Exercise Sheet 4

Kastoryano: Quantum Error Correction

November 6, 2018

1 Exercise 1: The 7 qubit code.

Consider the 7 qubit code defined by the following 6 stabilizer operators:

$$S_1 = Z_1Z_3Z_5Z_7 \qquad S_4 = X_1X_3X_5X_7 \tag{1}$$
$$S_2 = Z_2Z_3Z_6Z_7 \qquad S_5 = X_2X_3X_6X_7 \tag{2}$$
$$S_3 = Z_4Z_5Z_6Z_7 \qquad S_6 = X_4X_5X_6X_7 \tag{3}$$

Show that the 7 qubit code defined by the stabilizer operators $\{S_1, \cdots, S_6\}$ protects against arbitrary single qubit phase (Z) or flip (X) errors.

2 Exercise 2: The 5 qubit code.

Consider the 5 qubit code defined by the operators

$$S_1 = X_1Z_2Z_3X_4 \tag{4}$$
$$S_2 = X_2Z_3Z_4X_5 \tag{5}$$
$$S_3 = X_1X_3Z_4Z_5 \tag{6}$$
$$S_4 = Z_1X_2X_4Z_5 \tag{7}$$

Exercise 1.1 Argue that the stabiliser operators are all mutually commuting.

Exercise 1.2 Show that $\bar{X} = X_1X_2X_3X_4X_5$ and $\bar{Z} = Z_1Z_2Z_3Z_4Z_5$ are logical operators of the code. Show that the distance of the code is at most 3.

Exercise 1.3 Show that the 5 qubit code protects against arbitrary single qubit flip or phase error (use may use Knill-Laflamme).