Coulomb Blockade and Transport in a Chain of 1-dimensional Quantum Dots*

Thomas Nattermann — Cologne
Michael M.Fogler — UCSD
Sergey Malinin — Detroit

*$e=h=k_B=1$

- Introduction
- Quantum wire as a chain of quantum dots
- Transport in short wires
- Experiments
- Long wires: Rare events
- Conclusions

Electrons in one dimension

Breakdown of Landau's Fermi liquid picture

$\Rightarrow$ density wave excitations

(a) 1D clean wire: Luttinger liquid $G = dJ/dV = \sigma/L = (K)\, e^2/h$

(b) single impurity : $K<1$ (>1): impurity (ir)relevant,

$\rightarrow G \sim (\max(T, eV))^{2/K-2}$

Kane & Fisher '92, Furusaki & Nagaosa ´92,93

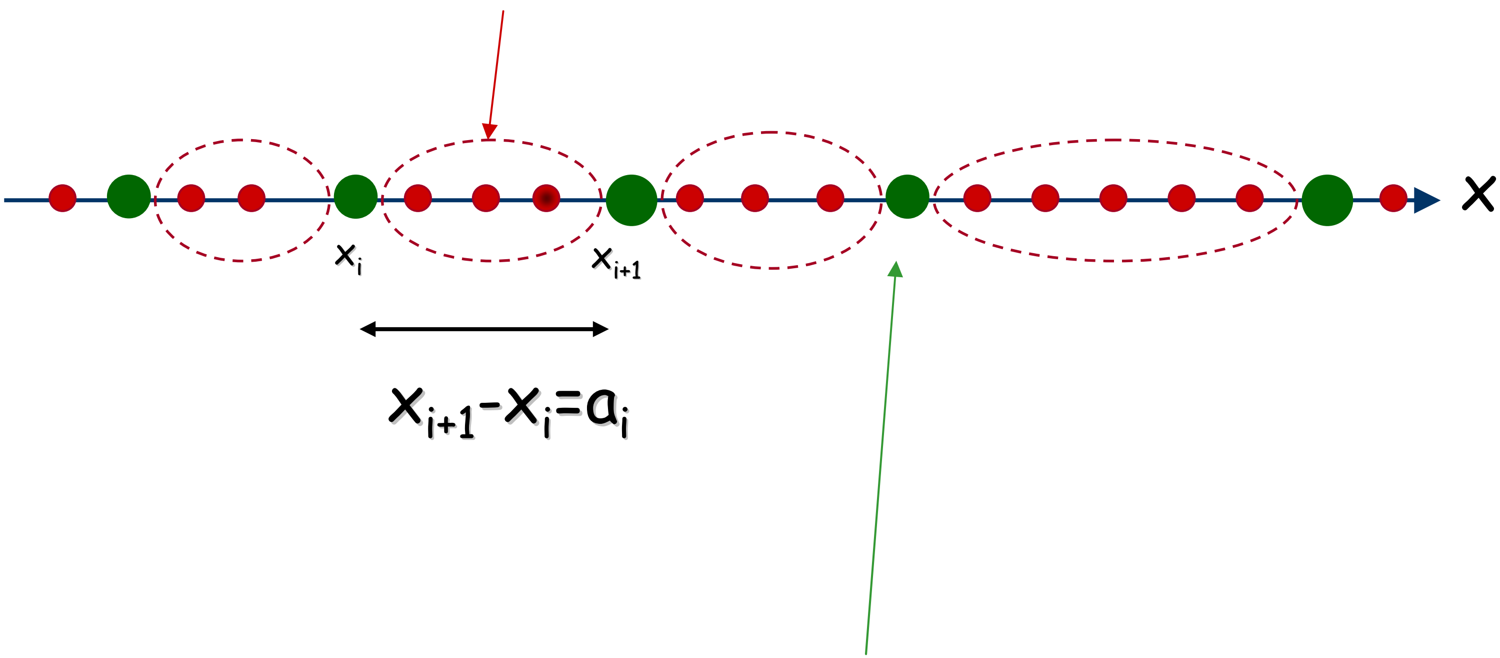
(c) Gaussian impurities: $\sigma \sim T^{2-2K(T)}$, $T \gg T_0$ Giamarchi &Schulz '87

$\sigma \sim \exp(-T_0/T)^{1/2}$ $T \ll T_0$ T.N., Giamarchi, Le Doussal 03

Quantum wire as a chain of quantum dots

Strong impurities:
quantum wire as a chain of quantum dots

"**quantum dot**" with integer number q_i of electrons



strong impurities, randomly distributed

Luttinger liquid with strong impurities:

charge density $\rho(x) = \frac{1}{\pi}(k_F + \partial_x \varphi)[1 + 2\cos(2\varphi + 2k_F x)]$

$$S = \frac{1}{2\pi K} \int_0^{\frac{v}{T}} d\tau \left[\int_0^L dx [(\partial_\tau \varphi)^2 + (\partial_x \varphi)^2] - \sum_{i=1}^{N} w \cos(2\varphi(x_i) + 2k_F x_i) \right]$$

K

$\lambda_T = v/T$ thermal de Broglie wavelength

$w \gg k_F$ pinning strength

$x_{i+1} - x_i = a_i$ impurity spacing

$\varphi(x_i) + k_F x_i = \pi N_i$

⇒ integer number $q_i = N_{i+1} - N_i$ of electrons between impurities

Classical ground state

$K, v \to 0$, compressibility $g = K/(\pi v)$ finite

$$S \to \frac{H}{T} \equiv \sum_j \frac{\Delta_j}{2T}(q_j - Q_j)^2$$

$q_i = N_{i+1} - N_i$

$\Delta_j = 1/(g a_j)$ charging energy of dot

$Q = k_F a_j / \pi$ "background charge"

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$Q_i = k_F a_j / \pi$ "background charge"

$T \ll \Delta_j \quad \to \quad q_j = [Q_j]_G$,

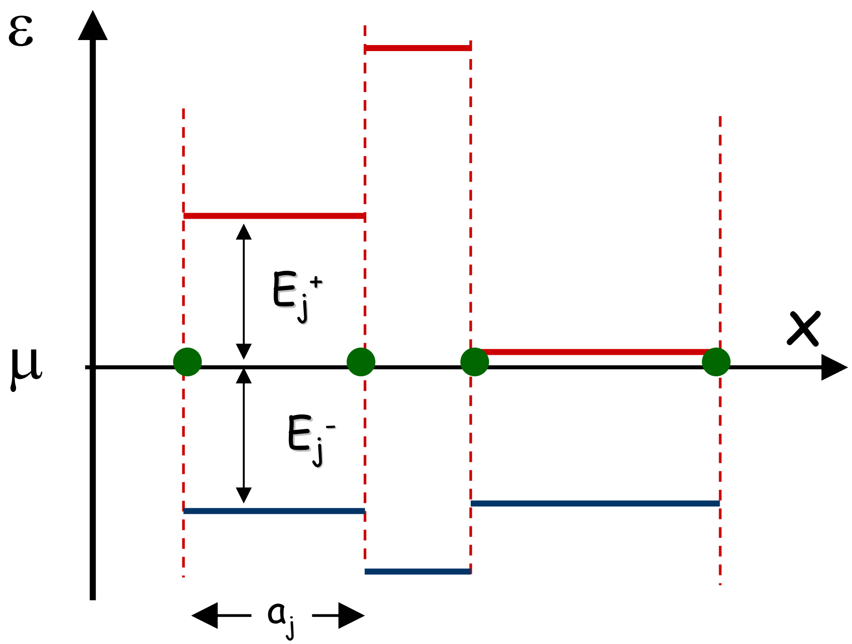
bifurcation dots: $q_j - Q_j = \pm \frac{1}{2}$

Charged excitations

change number of electrons by ± 1

$$E_j^{\pm} = \Delta_j \{ 1/2 \pm (q_j - Q_j) \} \qquad \equiv \pm$$

$\varepsilon_j^{\pm}$



$$E_j^+ + E_j^- = \Delta_j$$

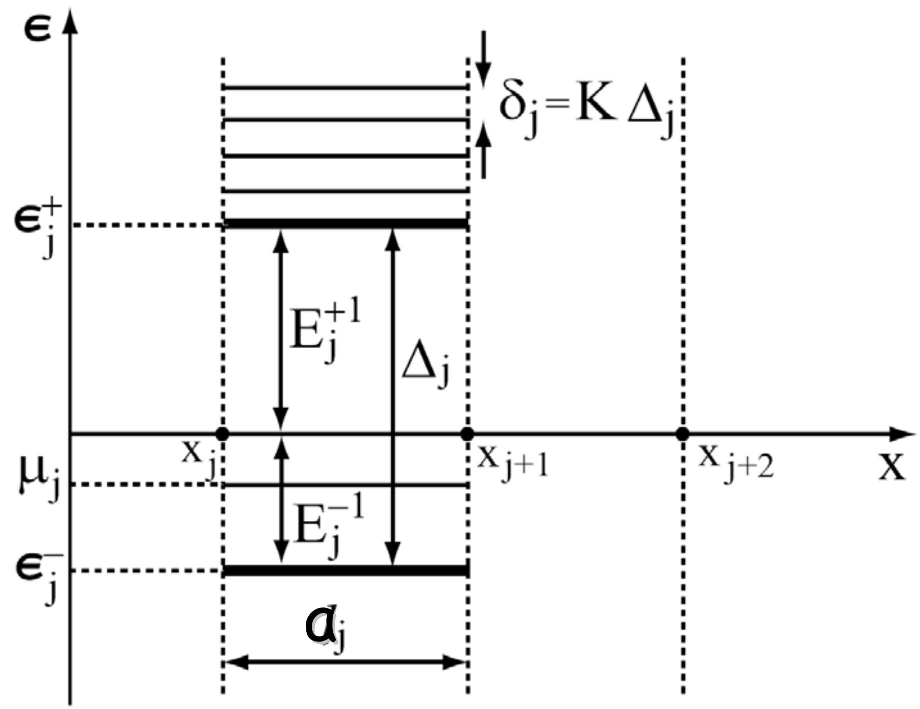
Coulomb blockade if $E_j^{\pm} \gg T$

Bifurcation dots:

$$q_j - Q_j = \pm \tfrac{1}{2} \quad \rightarrow \quad E^{\mp} = 0$$

Example: $Q=3/2$, $q=1$, $E^+=0$, $E^-=\Delta$
$Q=3/2$, $q=2$, $E^+=\Delta$, $E^-=0$

Neutral excitations



$\delta_j = \pi\, v/a_j = K\,\Delta_j$

$$S = \frac{1}{2\pi K} \sum_{j=0}^{N} \sum_{\omega_n} \frac{\omega_n}{\lambda_T} \left[\frac{|\varphi_{j+1,\omega_n} - \varphi_{j,\omega_n}|^2}{\sinh \omega_n a_j} + \left(|\varphi_{j,\omega_n}|^2 + |\varphi_{j+1,\omega_n}|^2 \right) \tanh \frac{\omega_n a_j}{2} \right]$$

$\omega_n = 2\pi n T \gg \delta_j\ ,\ \delta_{j+1}$: $\varphi(x_j)$ decoupled from neighbours

$\lambda_T \ll a_j$

Add external field: $H_F = -F\int dx\, \varphi(x)/\pi$

Discrete energy levels ($\delta_j = K\Delta_j$) :

$\Rightarrow$ energy conservation forbids tunneling except from

rare dots $\rightarrow$ $R \sim e^{\alpha L}$ Anderson et al (and many others)

$\Rightarrow$ conductivity vanishes

$\Rightarrow$ modification of the model:
weak coupling to a bath

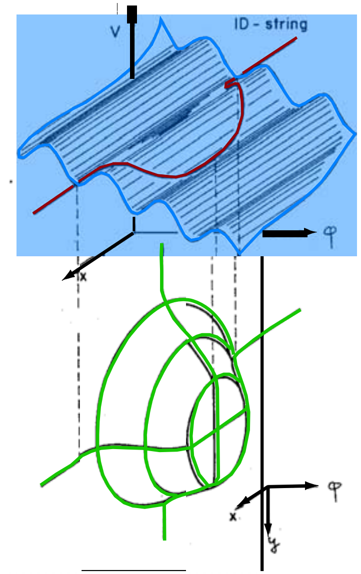
Bath: (i) electrons in the gate (ohmic)

$$S_d = \frac{\eta}{4\pi\lambda_T} \sum_j \sum_{\omega_n} |\omega_n| |\varphi_j(\omega_n)|^2$$

(ii) phonons

Transport in short wires

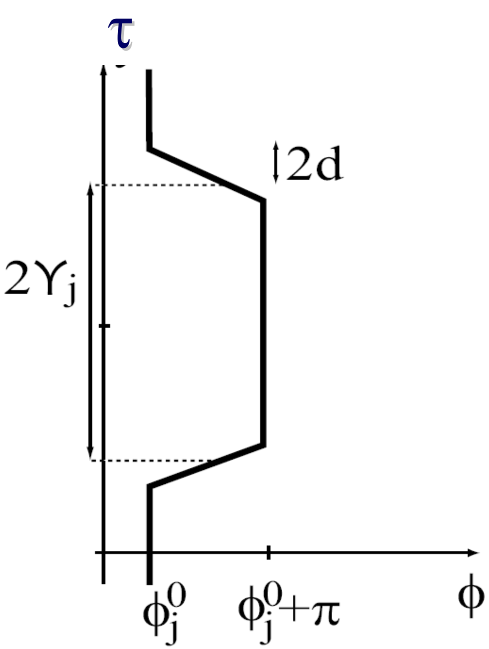
Transport at T=0

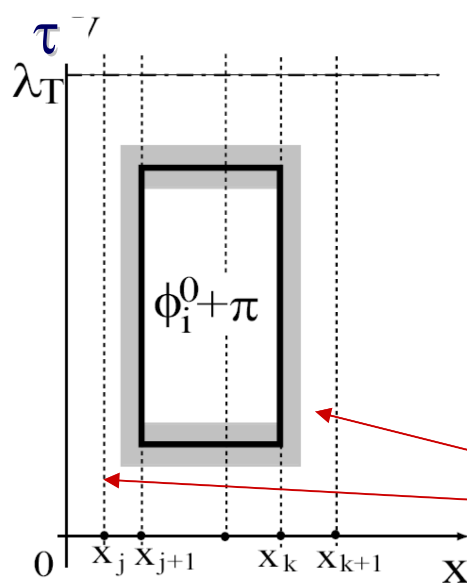


→ effective action $S\{\varphi(x_i,\tau)\}$ on impurity sites

→ classical metastale state $\varphi_i(\tau) \to \varphi_i = \pi\,(N_i + k_F x_i)$,

F>0 : Tunneling via instantons: $N_i \to N_i + 1$





bifurcation dots

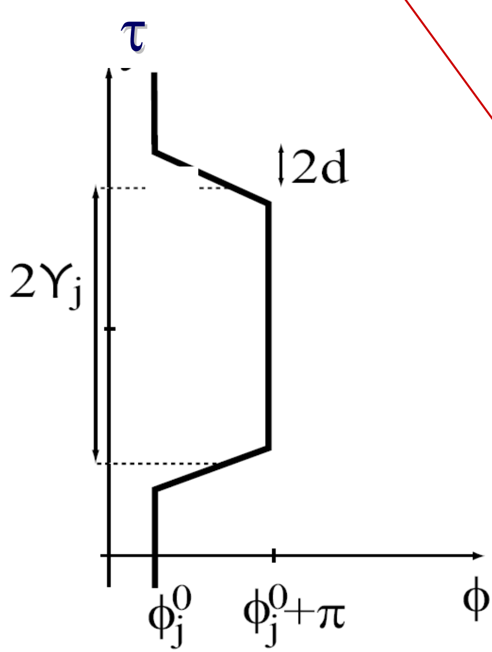
Transport at T=0

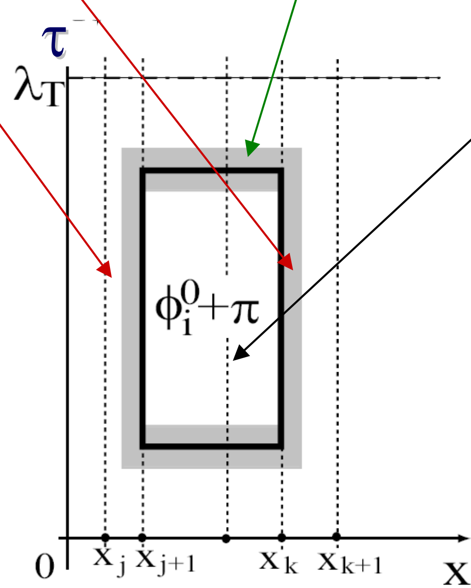
bifurcation dots:

$$s(V_1, T) \approx \frac{2}{K} \ln \left[\frac{D}{max(\delta, T, KV_1)}\right]$$

$$E_j^+ + E_{j+m}^- \approx \Delta / m$$

$$S \approx Y(E_j^+ + E_{j+m}^-)/\delta + m\,[s_0 + \ln \tanh (Y/2) - YV_1/\delta\,]$$



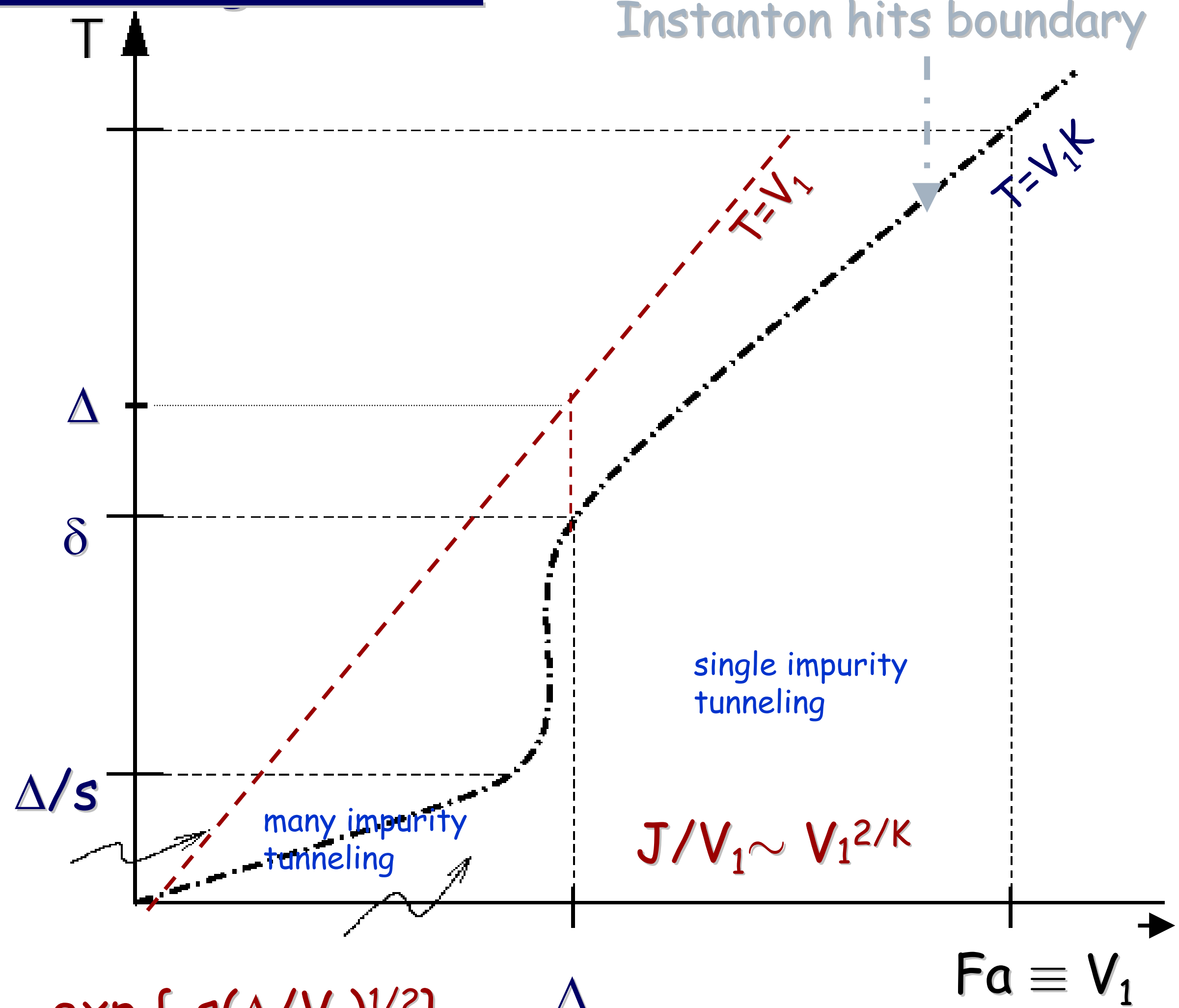


saddle point

$$J \sim \Gamma \sim \exp\{-S_{inst}/\hbar\}$$

Results from typical tunneling events:

(short wires)



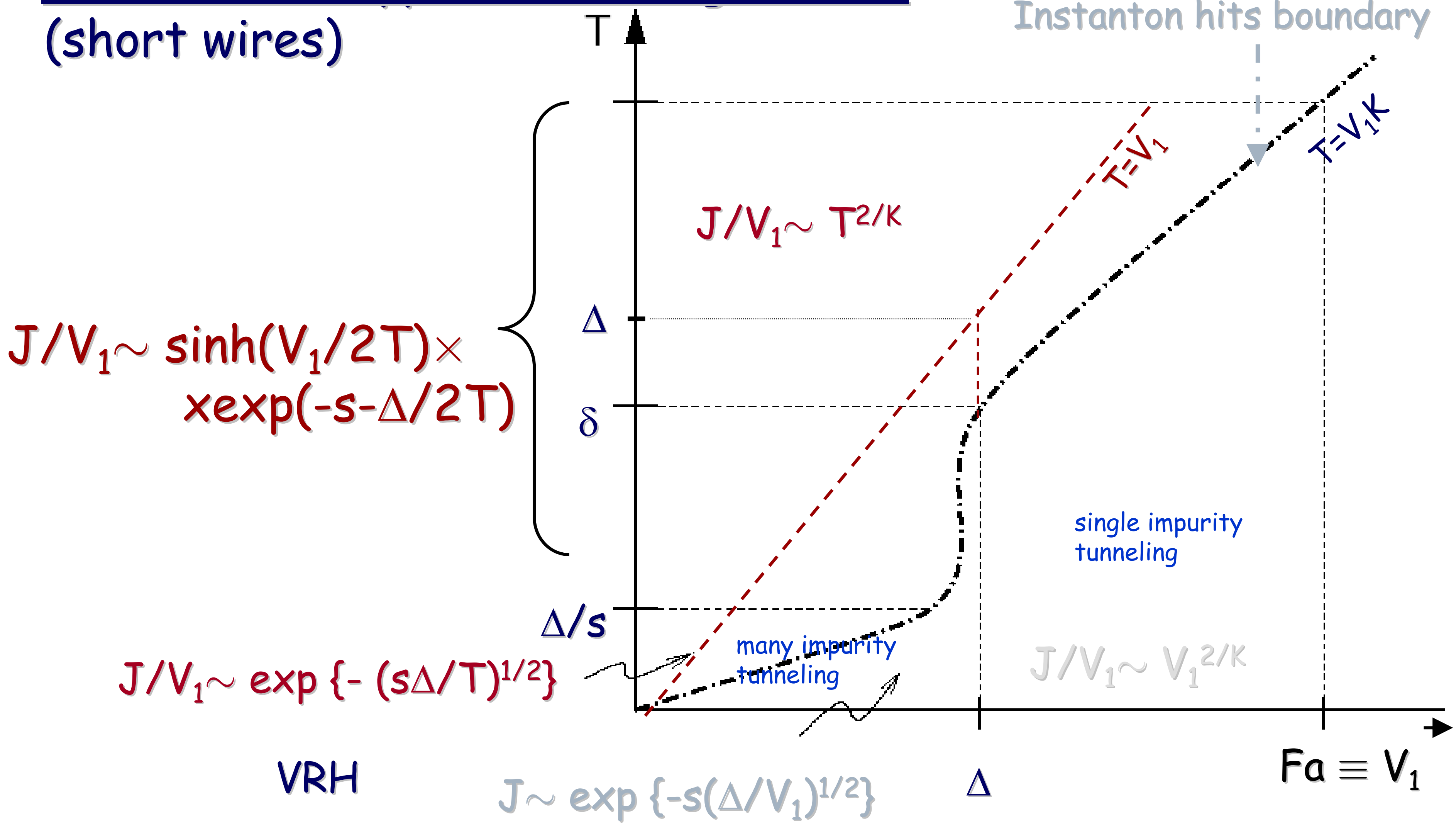
VRH

$J \sim \exp\{-s(\Delta/V_1)^{1/2}\}$

$\xi_{loc} \approx a/s$

Results from typical tunneling events:

(short wires)

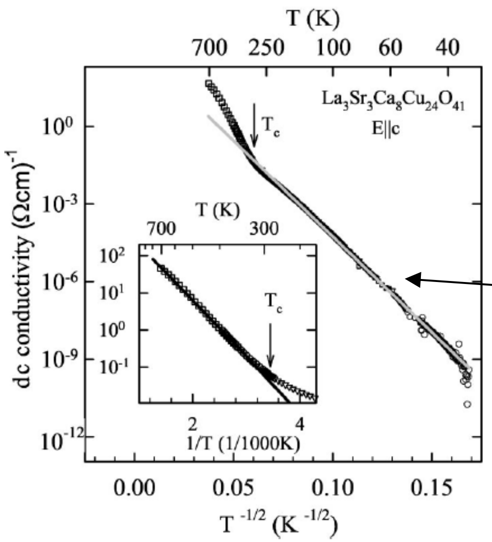


$$s(V_1,T) \approx \frac{2}{K} \ln\left[\frac{D}{max(\delta,T,KV_1)}\right]$$

$\xi_{loc} \approx a/s$

Experiments

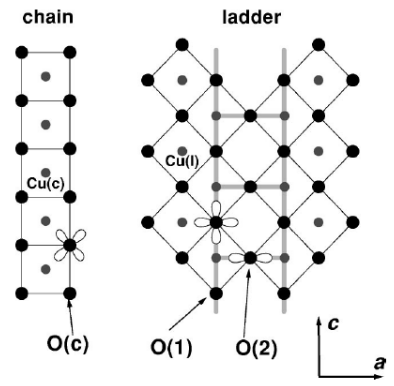
Different types of conductivity :
(i) variable range hopping

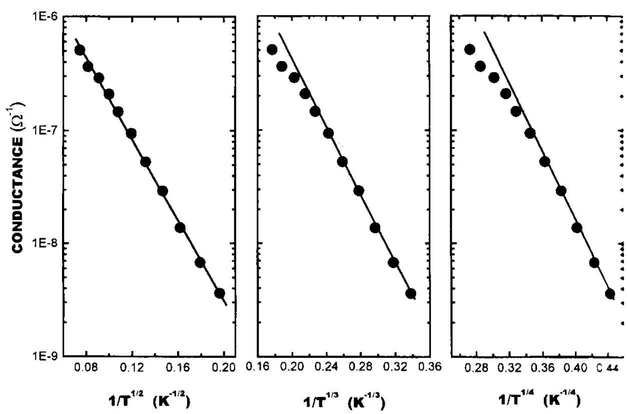


$La_3Sr_3Ca_8C_{24}O_{41}$

$T_0 \approx 3\ 10^4$ K

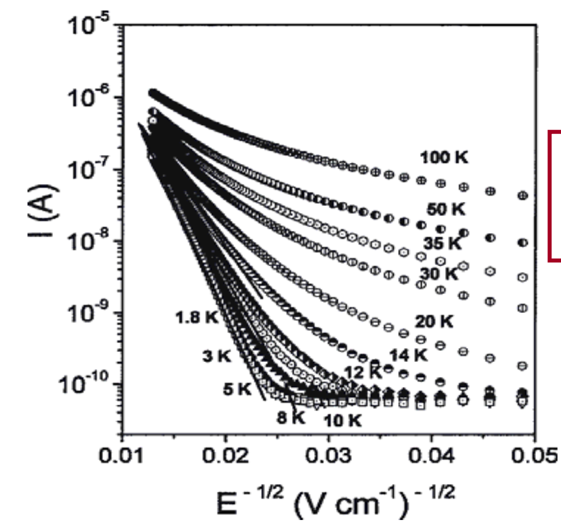
$\sigma \sim e^{-(T_0/T)^{1/2}}$





Carbon-nanotubes: Tang et al. 2000

$J \sim e^{-(T_0/T)^{1/2}}$



$\sigma \sim e^{-(E_0/E)^{1/2}}$

Polydiacetylen : Aleshin et al. 2004

Different types of conductivity :

(ii) Kane-Fisher behavior

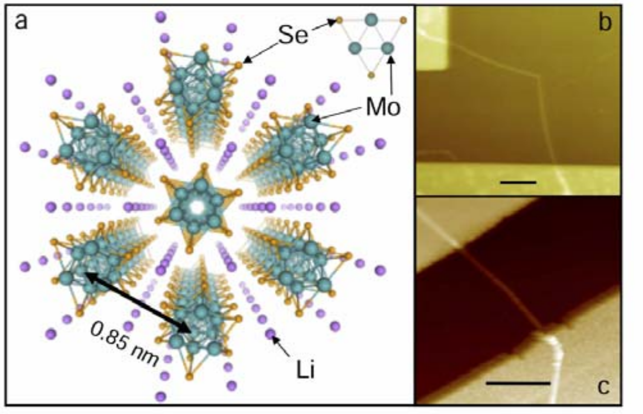
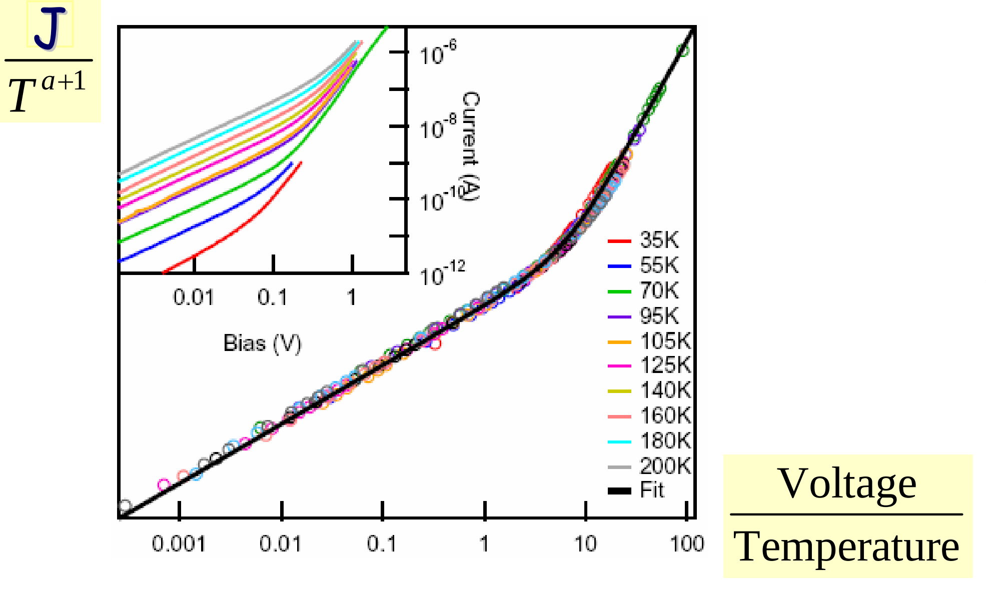


FIG. 1 (color online). (a) Structural model of a 7-chain MoSe nanowire along with the triangular Mo_3Se_3 unit cell. (b) and (c) AFM height images of MoSe nanowires between two Au electrodes. The wire heights are 7.2 nm and 12.0 nm, respectively. Scale bar = 500 nm.

- **MoSe Nanowires** Venkataraman, PRL (2006)

$$J/T^{\alpha+1} \sim \max\left(V/T,\ V^{\beta+1}/T^{\alpha+1} \right)$$



short wires (L ~ 1 μm):

"Temperature" Exponent α is close to "Voltage" Exponent β

Agrees with the conventional "Luttinger-liquid" picture with

$\alpha=\beta=2/K-2$

Different types of conductivity : (iii) new behavior

- Polymer nanofibers

long wires: 10μm

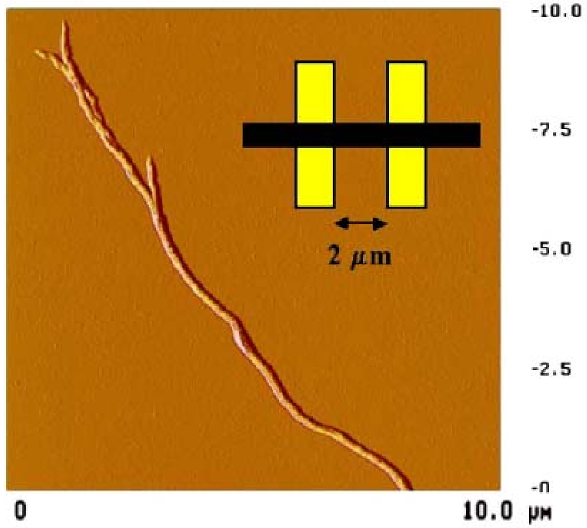
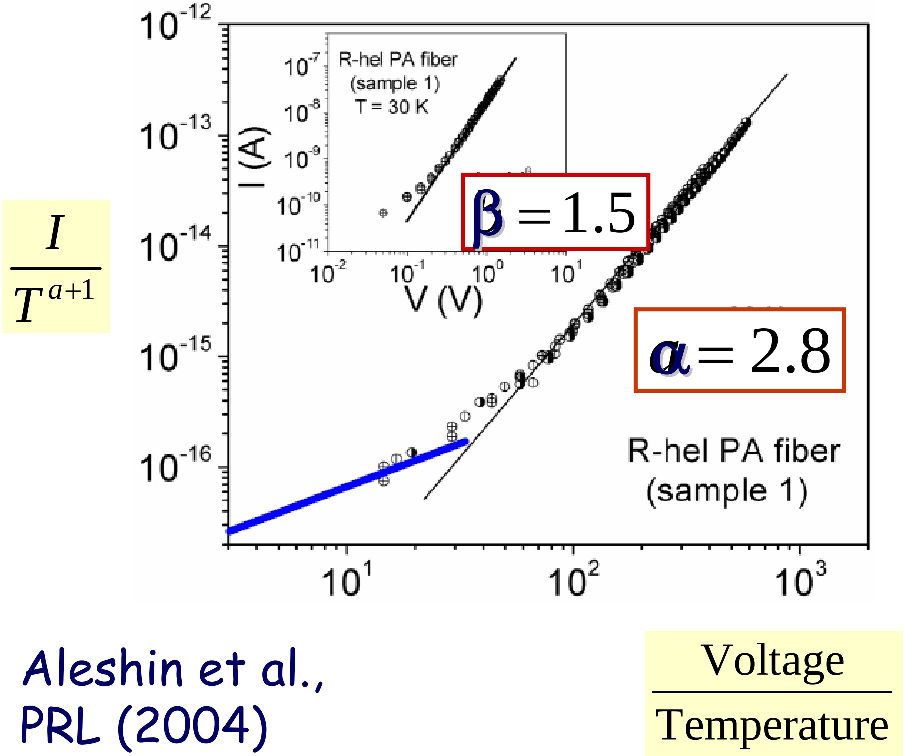


FIG. 1 (color online). AFM image of a *R*-hel PA fiber; the inset shows the schematic of a two-probe device based on such a *R*-hel PA fiber on top of Pt electrodes.



"temperature" exponent **exceeds** "voltage" exponent!

$$J / T^{\alpha+1} \sim \max (V/T, V^{\beta+1}/T^{\alpha+1})$$

Aleshin et al., PRL (2004)

Disagrees with the conventional Luttinger-liquid picture

Observed Power-Law Exponents

L ~10 μm polymers
Aleshin et al.,
PRL (2004)

Sample	1	2	3	4	5	6
α	2.8	5.5	7.2	5.6	5.0	4.1
β	1.5	3.8	4.7	1.0	1.1	1.8

L ~ 100 μm InSb wires
Zaitsev-Zotov et al.,
JPCM (2000)

Sample	1	2	3	4
α	2.3	3.4	4.5	4.6
β	1.3	3.4	2.8	2.0

- In long wires T-exponent exceeds V-exponent: $\alpha > \beta$
- Exponents are sample-dependent

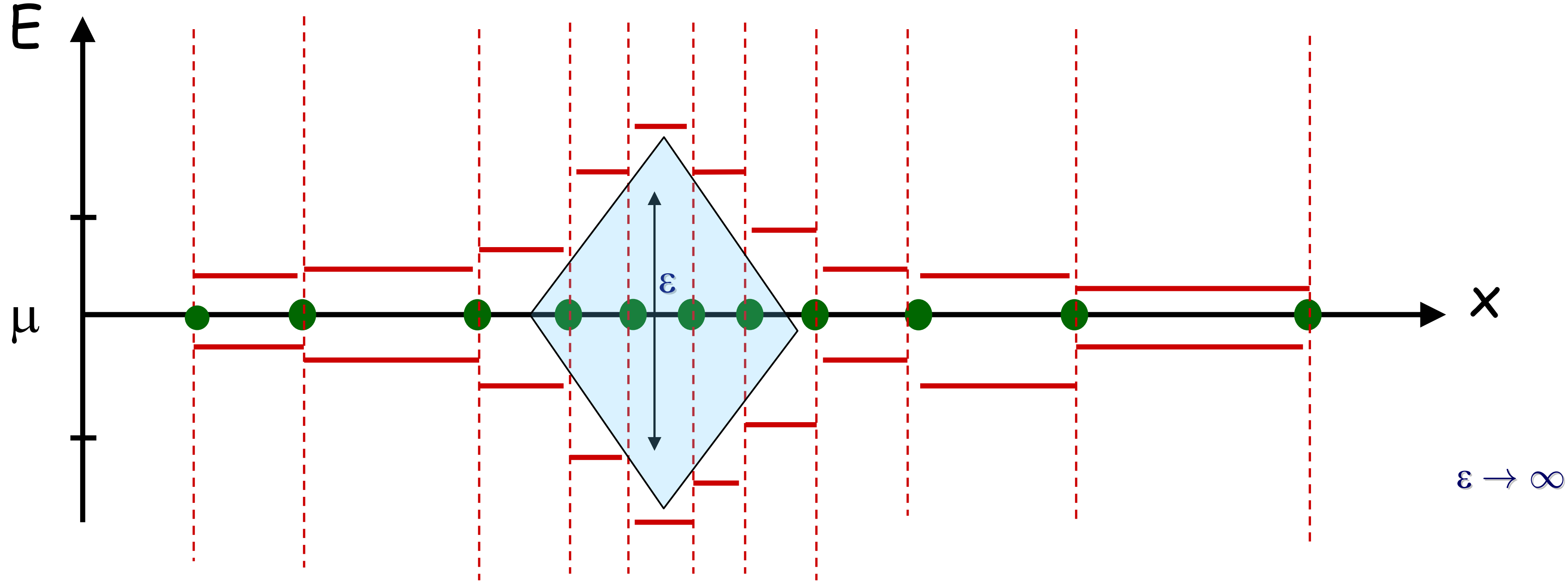
Long wires:
Rare events

4. Long wires: rare events

So far: typical quantum dots with $a_i \approx a$

Now: consider regions with many narrow dots with $a_i \ll a$

"Break" : sequence of narrow dots



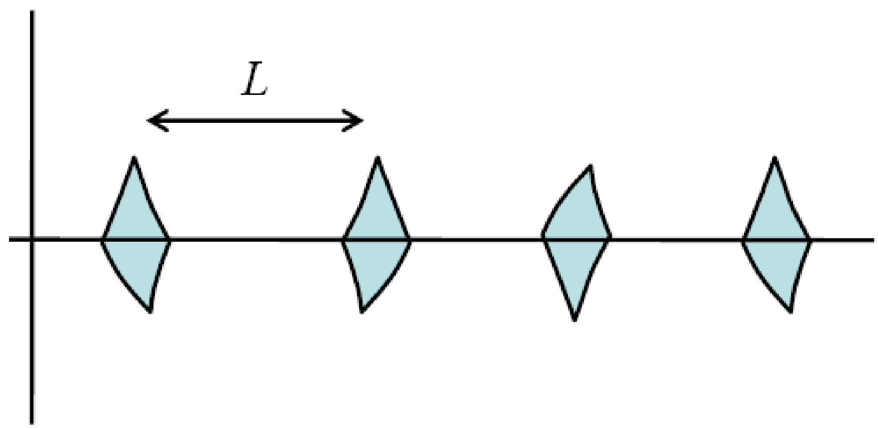
Probablity that no charge excitation in energy interval ε

$$P_\varepsilon(\varepsilon) = 1 - (\varepsilon/\Delta)(1 - e^{-\Delta/\varepsilon}) \approx \begin{cases} 1-\varepsilon/\Delta, & \varepsilon/\Delta \ll 1 \\ \Delta/(2\varepsilon) & \varepsilon/\Delta \gg 1 \end{cases}$$

Wire: non-overlapping breaks + low resistance connecting pieces*

break resistance $R_0 e^u \rightarrow$

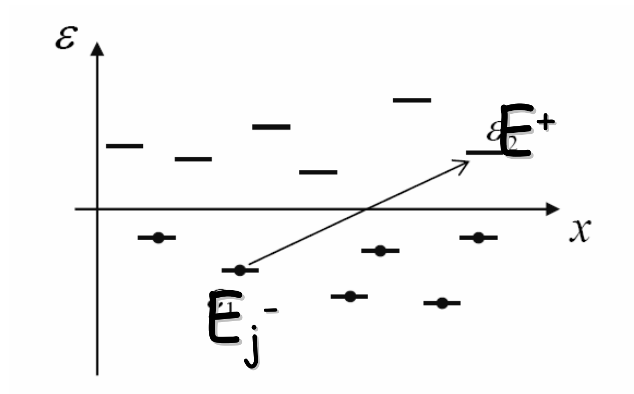
total resistance $R = L \int P_u(u)\, R_0 e^u\, du$



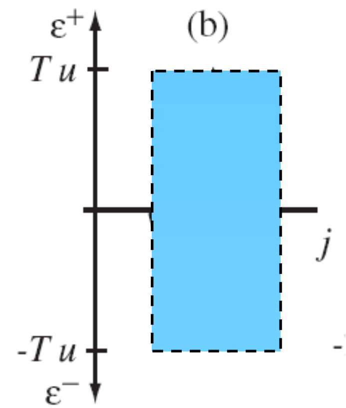
$P_u(u)$: density of pieces with resistance e^u

* Ruzin and Raikh 1988

(a) Ohmic break of |j-k| dots



$$R_{j\to k} \approx R_0 e^{s|j-k|} e^{\frac{1}{T}E_{kj}} \geq R_o e^u$$



simplification: rectangular break $s|j-k| \gtrsim u$,

$E_{jk}/T \gtrsim u$

$P_u(u) \sim P_\varepsilon(\varepsilon)^{|j-k|} \sim P_\varepsilon(uT)^{u/s} \sim$

- $\ln P_u(u) \sim - u^2 T/\Delta\, s$ if $uT \ll \Delta$
- $\ln P_u(u) \approx - u/s \ln (2uT/\Delta)$ if $uT \gg \Delta$

Infinite wire

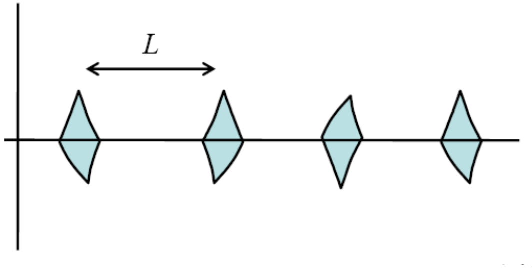
$R = L \int P_u(u)\, R_0 e^u \, du \;\rightarrow$ saddle point solution

$s \ll 1$: $\rho \sim \exp(s\Delta/4T)$ Raikh Ruzin '89

$s \gg 1$: $\rho \sim \exp(\Delta\, e^s/4sT)$ new

break size $a_b \approx a\, \Delta\, e^s/(4sT) \gg a$

break distance $L_b \approx a_b \exp[s a_b/a]$



Example: $s=5,\ \Delta \approx T$

$\rightarrow a_b/a \approx 7{,}4$

$L_b/a \approx 10^{16}$

Finite wire: $P_u(u)(L/a_b) \approx 1 \quad \to \quad u_{max}$

$sT \ln(L/a_b) \ll \Delta$: $\ln R \sim [s\Delta \ln(L/a) / T]^{1/2}$ VRH

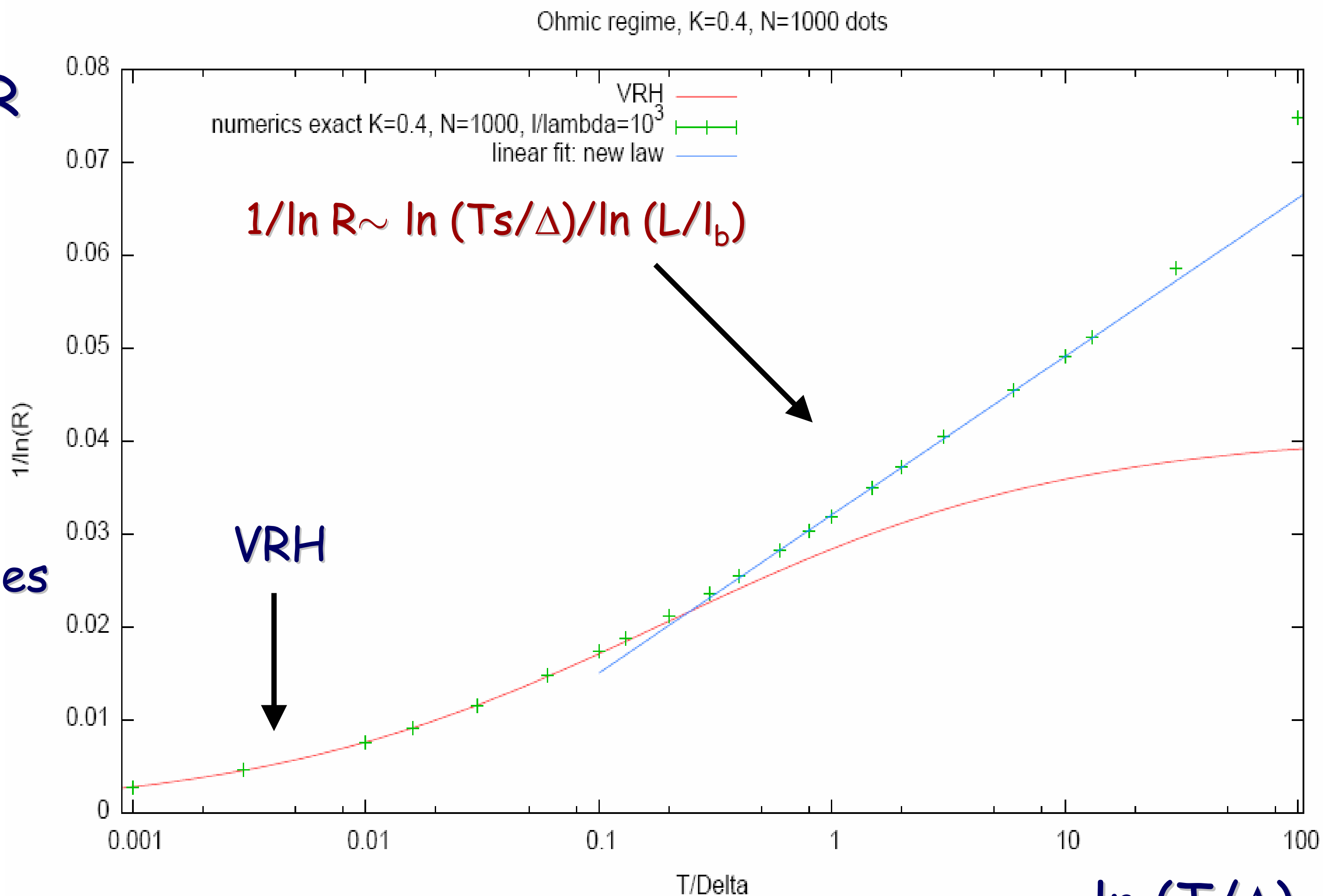
Raikh Ruzin '89

$sT \ln(L/a_b) \gg \Delta$: $\ln R \sim s \ln(L/a_b) / \ln(Ts/\Delta) \sim \alpha \ln T$

$\alpha = s(T) \ln(L/a_b) / \ln^2(Ts/\Delta)]$

Numerics: Christophe Deroulers (unpublished)

$1/\ln R$



$1/\ln R \sim \ln(Ts/\Delta)/\ln(L/l_b)$

VRH

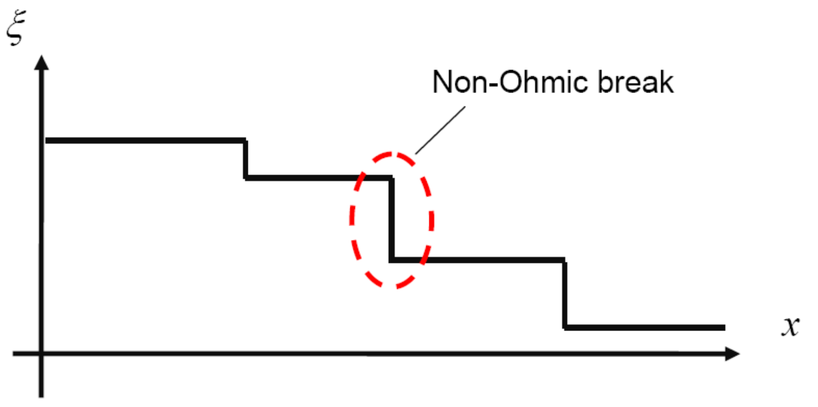
K=0.4

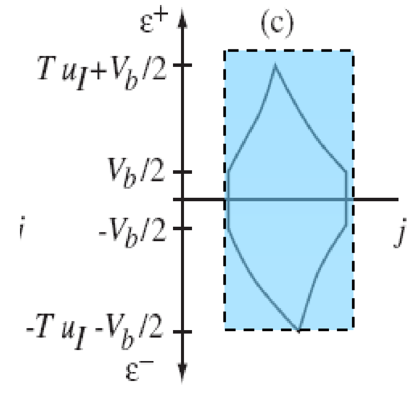
1000 impurities

$ak_F=10^3$

$\ln(T/\Delta)$

(b) Non-Ohmic breaks of m dots





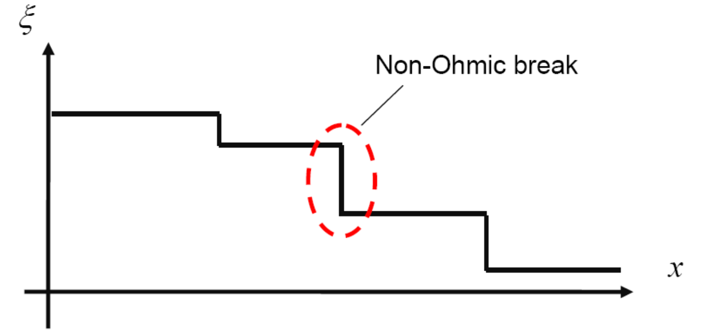
$$J_{j\to k} \approx J_0 e^{-s|j-k|-\frac{1}{T}\varepsilon_{kj}}\ \sinh[(\zeta_j-\zeta_k)/T]$$

assumption: largest voltage drops $V_b=\xi_j-\xi_k \gg T$ at a few breaks

constant $I = I_0\, e^{-u}$ everywhere

$$u \le \underbrace{s|j-k|}_{\ge u} + \underbrace{\varepsilon/T - V_b/T}_{\ge u}$$

(b) Non-Ohmic breaks



prob. distribution

$P_V(V_b) \approx$ P_u(uT+V_b/2) $\sim P_u(u) \exp[-V_b/s\ T]$

$V = L\int dV_b\, P_V(V_b) = V\ (u(J)) \quad \rightarrow \quad J(V,T)$

Different regimes

(i) V_b, $uT \ll \Delta$ Fogler&Kelly (2005)

(ii) $uT \gg \Delta \gg V_b \gg T$

(iii) $uT \gg V_b \gg \Delta \gg T$

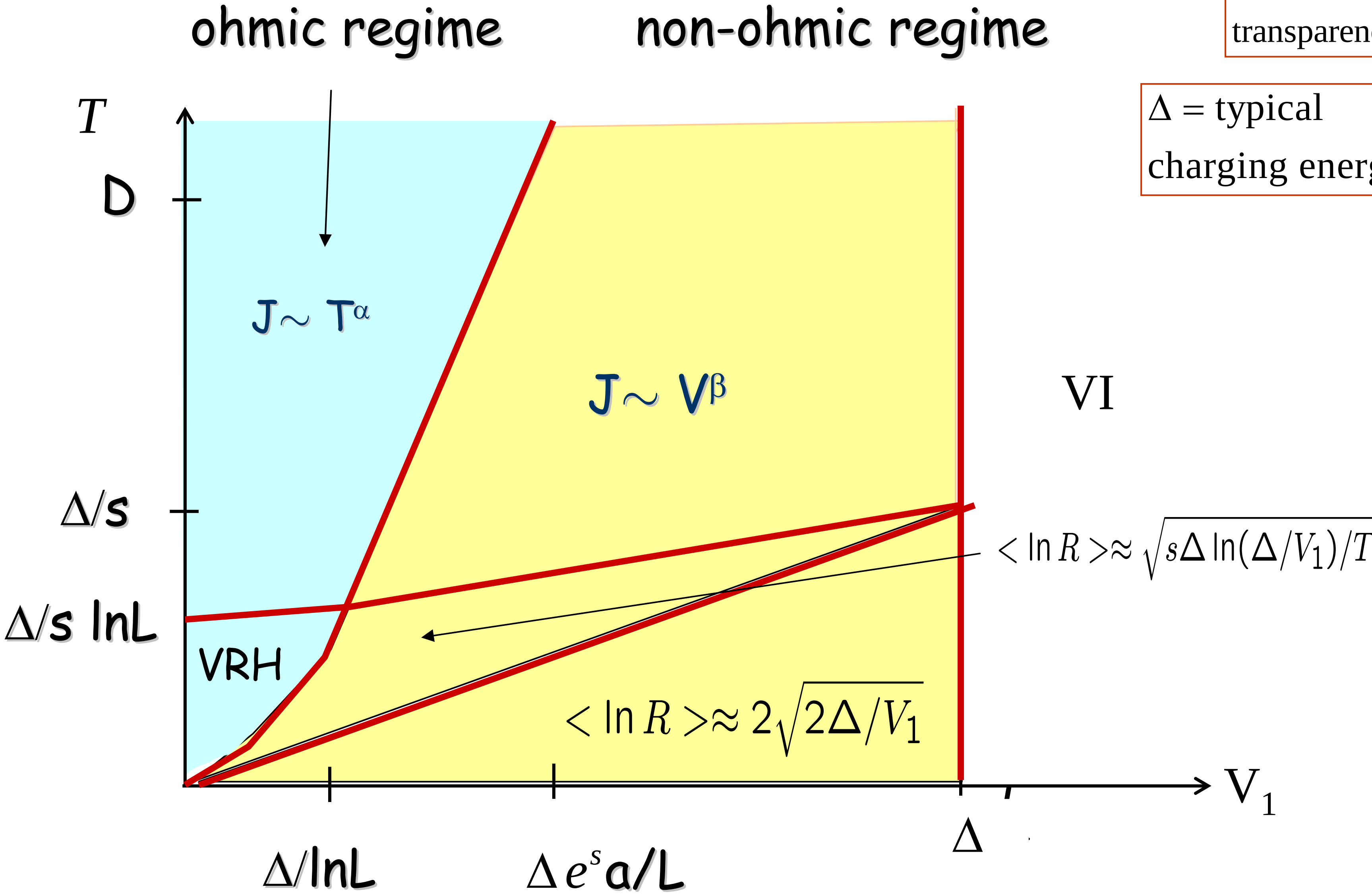
$u = -\ln (J/J_0) = s \ln (2T/V_1) / \ln(Tu/\Delta)$

$J \sim V^\beta$, $\beta = s / \ln (Ts/\Delta)$, $\alpha/\beta = \ln(L/a_b) / \ln(Ts/\Delta) > 1$

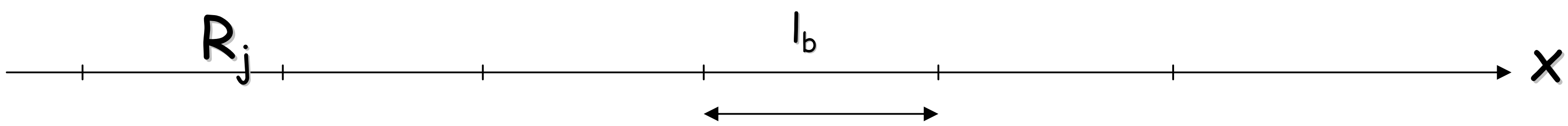
Regime diagram

s = log of tunneling transparency

Δ = typical charging energy



$R=\sum_{j=1}^{N} \underbrace{R_0 \exp u_j}_{R_j}$



$$p(R_j) \sim R_j^{-1-\kappa}, \kappa = s^{-1} \ln[\frac{TR}{\Delta R_j}]$$

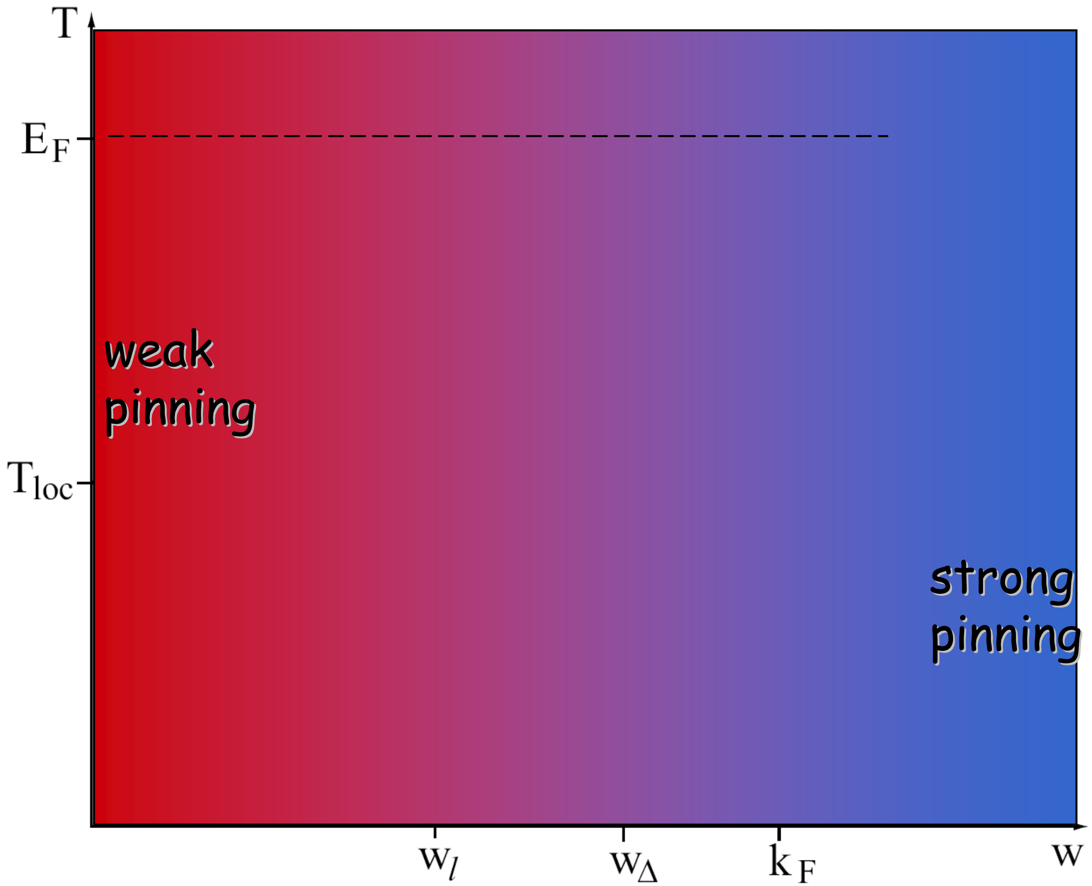
$\sim$ power law $\rightarrow$ R_j Levy random walk

P(R) Levy distribution , $\kappa \ll 1$: Frechet distribution

$$P_R(R) \sim R^{-1-\kappa} \exp[-(R^*/R)^\kappa]$$

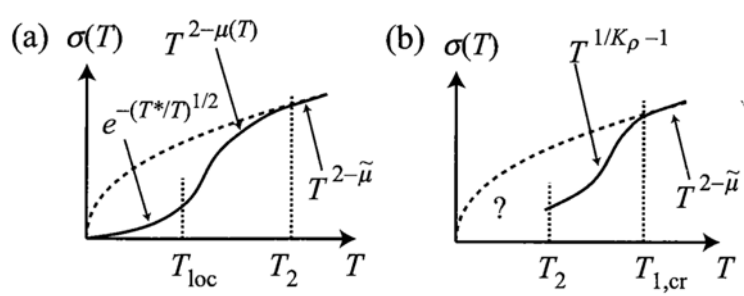
$$\kappa^* \ln(R^*/R_0) = \ln(l_b/\kappa^*), \quad \kappa^* = \kappa(R^*)$$

Weak pinning: $w, a \rightarrow 0$, $\xi_0 = (a/w^2)^{1/3}$ fixed



$a = \xi_0^3 w^2$,

Giamarchi, 2004:
"Quantum Physics in One Dimension"



weak strong

$T > T_{1,cr}$: single impurity weak

$T < T_2$: collective effects

Conclusions:

- linear and non-linear conductivity , field and temperature cross-over between single and many impurity tunneling
- low field and temperature: Mott-Shklovskii-VRH
  larger V,T: Kane-Fisher - power law behavior
- Kane-Fisher "single-dominant-barrier" theory is not valid in long wires that contains many (> 100 ?) impurities
- true power-law exponents exceed the single-barrier ones by a "large" log-factor (perhaps, by 2 or 3 in practice)!
- resistance is controlled by "breaks" –dense clusters of impurities
- global weak/strong pinning regime diagram

Model: Bosonization:

displacement from perfect order;

Charge density $\rho(x) = \sum_n \delta(x - x_n)$ $x_n = n\,\pi/k_F - \phi(x_n)/k_F$

$$\rho(x) = \Psi^+\Psi = (k_F + \partial_x\phi) \sum_n \delta(k_F x + \phi(x) - \pi\, n) = \pi^{-1}(k_F + \partial_x\phi)\sum_{-\infty}^{\infty} e^{2mi(k_F x + \phi)}$$

$$\Psi^+_F(x) = [\pi^{-1}(k_F + \partial_x\phi)]^{1/2} \sum_{m\ \mathrm{odd}} e^{im(k_F x + \phi)}\, e^{i\theta(x)}\ ;\quad 2[\partial_x\theta(x), \phi(x')] = i\pi\,\delta(x - x')$$

$$H = (\hbar^2/2m)\int dx\, |\nabla\Psi|^2 + 1/2\int\int dx\, dy\, V(x-y)\rho(x)\rho(y)$$

$$\Rightarrow \quad H = (\hbar/2\pi)\int dx\, \{v_J(\partial_x\theta)^2 + v_N(\partial_x\phi)^2\}$$

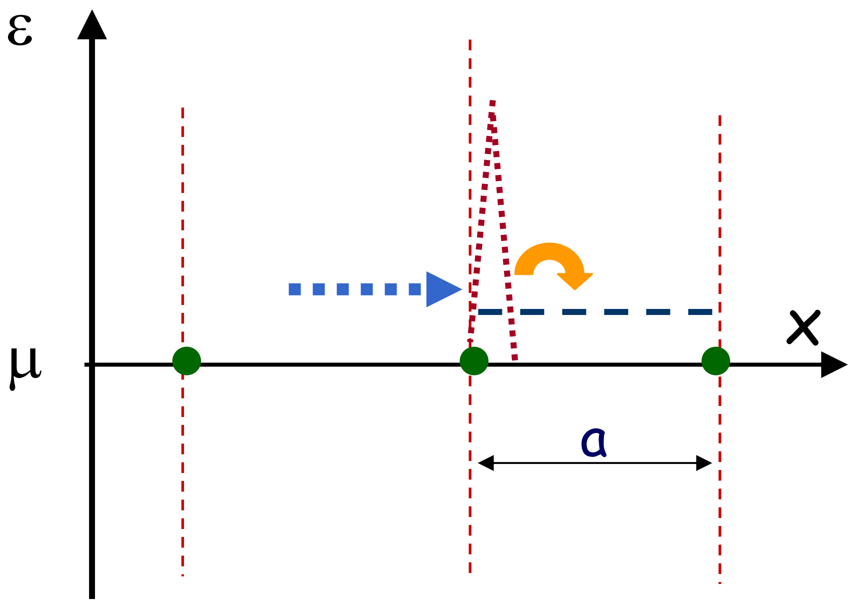
$v_J = \hbar k_F/m \equiv Kv$ $v_N = (\hbar\pi\,\kappa)^{-1} \equiv v/K$

$K=1$: free electrons

$K \sim 10^{-1} \ldots 10^{-2}$ CDWs, SDWs

$$S/\hbar = (2\pi K)^{-1} \int dx \int d\tau\ \{v^{-1}(\partial_\tau\phi)^2 + v(\partial_x\phi)^2 + 2KF\,\phi(x)\}$$

Large Voltage/temperature: power laws



Larkin and Lee '78

Tunneling action dominated by spreading of charge:

$$S_{tun} \sim \int d\tau\, E(\tau) \sim \int d\tau\, (gx(\tau))^{-1} \sim -K^{-1} \ln (E_{final}/E_{initial})$$

$$s \rightarrow s_{eff} \approx s + 2K^{-1} \ln [\min(1, K\Delta/T, \Delta/Fa)]$$

$\rightarrow T > \Delta$: $J_{k,k+1} \sim (\max(T,V)/\Delta)^{2/K}$ Kane-Fisher 1992

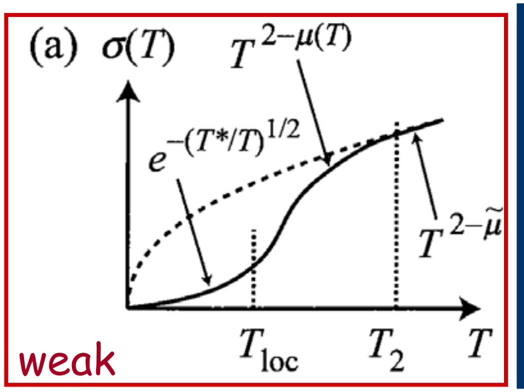
Strong and weak pinning

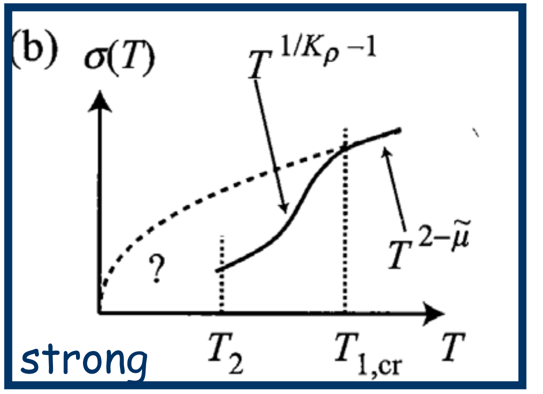
$T > T_{1,cr}$: single impurity weak

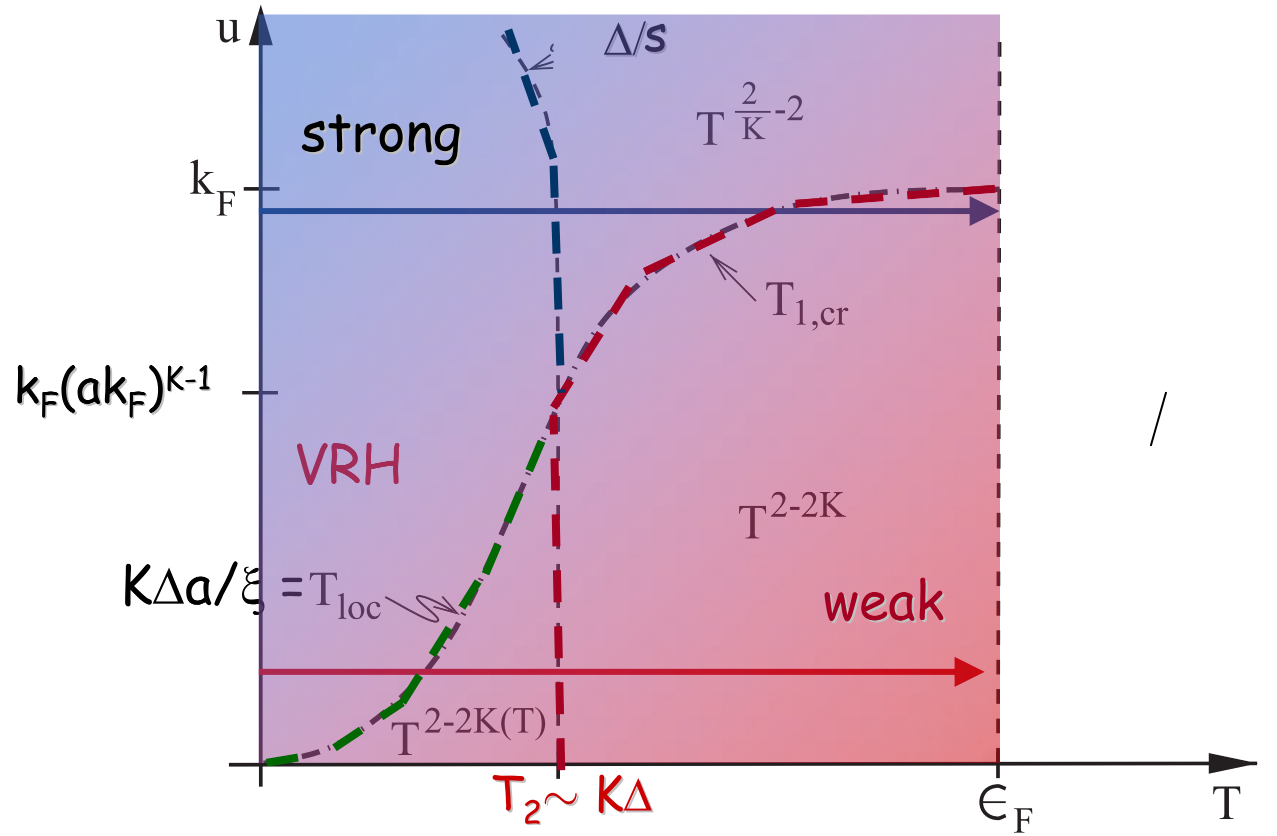
$T < T_2$: collective effects

$u < k_F$: SCHA $u \rightarrow u_{eff}$

Giamarchi, 2004: "Quantum Physics in One Dimension"







Experiments: sample dependence

SWCNT = single-wall carbon nanotube, MWCNT = multiple-walls carbon nanotube.

System	Diameter	Length	e^- mean free path	$g = K$	Size l of dots	Nb. N of dots
MoSe nanowires PRL **96** 076601 (2006)	1-20 nm 2-620 channels	1 μm	0.3-0.6 μm	0.15		
polyactelyne nanofibers PRB **72** 153302 (2005) PRL **93** 196601 (2005)	40-60 nm high $\times$ 100-300 nm	2 μm			l =10 μm	$N = 200$
polydiacetylene (PDA) PRB **69** 214203 (2004)		25 μm	hopping length: 10 nm at 120-300 K, 30 nm at 30 K			$N = 2500$
SWCNT NanoLetters **4** 2003 (2004)	$\leq$ 5 nm	0,4 cm	1 μm (defect-free)			
SWCNT Physica B **279** 200 (2000)	1 nm	300 μm				
MWCNT PRL **93** 086801 (2004)	5-25 nm	1.1-2.6 μm	25-250 nm			
InSb nanowires J Phys: Condens Matter **12** L303 (2000)	0.5 nm	0.1-1 mm			l=1-10 μm	N=2000-5000
CDW nanowires ($NbSe_3$) PRL **93** 176602 (2004)	30-300 nm 220 to 25640 chains	2-20 μm				
2 parallel GaAs nanowires Science **295** 825 (2002) PRB **68** 125312 (2003)	20-30 nm	2-10 μm	6 μm	0.67 ± 0.07 0.59 ± 0.03		

Tunneling rate

$$Z = Z_0 + iZ_1 = \int \mathcal{D}\varphi(x,\tau) e^{-1/\hbar \int_{-K_{\text{eff}}/2T}^{K_{\text{eff}}/2T} dy\, \mathcal{L}(\{\varphi\}, f)}$$

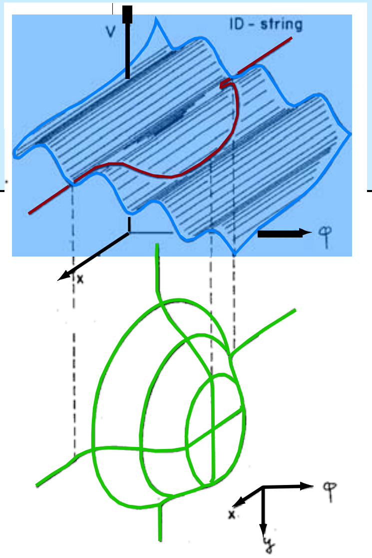
$$\hbar\Gamma = -2\,\mathrm{Im}\,F = 2\beta^{-1}\,\mathrm{Im}\ln Z,$$

$$\Gamma \approx 2(\beta\hbar)^{-1}\frac{Z_1}{Z_0}.$$

$$\frac{Z_1}{Z_0} \propto \mathrm{Im}\int \sqrt{\frac{S_{\text{inst}}}{\hbar}} dR\, e^{-S(R)/\hbar}$$

$$\propto e^{-S_{\text{inst}}/\hbar}\,\mathrm{Im}\int_{-R_c}^{\infty} dr \sqrt{\frac{S_{\text{inst}}}{\hbar}} e^{2\pi f r^2/(pK_{\text{eff}})}.$$

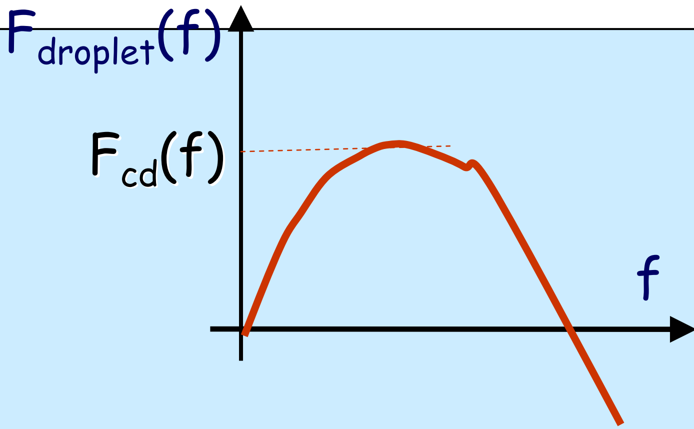
Quantum creep: Tunneling in applied field



A) Classical creep

nucleation = formation of a critical droplet
of free energy $F_{droplet}(f) = \sigma A_{surface} - f \cdot V_{droplet}$

$\Rightarrow$ creep velocity $\sim \tau^{-1} \sim e^{-F_{cd}(f)/T}$



B) Quantum creep

(see Coleman "fate of the false vacuum")

nucleation in space-time = formation of an instanton of action

$$S_{instanton} = \sigma A_{surface} - f \cdot V_{instanton}$$

$\Rightarrow$ creep velocity $\sim \tau^{-1} \sim e^{-S_{inst}(f)/\hbar}$