Time (a-)symmetry
in a recollapsing quantum universe

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Abstract: It is argued that Hawking's 'greatest mistake' was no mistake: in the canonical theory of quantum gravity for Friedmann type universes all time arrows must be correlated with that of the expansion. For recollapsing universes this seems to be facilitated partly by quantum effects at the turning point. Because of the resulting thermodynamical symmetry between expansion and (formal) collapse, black holes must formally become 'white' during the collapse phase (while physically only expansion of the universe and black holes can be observed). It is conjectured that the quantum universe remains singularity-free in this way (except for the homogeneous singularity) as a consequence of an appropriate *intrinsic* initial condition for the wave function.

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1. Conditioned entropy in quantum cosmology

Invariance under reparametrization of time may be considered as a specific consequence of Mach’s principle (the absence of any preferred or ‘absolute’ time parameter). In quantum theory it leads to a time-independent Schrödinger equation (Hamiltonian constraint), since any reparametrization of physical time (‘clocks’) would depend on the considered orbit. For example, in canonical quantum gravity the wave function of the universe is dynamically described by the ‘stationary’ Wheeler-DeWitt equation $H\Psi_{universe} = 0$ in superspace (the configuration space of geometry and matter). Although the conventional time dependence has then simply to be replaced by the resulting quantum correlations between all dynamical variables of the universe (which have to include all physical clocks, in particular the spatial metric -- see Page and Wootters, 1983), this leaves open the problem of how to describe the asymmetry in time which is manifest in most observed phenomena.

For example, entropy as the thermodynamical measure of time asymmetry is in quantum theory defined as a functional of the density matrix ρ,

$$S = \mathrm{Trace}\,\{\, P\rho \ln (P\rho)\,\} \quad . \qquad (1)$$

It requires an appropriate ‘relevance concept’ or ‘generalized coarse graining’ which is represented by a ‘Zwanzig projection’ P (an idempotent operator on the space of density matrices -- cf. Zeh, 1989). Well known examples are Boltzmann's neglect of particle correlations, or the relevance of locality, $P_{local}\rho := \Pi_i \rho_{\Delta V i}$. This neglect of all long range correlations (quantum or classical) gives rise to the concept of an entropy *density*. The density matrix in (1) may then even represent a pure (‘real’) state, $\rho = |\varphi\rangle\langle\varphi|$, which should, however, depend on an appropriate time variable (in order to allow the entropy to grow).

Since the physical entropy is in contrast to the entropy of information objectively defined as a function of the macroscopic variables (like volume and temperature -- regardless of whether these are known), φ cannot be identified with $\Psi_{universe}$ (which is a superposition of macroscopically different states), but must instead represent some ‘relative state’ (conditioned wave function) of the microscopic degrees of freedom with respect to ‘given’ macroscopic variables of the universe (including the clocks). This state is usually understood as the ‘collapse component’ or the ‘Everett branch’ that has resulted from all measurements or measurement-like processes which according to von Neumann's dynamical description would have led to superpositions of macroscopic ‘pointer positions’. These components can be considered as dynamically decoupled from one another once they have decohered. Measurements and decoherence represent the quantum mechanical aspect of time asymmetry (Joos and Zeh, 1985; Gell-Mann and Hartle, contributions to this conference) that also has to be derived from the Wheeler-DeWitt wave function.

A procedure for deriving an approximate concept of a time-dependent wave function $\varphi(t)$ has been proposed by means of a WKB approximation (geometric optics) for part of the dynamical variables of the universe. They may be those describing the spatial geometry (Banks, 1985), those forming the ‘mini superspace’ of all monopole amplitudes on a Friedmann sphere (Halliwell and Hawking, 1985), or all macroscopic variables which define an appropriate ‘midi superspace’. For example, Halliwell and Hawking assumed that the wave function of the universe can approximately be written as a sum of the form

$$\Psi_{\text{universe}} \approx \sum_r e^{i\,S_r(\alpha,\Phi)} \varphi_r(\alpha,\Phi;\{x_n\}) , \tag{2}$$

where $\alpha = \ln a$ is the logarithm of the expansion parameter, Φ is the monopole amplitude (homogeneous part) of a massive scalar field which represents matter in this model, and the variables x_n (with $n > 0$) represent all multipole amplitudes of order n. The exponents $S_r(\alpha,\Phi)$ are Hamilton-Jacobi functions with appropriate boundary conditions, while the relative states $\varphi_r(\alpha,\Phi;\{x_n\})$ are assumed to depend only weakly on α and Φ. If the corresponding orbits of geometric optics in mini superspace are parametrized in the form $\alpha(t_r)$, $\Phi(t_r)$, one may approximately derive, from the Wheeler-DeWitt equation, a unitary evolution

$$i\frac{\partial}{\partial t_r}\, \varphi_r(t_r,\{x_n\}) = H_x\, \varphi_r(t_r,\{x_n\}) \tag{3}$$

for the ‘relative states’ $\varphi_r(\alpha(t_r),\Phi(t_r),\{x_n\}) = \varphi_r(t_r,\{x_n\})$. It applies along all orbits of each WKB sheet $S_r(\alpha,\Phi)$. In order to be acceptable as describing the quantum dynamics of the observed world (within these approximations), this equation must include the description of the above-mentioned measurements and measurement-like interactions in von Neumann’s form

$$\varphi_r(t_r) \;\propto\; \left(\sum_k c_k\, \varphi_k^S\right) \varphi_0^A \;\underset{t_r}{\rightarrow}\; \sum_k c_k\, \varphi_k^S\, \varphi_k^A \tag{4}$$

in the direction of 'increasing time'. For proper measurements the 'pointer positions' ${\varphi^A}_k$ of the 'apparatus' A must decohere through further 'measurements' by their environment, and thus lead to newly separated branches with their corresponding 'conditioned (physical) entropies'. The formal entropy corresponding to the apparent ensemble of different values of k would instead have to be interpreted as describing 'lacking knowledge'.

This asymmetry with respect to the direction of the orbit parameter t_r means that (4) may be meaningfully integrated, starting from the wave function representing some instantaneous state of the observed world (i.e. from the actual world *branch)*, only into the 'future' direction of t_r (where it will describe the entangled superposition of all outcomes of future measurements). In the 'backward' direction of time this calculation would *not* reproduce the correct quantum state, since the unitary predecessors of the non-observed components would be missing. This would be particularly important if the orbits were continued into the inflationary era, or even into the Planck era, where different orbits in mini superspace (and, in the case of recollapsing universes, also both of their 'ends') have to interfere with one another to form the complete boundary condition for the total Wheeler-DeWitt wave function at $a \to 0$. Entropy is expected to grow in the same direction of time as that describing measurements. Any such asymmetry requires a very *special cosmic initial condition;* the existence of measurement-like processes in the quantum world requires essentially a non-entangled initial state (Zeh, 1989). Since the unitary dynamics was derived from the Wheeler-DeWitt equation, this initial condition for φ_r must then also be a consequence of the general structure of Ψ_{universe}.

In order to describe an appropriate asymmetry of the Wheeler-DeWitt wave function, it will be assumed in accordance with existing models that the Wheeler-DeWitt Hamiltonian for the gauge-free multipoles of Friedmann type universes is of the form

$$2e^{3\alpha}\,H = +\frac{\partial^2}{\partial\alpha^2} - \frac{\partial^2}{\partial\Phi^2} - \sum_n \frac{\partial^2}{\partial x_n^2} + V(\alpha, \Phi, \{x_n\})\,, \qquad (5)$$

with a potential that becomes 'simple' (e. g. constant) in the limit $\alpha \to -\infty$. In his talk, Julian Barbour gave an example of how complicated the effective potential in configuration space becomes instead, once a particle concept has emerged from the general state of the quantum fields. The hyperbolic nature of (5) defines an initial value problem with respect to α which then allows one also to choose a 'simple' (or symmetric) initial condition (SIC!) for $\Psi_{universe}$ in the same limit of small a. For example, it may be assumed to be of the form (Conradi and Zeh, 1991)

$$\Psi(\alpha,\Phi,\{x_k\}) \to \frac{1}{(-V)^{1/4}} \exp\left[\int_{-\infty}^{\alpha} \sqrt{-V(\alpha',\Phi,\{x_k\})}\, d\alpha'\right] \to \Psi(\alpha) \qquad (6)$$

If the initial simplicity of the relative states φ_r of (2) can be derived from this or some similar simple structure of the total wave function close to the singularity, this would mean that 'early times' must correspond to small values of a. Unfortunately, orbits in the mini superspace formed by α and Φ must return to small values of a for appropriate values of the cosmological constant (even though they are not symmetric in the generic case -- see Fig. 1). How, then, can one distinguish between the Big Bang and the Big Crunch? Or is that distinction really required for the definition of an arrow of time?

The contributions of Murray Gell-Mann, Jim Hartle and Larry Schulman to this conference indicate that this need not be the case, provided the considered universe is very young compared to its total lifetime. A symmetric (double-ended) low entropy condition might be possible even if the total $\Psi_{universe}$ obeyed a unitary time dependence – although it would then represent a very strong constraint. In quantum gravity, however, where there is no fundamental time parameter, one has to conclude that a simple condition for φ_r may be *derived* from $\Psi_{universe}$ either at both ends of an orbit in mini superspace or at none. (Any asymmetric selection criterion for orbits or their relative states -- for example by means of a time-directed probability interpretation -- would *introduce* an absolute direction of time.) In the second case the asymmetry of the world could only be understood as a 'great initial accident' (at one end) -- if it is possible at all. In the first case all arrows of time have to reverse their direction when the universe reaches its maximum extension. The approximately derived unitary dynamics for ϕ_r may then only be applied in the direction of growing values of a. In particular, using the cosmic inflation for explaining a low entropy state at *one* end only would be equivalant to *presuming* the arrow of causality to apply in a certain direction of the orbit (instead of deriving this asymmetry as claimed).

Notice that in quantum gravity there is no problem of consistency between the lifetime of the recollapsing universe and its supposedly much longer Poincaré cycles (or mean time intervals between two *statistical* fluctuations of cosmic size), as it has to be expected to arise with the mentioned double-ended boundary conditions under deterministic (such as unitary) dynamics. The exact dynamics $H\Psi_{universe} = 0$, understood as an intrinsic initial value problem in the variable α, constitutes a well-defined one-ended condition, while the reversal of the arrows of time described by the dependence $\varphi_r(t_r)$ is facilitated by the *corrections* to the derived unitary dynamics. These corrections have to describe <u>re</u>coherence and inverse branchings on the return leg.

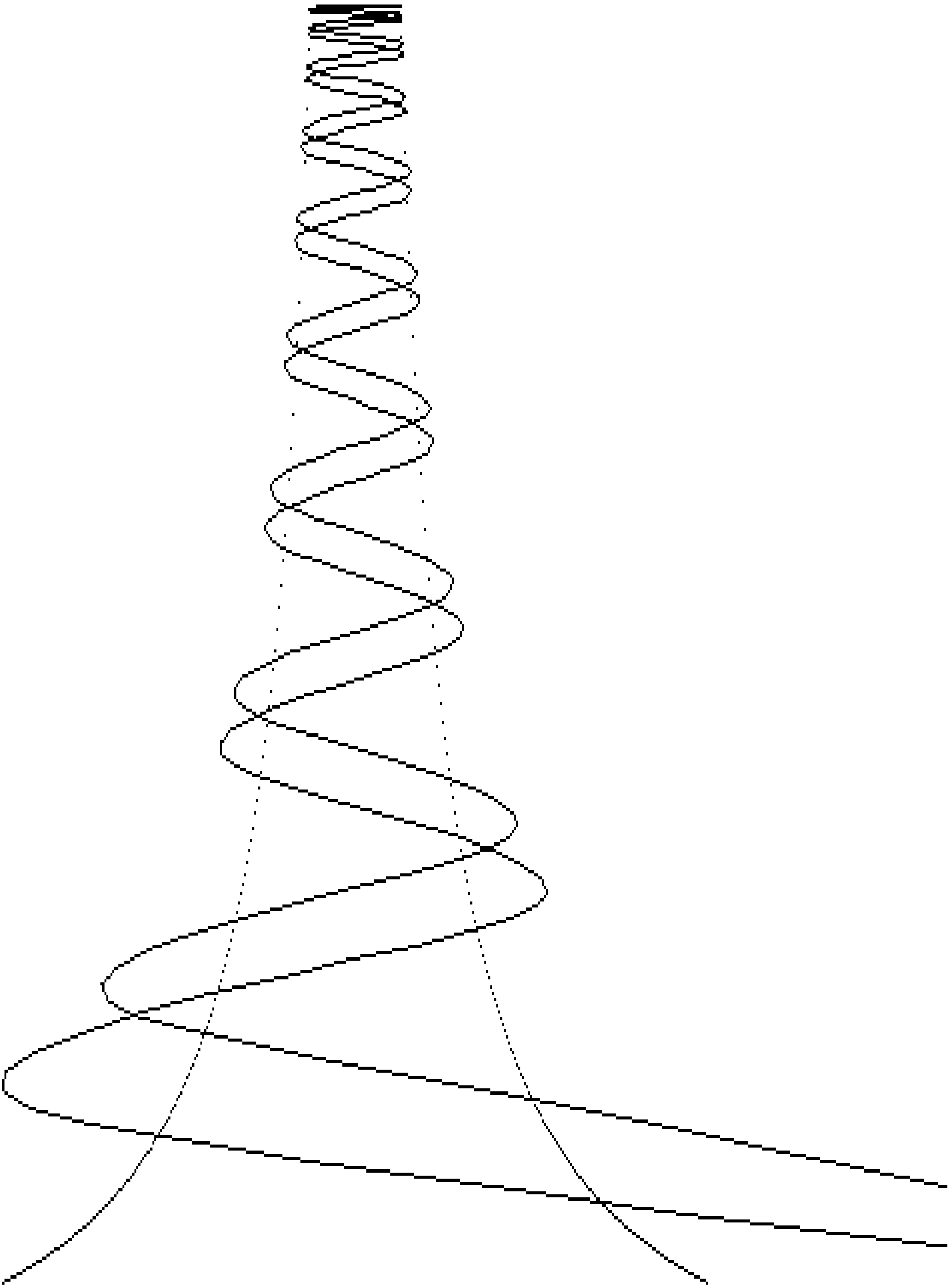

Fig. 1: Asymmetric classical orbit in mini superspace. (After Hawking and Wu, 1985 -- see also Laflamme, this conference). a is plotted upwards, Φ from left to right. Dotted curve corresponds to $V = - a^4 + m^2a^6\Phi^2 = 0$.

I am thus trying to convince Stephen Hawking that he did *not* make a mistake before he changed his mind about the arrow of time! The asymmetry of individual orbits in mini superspace (pointed out by Don Page, 1985) does not appear to be sufficient for deriving much stronger *thermodynamic* conclusions. They would instead require a demonstration that most orbits in mini superspace enter the Planck era in two different regions corresponding to to ‘relative wave functions’ of extremely different structure.

2. Reversal of the expansion of the universe in quantum cosmology

Within the canonical quantum theory of gravity it appears therefore impossible for the arrow of time to maintain its direction when the universe starts recollapsing. However, the picture described so far is not yet a sufficient representation of the exact situation defined by the Wheeler-DeWitt equation. As will be shown, the approximation of geometric optics does not justify the continuation of classical orbits in mini superspace through the whole history of a universe. For example, an orbit chosen to be compatible with the WKB approximation of the wave function at one end, and found to be incompatible with it at the other one, would merely demonstrate that the concept of orbits must have broken down in between.

Wave-mechanically, orbits must be represented by wave packets. The exact dynamics for $\Psi_0(\alpha,\Phi)$ (now replacing the approximation $e^{iS(\alpha,\Phi)}$ in mini superspace) is described by the wave equation

$$2e^{3\alpha}\, H\, \Psi(\alpha,\Phi) \;=\; \frac{\partial^2\Psi}{\partial\alpha^2} \;-\; \frac{\partial^2\Psi}{\partial\Phi^2} \;+\; [\,- e^{4\alpha} + m^2e^{6\alpha}\Phi^2\,]\,\Psi(\alpha,\Phi) \;=\; 0\ . \quad (7)$$

The α-dependent oscillator potential in Φ suggests the ansatz

$$\Psi(\alpha,\Phi) = \sum_n c_n(\alpha)\, \Theta_n\!\left(\sqrt{me^{3\alpha}}\,\Phi\right) , \quad (8)$$

where the functions Θ_n are the oscillator eigenfunctions. In adiabatic approximation the coefficients $c_n(\alpha)$ decouple dynamically,

$$\frac{d^2 c_n(\alpha)}{d\alpha^2} + [\, - e^{4\alpha} + (2n + 1)\, m\, e^{3\alpha}\,]\, c_n(\alpha) \;=\; 0\ . \tag{9}$$

In this case, coherent oscillator wave packets exhibit the least possible dispersion, and may therefore be expected to resemble the orbits of geometric optics best.

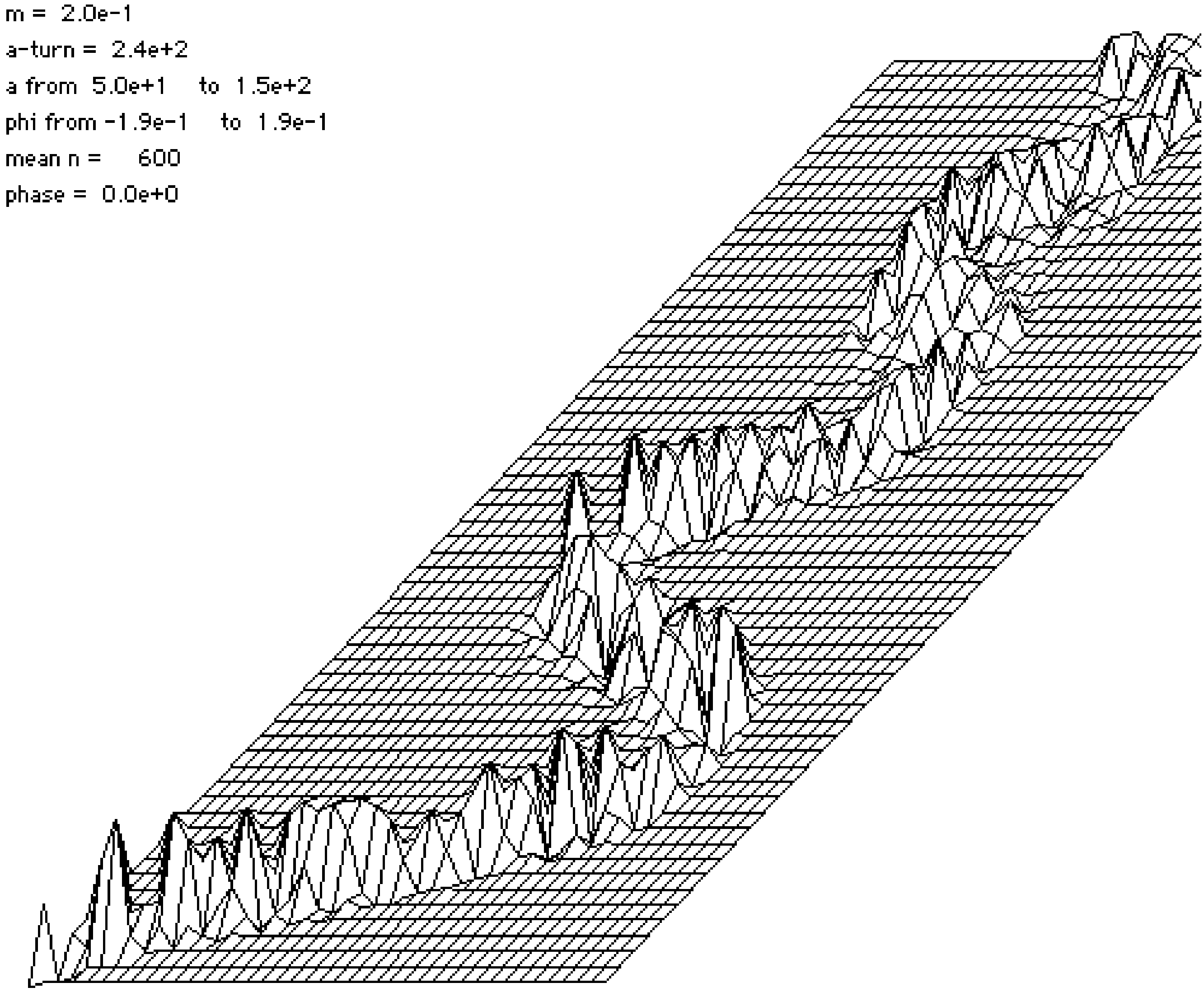

Fig. 2: Wave packet representing the orbit of an expanding universe (first cosine of Eq. (10) only). Plot range from left to right is $-0.19 < \Phi < 0.19$, from bottom to top $50 < a < 150$. The intrinsic structure of the wave packet is not resolved by the chosen grid size

As was demonstrated by Kiefer (1988), the usual (here ‘final’ with respect to the intrinsic wave dynamics) condition of square integrability in $\alpha \to +\infty$ leads to the classically expected reflection of quasi-orbits from the repulsive curvature-induced potential $e^{4\alpha}$. For example, a further WKB approximation to (9) leads to solutions of the form

$$
\begin{aligned}
c_n(\alpha) \;&\propto\; \cos[\phi_n(\alpha) + n\Delta\phi] \;+\; \cos[\phi_n(\alpha) - n\Delta\phi + \delta_n] \qquad (10)\\
&= \text{'expanding universe'} \;+\; \text{'collapsing universe'}\;,
\end{aligned}
$$

where the ϕ_n’s are monotonic functions of α, approximately proportional to n, while $\delta_n = (\pi/4)m^2(2n+1)^2$ is the ‘scattering’ phase shift enforced by the ‘final’ condition. The two cosines correspond to the expanding and the recollapsing parts of the histories of classical universes in mini superspace. $\Delta\phi$ is the phase of the classical Φ–oscillation at the point of maximum a (describing the asymmetry of the orbit). However, if the constant factors for the rhs of Eq. (10) are chosen to form coherent states from the first cosine, the phase relations which would also be required to form coherent states from the second cosine are completely destroyed by the large phase shift differences $\delta_n - \delta_{n-1} \propto n$. While the term representing the expanding universe (Fig. 2) may then nicely resemble the corresponding part of a classical orbit (Fig. 1), the reflected wave is smeared out over the whole allowed area (Fig. 3). From a sharp (n-independent) potential barrier, the wave packets would instead be reflected without any dispersion.

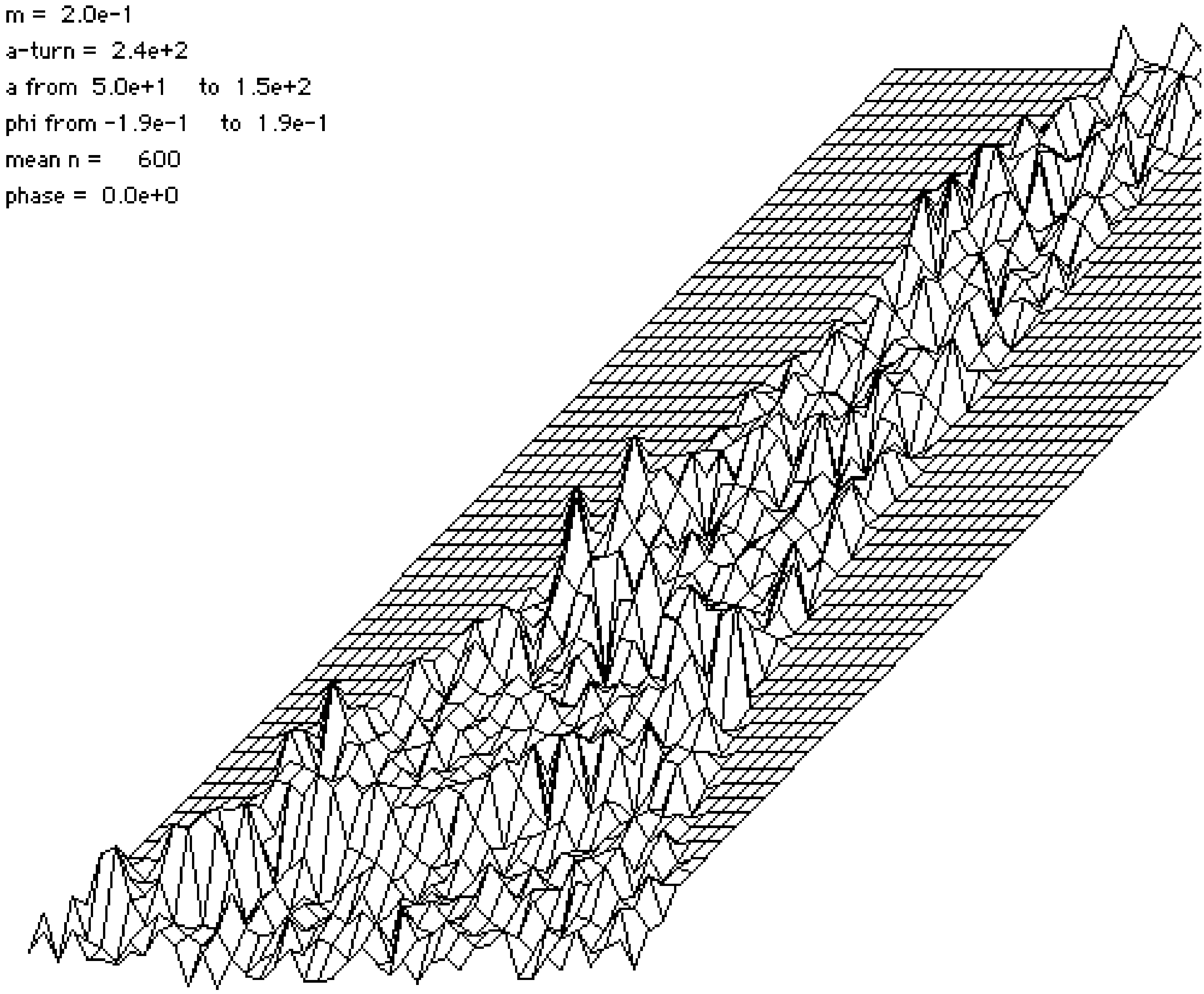

Fig. 3: Same wave packet as in Fig. 2 with recollapsing part (second cosine) added. The part of the wave packet representing the expanding universe of Fig. 2 is still recognizable.

This dispersion will become even more important for more ‘macroscopic’ universes (higher mean oscillator quantum numbers ñ), since the phase shift *differences* are proportional to n. The result depicted in Fig. 3 may therefore be expected to represent a generic property of Friedmann type quantum universes. Quasi-classical orbits must then not be continued beyond the turning point. The wave mechanical continuation leads instead to a *superposition of many* ‘recollapsing’ universes (which cannot be intrinsically distinguished from expanding ones). Cosmological quantum effects of gravity thus seem to be essential not only at the Planck scale! The phase relations of the resulting superpositions of quasi-orbits on the return leg in mini superspace are however destroyed by decoherence -- now ‘irreversibly’ acting in the opposite direction of the orbit because of the (formally) final condition. (The phase shifts δ_n could as well have been put into the first cosine with a negative sign.) This demonstrates again that the unitary dynamics (3) for the relative states cannot in fact be continued beyond the turning point.

Although wave packets solving the Wheeler-DeWitt equation in mini superspace can be *defined* intrinsically asymmetric, they are physically determined (as separate Everett branches) by their decoherence from one another -- a mechanism that must work symmetrically on both legs. Wave packets in the *complete* configuration space are not to describe the whole 'quantum world', but merely the causal connections which seem to define its quasi-classical 'branches'.

3. Black-and-white holes

The formal reversal of the arrow of time (with or without the importance of quantum effects near the turning point of the universal expansion) must also drastically influence the internal structure of black holes (Zeh, 1992). Consider a black hole that forms during the expansion of the universe, but that is massive enough to be able to survive the turning point (cf. Penrose's diagram depicting a time-asymmetric universe in Fig. 4). If the arrow of time is however formally reversed along an orbit through mini or midi superspace, this black hole cannot continue 'losing hair' any further by radiating its higher multipoles *away* (by means of retarded radiation). It must instead grow hair again by means of the now *coherently* incoming (advanced) radiation that has to drive the matter apart again.

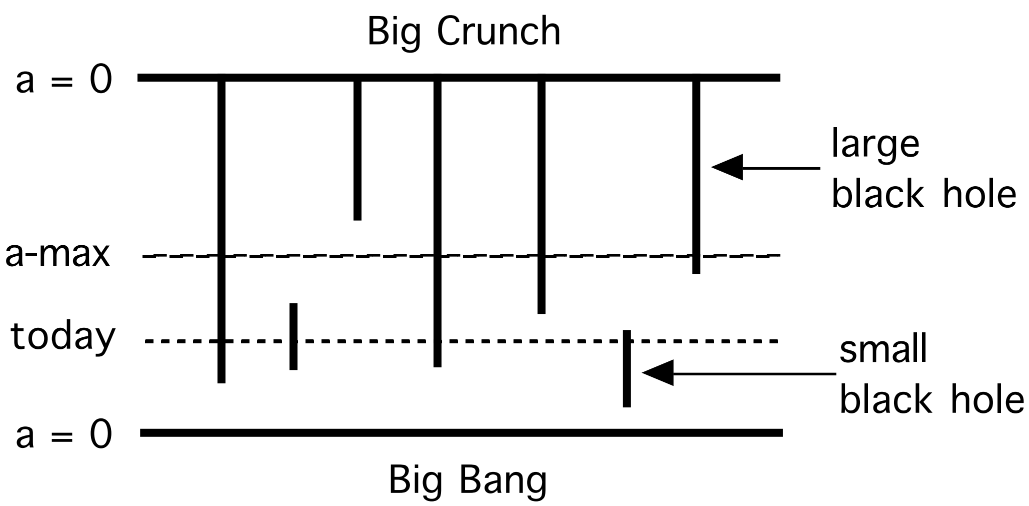

Fig. 4: Time-asymmetric universe with a homogeneous Big Bang (Penrose, 1981)

The reversal of the arrows of time of course has to include the replacement of time-directed 'causality' by what would formally appear as a 'conspiracy'. A mere reversal of the expansion would not be sufficient to 'cause' a reversal of the thermodynamic or radiation arrow without simultaneous reversal of the time-direction of this causation. The (fork-like) causal structure (see Zeh, 1992) must thus be contained in the dynamical structure of the universal wave function that results from the intrinsic initial condition by means of the Wheeler-DeWitt equation. The black hole must therefore formally disappear as a white hole during the recollapse phase of the universe.

This unconventional behaviour of black holes seems to become interesting only in the very distant future (after an horizon and a singularity might be expected to have formed). However, our simultaneity with a black hole is not well defined because of the time translation invariance of the Schwarzschild metric. Fig. 5 shows a spherical black hole in Kruskal-type coordinates (a modified Oppenheimer-Snyder scenario) after a translation of the Schwarzschild time coordinate t such that the turning point is now at $t_{turn} = 0$ (hence also at $v_{Kruskal} = 0$). The 'black-and-white hole' must then appear *thermodynamically symmetric* (although by no means symmetric in non-conserved details -- microscopic or macroscopic). If past horizons and singularities can in fact be excluded by an appropriate condition to the Big Bang, the same conclusion must hold in quantum gravity for future horizons and singularities. So we may conjecture a singularity-free quantum world.

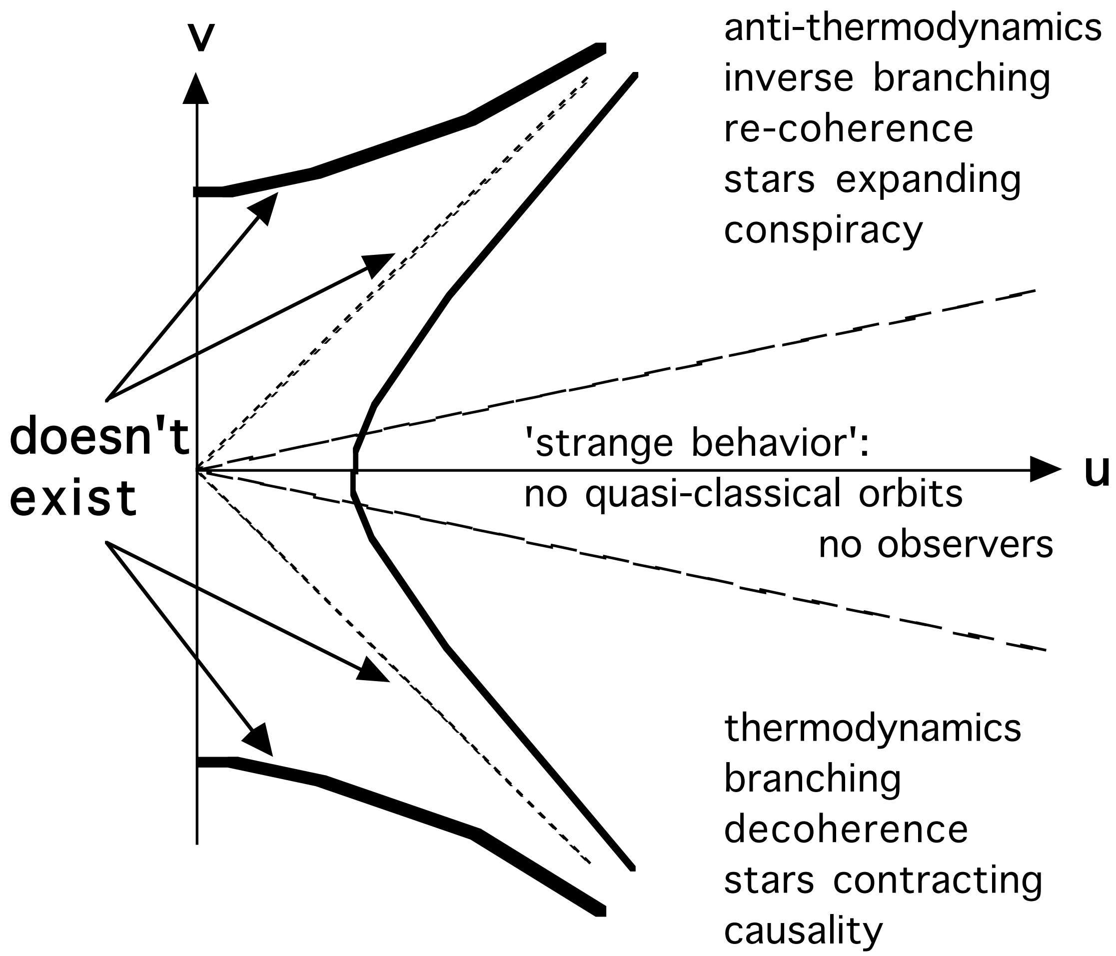

Fig. 5: ‘Black-and-white hole’ originating from a thermodynamically active and collapsing spherical matter distribution, with the Kruskal time coordinate v = 0 chosen to coincide with the time of maximum extension of the universe. This classical picture itself is not meaningful in the region of ‘quantum behaviour’ around v = 0.

Fig. 6 shows the same situation as Fig. 5 from our perspective of a young universe (after an inverse translation of the Schwarzschild time coordinate). In this diagram, $t = t_{turn}$ appears to be very ‘close’ to where one would expect the future horizon to form. From this perspective, the occurring strange thermodynamic and quantum effects also appear to be located close to the future horizon, thereby preventing it to form.

Because of the extreme time dilatation, this reversal of the gravitational collapse cannot be seen by an asymptotic observer, although it could be experienced by suicidal methods within relatively short proper times. If the black-and-white hole is massive enough, this kind of ‘quantum suicide’ must be quite different from the classically expected one by means of tidal forces. In a classical picture, travelling through a black-and-white hole considerably shortens the proper distance between the Big Bang and the Big Crunch, but unfortunately we could not survive as information-*gaining* systems.

This consideration should at least demonstrate that the classical (Kruskal-Szekeres) continuation of the Schwarzschild metric beyond the horizon is very doubtful!

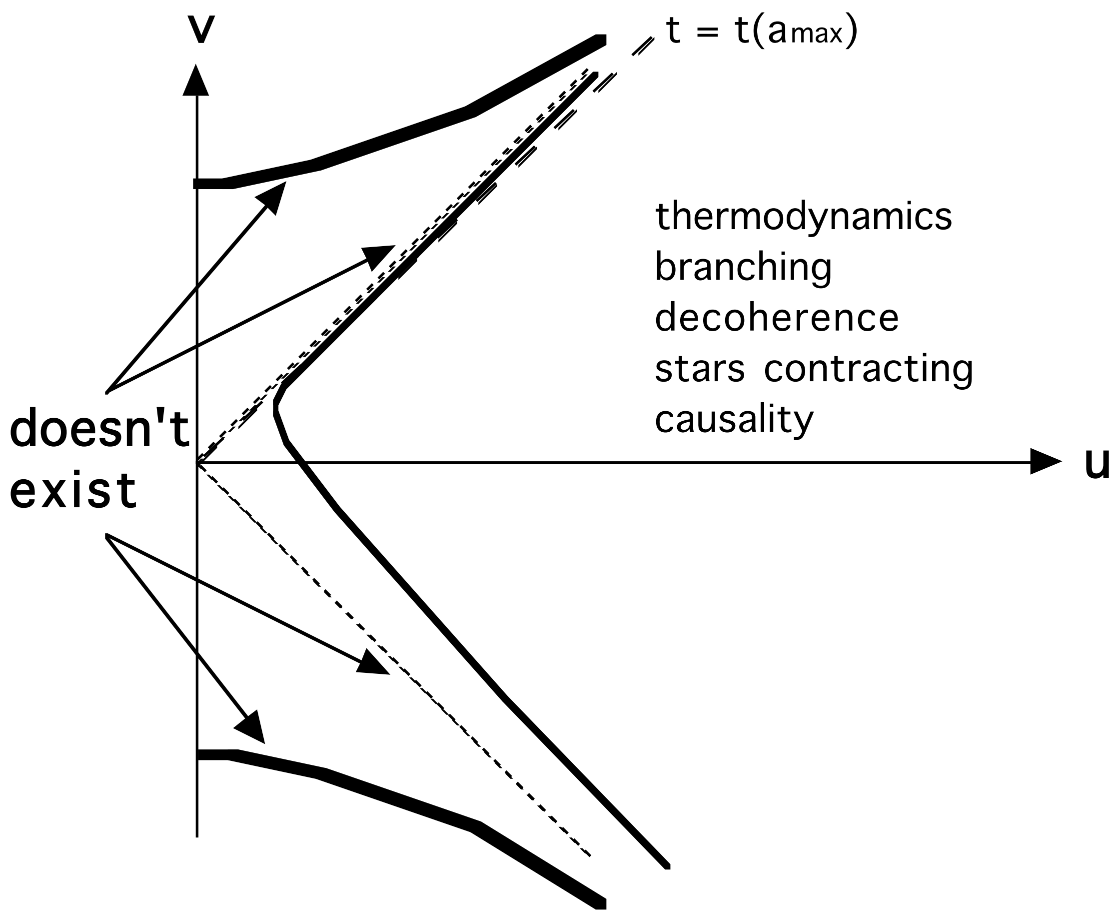

Fig. 6: Same black-and-white hole as in Fig. 5, considered from our perspective of a young universe.

Before Stephen Hawking changed his mind about the time arrow in a recollapsing universe, he had conjectured (Hawking, 1985) that the arrow would reverse *inside* the horizon of a black hole, since "it would seem just like the whole universe was collapsing around one" (cf. also Zeh, 1983). However, this picture would not yet describe a thermodynamically time-symmetric universe.

Penrose's black holes hanging like stalactites from the ceiling (the Big Crunch) now become symmetric as shown in Fig. 7. Black-and-white holes in equilibrium with thermal radiation (as studied by Hawking, 1976) would instead consist of thermal radiation at both ends. They would possess no 'hair' to lose or grow. The classically disconnected upper and lower halves of Fig. 7 should rather be interpreted as two of the many Everett branches, each of them representing an *expanding* universe, which interfere destructively above the turning point.

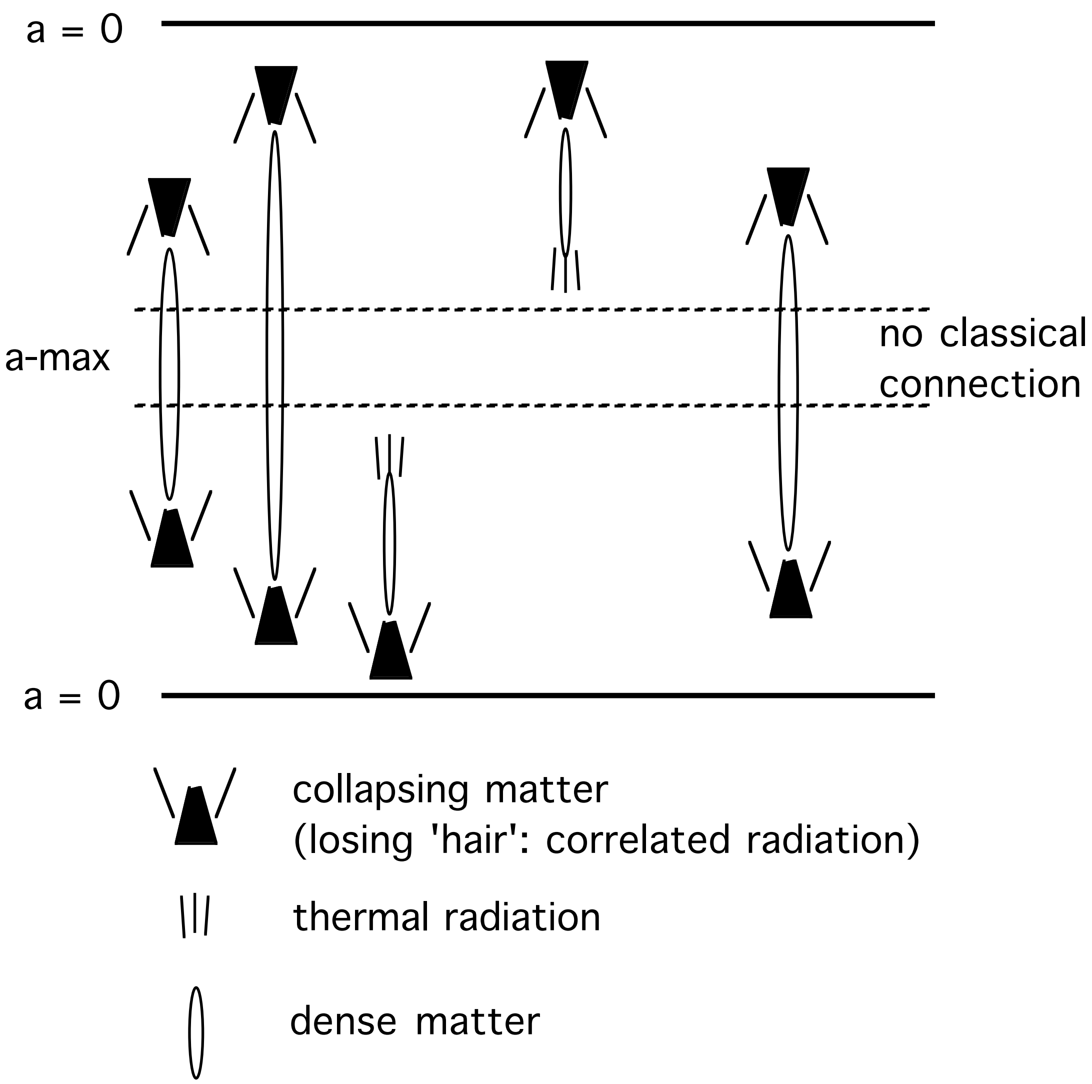

Fig. 7: Time-symmetric and singularity-free universe with black-and-white holes and small black or white holes.

The absence of singularities in this quantum universe thus appears to be a combined thermodynamic and quantum effect. However, one may equivalently interpret the result as demonstrating that in quantum cosmology the thermodynamic arrow is a *consequence* of the absence of inhomogeneous singularities -- a generalization (or rather, a symmetrization) of Penrose’s Weyl tensor condition.

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Discussion:

Hawking: Your symmetric initial condition for the wave function is wrong!

Zeh: Do you mean that it does not agree with the no-boundary condition?

Hawking: Yes.

Zeh: It was not meant to agree with it, although we found it to be very similar to the explicit wave functions you gave in the literature for certain regions of mini superspace. This is however not essential for my argument. It requires only that the multipole wave functions φ_r become appropriately ‘simple’ (low-entropic and factorizing) for small values of a (as you too seem to assume, although only at that end of the orbit where you start your computation).

Barbour: Did I understand you correctly to say that the criteria Kiefer used to obtain his solution was of the kind I call Schrödinger type, namely that there should be no blowing up of the wave function anywhere in the configuration space?

Zeh: Yes - if by blowing-up solutions you mean the exponentially increasing ones. Otherwise you would not be able to describe reflection (turning orbits) by means of wave packets. I think this assumption corresponding to the usual normalizability is natural (or ‘naive’ according to Kuchar) if the expansion parameter a is considered as a dynamical quantum variable (as it should in canonical quantum gravity).

Barbour: Could it be that worries about the turning point are an artifact of the extreme simplicity of the model? Consider in contrast a two-dimensional oscillator in a wave packet corresponding to high angular momentum!

Zeh: The described quantum effects at the turning point are due to the specific Friedmann potential with an oscillator constant for Φ exponentially increasing with α. They do not seem to disappear if added degrees of freedom possess similarly ‘normal’ potentials (e.g. polynomials multiplied by positive powers of a). This seems to be the case in Friedmann-type models.

Kuchar: Did you study decoherence between φ’s corresponding to one S, or also the decoherence corresponding to different S’s?

Zeh: I expect decoherence to be effective between different orbits in mini superspace (cf. Kiefer, 1987), between macroscopically different branches of the multipole wave functions φ_r along every orbit, and between different WKB sheets corresponding to different S’s (cf. Halliwell, 1989; Kiefer and Singh, 1991).

Griffiths: In ordinary quantum mechanics of a closed system, I do not know how to make any sense out of it using the “wave function of the closed system”. I need the unitary transformations that take me from one time to another. Is there any analogy of this in quantum gravity? For if not, it is hard to see how quantum gravity can be used to produce a sensible description of something like the world we live in.

Zeh: Your question seems to apply to quantum gravity in general. I think that it is sufficient for the "wave function of the universe" to contain correlations between all physical variables - including those describing clocks. In classical theory they would be unique, and would correspond to the orbits in the complete configuration space after eliminating any time parameter. In quantum theory there are no orbits that could be parametrized. The quantum correlations must of course obey 'intrinsic' dynamical laws as they are described by the Wheeler-DeWitt equation. From them one tries to recover the time-dependent Schrödinger equation (which has to describe the "observed world") as an approximation when spacetime (the history of spatial geometry) is recovered as a quasi-classical concept.

Lloyd: Could you clarify how black holes would grow hair in the contraction phase? Is it through interference between incoming radiation and the Hawking radiation.

Zeh: Only the incoming (advanced) radiation is essential, since black holes can form by losing hair even if Hawking radiation is negligible. This is a pure symmetry consideration. A final condition which is thermodynamically and quantum mechanically (although not necessarily in its details) the mirror image in time of an initial condition that leads to black holes must consequently lead to their time-reversed phenomena. If Hawking radiation is essential (for small mass), the black hole may disappear earlier than $t(a_{max})$, again before an horizon forms.

Hawking: The no-boundary condition can only be interpreted by means of semi-classical concepts such as the saddle point method.

Zeh: I would prefer to understand such a fundamental conclusion as the arrow of time in terms of the exact (even though incomplete) description. In particular, your opposite conclusion might be induced by the direction of computation (along the assumed orbits) by using approximations -- similar to how it is often wrongly argued in chaos theory in the form of 'growing' errors as an explanation of increasing entropy!

-- Did I understand you correctly during your talk that you -- at the time when you made what you call your 'mistake' -- also expected black holes to re-expand during the recollapse of the universe?

Hawking: Yes. I did not understand black holes sufficiently until I changed my mind.