Luttinger liquids: dissipation and spin dependent transport

Thomas Nattermann, Zoran Ristivojevic and Leiming Chen

University of Cologne

PRL **101**, 016405 (2008)
PRB **80**, 195102 (2009)
PRL **104**, 076401 (2010)

Outline

Realizations Luttinger liquids

Haldane's Bosonization of fermions

Some old results

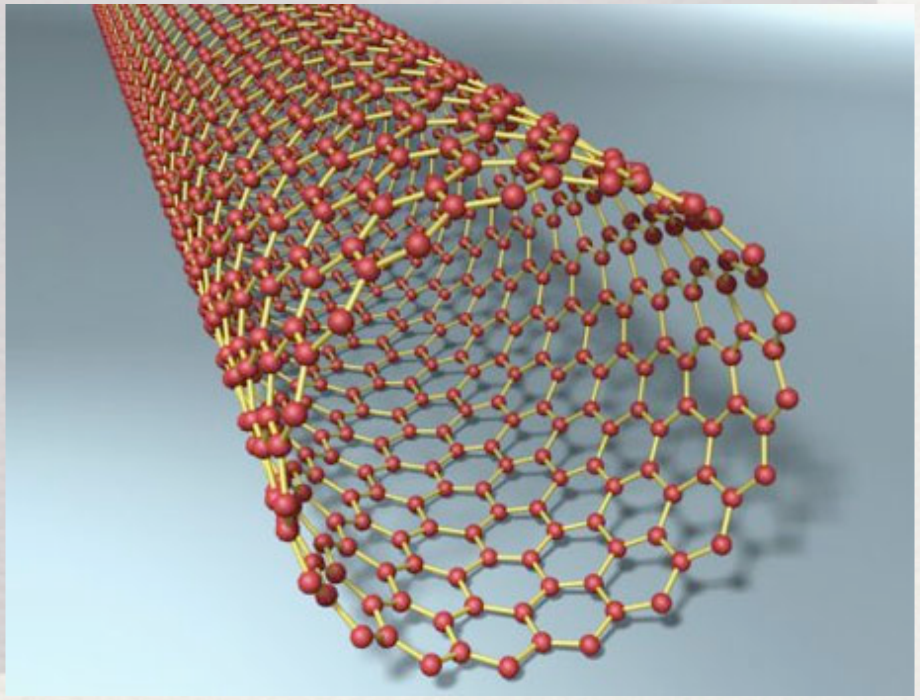
Dissipation in Luttinger liquids : 1 impurity
2 impurities

Spin dependent transmission through a double barrier

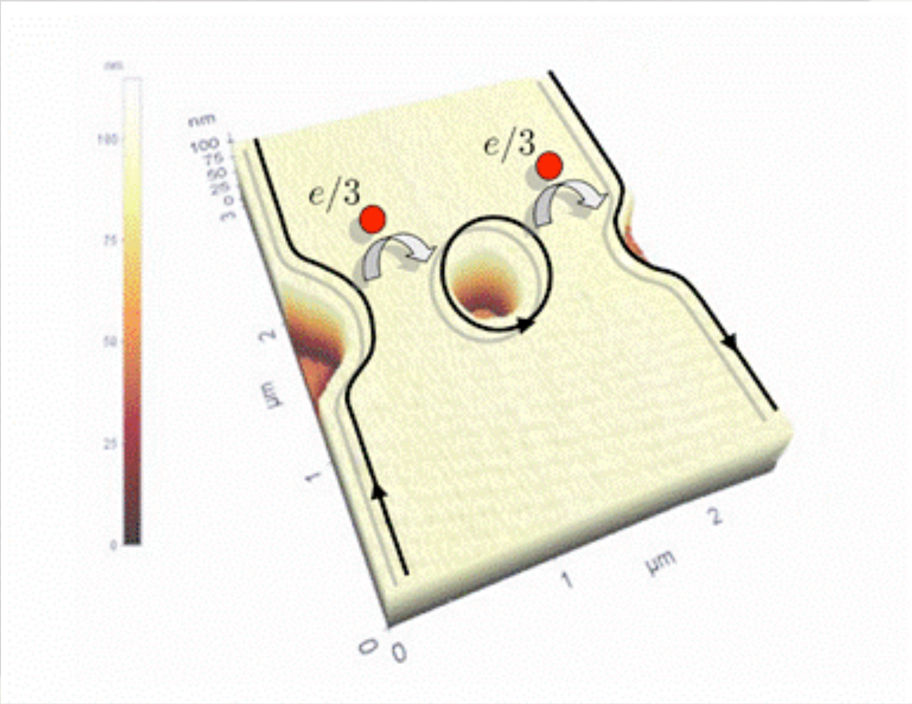
Summary

Realization of Luttinger liquids

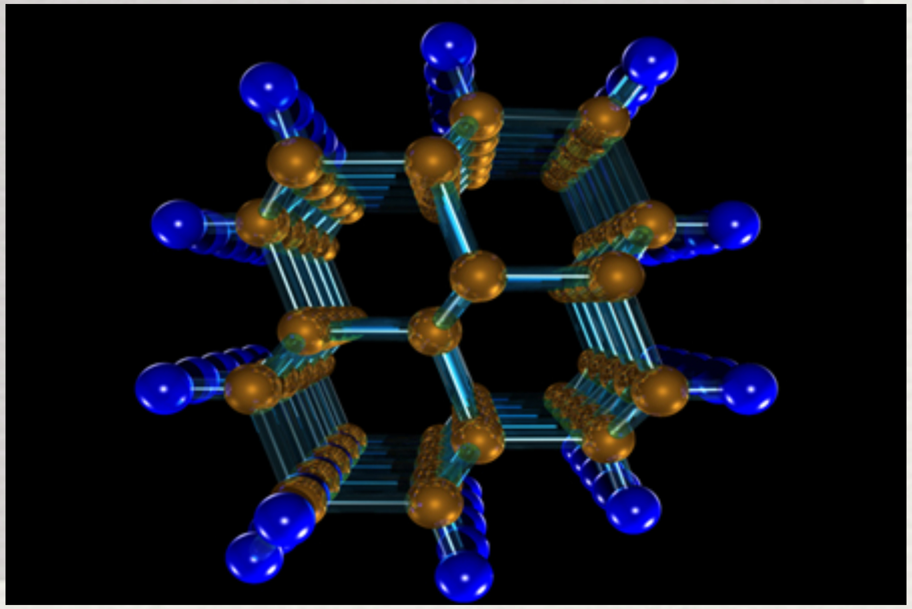
Carbon nanotubes



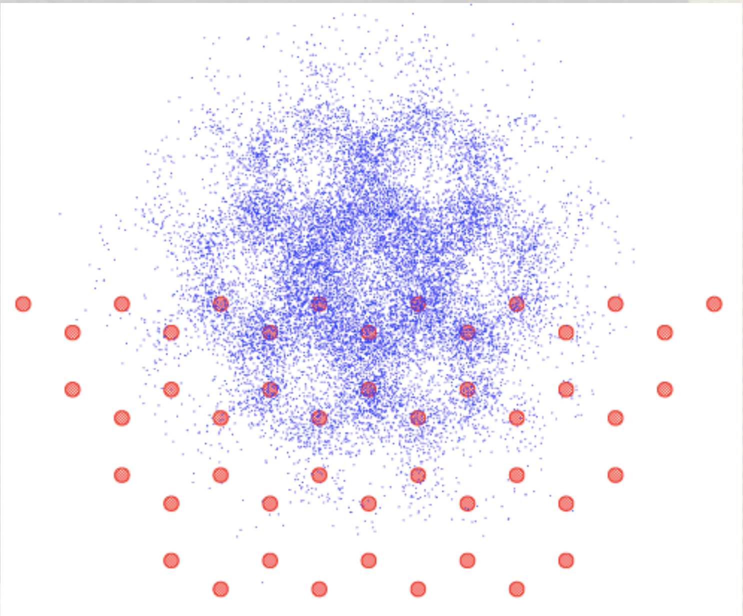
Quantum-Hall edge states,...



Quantum nanowires, organic conductors, chain-like compounds



Screw dislocation in hcp He 4



Haldane's bosonization

$$\Psi_F^+(x) = [\pi^{-1}(k_F + \partial_x\varphi)]^{1/2} \sum_{m\ odd} e^{im(k_F+\varphi)} e^{i\vartheta} \qquad 2[\partial_x\hat{\vartheta}(x), \hat{\varphi}(x')] = i\pi\delta(x - x')$$

Density

$$\rho(x) = \sum_n \delta(x - x_n) \approx \pi^{-1}(k_F + \partial_x\varphi)[1 + 2\sum_m \cos(2m(\varphi + k_F x))]$$

Path integral formulation

$$\frac{S}{\hbar} = \frac{1}{2\pi K}\int dx \int d\tau \left(\frac{1}{v}(\partial_\tau\varphi)^2 + v(\partial_x\varphi)^2\right)$$

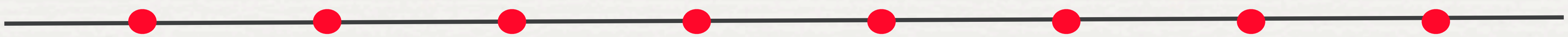
$$\frac{\hbar k_F}{m} = Kv, \quad \frac{1}{\hbar\pi\kappa} = \frac{v}{K}$$

Excitations: phonons

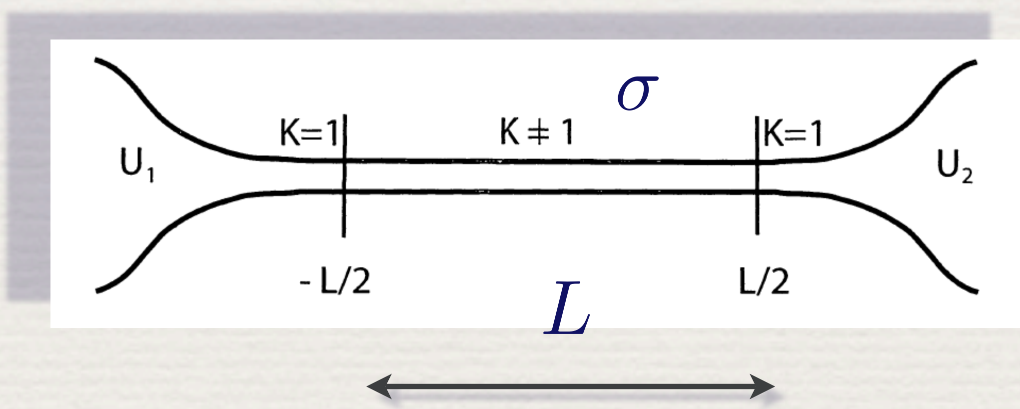
marginal supersolid: power law decay of spatial and superfluid correlations

K<1 (>1): repulsive
(attractive) interaction

Conductance of clean Luttinger liquids



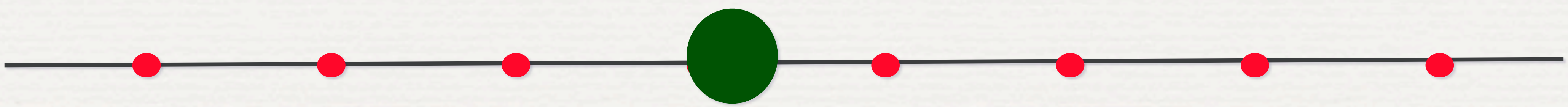
Clean wire $$G = dJ/dV = \frac{\sigma}{L} = (K)\frac{e^2}{h}$$



$$G^{-1} = \frac{h}{e^2} + \frac{L}{\sigma}$$

$$\sigma(\omega) = \frac{ie^2Kv}{\pi\hbar(\omega + i\delta)}$$

Conductance of a dissipative Luttinger liquids with a single impurity (1)



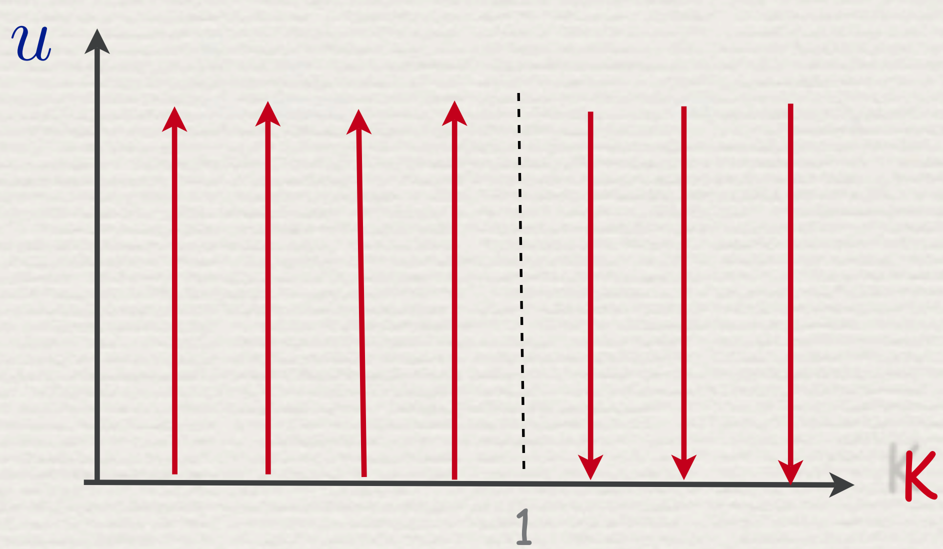
$$\frac{S_{imp}}{\hbar} = \frac{1}{2\pi K}\int d\tau u_0 \rho(0) \approx u \int d\tau \cos(2\varphi(0))$$

integrate over degrees outside impurity

$$\frac{S_0}{\hbar} = \frac{1}{\pi K}\int_\omega |\omega||\varphi_\omega|^2$$

<u>weak impurity</u>: RG for u

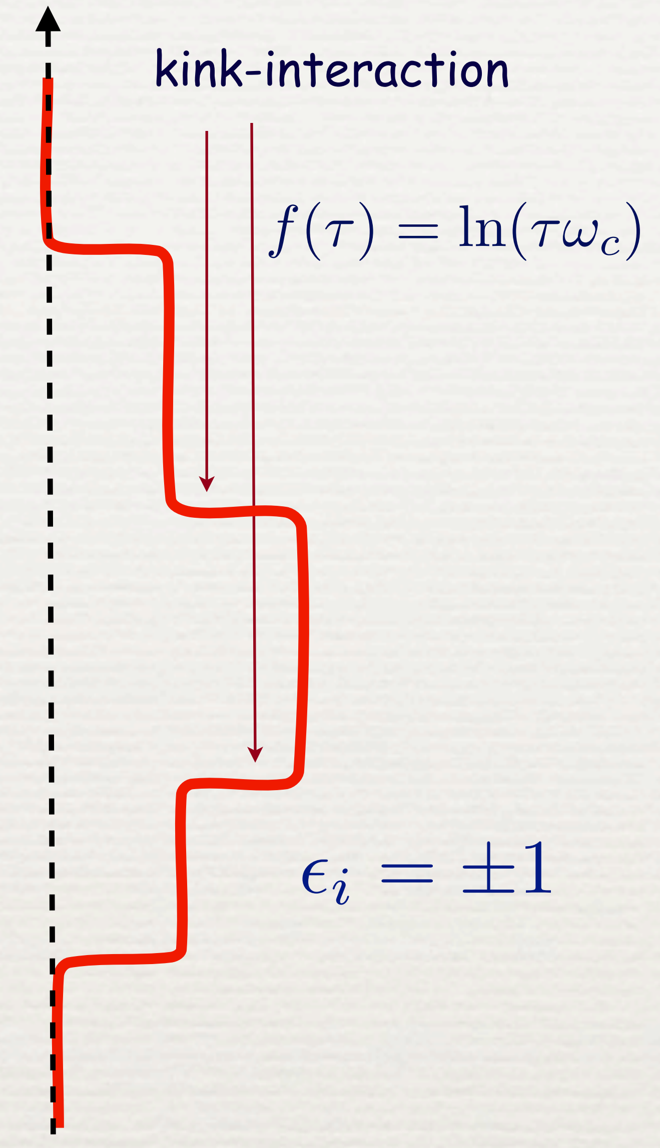
$$\frac{du}{d\ell} = (1-K)\,u$$



K<1 (>1) impurity (ir)relevant

Kane-Fisher '92

Conductance of Luttinger liquids with a single impurity (2)



τ

<u>strong impurity:</u> RG for tunneling transparency

$$\varphi(\tau) = \pi \sum_{i=1}^{2n} \epsilon_i \Theta(\tau - \tau_i)$$

$$Z = \sum_{n=0}^{\infty} \sum_{\epsilon_j = \pm 1} \frac{t^{2n}}{2n!} \int d\tau_1 ... d\tau_{2n} e^{2/K \sum \epsilon_i \epsilon_j f(\tau_i - \tau_j)}$$

$$t = e^{-S_{kink}/\hbar} \sim 1/u$$

$$Z = \sum_{n=0}^{\infty} \sum_{\epsilon_j = \pm 1} \frac{t^{2n}}{2n!} \int d\tau_1 ... d\tau_{2n} e^{-\frac{\pi}{4K} \int_\omega |\omega|^{-1} | \sum_{j=1}^{2n} \epsilon_j e^{i\omega\tau_j}|^2}$$

$$e^{-\frac{\pi}{4K} \int_\omega |\omega|^{-1} | \sum_{j=1}^{2n} \epsilon_j e^{i\omega\tau_j}|^2} = Z_0^{-1} \int D\vartheta e^{-\frac{K}{\pi} \int_\omega |\omega| |\vartheta(\omega)|^2 + i \sum_{j=1}^{2n} 2\vartheta(\tau_j)\epsilon_j}$$

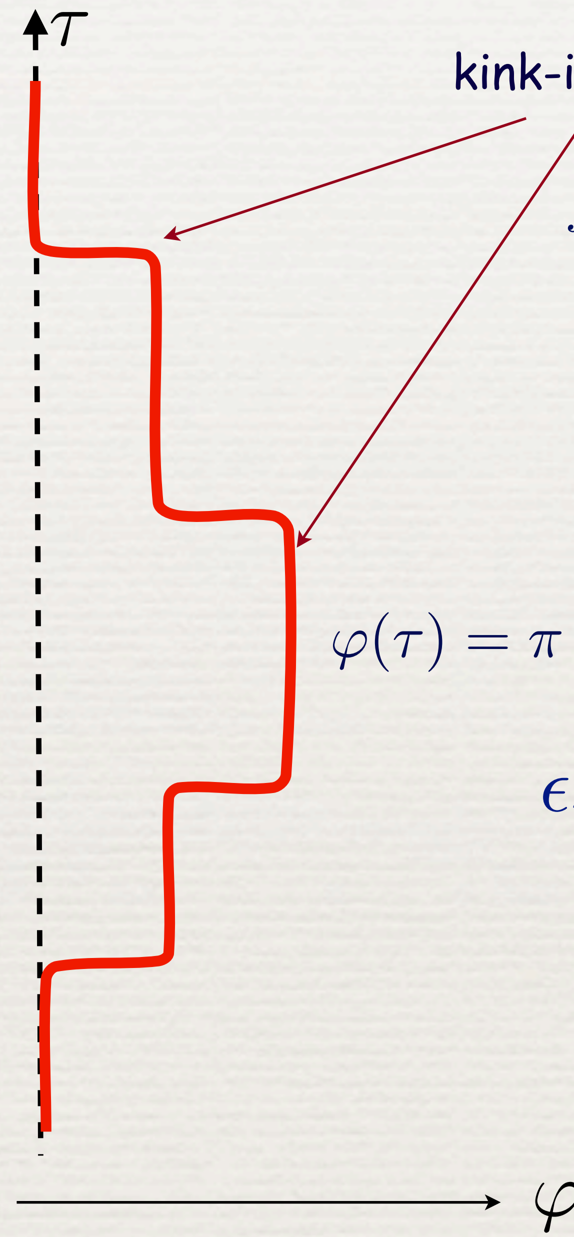
$$Z_0 = \int D\vartheta e^{-\frac{K}{\pi} \int_\omega |\omega| |\vartheta(\omega)|^2}$$

$$Z = Z_0^{-1} \int D\vartheta e^{-\frac{K}{\pi} \int_\omega |\omega| |\vartheta(\omega)|^2} \sum_{n=0}^{\infty} \frac{1}{2n!} \int d\tau_1, ... d\tau_{2m} (2t)^{2n} \cos(2\vartheta(\tau_1)) ... \cos(2\vartheta(\tau_{2m}))$$

Conductance of Luttinger liquids with a single impurity (2)

<u>strong impurity:</u> RG for tunneling transparency

$$t = e^{-S_{kink}/\hbar} \sim 1/u$$



$$f(\tau) = \ln(\tau\omega_c)$$

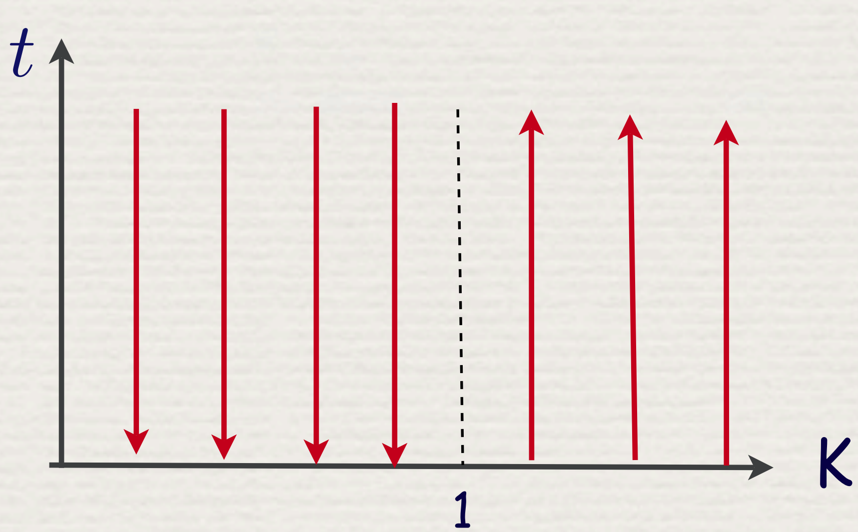
$$\varphi(\tau) = \pi \sum_{i=1}^{2n} \epsilon_i \Theta(\tau - \tau_i)$$

$$\epsilon_i = \pm 1$$

$$Z = \sum_{n=0}^{\infty} \sum_{\epsilon_j = \pm 1} \frac{t^{2n}}{2n!} \int d\tau_1 ... d\tau_{2n} e^{2/K \sum \epsilon_i \epsilon_j f(\tau_i - \tau_j)}$$

mapping on a dual model (Schmid '83):

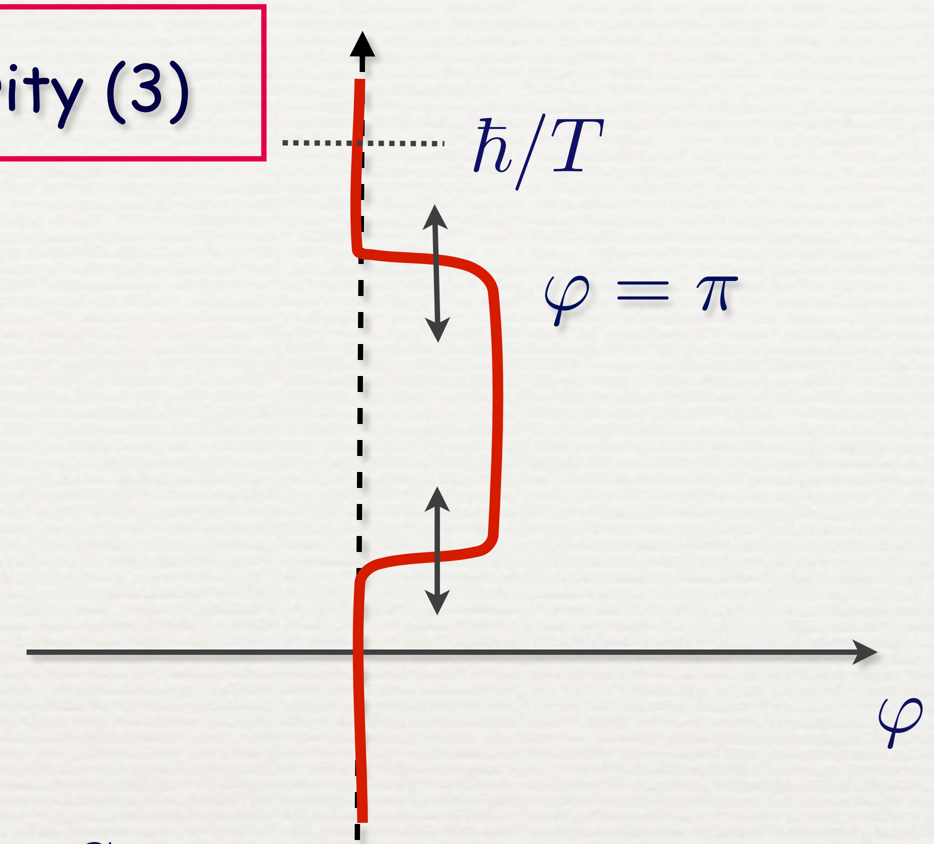
$$S_{eff} = \frac{1}{\pi} \int_\omega K|\omega||\vartheta_\omega|^2 - 2t \int d\tau \cos 2\vartheta(\tau)$$



$$\frac{dt}{d\ell} = (1 - K^{-1})t$$

→K<1 (>1) impurity (ir)relevant

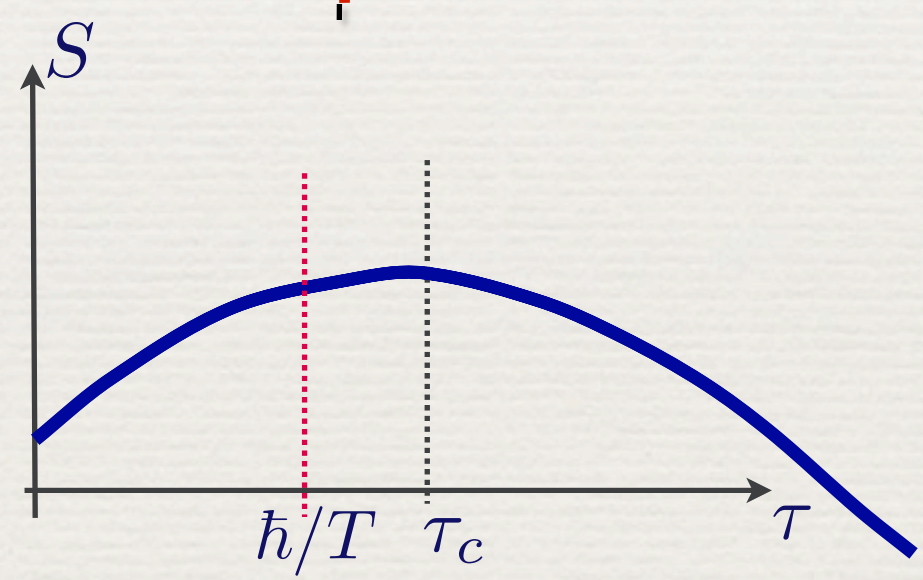
Conductance of Luttinger liquids with a single impurity (3)



K<1 -> strong impurity: instanton

Apply voltage V-> metastable state: decay rate

$$S(\tau)=\frac{2}{K}f(\tau)+2S_{kink}-eV\tau-\ln\tau$$



$$\Gamma=-2\frac{T}{\hbar}\mathrm{Im}\ln Z\approx-2\frac{T}{\hbar}\frac{\mathrm{Im}Z_1}{Z_0}$$

$$Z_1\approx i|S''(\tau_c)|^{-1/2}e^{-S(\tau_c)}$$

finite T: $\tau_c>\hbar/T \quad eV\to T$

kink-interaction $f(\tau)=\ln(\tau\omega_c)$

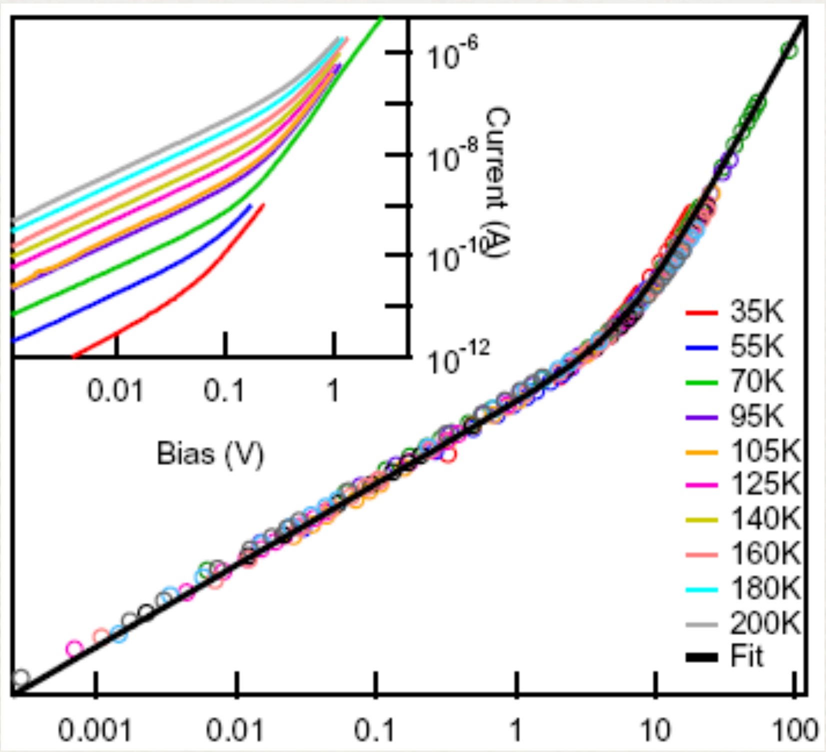
$$G=\frac{J}{V}\sim e^{-S(\tau_c)}\sim\left(\frac{max(eV,T)}{\hbar\omega_c}\right)^{2/K-2}$$

Kane-Fisher '92

Experiment: short wires $I / T^{a+1} \sim \max (V/T, V^{b+1}/T^{a+1})$

• MoSe Nanowires, $L \sim 1$ μm

$\frac{I}{T^{a+1}}$



Venkataraman, PRL (2006)

$\frac{\text{Voltage}}{\text{Temperature}}$

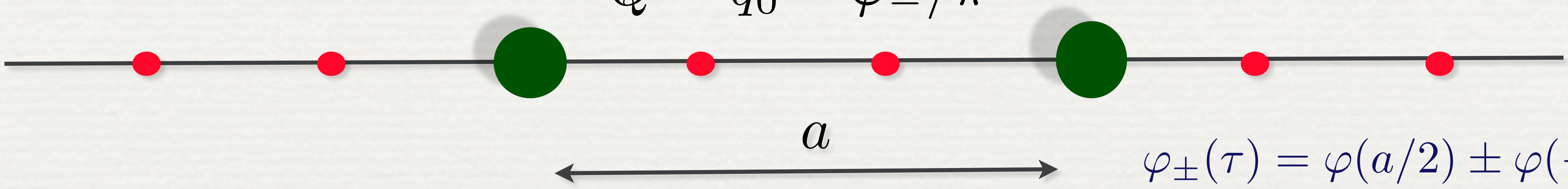
"Temperature" Exponent (a)
is close to
"Voltage" Exponent (b)

$$a \approx b$$

Agrees with the conventional "Luttinger-liquid" picture with a=b=2/K-2

Conductance of Luttinger liquids with two impurity (1)

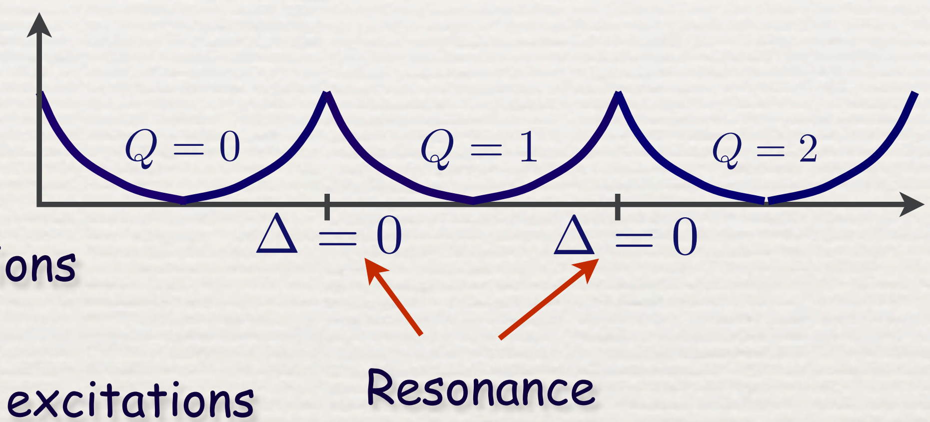
$$Q = q_0 - \varphi_-/\pi$$



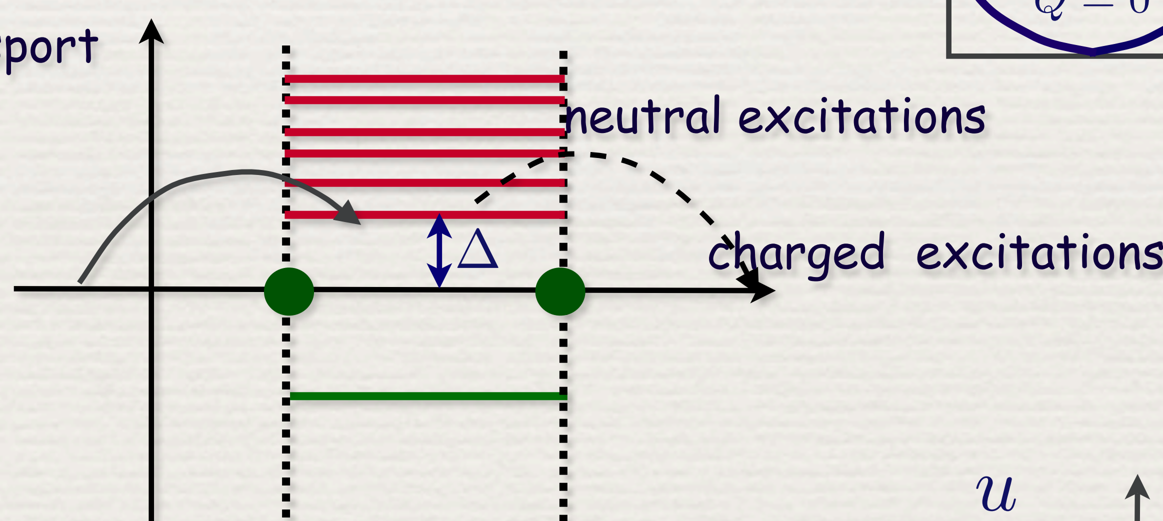
$$\varphi_\pm(\tau) = \varphi(a/2) \pm \varphi(-a/2)$$

Coulomb blockade :

$$\frac{S_{class}}{\hbar} = \frac{E_c}{2T}(Q-q_0)^2 + \frac{2k_f u_0}{\pi T}\cos\varphi_+ \cos(\pi Q)$$



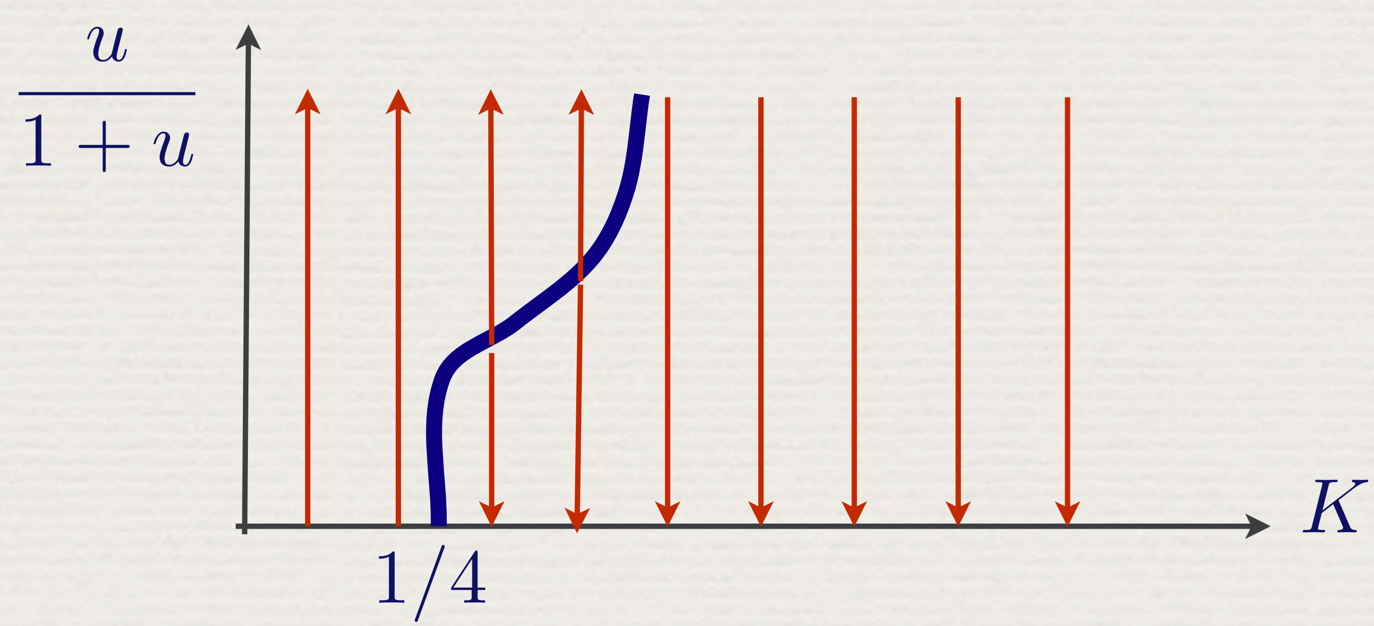
$$q_0 = k_F a/\pi = n + \frac{1}{2} - \Delta$$



$$E_\pm = \frac{E_c}{2}[1 \pm 2(Q-q_0)]$$

$$E_c = \frac{1}{\kappa a}$$

RG flow at resonance

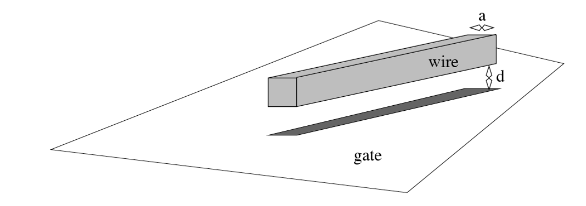


Dissipative Luttinger liquids

Couple LL to a gate with normal electrons

(Cazalilla et al. '06) ->

integrate over gate electrons -> dissipation for $K < \frac{2-s}{2p^2}$



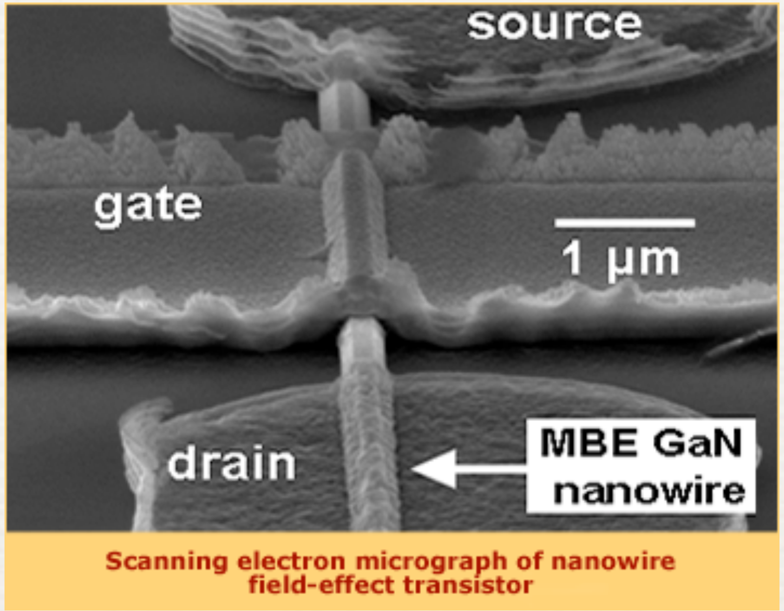
$$S_{diss} = \pi K \eta \int d\tau d\tau' dx \frac{(\varphi(x,\tau) - \varphi(x\tau'))^2}{(\hbar\beta \sin[\pi(\tau-\tau')/\hbar\beta])^{1+s}}$$

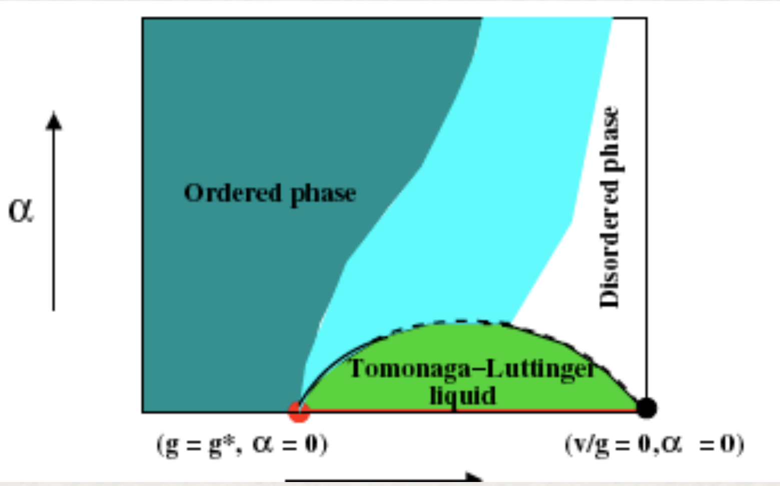
Diffusive length scale $L_\eta = (K\eta)^{-1}$

Coupling to 2D gate $L_\eta \sim a\sigma_{2D}/K$

-> on larger scales plasmons are diffusive,
Wigner crystal restored

-> finite conductivity $\sigma = 2KL_\eta e^2/h$

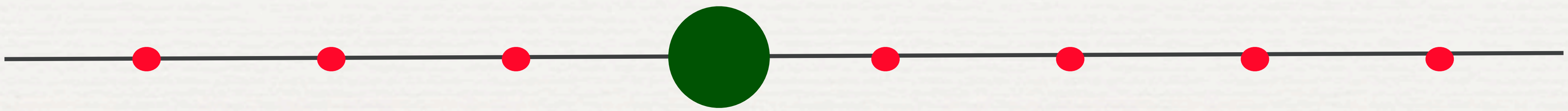




$$G^{-1} = \frac{h}{e^2}\left(1 + \frac{L}{2KL_\eta}\right)$$

$\Gamma = \hbar v K \eta$ plasmon damping

Conductance of **dissipative** Luttinger liquids with a single impurity (1)

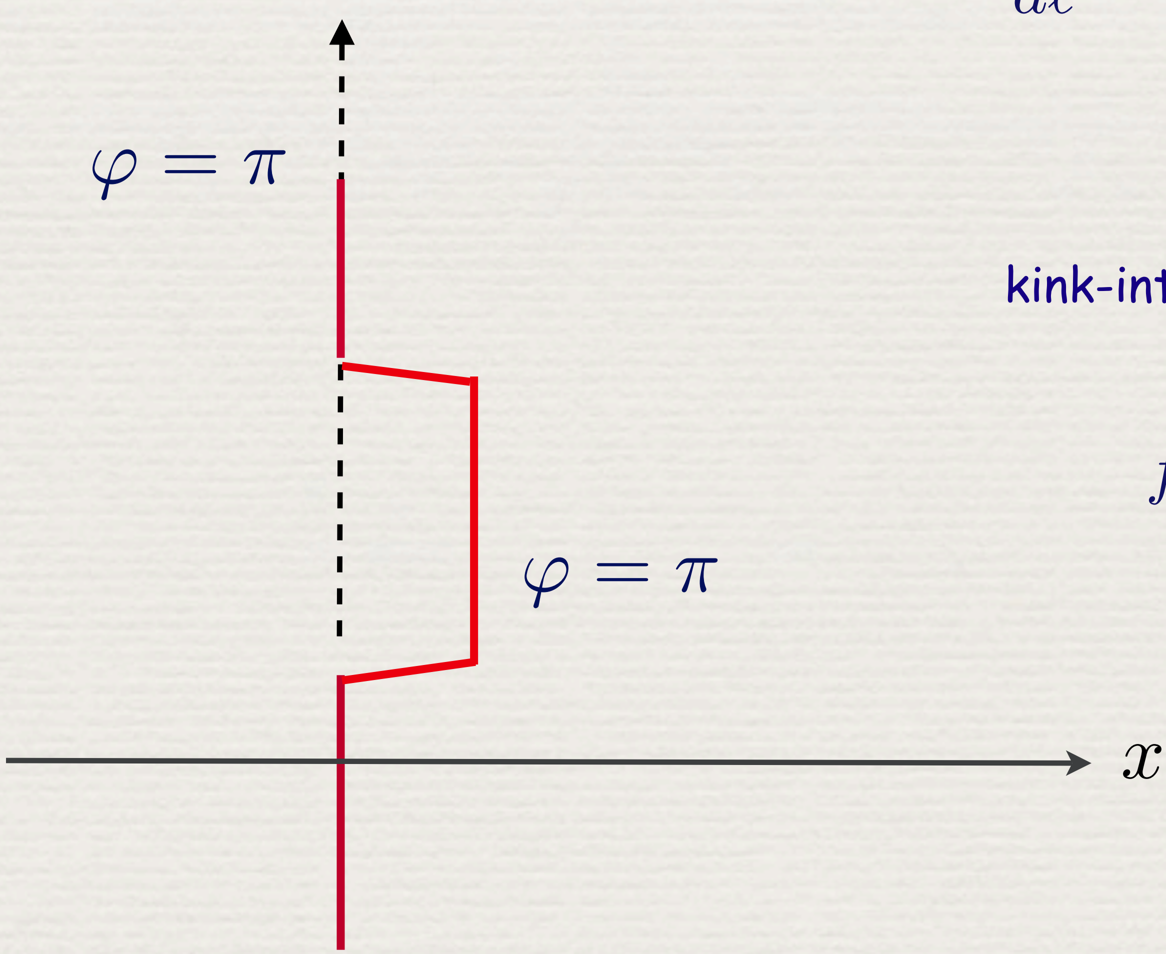


weak impurity: RG for u

$$\frac{du}{d\ell} = \left(1 - \frac{K}{\sqrt{1+\eta K e^{\ell}/\Lambda}}\right) u$$

strong impurity: RG for tunneling transparency => impurity always relevant for non-zero dissipation

$$\frac{dt}{d\ell} = \left(1 - \frac{\sqrt{1+\eta K e^{\ell}/\Lambda}}{K}\right) t$$

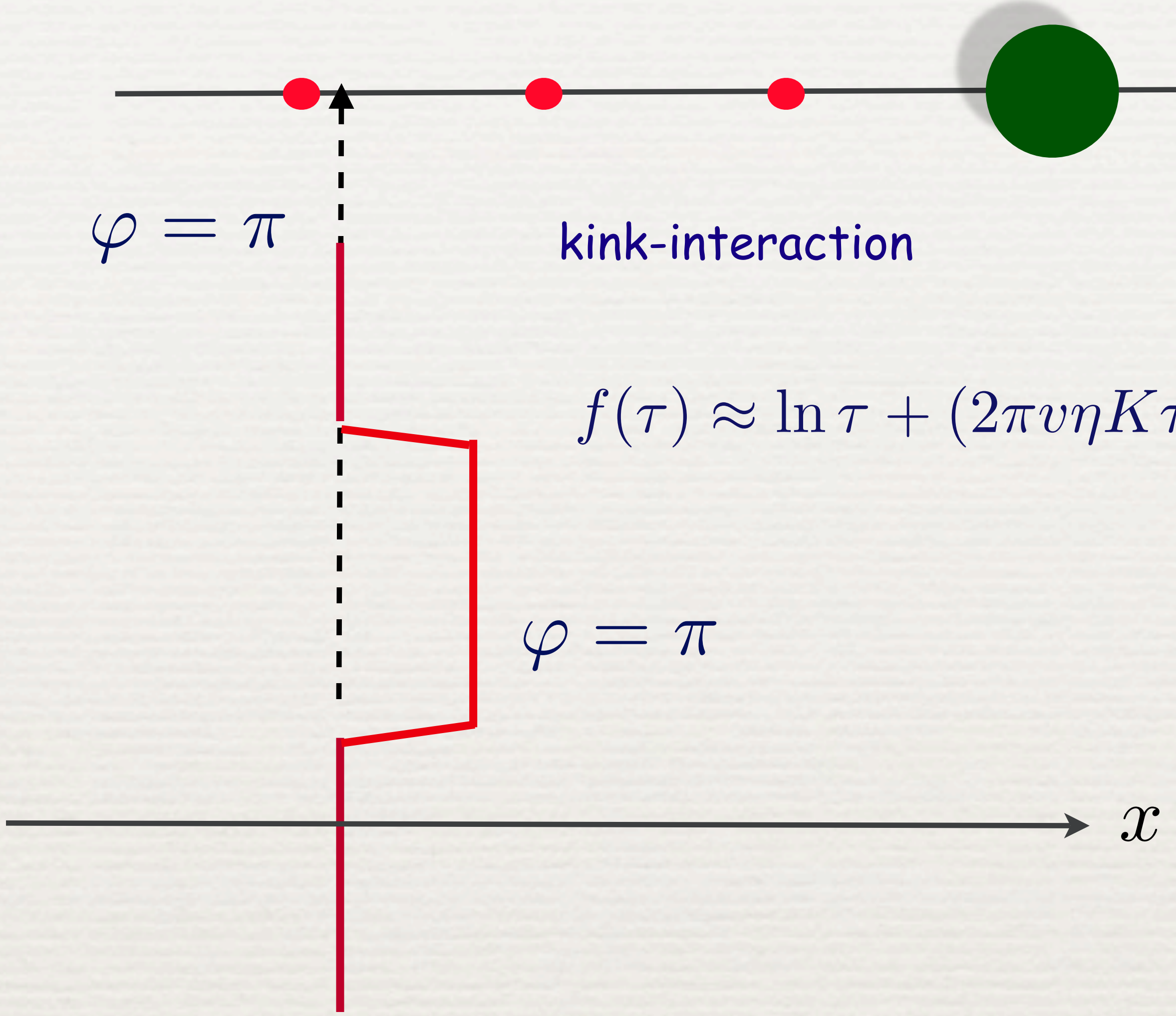


kink-interaction

$$f(\tau) = \int_0^{\omega_c} d\omega [1-\cos(\omega\tau)] \frac{\sqrt{\omega^2 + vK\eta|\omega|}}{\omega^2}$$

$$f(\tau) \approx \ln\tau + (2\pi v\eta K\tau)^{1/2}$$

Conductance of **dissipative** Luttinger liquids with a single impurity (2)



$$f(\tau) \approx \ln \tau + (2\pi v \eta K \tau)^{1/2}$$

$$J \sim A(V) t^2 e^{-4\pi\Gamma/(K^2 eV)}$$

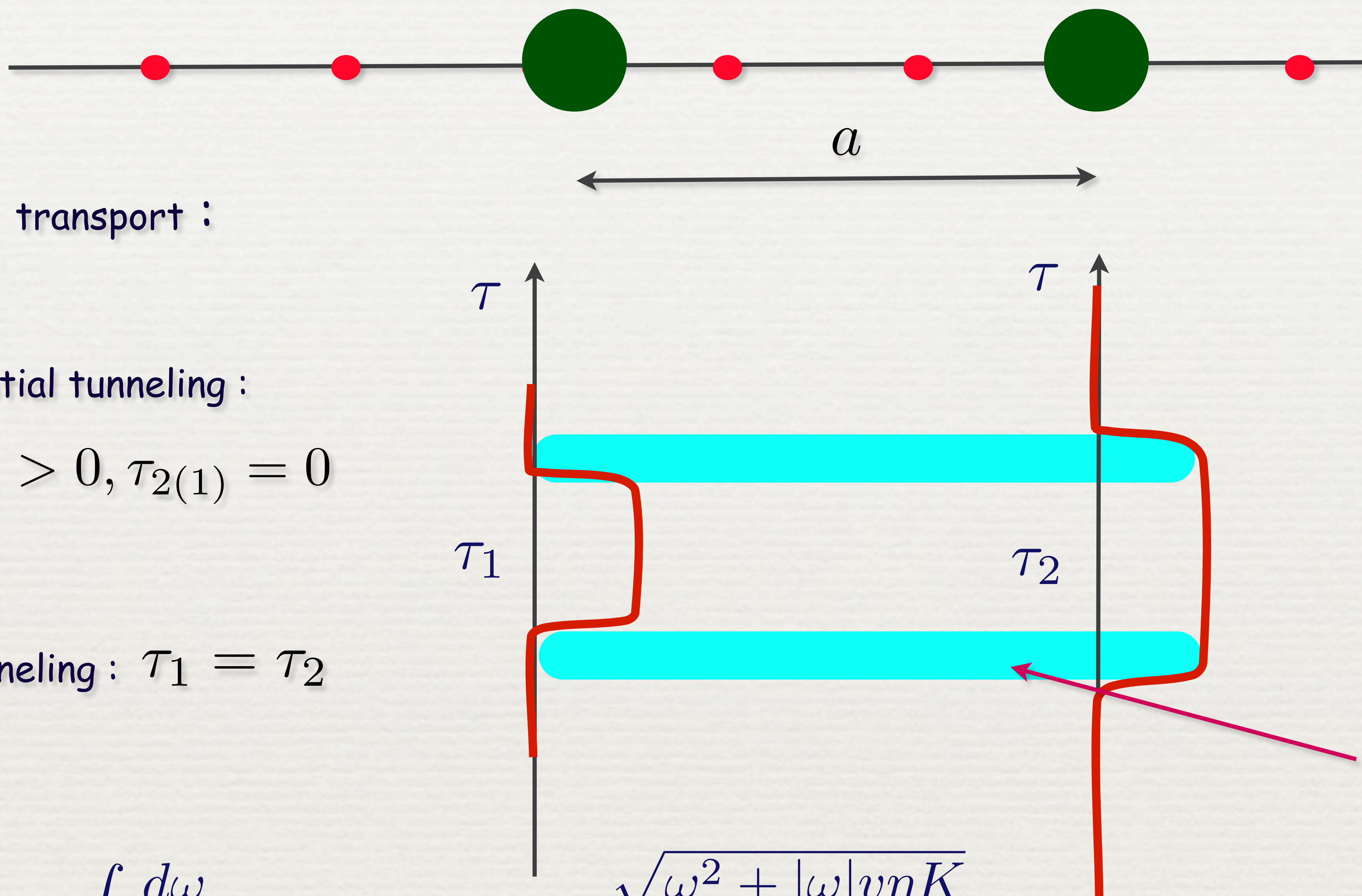
$$J \sim B(T) t^2 e^{-\sqrt{4\Gamma/(TK^2)}}$$

Cross-over to dissipation free behavior at $eV/\hbar \approx K\eta v \qquad \kappa T \approx C\eta$

Friedel-Oscillations

$$\langle \rho(|x| \gg L_\eta) \rangle \approx \frac{\pi^{-1} k_F \cos(2k_F x)}{(2\sqrt{y^2+y}+2y+1)^K}\left(1+\frac{KL_\eta}{|x|}\right)$$

Conductance of **dissipative** Luttinger liquids with two impurity



transport :

sequential tunneling :

$$\tau_{1(2)} > 0, \tau_{2(1)} = 0$$

co-tunneling : $\tau_1 = \tau_2$

Coulomb blockade:

$$f(\tau) = \int \frac{d\omega}{\omega^2}(1 - cos(\omega\tau))\frac{\sqrt{\omega^2 + |\omega| v\eta K}}{1 + e^{-a/v\sqrt{\omega^2+|\omega|v\eta K}}}$$

$$\frac{S_{\text{inst}}}{\hbar} = \frac{2S_{\text{kink}}}{\hbar}[\Theta_{H,\delta}(\tau_1) + \Theta_{H,\delta}(\tau_2)] + \frac{2}{K}[f(\tau_1) + f(\tau_2)] - \frac{eV_0}{2\hbar}(\tau_1 + \tau_2) + |\tau_1 - \tau_2| E_{\text{sign}(\tau_1 - \tau_2)}\frac{1}{\hbar},$$

Regime diagram at zero temperature

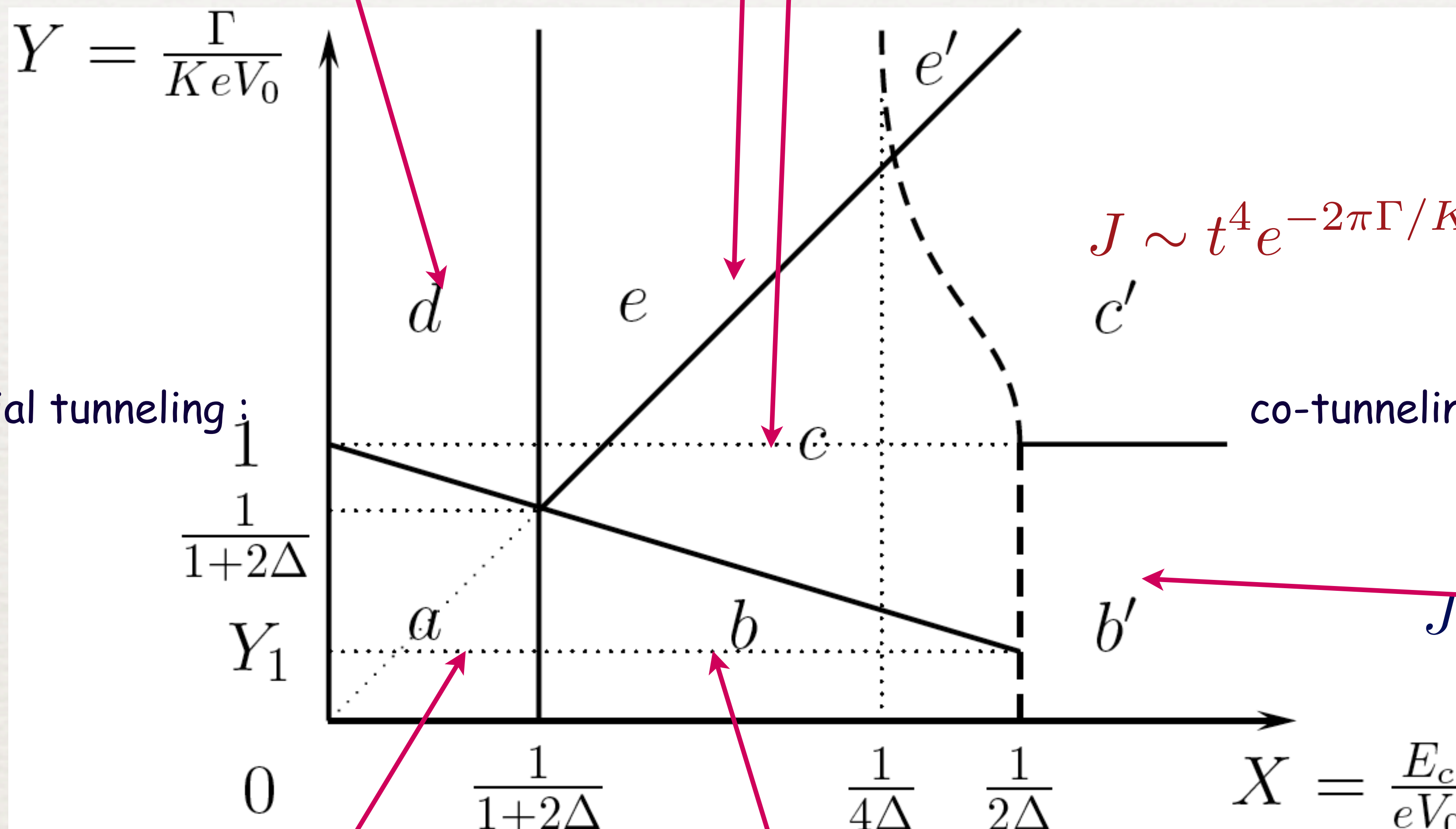
$J \sim t^2 e^{-4\pi\Gamma/(K^2 eV)}$

$J \sim t^2 e^{-\pi\Gamma/(K^2(eV-2|\Delta|E_c))}$

$J \sim t^4 e^{-2\pi\Gamma/K^2 eV}$

sequential tunneling :

co-tunneling



$J \sim t^4 E_c^{2/K} (eV)^{2/K-1}$

$J \sim t^2 (eV)^{2/K-1}$

$J \sim t^2 E_C^{1/K} (eV - 2|\Delta|E_c)^{1/K-1}$

$\Gamma = \hbar v K \eta$ plasmon damping

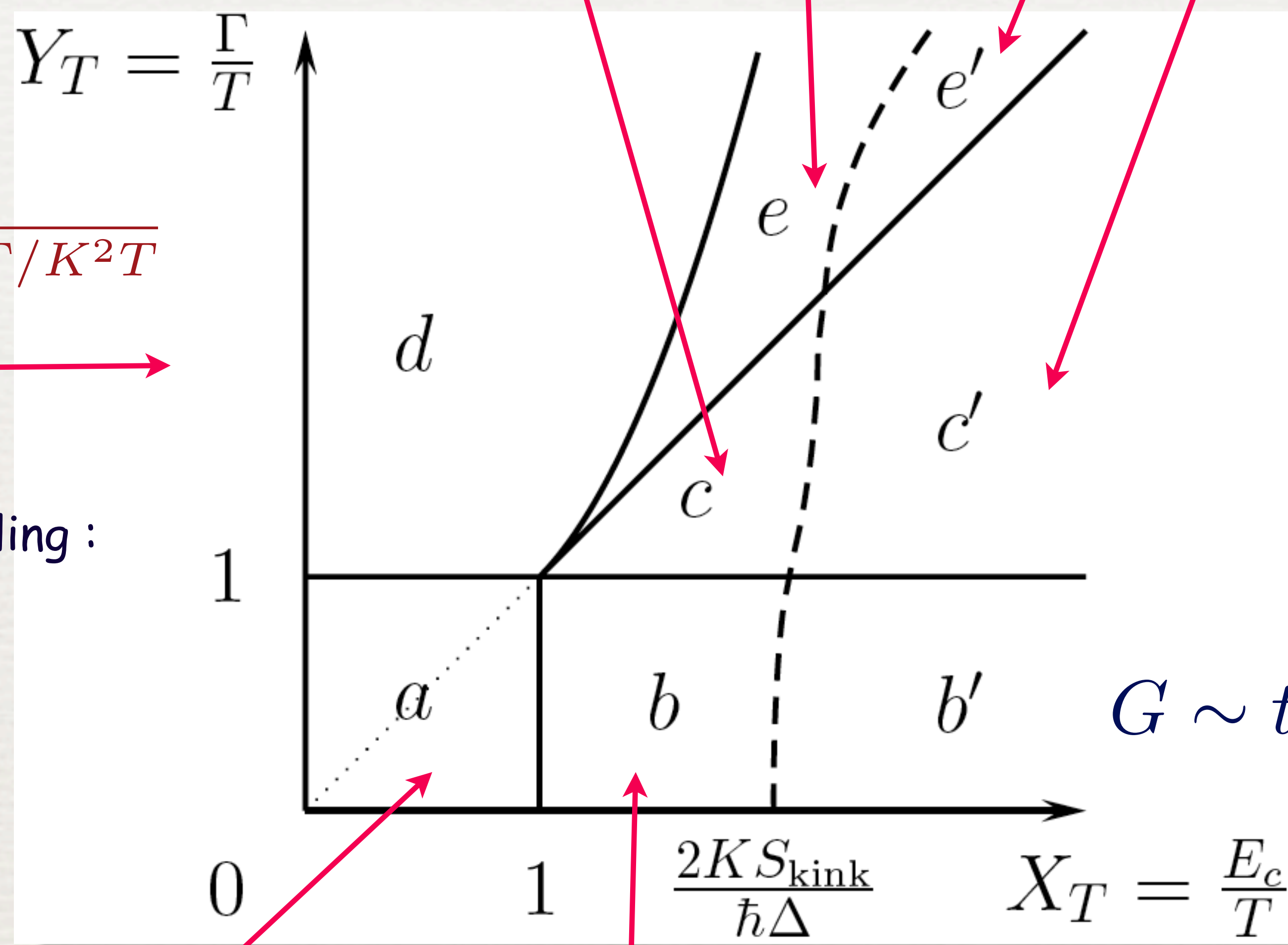
$G \sim t^2 e^{-E_c|\Delta|/T} e^{-\sqrt{2\pi\Gamma/K^2 T}} (e^{\sqrt{2\pi}\Gamma/E_c K} - 1)^{-1/K}$

$G \sim t^4 e^{-\sqrt{8\pi\Gamma/K^2 T}}$

$G \sim t^2 e^{-\sqrt{8\pi\Gamma/K^2 T}}$

sequential tunneling :

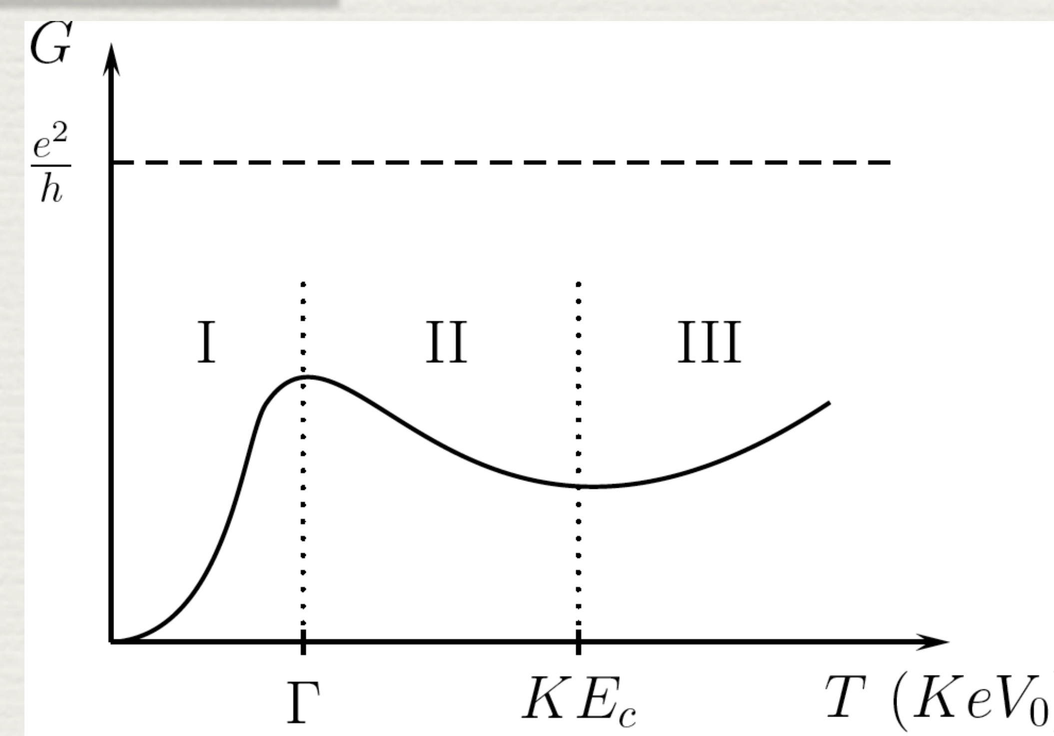
co-tunneling :



$G \sim t^4 T^{2/K-2}$

$G \sim t^2 T^{2/K-2}$

$G \sim t^2 E_C^{1/K} e^{-E_c|\Delta||T} T^{1/K-2}$



Spin dependent transport

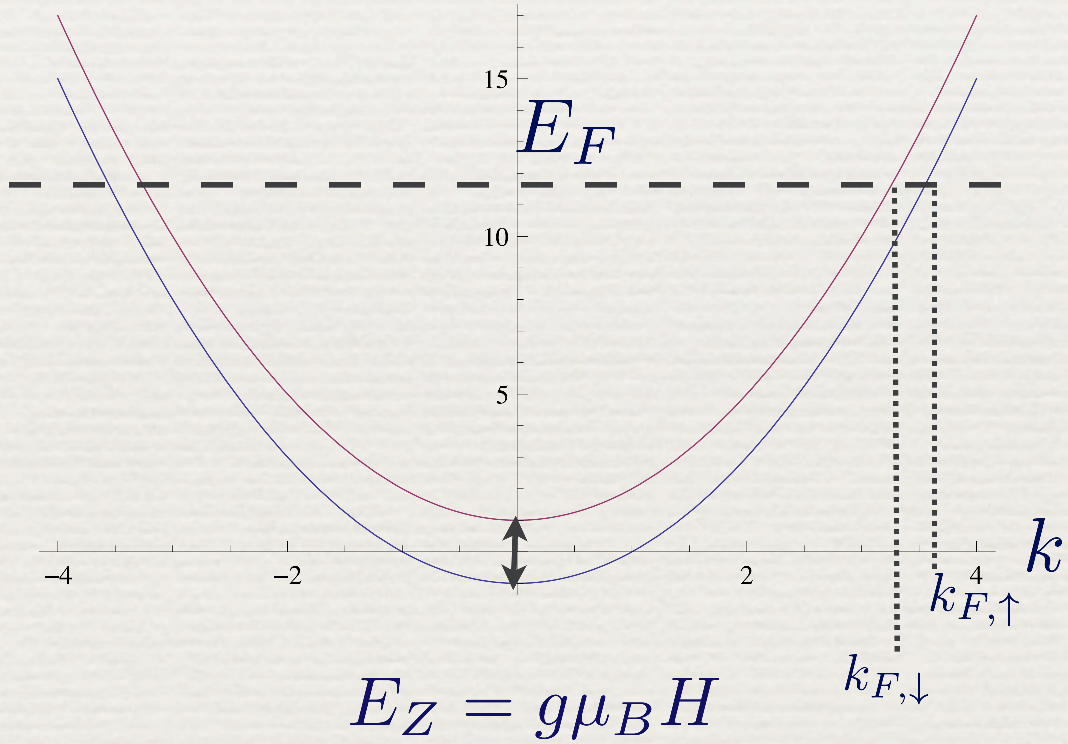
$$\varphi \to \varphi_s, \quad s = \uparrow, \downarrow$$

$$\rho(x) = \sum_n \delta(x - x_n) \approx \pi^{-1}(k_F + \partial_x \varphi)[1 + 2\sum_m \cos(2m(\varphi + k_F x))]$$

separate spin and charge

$$\varphi_\rho = (\varphi_\uparrow + \varphi_\downarrow)/\sqrt{2}$$

$$\varphi_\sigma = (\varphi_\uparrow - \varphi_\downarrow)/\sqrt{2}$$

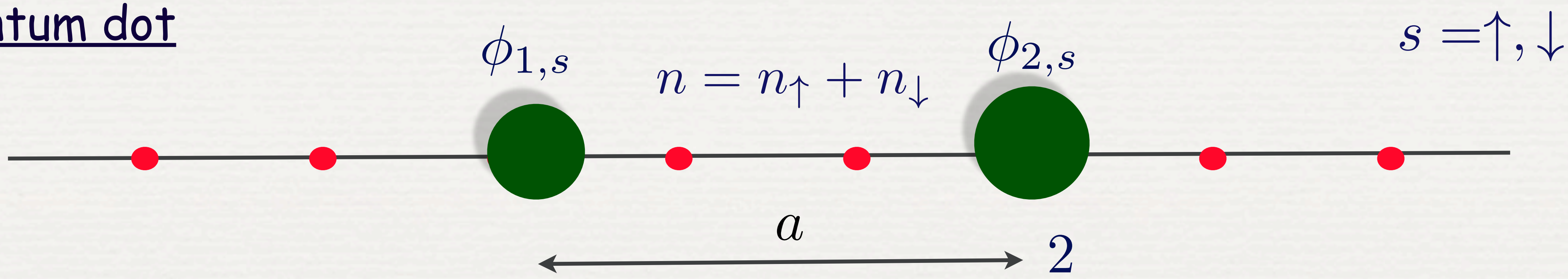


$$\frac{S}{\hbar} = \sum_{l=\rho,\sigma} \frac{1}{2\pi K_l} \int dx \int d\tau \left(\frac{1}{v_l} (\partial_\tau \varphi_l)^2 + v_l (\partial_x \varphi_l)^2 \right)$$

$$K_l = K/\sqrt{1 \pm \frac{K^2 W(0)}{\pi \hbar v_F}}, \quad l = \rho, \sigma$$

Magnetic field lifts degeneracy of Fermi points of spin up and down electrons

Quantum dot



Integrate degrees of freedom outside of impurities: -> 4 fields

$$\Phi_{l,k} = \varphi_{2\uparrow} + k\varphi_{1\uparrow} \pm (\varphi_{2\downarrow} + k\varphi_{1\downarrow}), \quad k = \pm, \quad l = \rho, \sigma$$

$$S_{eff} = \sum_{l=\rho,\sigma} \sum_{k=\pm} \int \frac{d\omega}{16\pi^2} \frac{\hbar|\omega|}{K_l} |\Phi_{lk}(\omega)|^2 + \int d\tau V_{eff}$$

$$V_{eff} = \sum_{l=\rho,\sigma} \frac{1}{2} U_l \Phi_{l-}^2 + \sum_{s=\uparrow,\downarrow} V_s [\cos(2\varphi_{1s} + k_{Fs}a) + \cos(2\varphi_{2s} - k_{Fs}a)]$$

Charging and magnetic energy of the dot: $U_l = \frac{1}{\kappa_l a}, \quad l = \rho, \sigma$

$\Phi_{\rho-} = \text{total extra charge}$

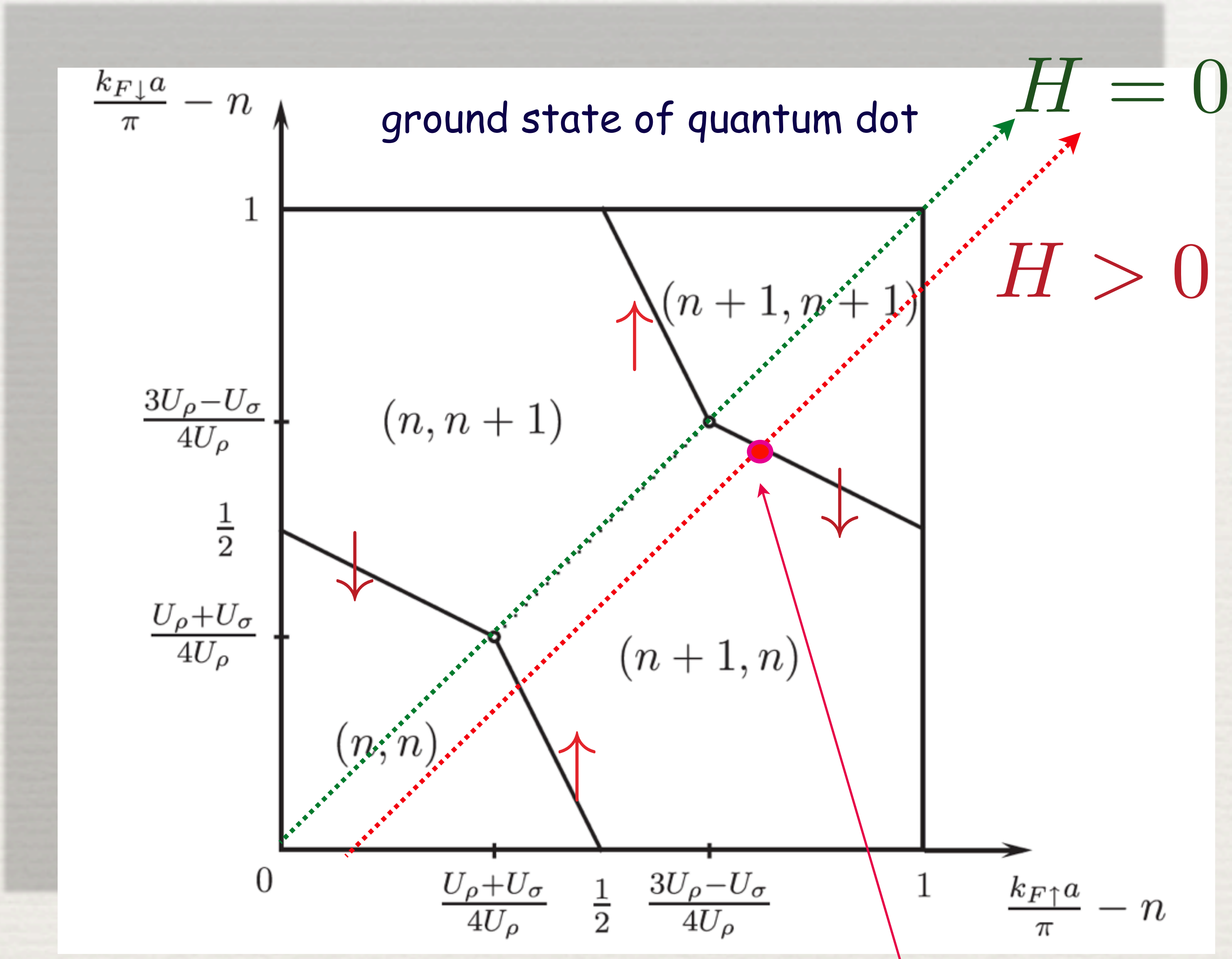
$\Phi_{\sigma-} = \text{total extra spin}$

$\frac{d}{dt} e\Phi_{\rho+} \sim \text{electric current}$

Strong impurities: -> number of electrons in the dot is integer

$V_i \gg U_{c,\rho}, U_{c,\sigma}$ -> minimization of classical action -> unique ground state

-> ground state degeneracy at particular lines -> resonance



spin-down electrons can tunnel sequentially =>

spin selective tunnel barrier

Strong impurities- continuation : consider resonance for spin down electrons

$$n_{\uparrow} = n_{2\uparrow} = n_{\uparrow} \quad fixed$$

->

$$S_{eff} = \int \frac{d\omega}{4\pi^2} \frac{\hbar|\omega|}{K_{eff}} (|\varphi_{1\downarrow}|^2 + |\varphi_{2\downarrow}|^2) + \int d\tau V_{eff}\left(-\frac{k_{F\uparrow}a+\pi}{2}, \frac{k_{F\uparrow}a+\pi)}{2}, \varphi_{1\downarrow}, \varphi_{2\downarrow}\right)$$

$$K_{eff} = \frac{2K_\rho K_\sigma}{K_\rho + K_\sigma} \qquad \frac{dt_\downarrow}{d\ell} = \left(1 - \frac{1}{2K_{eff}}\right) t_\downarrow$$

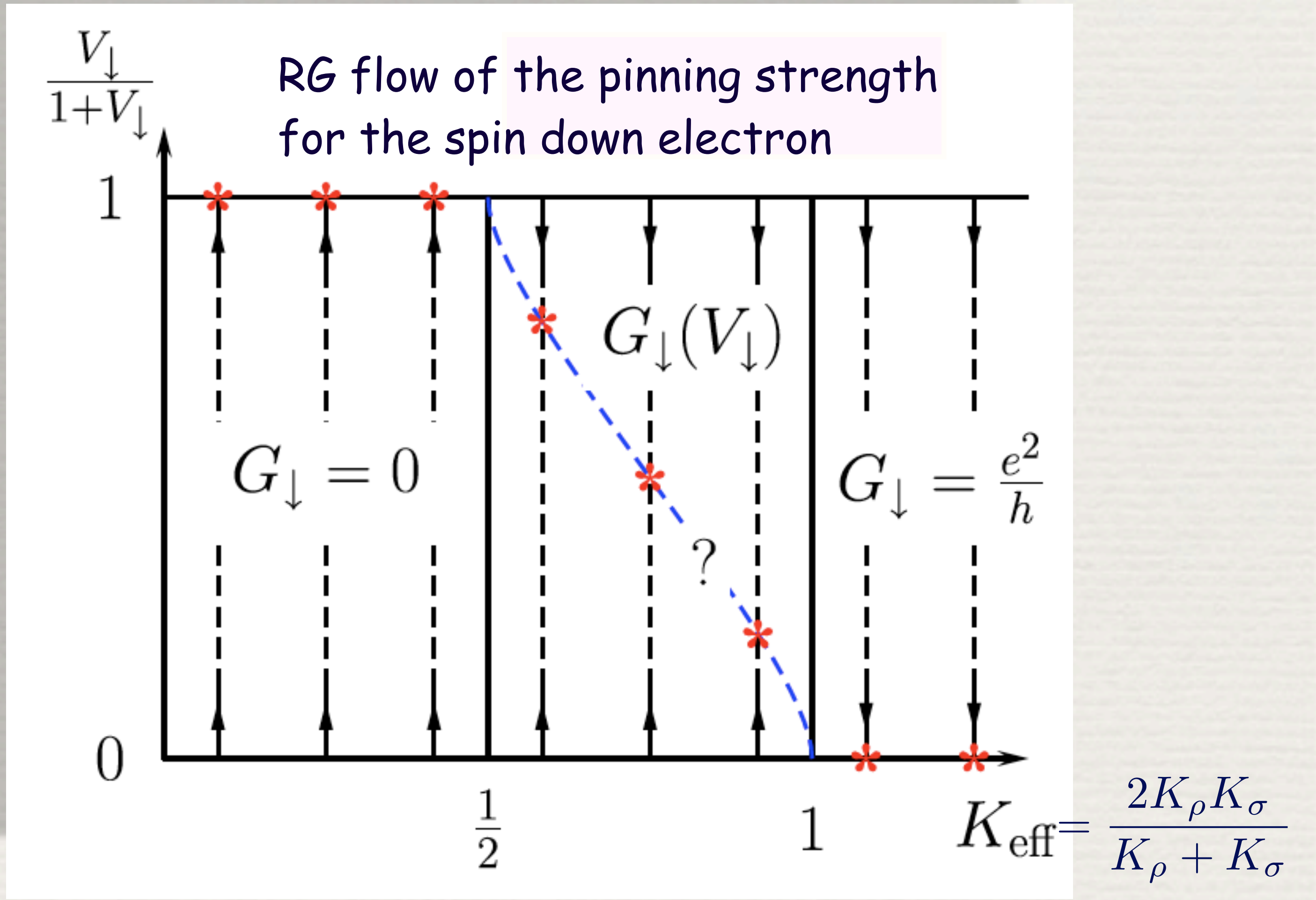
Off resonance: $t_\downarrow \to 0$

Weak impurities:

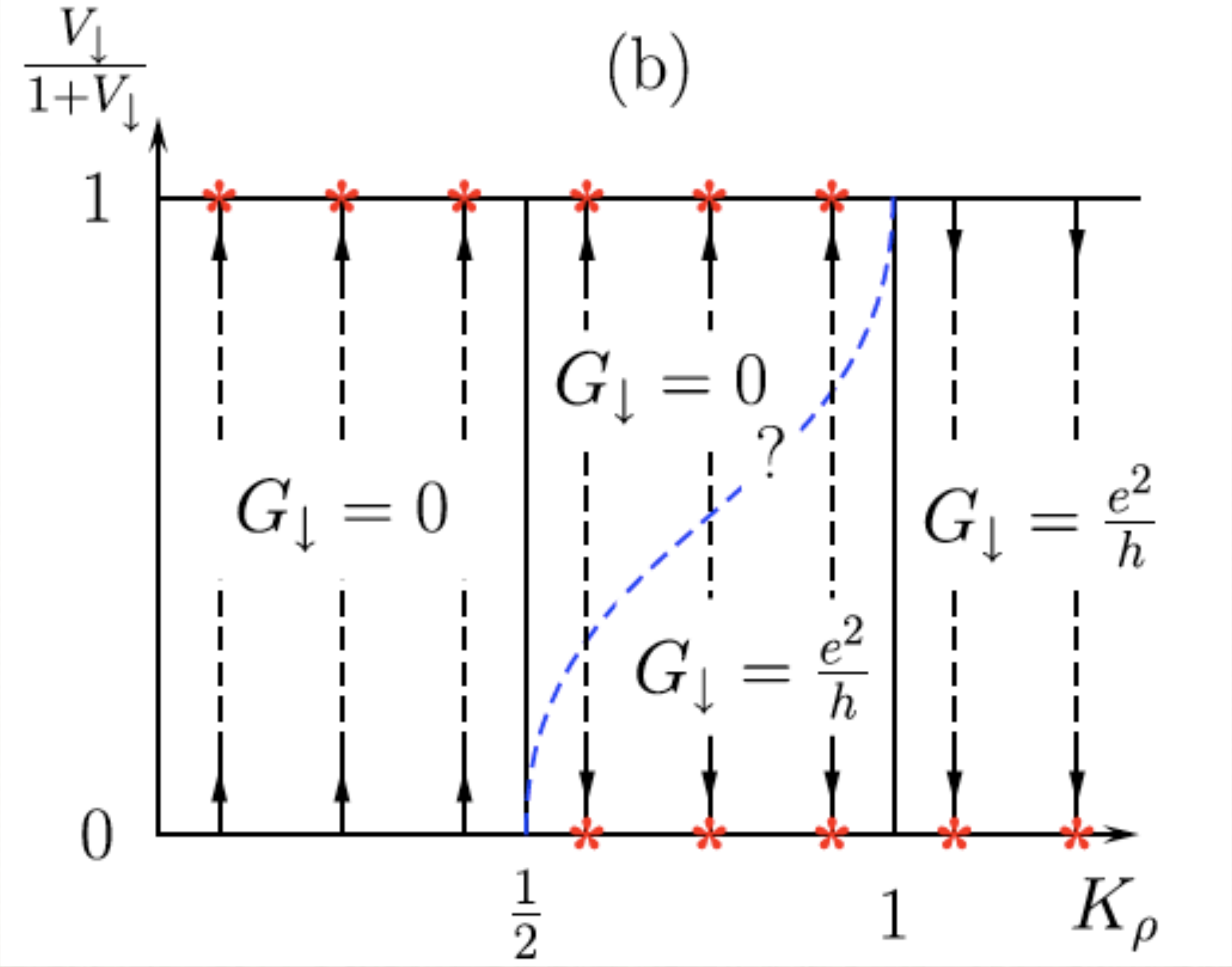
$$\frac{d}{dl} V_\downarrow = \left(1 - \frac{K_\rho + K_\sigma}{2}\right) V_\downarrow$$

$$G_\uparrow \sim (eV)^{2/K_{eff}-2}$$

$$G_\downarrow \sim \frac{e^2}{\hbar}(1 + [\pi V_\downarrow^* / E_F]^2)^{-1}$$



Weak impurities: resonance condition $\cos(k_{Fs}a) = 0$



Conclusions:

Luttinger liquids show dissipation when coupled to a gate.

Dissipation stabilizes Wigner crystal, strongly reduces the tunneling probability through impurities.

For tunneling through quantum dots in the presence of dissipation even under resonance conditions transmission is not perfect. Transmission is exponentially small when dissipation is sufficiently big.

In the absence of dissipation and under the influence of an external magnetic field the resonance condition is fulfilled either for up or down spin electrons which can act as a spin filter.