Exercise Sheet 7

Kastoryano: Quantum Error Correction

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1 Exercise 1: Commuting projector codes

Let $\mathcal{C}$ be a local commuting projector code on a finite two dimensional lattice Λ. I.e. there exist commuting projectors $\{P_j\}$ such that $[P_j, P_k] = 0$ and $P_J^2 = P_j$ for all $j, k \in \Lambda$ and each P_j acts within a finite ball around site j of radius r. Then, by definition, for all $|\varphi\rangle \in \mathcal{C}$, $P_j|\varphi\rangle = |\varphi\rangle$. Although this definition is very similar to that of stabilizers, we here do not require that the P_j's are products of Pauli operators.

Exercise 1.1: Show that if for any orthogonal $\varphi_j, \varphi_j \in \mathcal{C}$, $\langle\varphi_j|O|\varphi_k\rangle = 0$ and $\langle\varphi_j|O|\varphi_j\rangle = \langle\varphi_k|O|\varphi_k\rangle$ for all j, k, then

$$P_{\mathcal{C}} O P_{\mathcal{C}} = c(O) P_{\mathcal{C}} \tag{1}$$

Exercise 1.2: Suppose that for a state $\rho \in \mathcal{C}$, region A is correctible, and region B is correctible, and the distance between regions A and B is larger than r. Show that $A \cup B$ is correctible. (Hint: use Eqn. (1) as your definition of correctability).

Exercise 1.3: In class we showed that for a commuting projector code $[[n, k, d]]$ that is local on a 2D lattice, the bound $\frac{kd^2}{n} \leq c$ holds for some constant c and any system size n. Show that on a D-dimensional lattice, the bound $\frac{kd^{2/(D-1)}}{n} \leq c$ holds for some constant c.

NOTE: do not re-derive Lemmas 1,2,3 or Fact 2, just use them to show the theorem.