The Niels Bohr International Academy

UNIVERSITY OF COPENHAGEN

DISSIPATIVE QUANTUM WALKS AND TOPOLOGY

M. J. Kastoryano and M. Rudner

New Trends in Strongly Entangled Many-Body Systems
UCL, November 2015

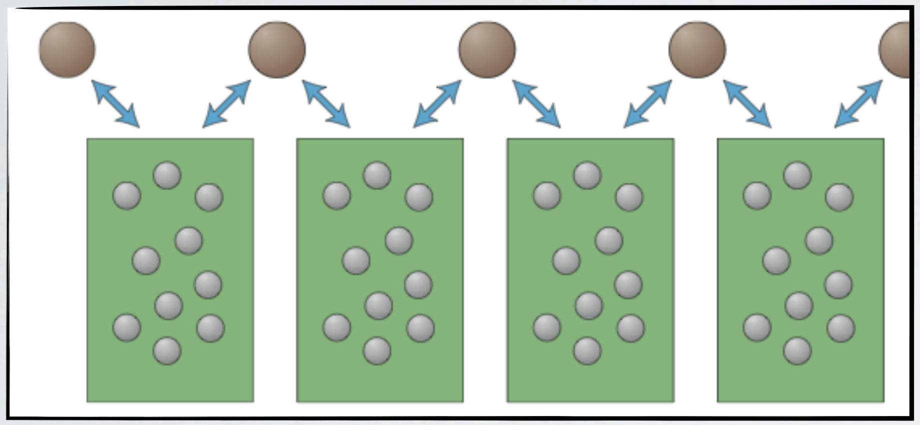
VILLUM FONDEN

CARLSBERGFONDET

TOPOLOGY AND DISSIPATION

Dissipative engineering

Computation



F. Verstraete et al.,Nat. phys. 5.9 (2009): 633-636.

MJK et. al., Phys. Rev. Lett. 110, 110501

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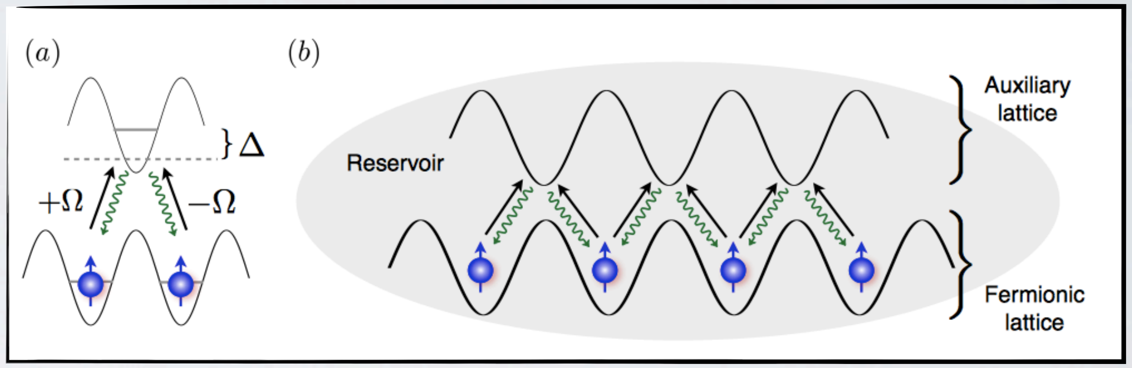
State engineering



S. Diehl *et. al.*, Nat. Phys. 7 (2011), pp. 971–977.

S. Diehl *et. al.*, Nat. Phys. 4.11 (2008): 878-883

S. Diehl *et. al.*, PRL 105.1 (2010): 015702.

MJK *et. al.*, PRL 106.9 (2011): 090502

H. Krauter *et. al.*, PRL 107 (2011): 080503

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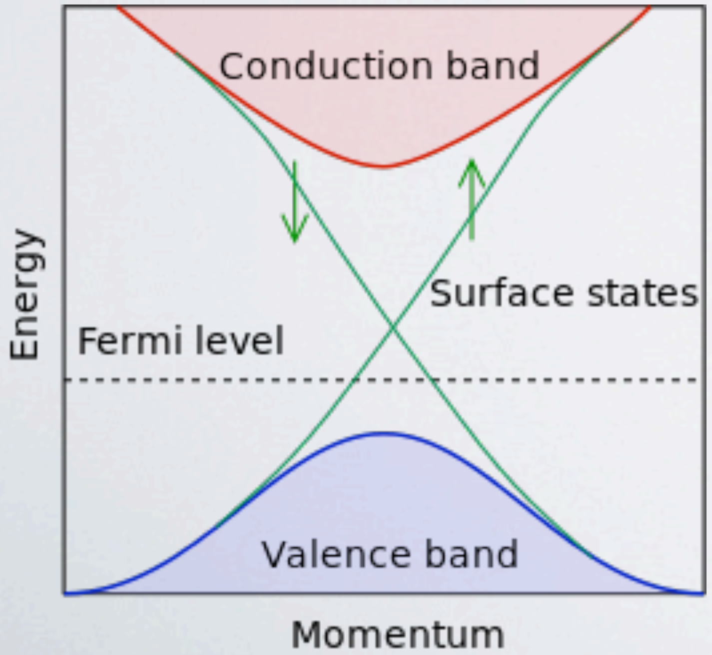
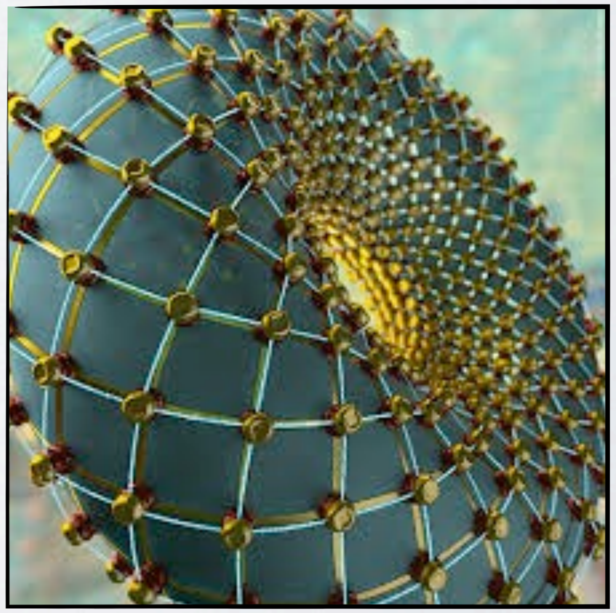
Topological systems

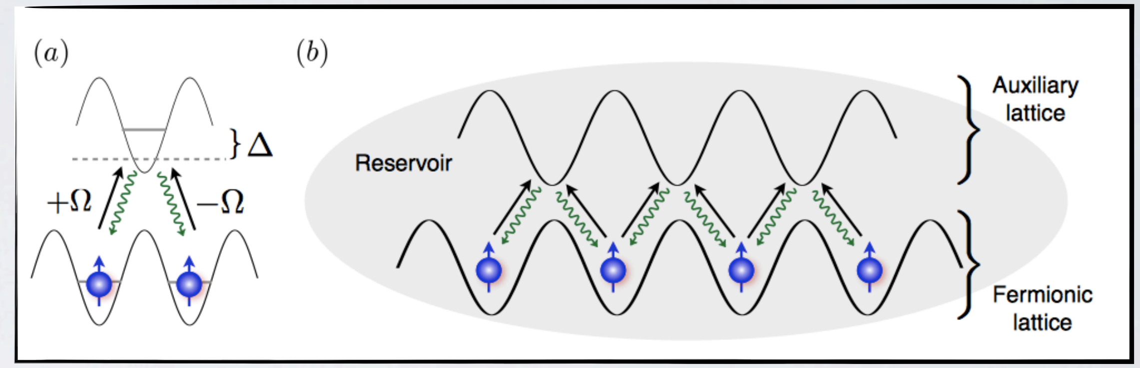




TOPOLOGY AND DISSIPATION

Dissipative engineering

State engineering



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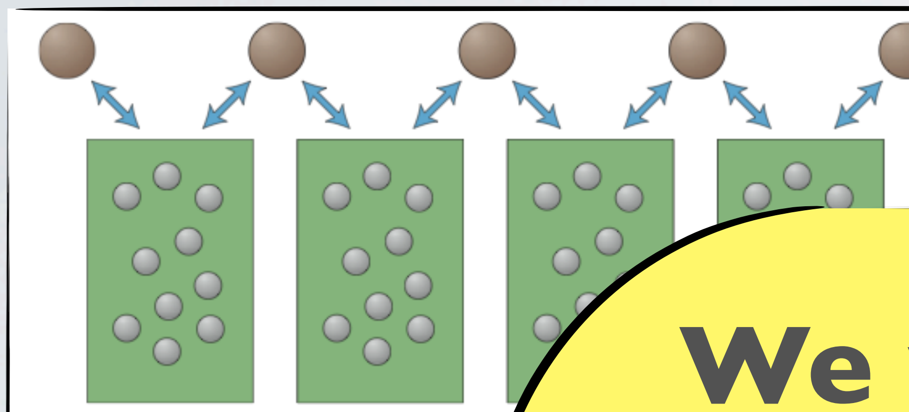

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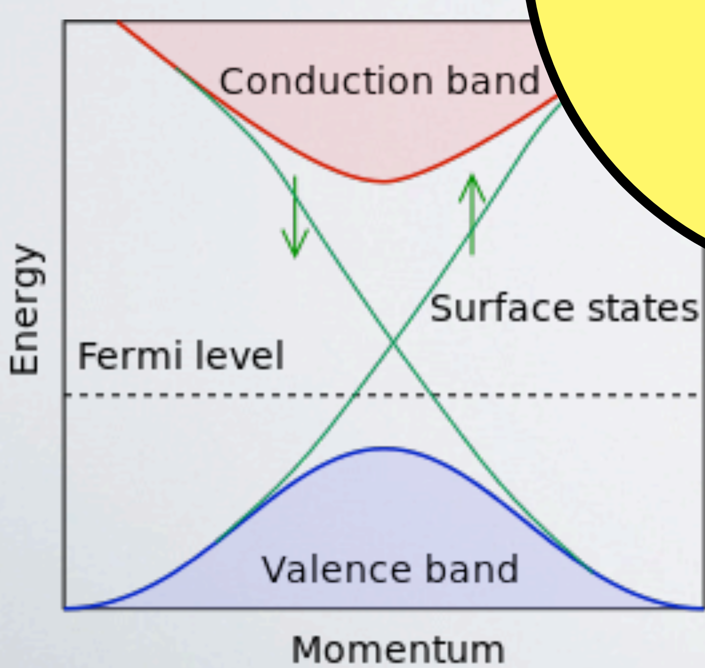
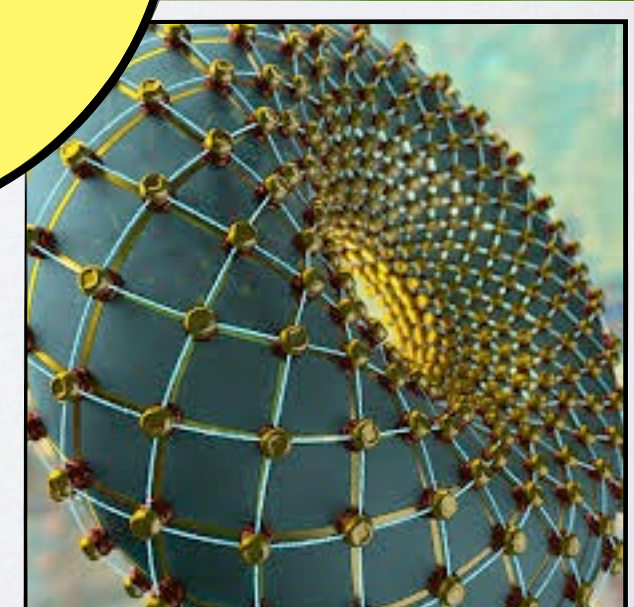




We want to find genuinely new open system behavior

DISSIPATIVE SYSTEMS

Master equation

$$\mathcal{L}(\rho) = i[H,\rho] + \sum_m L_m \rho L_m^\dagger - \frac{1}{2}\{L_m^\dagger L_m, \rho\}_+$$

Stationary states

$$\mathcal{L}(\rho_{\rm ss}) = 0$$

Is the stationary state unique?

Dark states

$$L_m|\psi\rangle = 0 \quad H|\psi\rangle = \lambda|\psi\rangle \qquad \text{all } m$$

Spectral properties

Eigenvalues of $\mathcal{L}$ have non-positive real part.

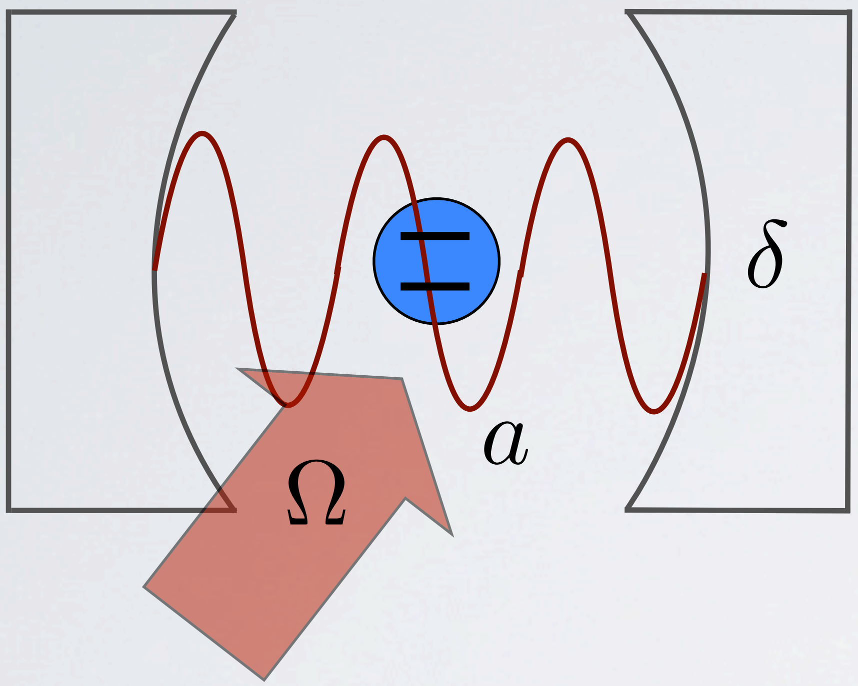
Zero eigenvalue corresponds to the stationary states

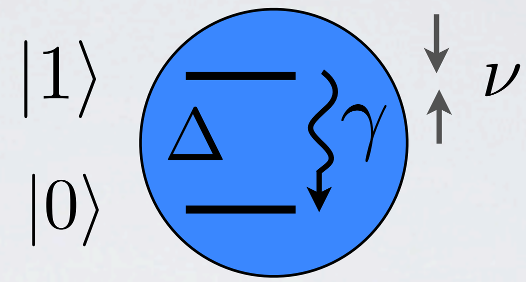
Relaxation rate is related to the inverse of the gap of $\mathcal{L}$.

Gap of $\mathcal{L}$: minimum real part of an eigenvalue of $\mathcal{L}$.

MOTIVATING EXAMPLE

Cavity QED





set $-\nu = \delta$

$$H_{JC} = \frac{1}{2}(\Delta - \nu)\sigma^z + (g\sigma^+ a + \Omega\sigma^+ + h.c.)$$

$$\dot{\rho} = i[H_{JC}, \rho] + \gamma(\sigma^- \rho \sigma^+ - \frac{1}{2}\{\sigma^+\sigma^-, \rho\}_+)$$

Assume no cavity decay!

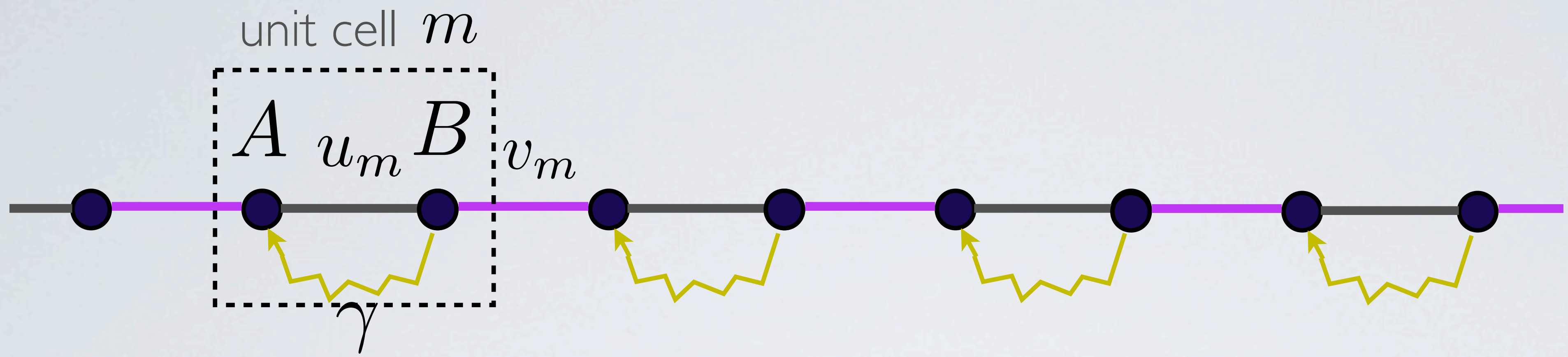
If $\Omega > g$ stationary state is $$|\psi_{\mathrm{ss}}\rangle = e^{-(\Omega/g)^2} \sum_{j=0}^{\infty} \frac{(-\Omega/g)^j}{\sqrt{j}} |0, j\rangle$$ coherent state

independent of (Δ, γ, ν).

always pure!

Bound topological edge state

THE MODEL



periodic bc's

$$H = \sum_m u_m |A,m\rangle\langle B,m| + v_m |A,m+1\rangle\langle B,m| + h.c.$$

$$\mathcal{L}(\rho) = i[H,\rho] + \sum_m L_m \rho L_m^\dagger - \frac{1}{2}\{L_m^\dagger L_m, \rho\}_+$$

Lindblad (jump) operators: $L_m = \sqrt{\gamma}|A,m\rangle\langle B,m|$

Properties:

unique steady state

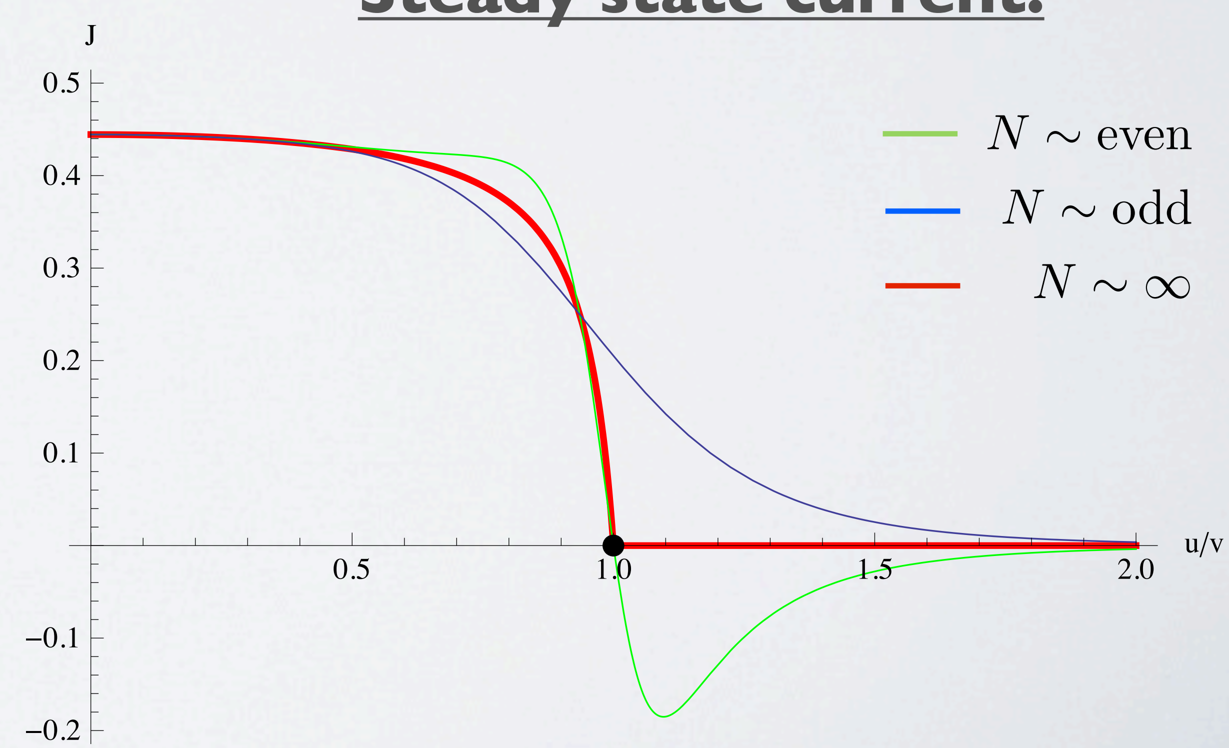
can be extracted analytically

pure (dark) steady state iff u=v

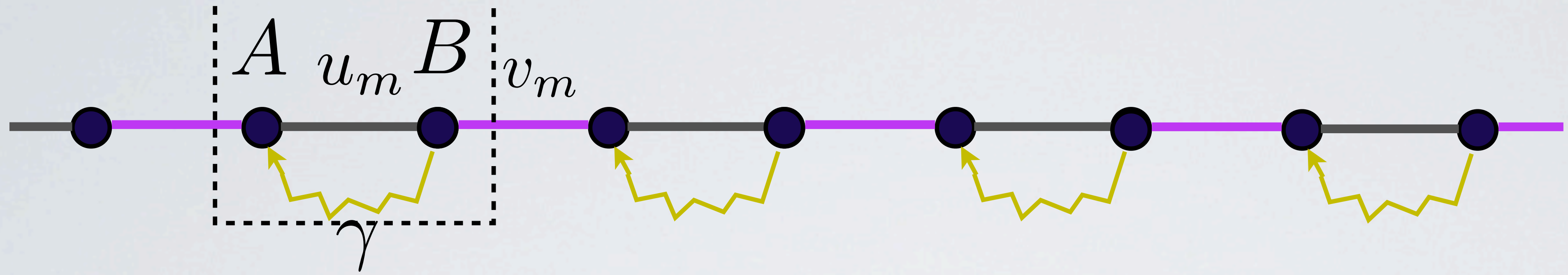
Current: $J = 2i\mathrm{tr}[\frac{dH_s}{ds}\rho]$ $\quad v \to e^{is}v$

in general currents in open systems are ambiguous, as there are usually no conservation laws

Steady state current:

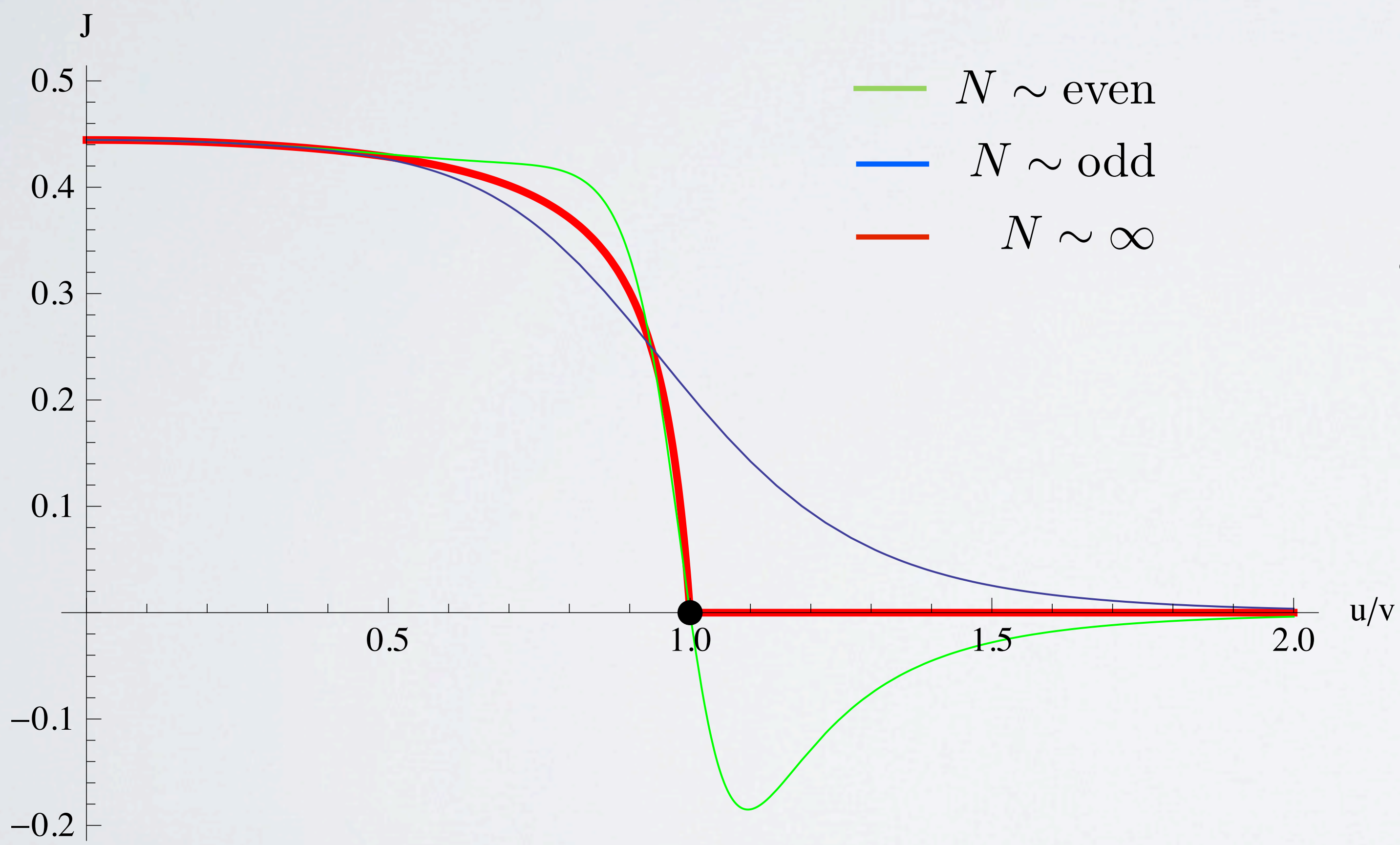


THE CURRENT



periodic bc's

Steady state current:



Current

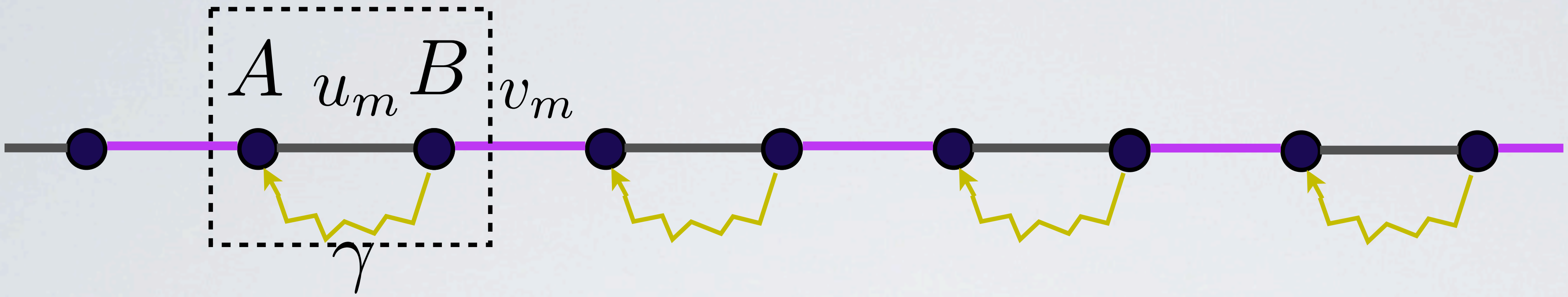
$$J_L = \frac{4v^{L+1}(u^2 - v^2)\gamma}{8(u^2 - v^2)(u^L - v^L) + (u^L + v^L)(\gamma^2)}$$

strictly zero for u>v

transition is independent of dissipative strength and on-site energy!

What is the origin of this sharp transition?

THE CURRENT



periodic bc's

Translation invariance, steady state is diagonal in the momentum basis

$$\rho^k_{\text{ss}} = \frac{1}{Z}\begin{pmatrix} 1+\frac{\gamma^2}{4|c_k|^2} & \frac{i\gamma}{2\bar{c}_k} \\ \frac{-i\gamma}{2c_k} & 1 \end{pmatrix} \qquad c_k = u + e^{ik}v$$

Can this form be derived in general?

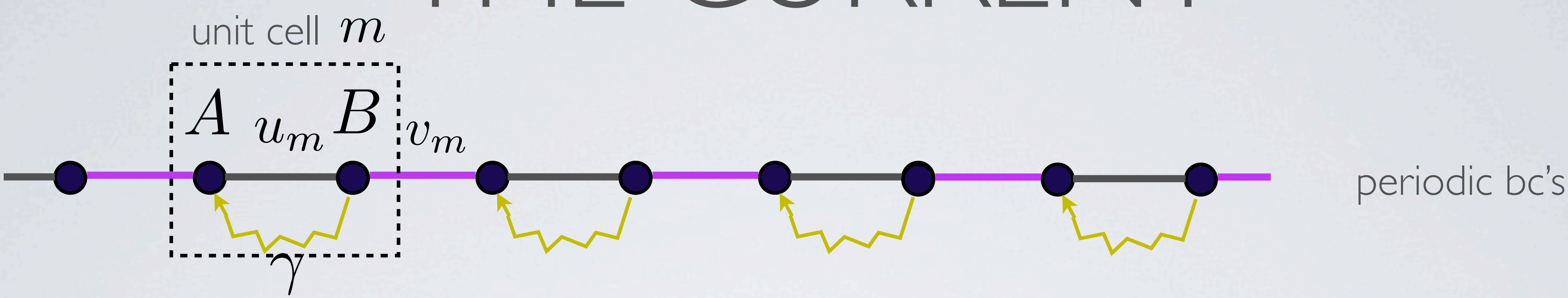
k independence of B site population

Steady state is reached when probability of entering and exiting momentum k shell is equal.

only depends on B population, therefore constant for all k.

Steady state can generally be extracted from momentum conserving master equation!

THE CURRENT



replace $L_m = \sqrt{\gamma}|A,m\rangle\langle B,m|$

with $L_k = \sqrt{\gamma}|A,k\rangle\langle B,k|$

$$\mathcal{L}(\rho) = i[H,\rho] + \sum_k L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\}_+$$

$$H_k = \begin{pmatrix} 0 & c_k \\ \bar{c}_k & 0 \end{pmatrix} \qquad c_k = u + e^{ik} v$$

$$\boxed{\rho_{\mathrm{ss}}^k = \frac{1}{Z}\begin{pmatrix} 1 + \frac{\gamma^2}{4|c_k|^2} & \frac{i\gamma}{2\bar{c}_k} \\ \frac{-i\gamma}{2c_k} & 1 \end{pmatrix}}$$

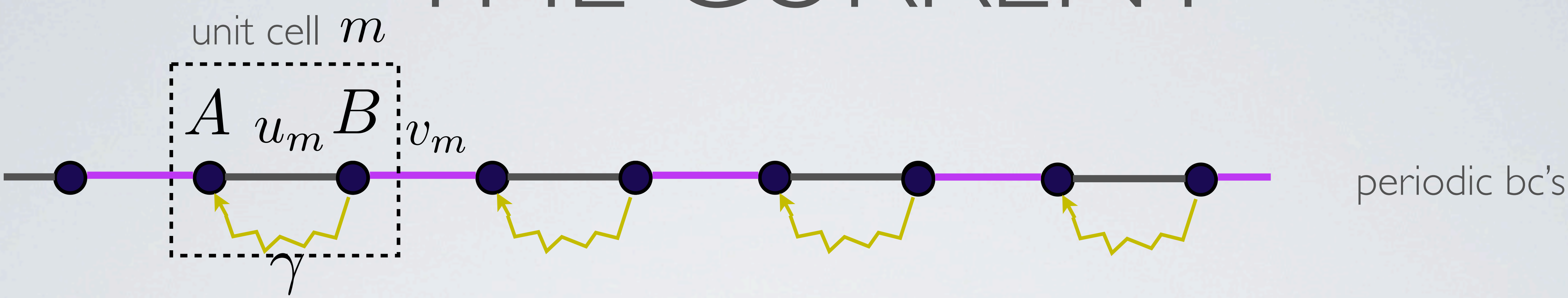
Reduced to solving a 2x2 matrix equation.

We obtain a solution for each momentum k.

Steady state of original system is the convex combination that are equal B-site population.

Solution is general!

THE CURRENT



$$H_k = \begin{pmatrix} 0 & c_k \\ \bar{c}_k & 0 \end{pmatrix} \qquad \rho^k_{\rm ss} = \frac{1}{Z}\begin{pmatrix} 1+\frac{\gamma^2}{4|c_k|^2} & \frac{i\gamma}{2\bar{c}_k} \\ \frac{-i\gamma}{2c_k} & 1 \end{pmatrix}$$

Steady-state current:

$$J_{\rm ss} = 2i\int dk \ {\rm tr}[\frac{dH_k}{dk}\rho_{\rm ss}]$$

$$= 2i\int dk \ {\rm tr}\left[\begin{pmatrix} 0 & \frac{dc_k}{dk} \\ \frac{d\bar{c}_k}{dk} & 0 \end{pmatrix}\begin{pmatrix} * & \frac{i\gamma}{2\bar{c}_k} \\ \frac{-i\gamma}{c_k} & 1 \end{pmatrix}\right]$$

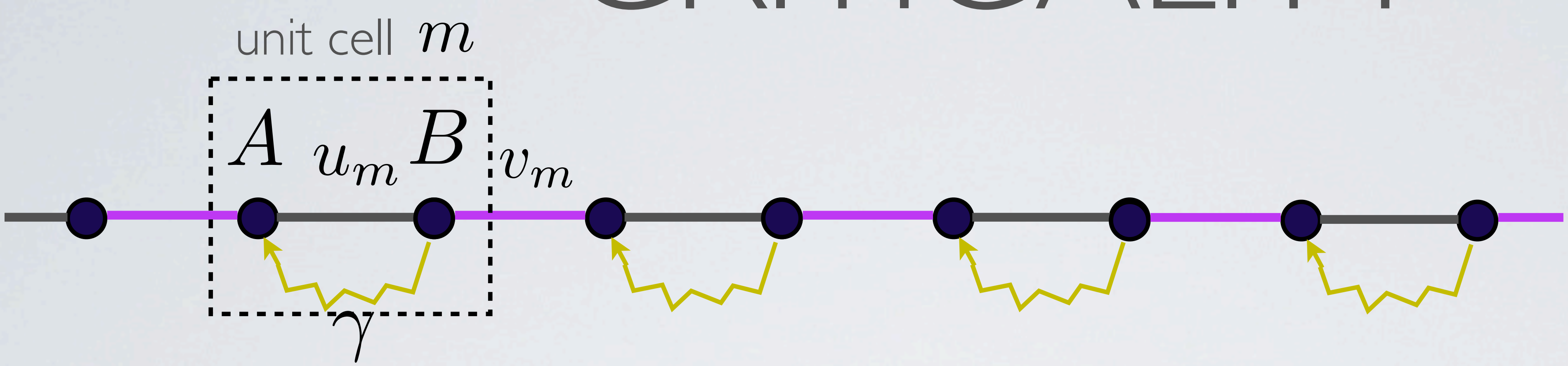
$$= \frac{2\gamma}{Z}\int dk \ {\rm Im}[\frac{d}{dk}\log c_k]$$

Winding number!

Discontinuity in the current is topological!

What about the pre-factor?

CRITICALITY



periodic bc's

Time to equilibrium: # of jumps required to reach equilibrium times the probability for a jump to occur.

$$J_{\text{ss}} = \frac{2\gamma}{Z}\int dk \; \text{Im}[\frac{d}{dk}\log c_k]$$

Probability of a jump = population at B sites.

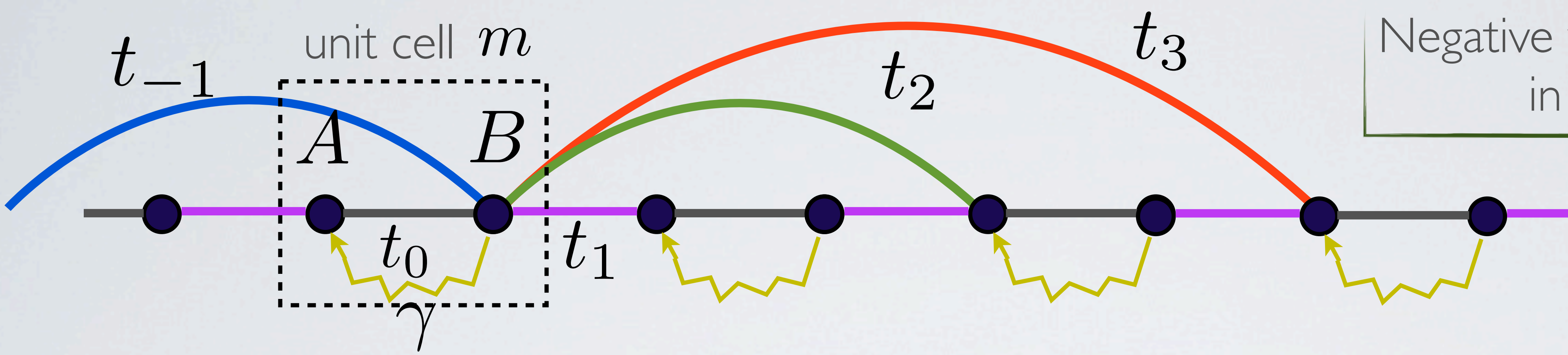
$1/Z$ gives the probability for a jump to occur

$$\rho^k_{\text{ss}} = \frac{1}{Z}\begin{pmatrix} 1+\frac{\gamma^2}{4|c_k|^2} & \frac{i\gamma}{2\bar{c}_k} \\ \frac{-i\gamma}{2c_k} & 1 \end{pmatrix}$$

$$\boxed{J_{\text{ss}} = \frac{\nu}{\tau}}$$

τ is the average time between jumps

GENERALITY



Negative winding number = current in opposite direction.

periodic bc's

$$\boxed{J_{\rm ss} = \frac{\nu}{\tau}}$$

$$\nu = \int dk \; \mathrm{Im}[\frac{d}{dk} \log c_k]$$

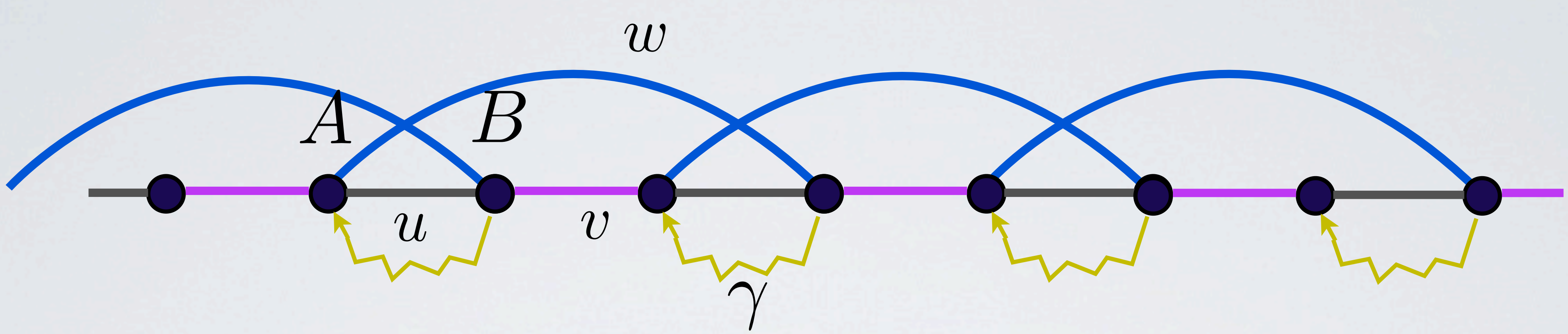
$$c_k = \sum_l t_l e^{ilk}$$

$$\tau = \frac{1}{2\gamma} \int dk \; (2 + \frac{\gamma^2}{4|c_k|^2})$$

Is this really topology?

τ diverges quadratically at the transition.

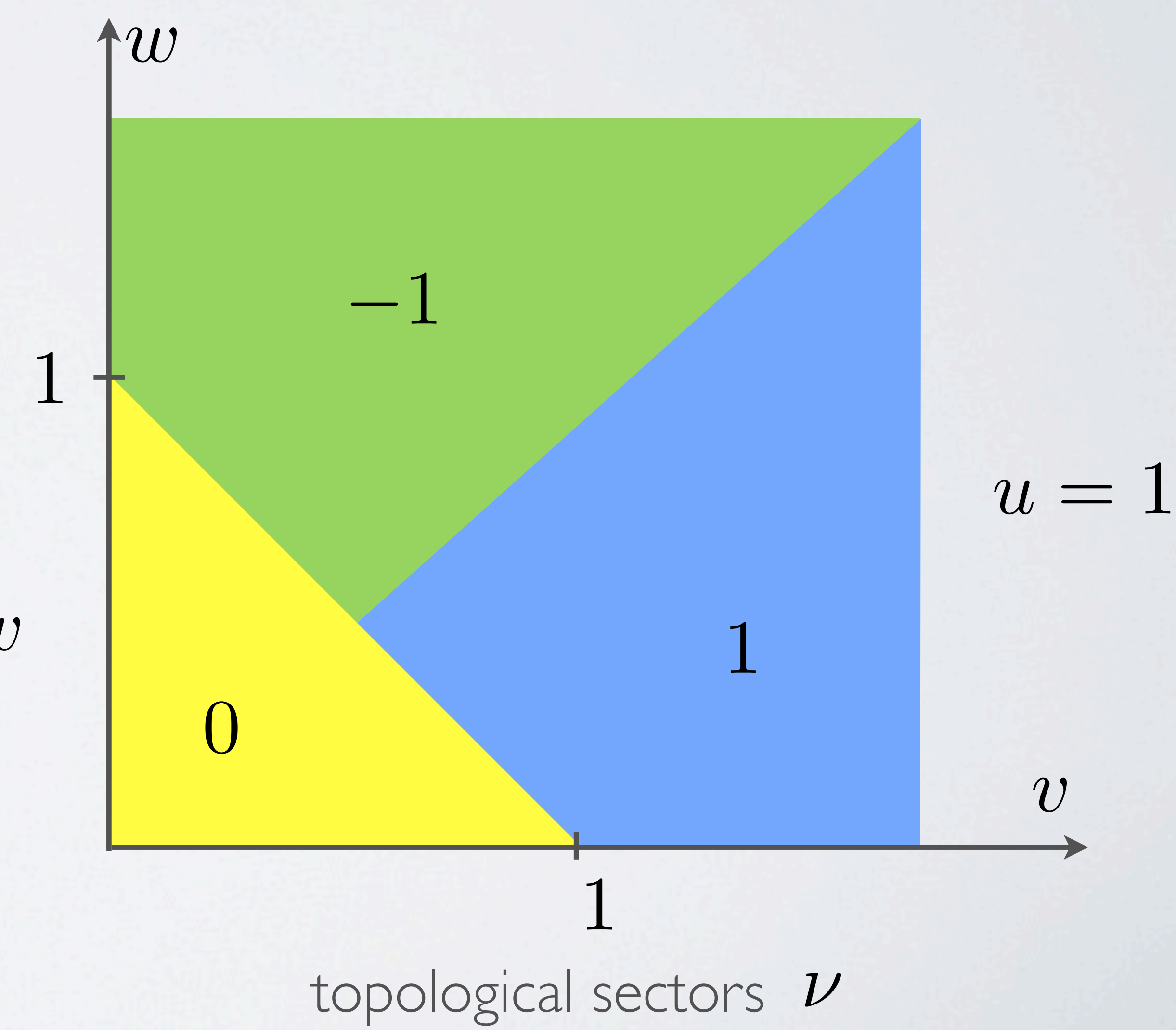
NEXT NEAREST NEIGHBOR



$$c_k = we^{-ik} + u + ve^{ik}$$

$$\nu = \begin{cases} 0, & \text{if } u > v + w \\ -1, & \text{if } u < v + w \text{ and } v > w \\ 1, & \text{if } u < v + w \text{ and } v < w \end{cases},$$

$$\tau \sim \begin{cases} 1/|u^2 - (v+w)^2|, & \text{around } u = v + w \\ 1/|v^2 - w^2|, & \text{around } v = w \end{cases}$$



EDGE STATES

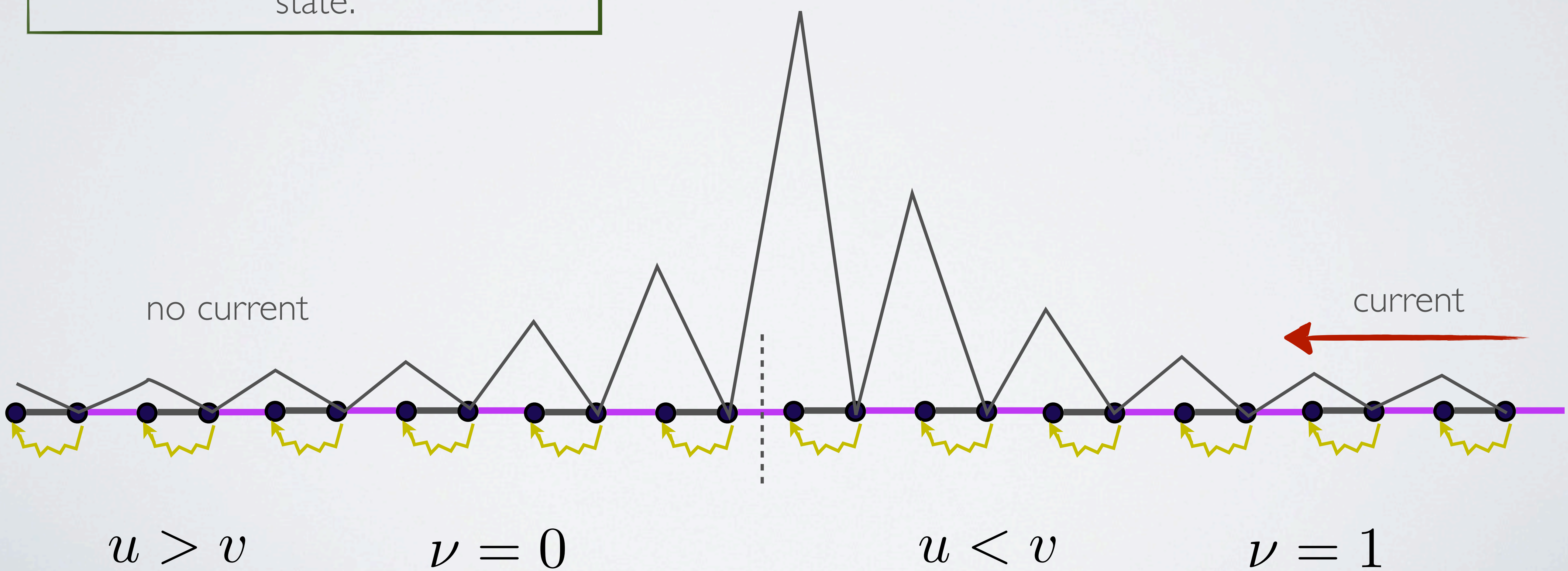
Back to nearest neighbor chain

Recall, in TI system, stationary state dark iff u=v.

As particle crosses between the two sectors, it gets caught in a dark state.

Boundary acts as a sink!

Stationary state must be dark!



EDGE STATES

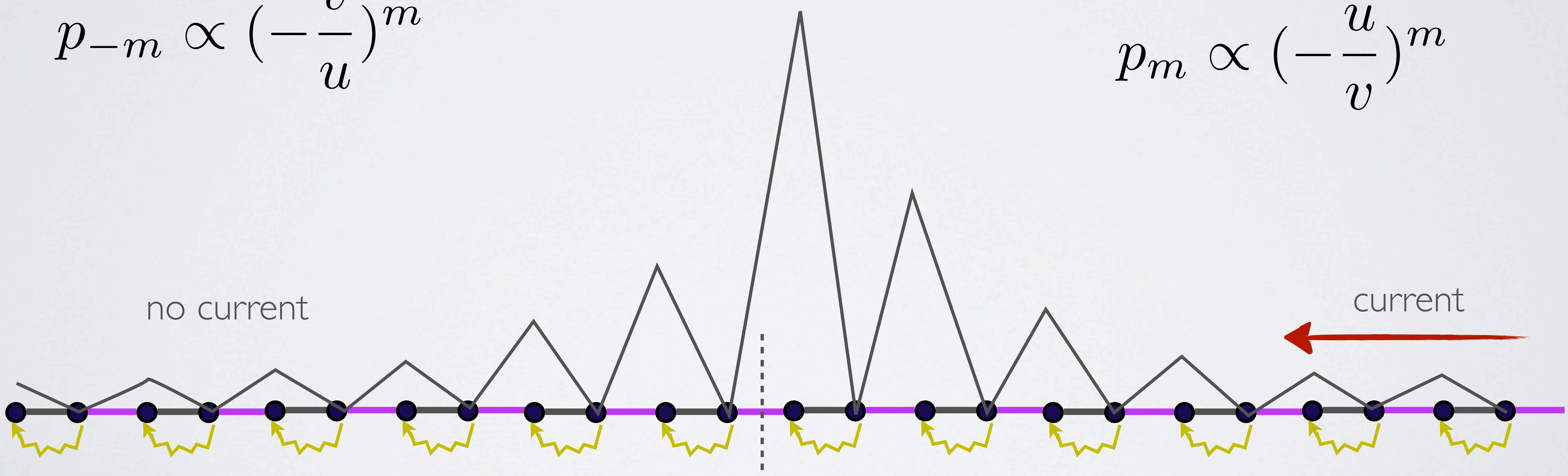
Assume: $|\psi_{\text{ss}}\rangle = \sum_m p_m |A, m\rangle$

$$H = \sum_m u_m |A, m\rangle\langle B, m| + v_m |A, m+1\rangle\langle B, m| + h.c.$$

$$H|\psi_{\text{ss}}\rangle = 0 \Rightarrow \quad p_m u_m + p_{m+1} v_m = 0$$

$$p_{-m} \propto (-\frac{v}{u})^m$$

$$p_m \propto (-\frac{u}{v})^m$$



$u > v$ $\quad \nu = 0$ $\quad u < v$ $\quad \nu = 1$

EDGE STATES

Assume: $|\psi_{\rm ss}\rangle = \sum_m p_m |A, m\rangle$

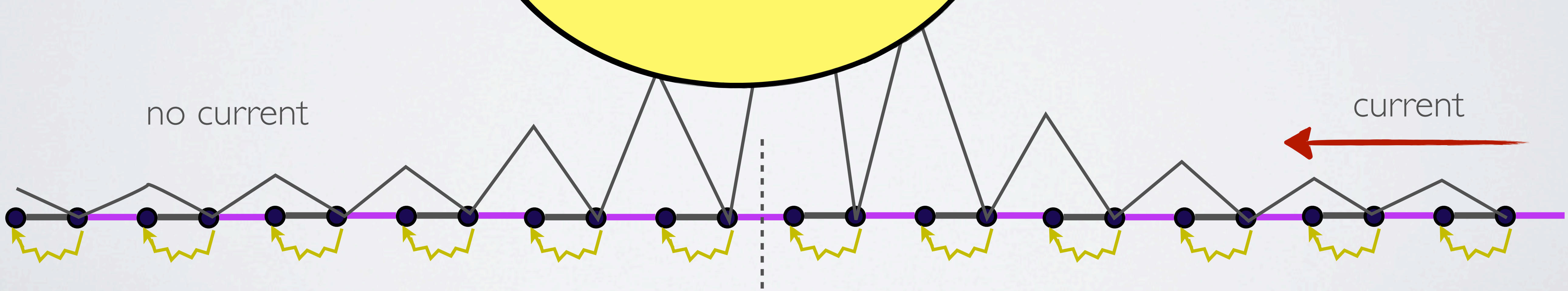
$$H = \sum_m u_m |A,m\rangle\langle B,m| + v_m |A,m+1\rangle\langle B,m| + h.c.$$

$$H|\psi_{\rm ss}\rangle = 0 \Rightarrow \quad p_m u_m + p_{m+1} v_m = 0$$

$$p_{-m} \propto \left(-\frac{v}{u}\right)^m$$

General phenomenon?

$$p_m \propto \left(-\frac{u}{v}\right)^m$$



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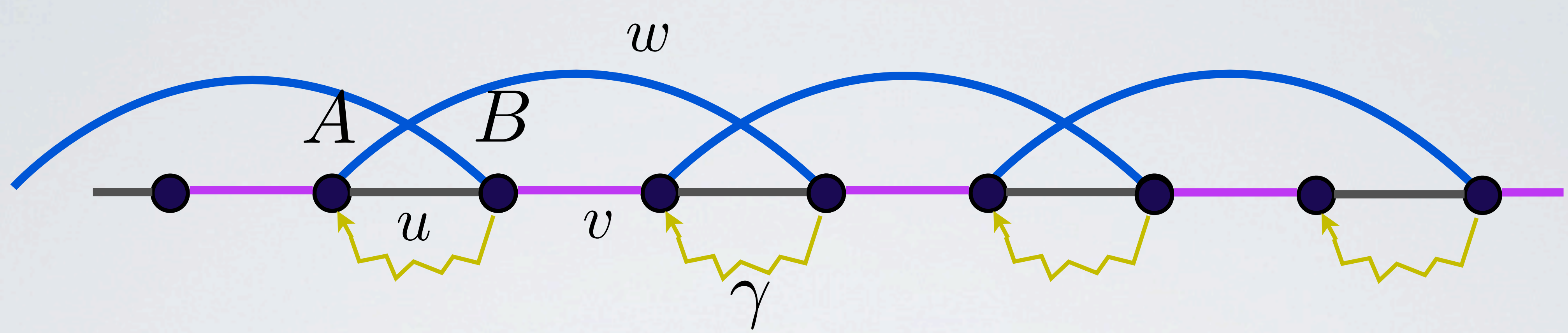

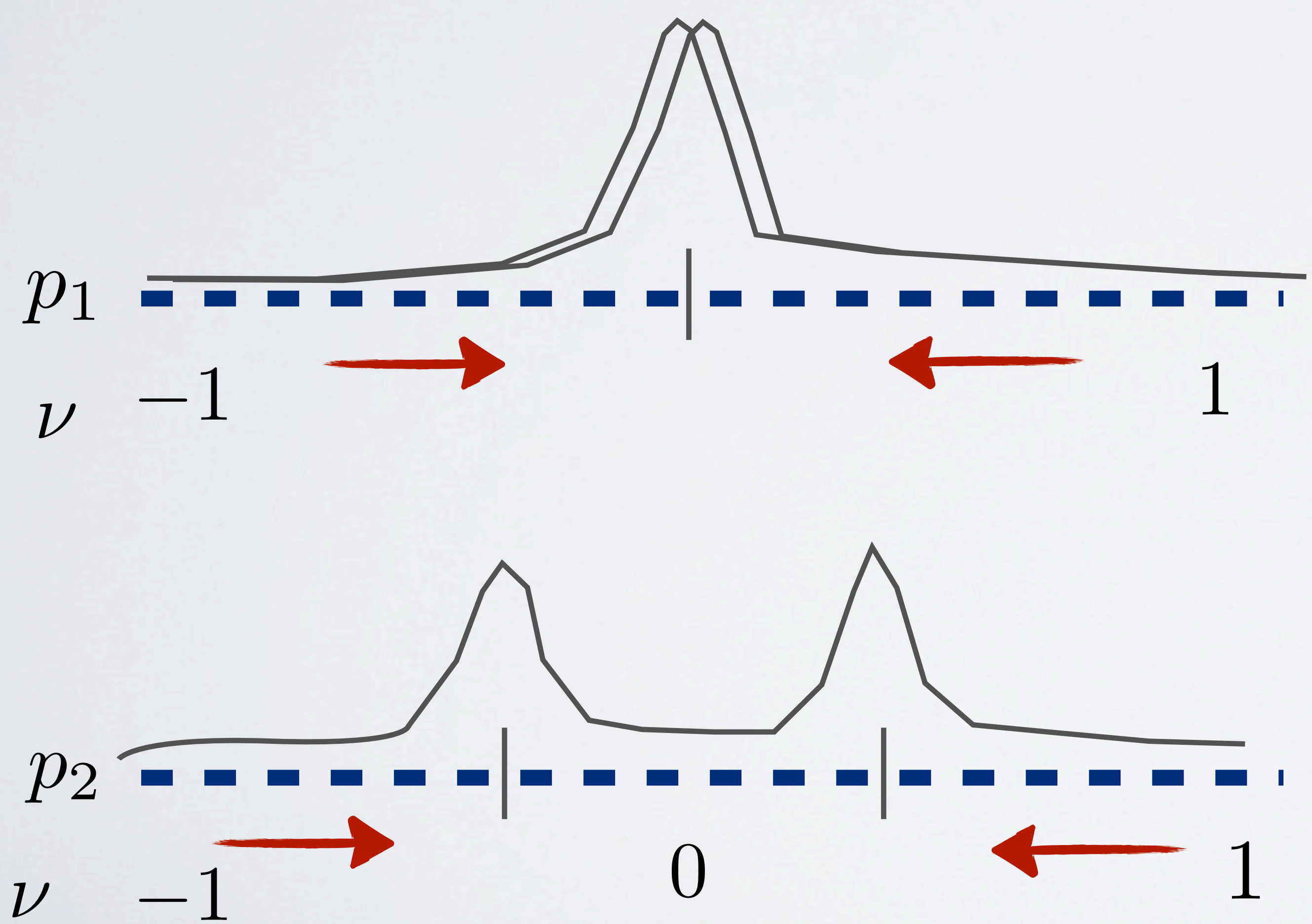

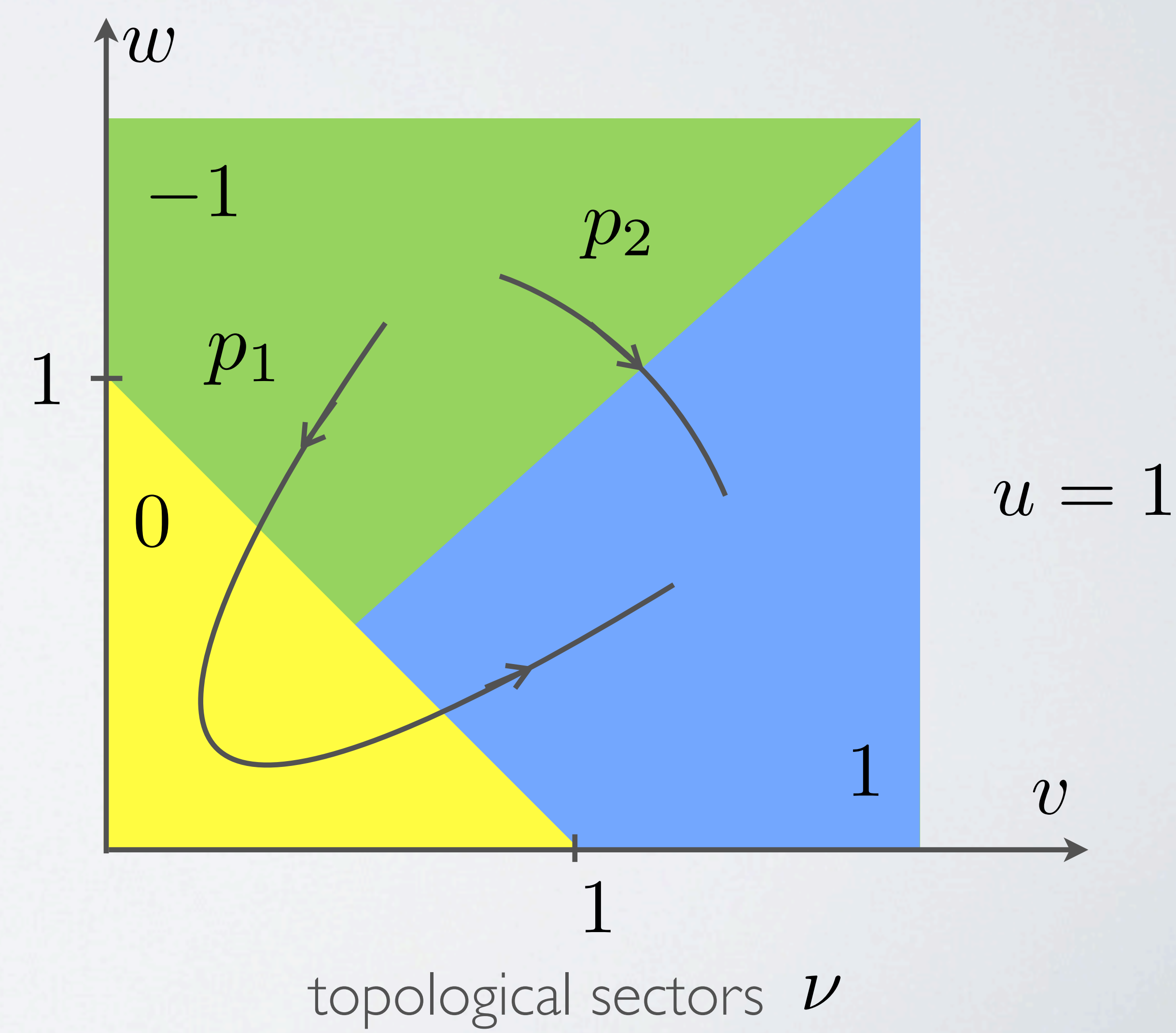

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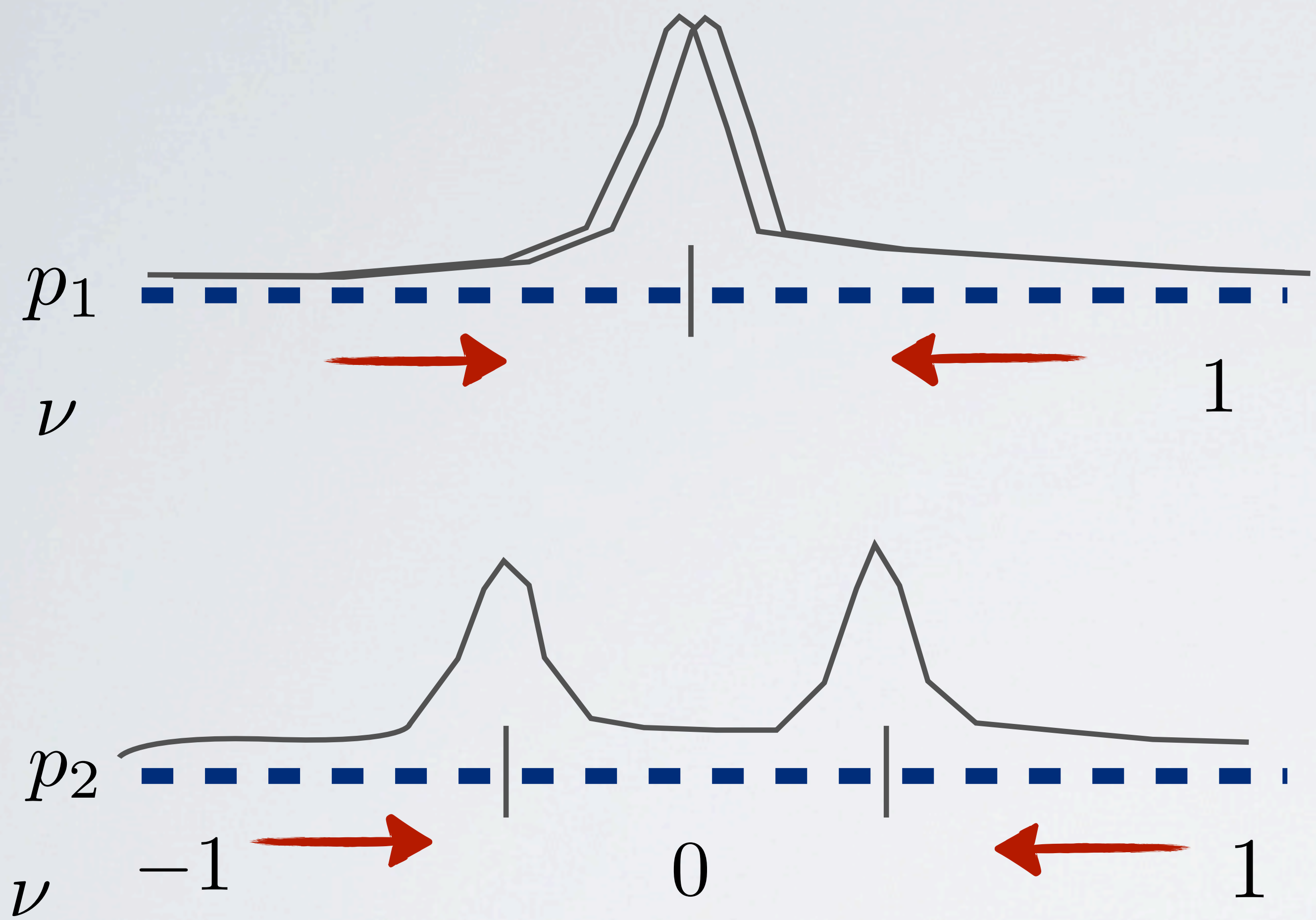
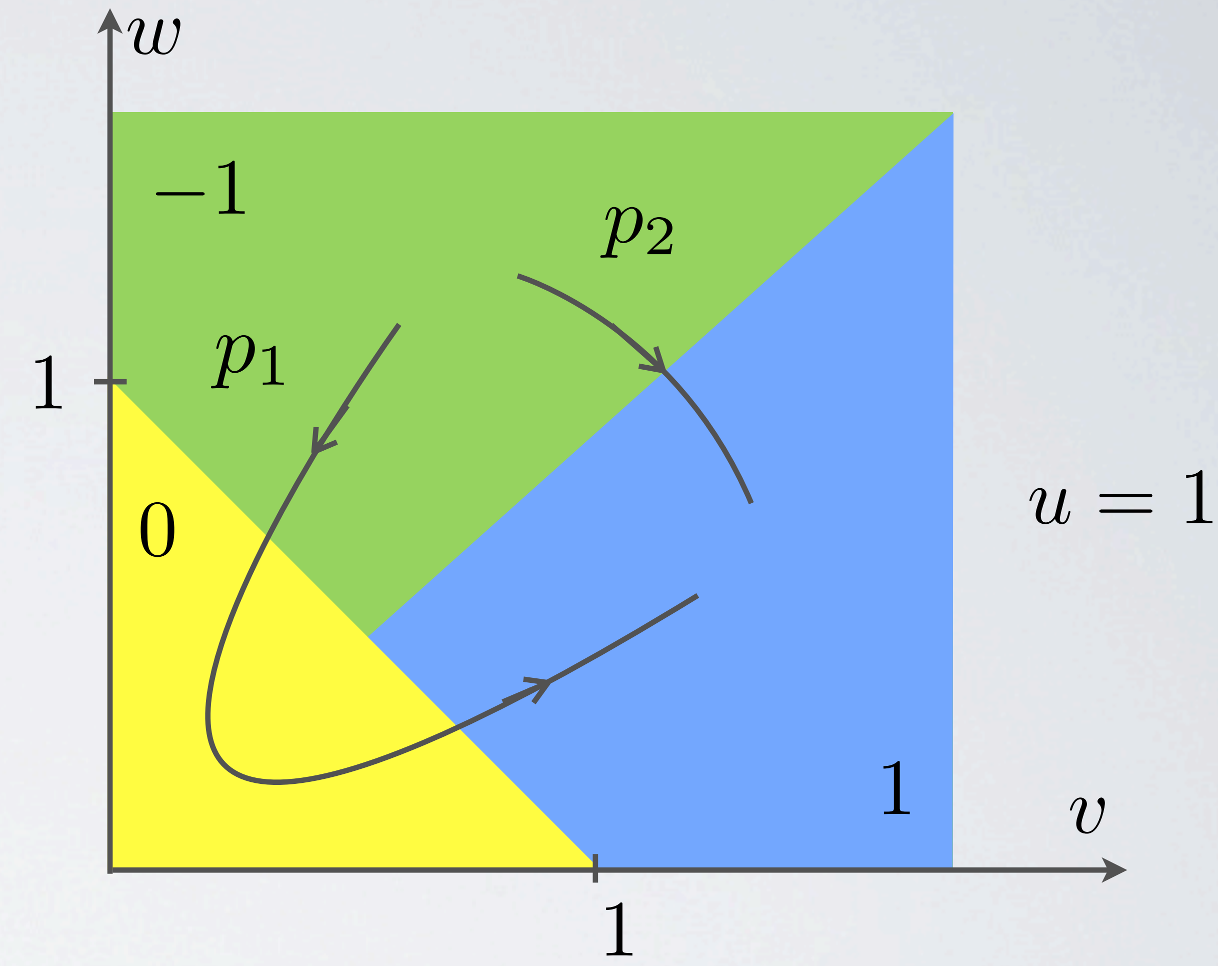




topological sectors ν

Each boundary can support a pure edge state!

What are the general rules for the bulk edge correspondence?

But all currents have to end in an edge!

How is it actually related to the topology?

TRANSFER MATRIX

General model

$$H = \sum_m \sum_{l=-L}^{L} t_l |A, m\rangle\langle B, m+l| + h.c.$$

Assume dark stationary states

$$|\psi_{ss}\rangle = \sum_m p_m |A, m\rangle$$

Recurrence relation

$$\sum_{l=-L}^{L} t_l \; p_{m+l} = 0$$

Transfer Matrix

$$T \begin{pmatrix} p_{m+L-1} \\ \dots \\ p_{m-L} \end{pmatrix} = \begin{pmatrix} p_{m+L} \\ \dots \\ p_{m-L+1} \end{pmatrix}$$

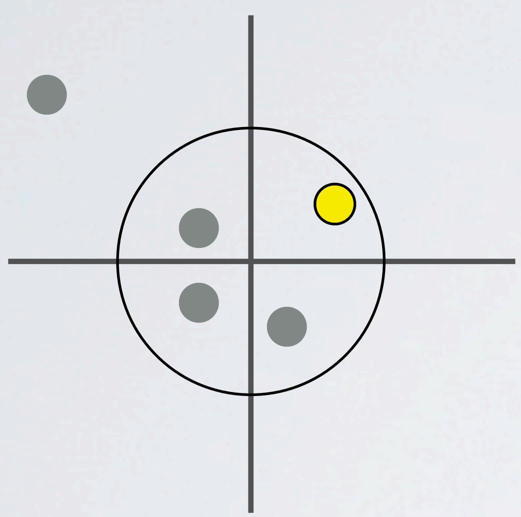
Physical states correspond to eigenvectors with eigenvalues of magnitude less than one.

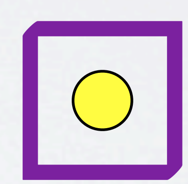
TRANSFER MATRIX

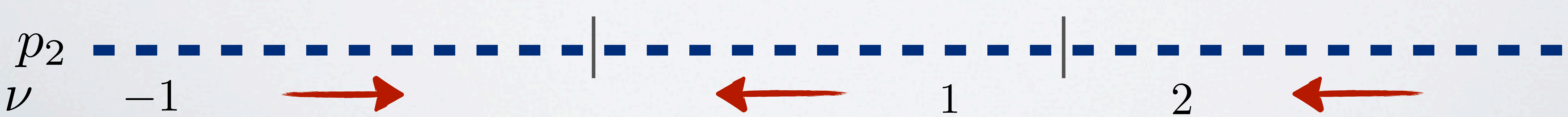
Physical states correspond to eigenvectors with eigenvalues of magnitude less than one.

However not all eigenvalues in the unit circle correspond to legitimate edge states.

Left and right moving transfer matrices must be bounded from any initial point!







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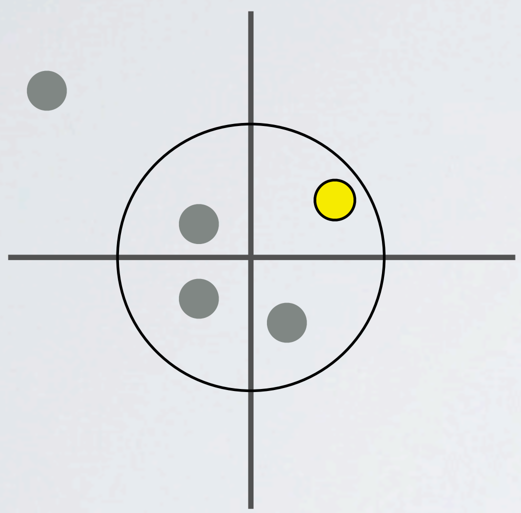

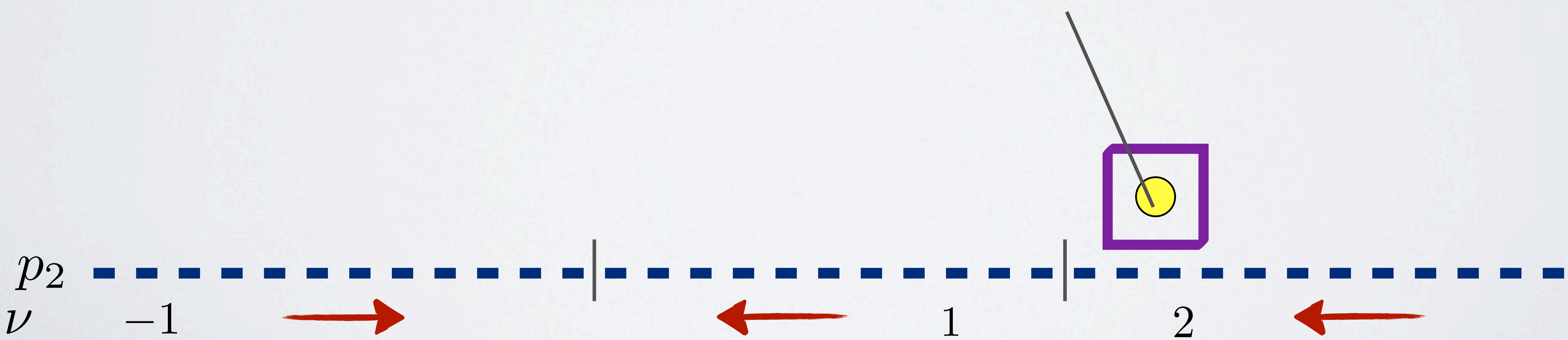

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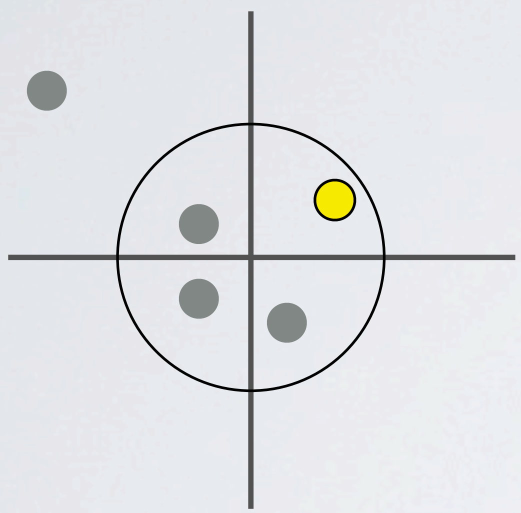

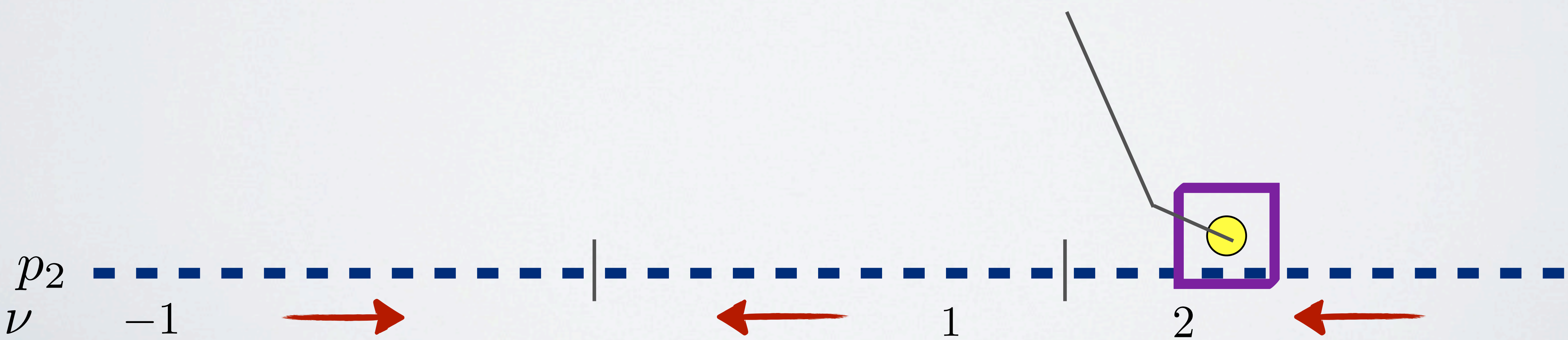

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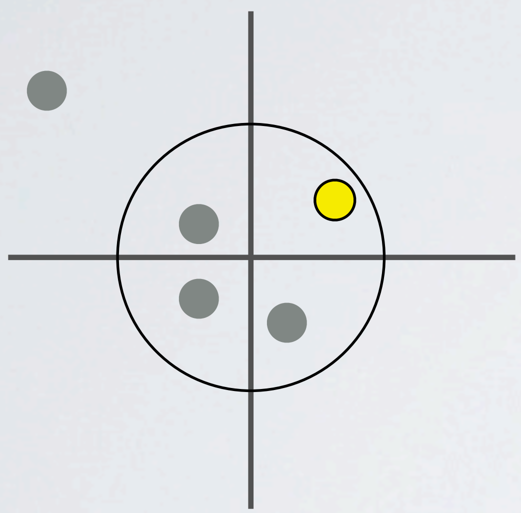

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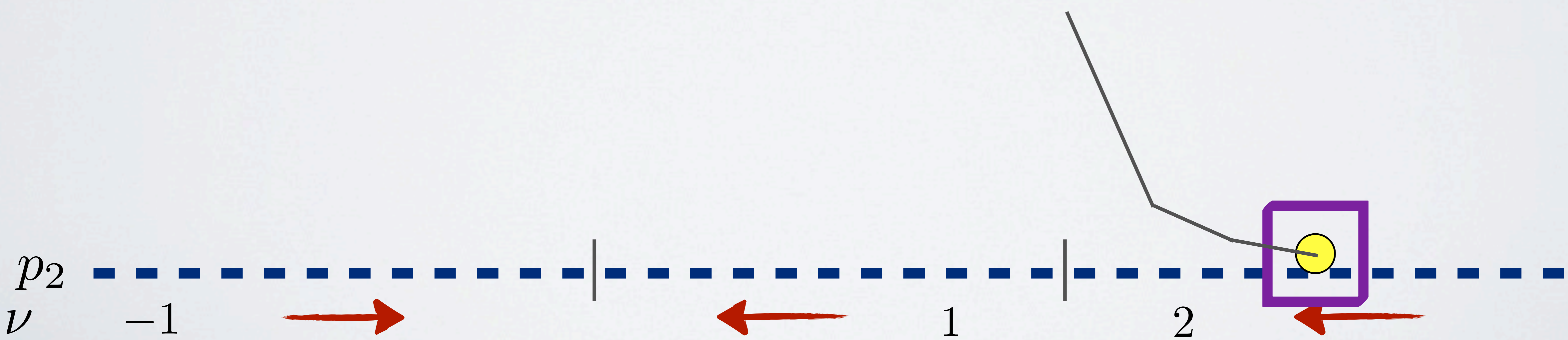

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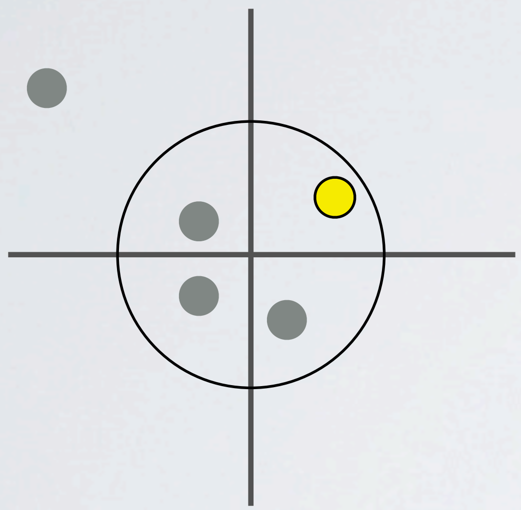

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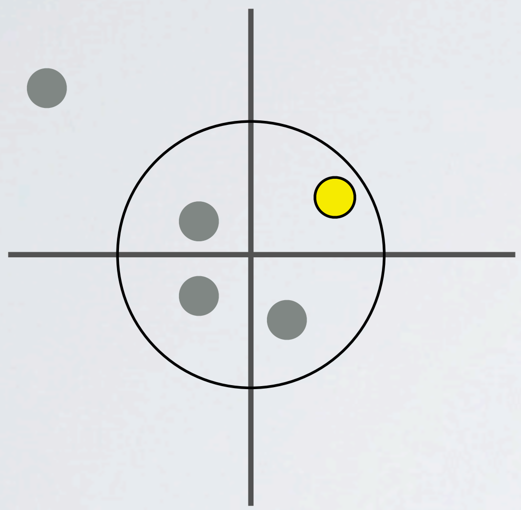

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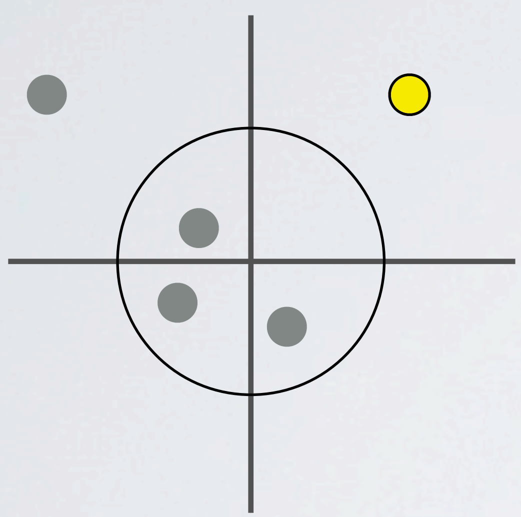

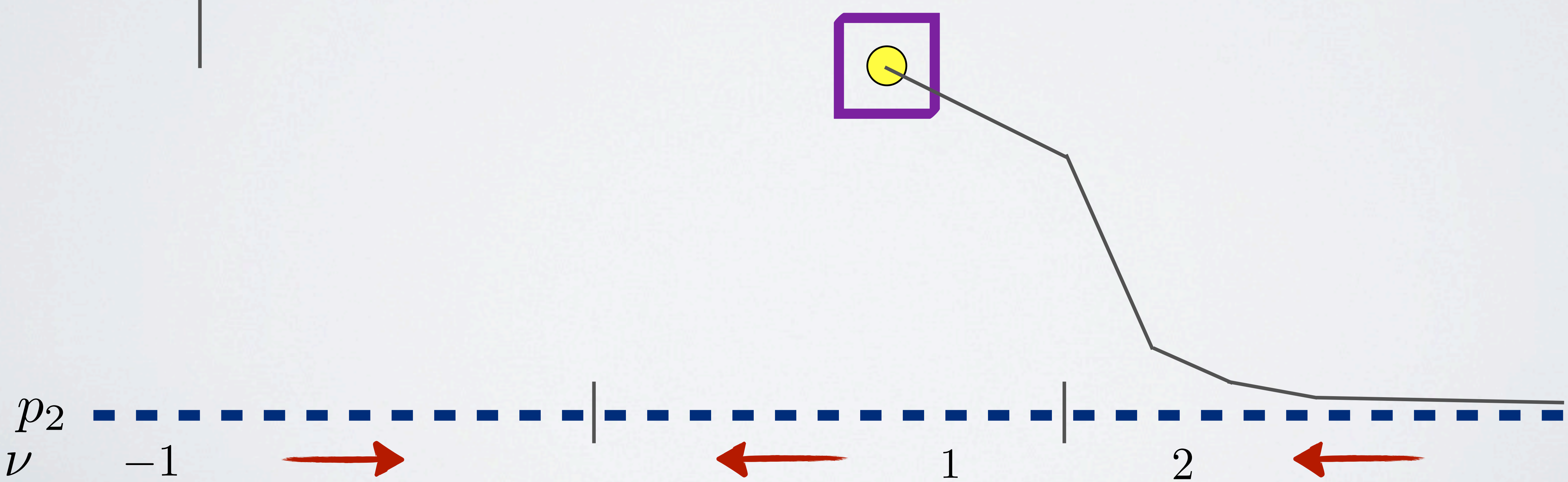

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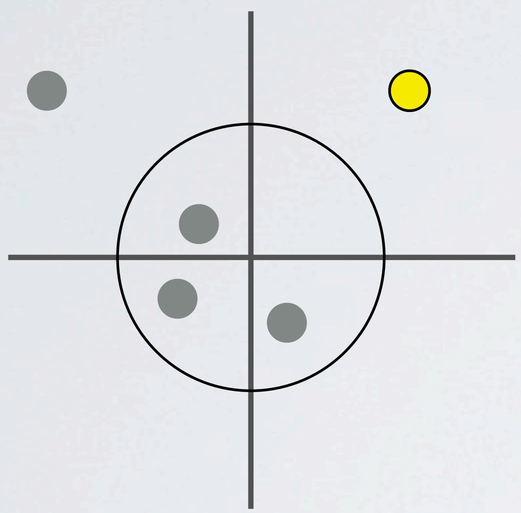

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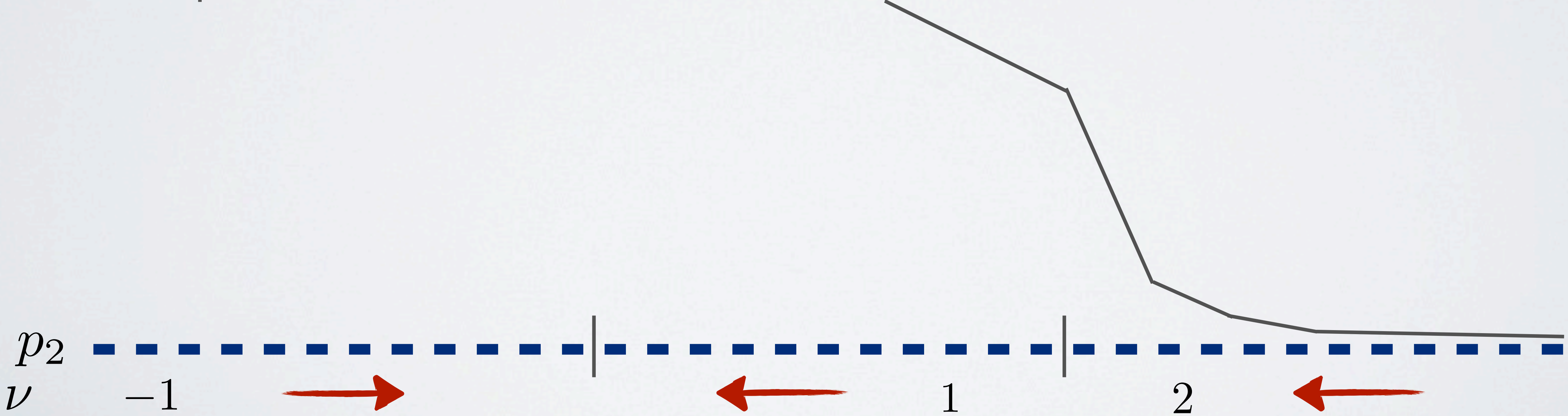

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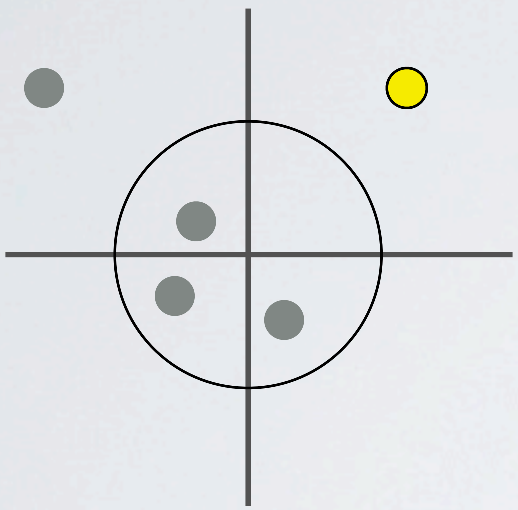
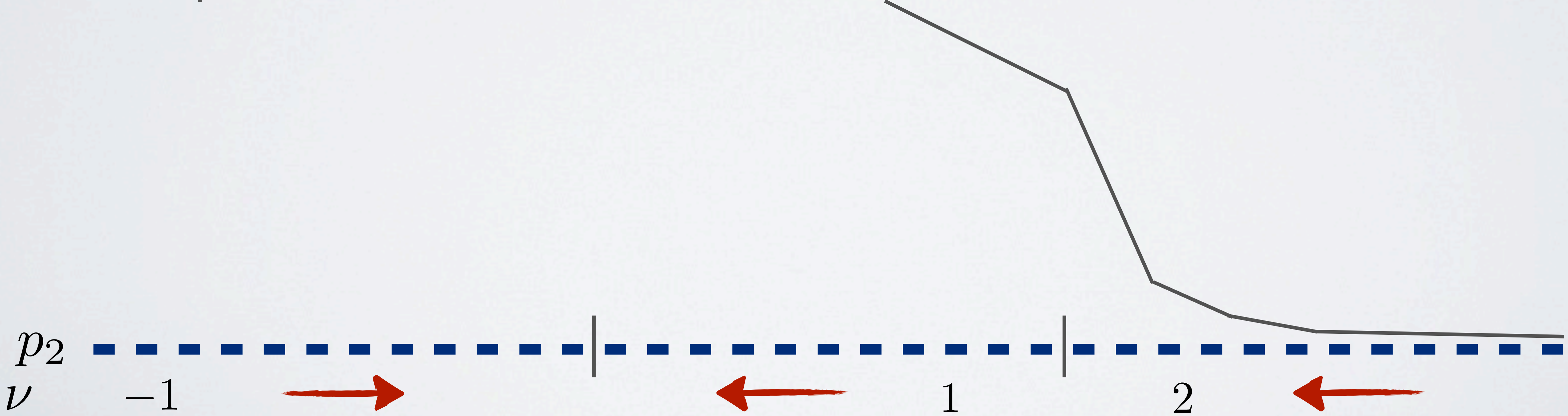
Left and right moving transfer matrices must be bounded from any initial point!





TRANSFER MATRIX

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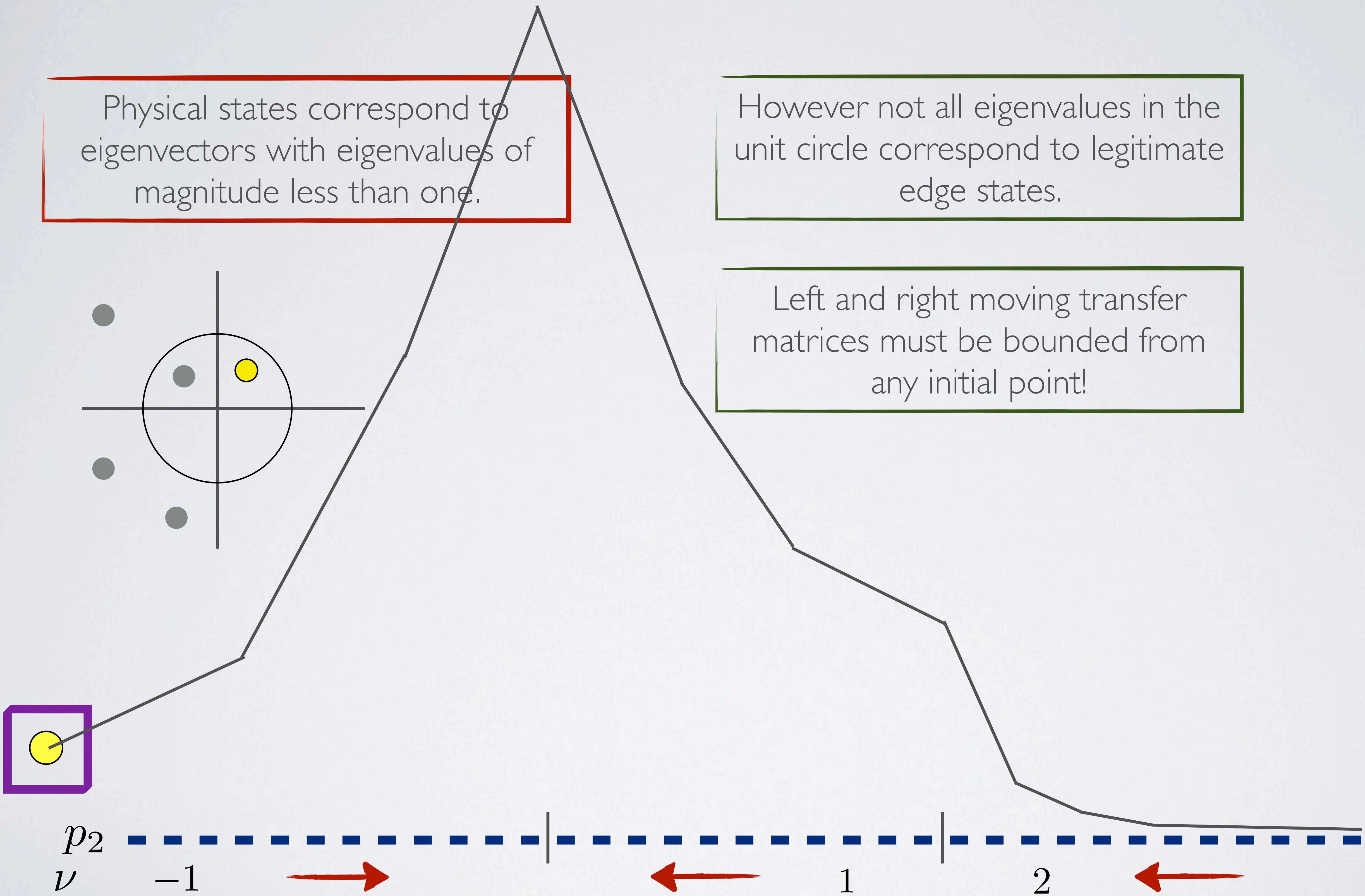


BULK EDGE CORRESPONDENCE

The eigenvalues of the transfer matrix are equal to the roots of the polynomial:

$$\sum_{l=-L}^{L} t_l x^{l+L} \quad \Leftrightarrow \quad c(k) = \sum_{l=-L}^{L} t_l \, e^{ikl}$$

Only roots inside the unit circle are edge states

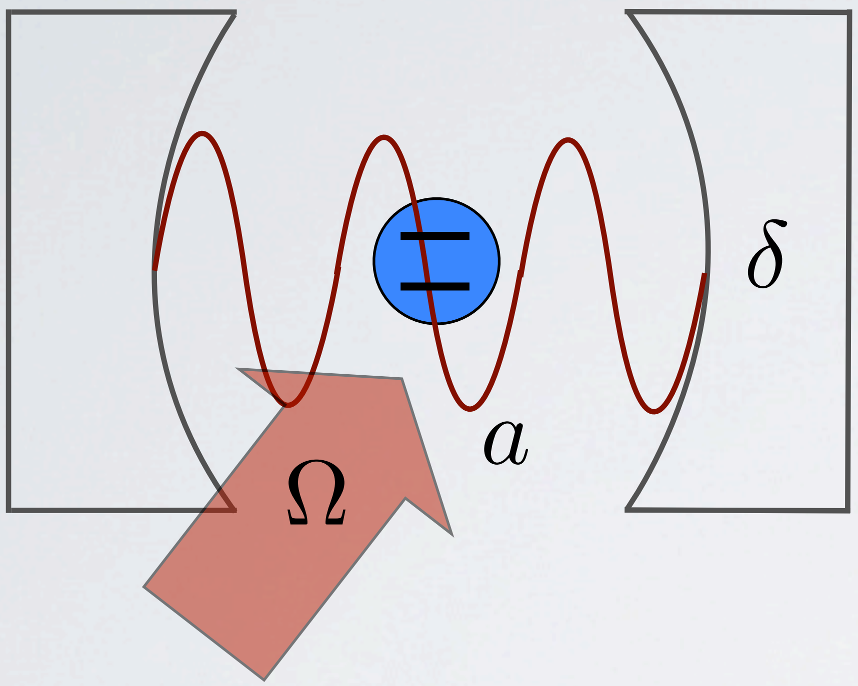
Winding number is simply the number of roots in the unit circle!

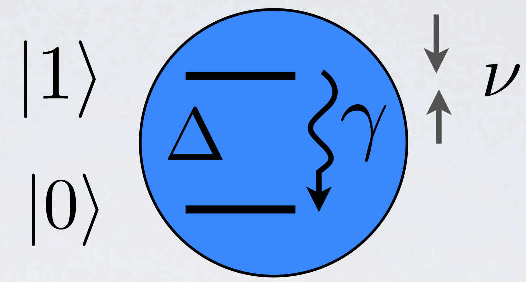
Number of edge states corresponds simply to the difference of winding number at a boundary!

But we have to be careful with ordering of the regions!

MOTIVATING EXAMPLE

Cavity QED





set $-\nu = \delta$

$$H_{JC} = \frac{1}{2}(\Delta - \nu)\sigma^z + (g\sigma^+ a + \Omega\sigma^+ + h.c.)$$

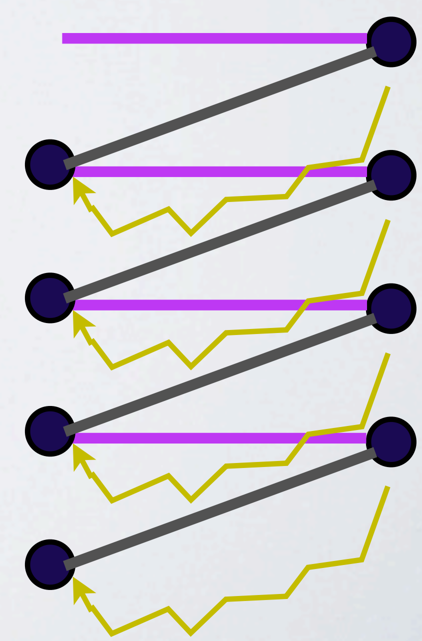
Discretize harmonic oscillator

Topological (dark) edge state
around $u = v_m$.

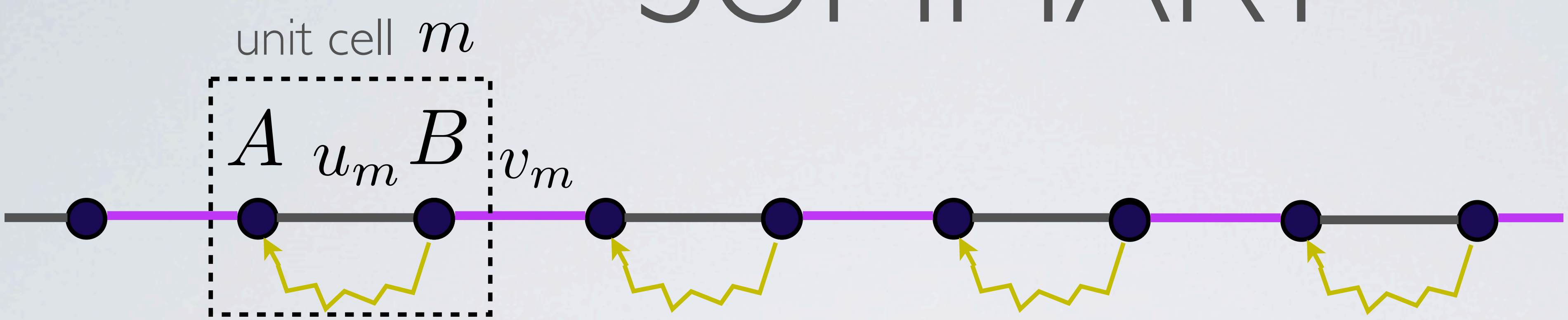
$$v_m = \sqrt{m}v_0$$

$$u_m = u$$

$$u >> v_0$$



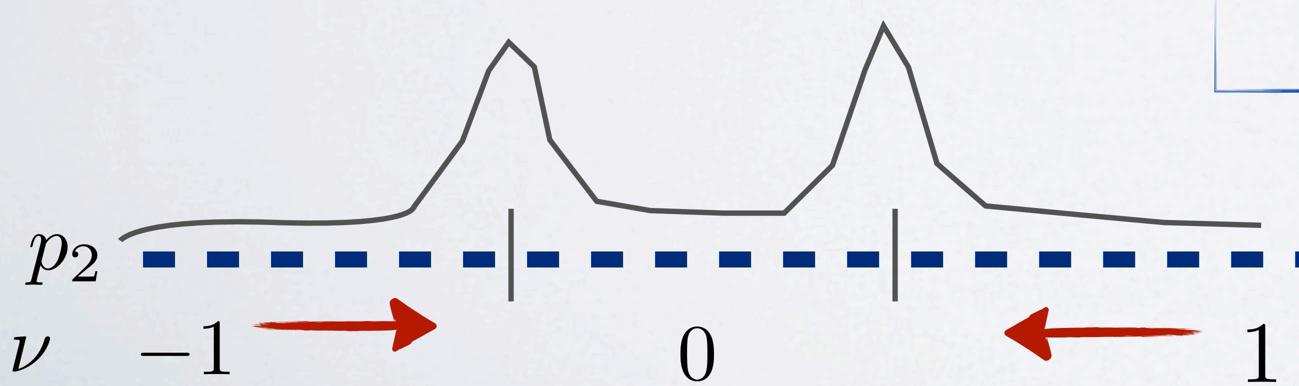
SUMMARY



Steady-state topology in the bulk

$$J_{\rm ss} = \frac{\nu}{\tau}$$

Bulk-edge correspondence



TAKE HOME MESSAGES

Dissipation can cause exotic behavior

Topological transition in very simple system

New type of topological phenomenon?