Foundations of Quantum Entanglement

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Seminar on disentangling quantum matter with quantum information theory

Summary

Quantum entanglement is an experimentally verified and widely accepted property of nature. It describes a phenomenon that occurs when two or more particles are correlated in a way such that the quantum state of each particle can not be described independently. The quantum state needs to be described for the complete system.
Measurement results on those entangled systems are found to be correlated. In most cases entangled particles exhibit properties that do not comply with the concept of local realism. This can be shown by performing a Bell experiment.

I. Introduction

Quantum states can be written as

$$|\Psi\rangle = \sum_{\mathrm{ij}} c_{\mathrm{ij}} |i\rangle_{\mathrm{A}} |j\rangle_{\mathrm{B}} \tag{1}$$

In general the coefficient does not factorize. If it does we obtain a

- **Product State**

$$|\Psi\rangle = \sum_{\mathrm{ij}} a_{\mathrm{i}} |i\rangle_{\mathrm{A}} \otimes b_{\mathrm{j}} |j\rangle_{\mathrm{B}} \tag{2}$$

Systems A and B are not correlated!
If c_{ij} does not factorize we obtain an

- **Entangled State** e.g. the Spin Singlet

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}} \left(|\uparrow\rangle_{\mathrm{A}} |\downarrow\rangle_{\mathrm{B}} - |\downarrow\rangle_{\mathrm{A}} |\uparrow\rangle_{\mathrm{B}} \right) \tag{3}$$

Systems A and B are correlated and influence each other to some extent.

II. Bell test

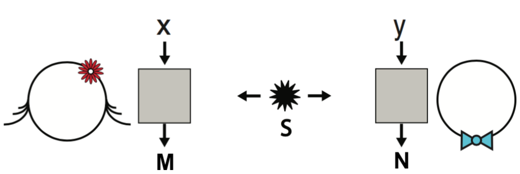

Figure 1: *Setup of a Bell experiment*

To explain correlations between entangled particles, *Einstein*, *Podolsky* and *Rosen* proposed to introduce local hidden variables.
John Bell proved that no physical theory of local hidden variables can ever reproduce all of the predictions of quantum mechanics.
By assuming **local realism** he derived

$$p(\mathrm{MN}|\mathrm{xy}) = \int \mathrm{d}\lambda \, q(\lambda) p(\mathrm{M}|\mathrm{x},\lambda) p(\mathrm{N}|\mathrm{y},\lambda) \tag{4}$$

to describe local correlations between particles.
If equation (4) holds, the *CHSH* inequality is upper bounded.

$$S = < M_1 N_1 > + < M_1 N_2 > + < M_2 N_1 > - < M_2 N_2 > \\ \leq 2$$

Bell showed that this **inequality is violated** for all pure maximally entangled quantum states, i.e. $S = 2\sqrt{2}$.
This means that the concepts of locality and realism do **not apply** for such quantum states! It remains unclear which of the two assumptions is violated.

Further Reading

[Brunner et al., 2014] Bell nonlocality
Rev. Mod. Phys. 86, 419 (2014), arXiv:1303.2849.

[Bell, 1964] On the Einstein Podolsky Rosen paradox
Physics I, p.195-200 (1964)