usual resonance factors

$$\rho_{RL}(t)\simeq\frac{8}{L^2}\sum_{k'}\frac{\sin^2[(E(k)-E(k'))\,t/2]}{(E(k)-E(k'))^2}|\tilde V(k,k')|^2 \tag{3.128}$$

where

$$\tilde V(k,k') := \int_{-L}^{L} dx\, V(x)\cos(kx)\cos(k'x). \tag{3.129}$$

For sufficiently large times the resonance factor in (3.128) may be replaced by $\pi t\delta(E(k)-E(k'))/2$ and the sum over k' for large L be approximated by an integral, $\sum_{k'}\to L/2\pi\int dk'\sigma(k')$ (with density of states $\sigma(k)$). This leads to the linear time-dependence

$$\rho_{RL}(t)\simeq\tfrac{1}{2}\left(1-\frac{a}{L}t\right)\simeq\frac{1}{2}\exp\left(-\frac{a}{L}t\right) \tag{3.130}$$

with $a=2|\tilde V(k,k)|^2\sigma(k)$. The coefficient vanishes with increasing L due to the normalization of the particle wave functions. For N independently scattered particles, with N being proportional to L (fixed density), the damping factor in (3.130) occurs N times and the L-dependence cancels in close analogy to (3.42) and (3.57).

However, the Born approximation can here be avoided in a simple way, as the stationary solutions $|\Phi_\pm\rangle$ corresponding to the chirality/spin states $|\varphi_R\rangle|\chi_1\rangle$, $|\varphi_L\rangle|\chi_2\rangle$ and $|\varphi_R\rangle|\chi_2\rangle$, $|\varphi_L\rangle|\chi_1\rangle$, respectively, can be found exactly. For example, for a local potential $V(x)=\gamma\delta(x)$ and an interval $-L\leqq x\leqq L$ they are given by

$$\Phi_\pm(x)=A_\pm\sin[k_\pm(x+L)] \qquad \text{for } x\leqq 0, \tag{3.131 a}$$

$$\Phi_\pm(x)=B_\pm\sin[k_\pm(x-L)] \qquad \text{for } x\geqq 0, \tag{3.131 b}$$

with eigenvalue equation

$$k=\mp\gamma\tan(kL) \tag{3.132}$$

derived from the continuity condition at $x=0$. The eigenvalues can be written as

$$Lk_n=(n-\tfrac{1}{2})\pi+\Delta_{n\pm}\,; \qquad n=1,2,3,\ldots \tag{3.133}$$

with $|\Delta_{n\pm}|<\pi/2$. The difference $\Delta_{n+}-\Delta_{n-}=:\Delta\Delta_n$ increases smoothly from 0 for small k to π for large k, where the Born approximation holds. For given momentum the quantum number n can be assumed to be of order L. The overlap integrals required for $\rho_{RL}(t)$ are calculated as

$$\langle\Phi_{n+}|\Phi_{m-}\rangle=(-)^{n+m}\frac{\sin(\Delta_{n+}-\Delta_{m-})}{(n-m)\pi+\Delta_{n+}-\Delta_{m-}} +(-)^{n+m}\frac{\sin(\Delta_{n+}+\Delta_{m-})}{(n+m)\pi+\Delta_{n-}+\Delta_{m-}}. \tag{3.134}$$

Although the denominator of the first term causes a resonance-like behaviour in $k-k'$ if n and m are replaced by $kL/2\pi$ and $k'L/2\pi$, the states can never be considered as dense within the resonance, and may contribute individually under a sum, critically depending on the values of $\Delta_{n+}-\Delta_{m-}$. The second term vanishes with $L\to\infty$.

The general time-dependence of $\rho_{RL}(t)$ is now given by expanding the initial state by means of the stationary states (3.131),

$$\begin{aligned}&(|\varphi_R\rangle+|\varphi_L\rangle)(|\chi_1\rangle+|\chi_2\rangle)|\Phi(0)\rangle\\ &=(|\varphi_R\rangle|\chi_1\rangle+|\varphi_L\rangle|\chi_2\rangle)\sum_n c_{n+}|\Phi_{n-}\rangle\\ &\quad+(|\varphi_R\rangle|\chi_2\rangle+|\varphi_L\rangle|\chi_1\rangle)\sum_n c_{n-}|\Phi_{n-}\rangle.\end{aligned} \tag{3.135}$$

With

$$c_{m-}=\sum_\mu c_{\mu+}(-)^{m-\mu}\frac{\sin(\Delta_{\mu+}-\Delta_{m-})}{(\mu-m)\pi+\Delta_{\mu+}-\Delta_{m-}} \tag{3.136}$$

one obtains from (3.126) for massless particles (with $E_{n\pm}=k_{n\pm}c$)

$$\begin{aligned}\rho_{RL}(t)=\tfrac{1}{2}\,\mathrm{Re}\sum_{n,m,\mu}c_{n+}c_{\mu+}(-)^{n+\mu}&\frac{\sin(\Delta_{n+}-\Delta_{m-})}{(n-m)\pi+\Delta_{n+}-\Delta_{m-}}\\ &\cdot\frac{\sin(\Delta_{m+}-\Delta_{\mu-})}{(m-\mu)\pi+\Delta_{m+}-\Delta_{\mu-}}\\ &\cdot\exp\{-i[(m-n)\pi+\Delta_{n+}-\Delta_{m-}]ct/L\}.\end{aligned} \tag{3.137}$$

In order to get a linear time dependence as in (3.130) one has to employ a similar initial condition. Choosing for simplicity $c_{n+}=\delta_{nl}$ gives

$$\rho_{RL}(t)=\frac{1}{2}\sum_{m=1}^{\infty}\frac{\sin^2(\Delta_{m+}-\Delta_{l-})}{[(m-l)\pi+\Delta_{m+}-\Delta_{l-}]^2}\cdot\cos[(m-1)\pi ct/L]. \tag{3.138}$$

assuming $\Delta_{m\pm}ct/L\ll 1$. In the Born approximation range ($\Delta_{m+}-\Delta_{L-}\simeq\pi$ for $m\simeq l$) the term with $m=l-1$ dominates for small t, leading to a quadratic time dependence. A linear time dependence cannot be obtained from a partial sum over m. It is therefore convenient to partition the sum into three parts,

$$\sum_{m=1}^{\infty}\simeq\sum_{m=-\infty}^{\infty}=\sum_{m=-\infty}^{l-\kappa-1}+\sum_{m=l-\kappa}^{l+\kappa}+\sum_{m=l+\kappa+1}^{\infty}, \tag{3.139}$$

thereby leaving the critical terms in the finite sum. The infinite sums can be replaced by integrals over

$k := (m-l)2\pi/L =: 2\pi\nu/L$ for sufficiently large κ,

$$2\rho_{RL}(t) \simeq \sin^2(\Delta\Delta_l)\left\{\sum_{\nu=-\kappa}^{\kappa} \frac{\cos(\nu\pi ct/L)}{[\nu\pi+\Delta\Delta_l]^2} + \frac{4}{L}\int_{2\pi\kappa/L}^{\infty} dk \frac{\cos(kct/2L)}{k^2}\right\}. \qquad (3.140)$$

Here again the smooth dependence of $\Delta_{l\pm} = \Delta_{\pm}(k_0)$ on $k_0 = 2\pi l/L$, and $|\Delta| < \pi$ together with $\kappa \gg 1$ have been taken into account. Integrating by parts and expanding the result for small values of $\kappa ct/L$ gives

$$2\rho_{RL}(t) \simeq \sin^2(\Delta\Delta(k_0))\left\{\sum_{\nu=-\kappa}^{\kappa} \frac{1}{[\nu\pi+\Delta\Delta(k_0)]^2} - \frac{1}{\pi^2\kappa} - \frac{ct}{4L}\right\}. \qquad (3.141)$$

As expected, this expression depends only linearily on time. The constant term has assumed an inconvenient form as a consequence of the approximations. It can, of course, be calculated exactly, and is known to be unity from the initial normalization of (3.126). Indeed, $\sum_{\nu=-\infty}^{\infty} (\nu\pi+\alpha)^{-2}$ is the pole expansion of $1/\sin^2(\alpha)$. The result up to first order in ct/L then reads

$$2\rho_{RL}(t) = 1 - \frac{ct}{4L}\sin^2(\Delta\Delta(k_0)) \simeq \exp\left[-\frac{ct}{4L}\sin^2(\Delta\Delta(k_0))\right]. \qquad (3.142)$$

Again, as in (3.130), the L-dependence cancels if the number of independently interacting particles is proportional to L. This result holds beyond the Born approximation (i.e. for arbitrary $\Delta\Delta(k_0)$) and may be generalized by means of (3.137) to include other initial conditions and times $t > L/c$.

IV. Summary and Conclusions

The microscopic behaviour of macroscopic systems is dynamically extremely sensitive to their environments. In particular certain phase relations will be destroyed locally by the specific nature of the interaction, which may be assumed to be entirely passive. If the system by itself is – for the usual purpose of calculating probabilities for future local measurements described by its density matrix, which in turn is diagonalized by an ensemble of states, the latter may turn out to appropriately characterize classical properties in a way similar to the wave packets originally used by Schrödinger to represent 'particles'. In this way classical properties are caused by the influence of a specific environment, and contrary to usual thought are not intrinsic to macroscopic systems. This demonstrates clearly that classical physics is not a limiting case of the quantum physics of isolated systems. On the other hand the origin of classical properties can be successfully analyzed within quantum mechanics. The 'particle' aspect - usually considered as the representation of spatial discontinuity - is thereby obtained as a *consequence* of the fundamental quantum measurement or collapse of the state vector (implicitely contained in the density matrix formalism). This is analogous to the possibility of eliminating as a fundamental process the concept of 'quantum jumps' in atoms – representing a discontinuity in time – and of replacing them by the time-dependent Schrödinger equation in connection with the collapse during measurements or observations [8]. The interaction with the environment – even if passive – leads to a non-unitary evolution in time of the local density matrix. This appears as a local indeterminism for the ensemble of diagonalizing states.

In detail the situation is different for discrete or continuous spectra of the considered degrees of freedom. The first case has been studied from various points of view for the example of a simple two-state system. If the internal dynamics is negligible, the resulting density matrix is diagonalized by the representation being 'measured' by the interaction. In the case of chiral molecules in a normal environment this basis will consist of the chirality states. Although an optically active molecule interacts with light so weakly that it could remain in its parity eigenstates if kept in a sufficient vacuum and free from collisions with the walls, it can hardly ever be produced in such a state. If exposed to collisions with air molecules, for example, chirality states will be preserved in contrast to parity states, although both pairs of states – if 'dressed' by virtual photons – may be energy eigenstates. If the internal dynamics is non-negligible, it may be suppressed by the watchdog-effect [6].

The situation of continuous degrees of freedom being measured by a continuous measurement scale is represented by the translational motion of small dust particles or large molecules interacting with photons or molecules. If the position is 'continuously measured', the density matrix resulting from an initial Gaussian wave packet is determined by an interplay of measurements with the dispersion of the wave packet. Asymptotically the effect of measurements dominates: The coherence length decreases whereas the spreading of the density matrix in space increases with time, thus locally mimicing an indeterministic motion. However, as the density matrix remains Gaussian, its eigenstates are eigenfunctions of the harmonic oscillator with arbitrary number of nodes and asymptotically increasing widths. They are inappropriate to represent localized particles. A reduction or Everett branching of the total state vector according to the corresponding Schmidt-representation (compare Eq. (3.1)) [11] would not describe the observed situation.

Measurements proper seem in any case to contain a discrete observation representation at some stage. Even if a photon is observed directly by the eye, it is registered by *discrete* visual cells. In such situations of 'measurements of continuous variables by discrete pointers' the unitary description of a measurement is (for measurements of the first kind)

$$\varphi(x)\,\Phi_0 \to \sum_n \varphi_n(x)\,\Phi_n, \tag{4.1}$$

where $\varphi_n(x) = P_n\varphi(x)$ is the restriction of $\varphi(x)$ onto the interval of x being registered by the pointer position Φ_n. Since in this case – in contrast to a continuum of pointer positions – the states Φ_n may be considered as mutually orthogonal after a measurement-like interaction, the local density matrix

$$\rho_\varphi = \sum_n \varphi_n(x)\,\varphi_n^*(x') \tag{4.2}$$

does not contain any interference between positions in different intervals. Hence, the eigenstates will be restricted to these intervals.

It is crucial for this argument that the discrete pointer by itself is not initially in a superposition $\sum_m c_m \Phi_m$ of different positions. This will in turn be achieved if the discrete pointer is macroscopic in being continuously measured by scattering photons or molecules as studied for two-state systems (chirality instead of pointer position). As demonstrated by the linearity of the master equation (3.59), the components $\varphi_n(x)\,\Phi_n$ will then in addition evolve in time independently of one another. (In spite of the linear Schrödinger equation this cannot always hold true, since the density matrix of a system interacting with its environment cannot in general obey a von Neumann equation [23]: Usually, factorizing states are not stationary.) This result allows to specify the reduction of the state vector equivalent to the Everett interpretation, provided an appropriate cosmological initial condition is assumed for the total Everett state [25].

Of course, no unitary treatment of the time dependence can explain why only one of these dynamically independent components is experienced. However, it does explain how macroscopic states – in particular the discrete ones like those occurring in measurement devices or in the central nervous system – will develop quantum correlations with macroscopic states of other systems, and form dynamically independent components (branches). For example, if $\sum_\alpha c_\alpha \varphi_\alpha(x)\,\Psi_\alpha$ describes the eigenstates φ_α of the density matrix of a dust particle (compare Sect. III.2.2), correlated to the states Ψ_α of the photon field, an observation of the dust particle as in (4.1) now leads to

$$\sum_\alpha c_\alpha \varphi_\alpha(x)\,\Psi_\alpha\,\Phi_0 \to \sum_{\alpha,n} c_\alpha P_n \varphi_\alpha(x)\,\Psi_\alpha\,\Phi_n \to \sum_{\alpha,n} c_\alpha P_n \varphi_\alpha(x)\,\Psi_{\alpha,n}\,\Phi_n. \tag{4.3}$$

where the second step corresponds to the scattering of photons off the discrete 'pointer' Φ. Since the states $\Psi_{\alpha,n}$ will be mutually orthogonal in each index due to the enormous information capacity of the photon field Ψ, the components differing in α and n will form dynamically independent branches. As this is required in order to render the reduction version of the measurement dynamically consistent with the Schrödinger equation (if Everett's interpretation [31] is accepted), this result is hoped to be an important step in understanding the measurement process.

This mechanism of producing classical properties would be effective also for theories like Bohm's [32, 33], that explicitely introduce classical variables in addition to a universal wave function. This may undermine the motivation for such theories [34].

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