Exercise Sheet 8

Kastoryano: Quantum Error Correction

December 12, 2018

1 Exercise 1: The Davies Master equation

Consider the derivation of the Davies master equation as outlined in Chapter 3.3 of "The theory of open quantum systems" by Breuer and Petruccione (chapter available on the course website). Let the system Hamiltonian be the Classical 1D Ising model with periodic boundary conditions:

$$H_S = \sum_{j=1}^{N} Z_j Z_{j+1} + h Z_j, \tag{1}$$

where $Z_{N+1} \equiv Z_1$. Consider the interaction Hamiltonian between the System and Environment to be

$$H_I = \sum_{j=1}^{N} \sum_{\alpha_j} \sigma_j^\alpha \otimes B_j^\alpha, \tag{2}$$

where $\sigma_j^\alpha \in \{X_j, Y_j, Z_j\}$ are the single site Pauli operators at site j, and B_{α_j} are operators on the environment.

The master equation is given explicitly in diagonal form by

$$\dot{\rho}_S = \mathcal{L}(\rho) = \sum_{\omega} \sum_{j=1}^{N} \sum_{\alpha_j} \gamma_{\alpha_j}(\omega) \mathcal{D}[A_{\alpha_j}(\omega)](\rho_S), \tag{3}$$

where

$$\mathcal{D}[X](\rho) = X\rho X^\dagger - \frac{1}{2}(X^\dagger X \rho + \rho X^\dagger X), \tag{4}$$

and the operators $A_{\alpha_j}(\omega)$ are defined by the equation

$$e^{-itH_S} \sigma_j^\alpha e^{itH_S} = \sum_{\omega} e^{-it\omega} A_{\alpha_j}(\omega). \tag{5}$$

We will not write the coefficients $\gamma_{\alpha_j}(\omega)$ out explicitly, but note that they satisfy $\gamma_{\alpha_j}(\omega) = e^{\beta\omega}\gamma_{\alpha_j}(-\omega)$.

Exercise 1.1 Calculate the $A_{\alpha_j}(\omega)$ explicitly for the X_j, Y_j, Z_j Paulis at a given site j. [HINT: eliminate all of the terms on the RHS of Eqn. (5) that cancel out, and expand the exponential of the remaining terms: $e^{itZ_jZ_{j+1}} = \cos(t)1 + i\sin(t)Z_jZ_{j+1}$] How many distinct values of ω are there?

Exercise 1.2 Show that $e^{-\beta H_S} A_{\alpha_j}(\omega) = e^{\beta\omega} A_{\alpha_j}(\omega) e^{-\beta H_S}$.

Exercise 1.3 Show that $\mathcal{L}(e^{-\beta H_S}) = 0$.