Exercise Sheet 1

Kastoryano: Quantum Error Correction

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1 Exercise 1: Classical Linear codes

Exercise 1.1 (from N&C) Show that the parity check matrix H and the generator matrix G for the same linear code satisfy $HG = 0$.

Exercise 1.2 (from N&C) Let H be a paritty check matrix such that any $d-1$ columns are linearly independent, but there exists a set of d linearly dependent columns. Show that the code defined by H has distance d.

Exercise 1.3 The Singleton bound: Show that an $[n, k, d]$ linear code must satisfy $n - k \geq d - 1$.

Exercise 1.4 The Gilbert-Varshamov bound: Show that for large n, there exists an $[n, k]$ linear error correcting code protecting against t bits for some k such that

$$\frac{k}{n} \geq 1 - H(\frac{2t}{n}), \tag{1}$$

where $H(x) = -x \log(x) - (1 - x) \log(1 - x)$ is the binary entropy function.

2 Exercise 2: Threshold for the repetition code

Show that the repetition code has an error correcting threshold at $p = 1/2$.

    Hint: first show that the logical failure rate $P_f(n, p)$ for n bits at physical error rate $0 \leq p \leq 1/2$ is given by

$$P_f(n, p) = \sum_{\ell = \lfloor n/2 \rfloor + 1}^{n} \binom{n}{l} (1 - p)^{n - \ell} p^{\ell} \tag{2}$$