Quantum Creep and VRH in 1D Disordered Electron Systems

Transport in Luttinger liquids and 1D CDWs: strong pinning and global regime diagram

T. Nattermann & S. Malinin
University of Cologne, Germany

B. Rosenow

T. Giamarchi — University of Geneva
P. Le Doussal — ENS Paris

Outline

- Systems and model
- Single impurity
- Gaussian impurities

- Weak and strong pinning
- Instantons for strong pinning
- T, E-dependence of (non-) linear conductivities
- Conclusions

Systems under consideration

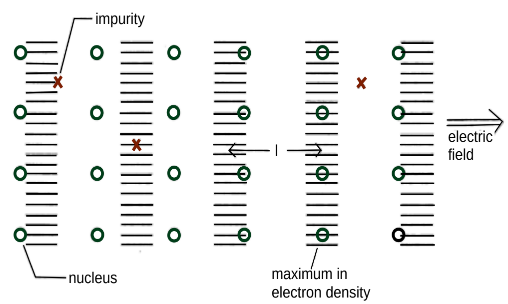
1D Quantum Fluids (Luttinger ´63, Haldane ´81)

$$\widehat{\psi}\dagger_B(x) = \sqrt{\widehat{\rho}(x)}e^{i\widehat{\theta}(x)}\,, \qquad \left[\partial_x\widehat{\theta}(x), \underline{\widehat{\phi}(x')}\right] = i\pi\delta(x - x')$$

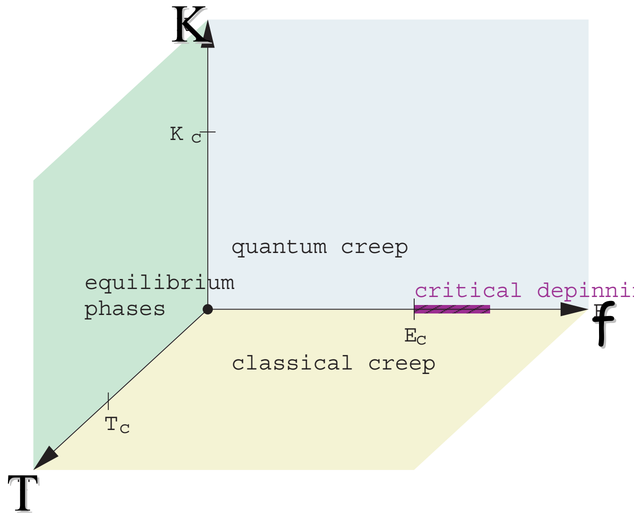
1D Mass-, Charge-, Spin-, Flux-Density Waves

$$\rho(\mathbf{x}, \phi) \sim \underline{\nabla\phi} + \rho_1 \cos\left(\mathbf{Q}\mathbf{x} + \underline{2\phi(\mathbf{x})}\right) + \ldots$$

charge density wave



Minimal model - energy densities



$$\rho(\mathbf{x},\phi) \sim \nabla\phi + \rho_1 \cos\left(\mathbf{Q}\mathbf{x} + 2\phi(\mathbf{x})\right) + \dots$$

- elastic energy: $\underline{\kappa^{-1}}\left(\nabla\hat{\phi}(\mathbf{x})\right)^2$

- kinetic energy: $\underline{K^2} \cdot \hat{P}^2(\mathbf{x})\,, \quad K \sim \frac{\hbar}{\sqrt{m}}$

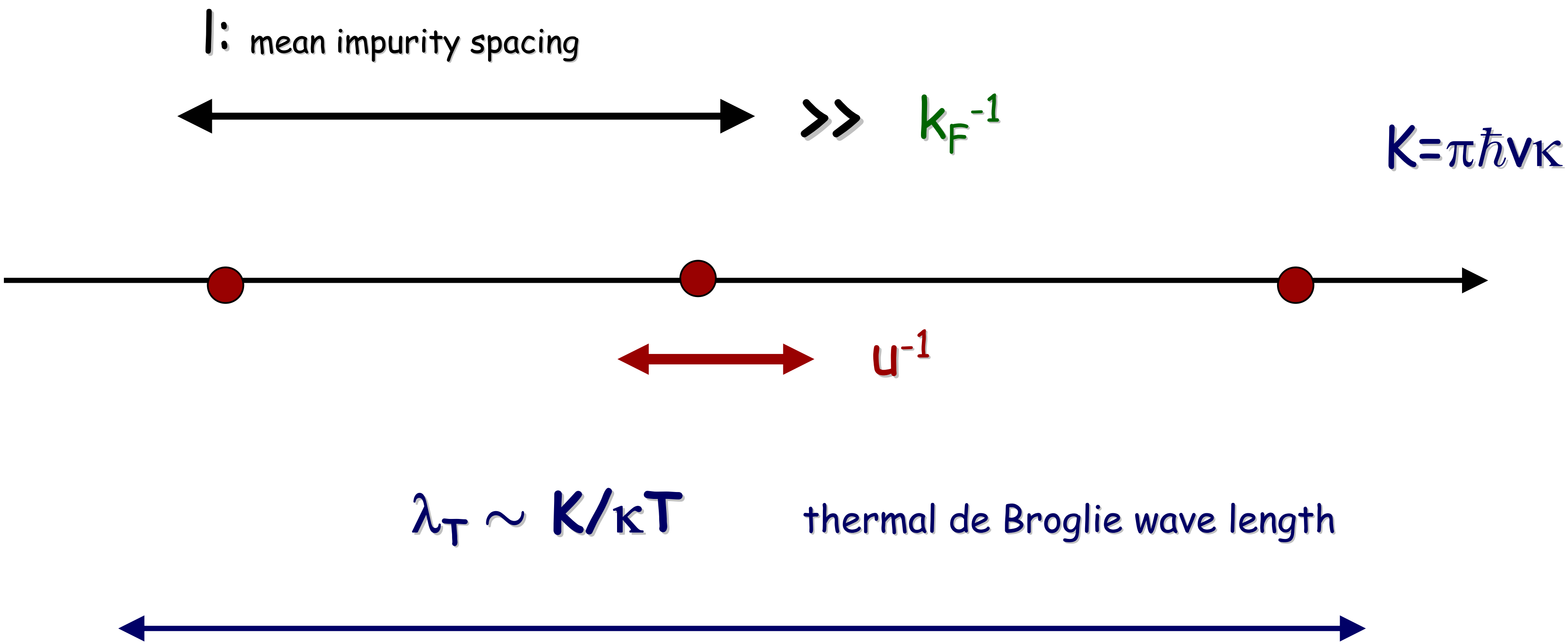
  $\left[\hat{P}(\mathbf{x}),\hat{\phi}(\mathbf{x}')\right] = \frac{\pi}{i}\delta(\mathbf{x}-\mathbf{x}')$

  K=1 : free electrons
  $K \sim 10^{-1} \dots 10^{-2}$ CDWs, SDWs

- random potential: $\underline{u} \sum_{i=1}^{N} \delta(\mathbf{x}-\mathbf{x_i}) \cdot \rho\left(\mathbf{x},\hat{\phi}\right)$

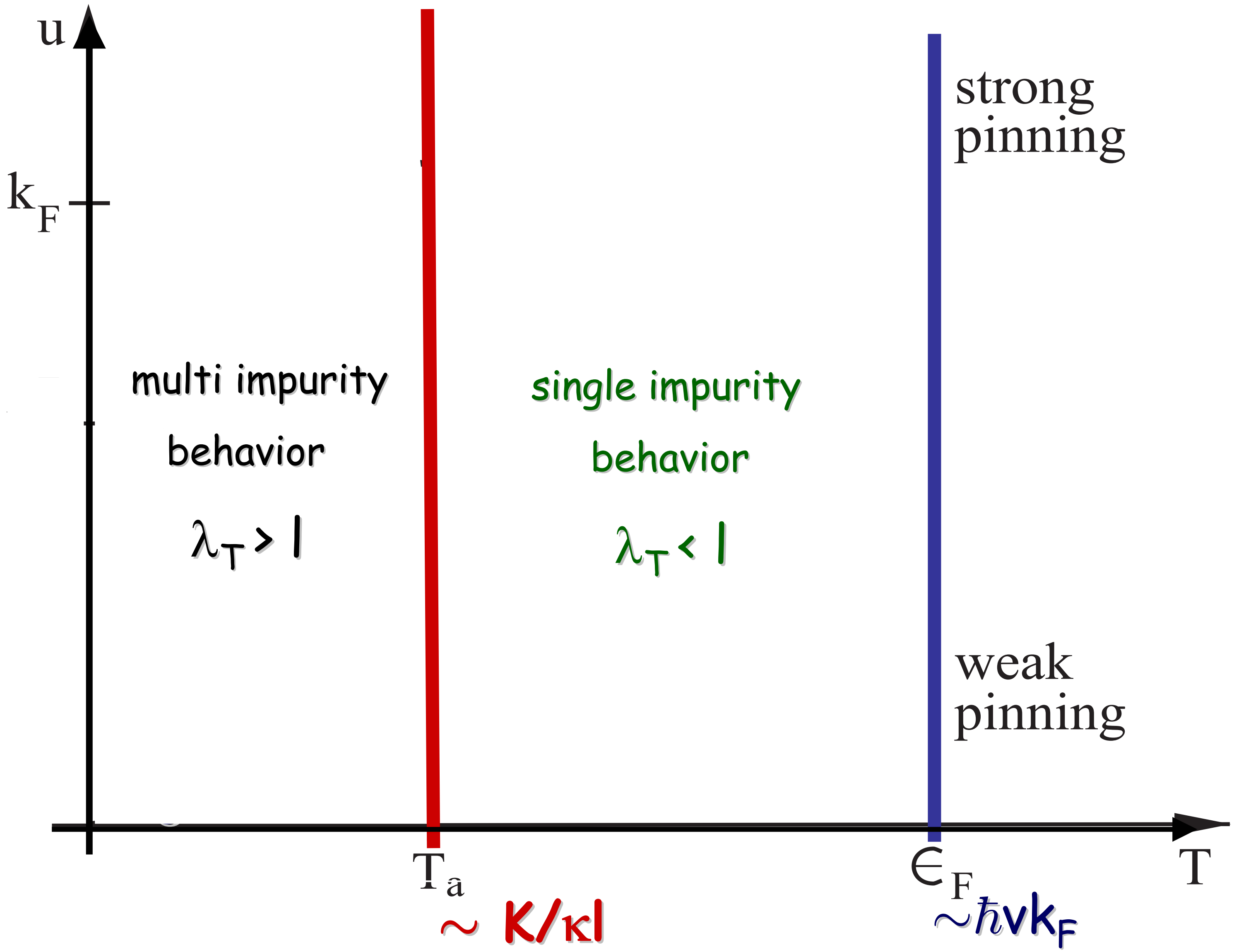
- driving force: $\underline{-f} \cdot \hat{\phi}(\mathbf{x})\,, \quad f \sim E, H, j, \dots$

Length scales

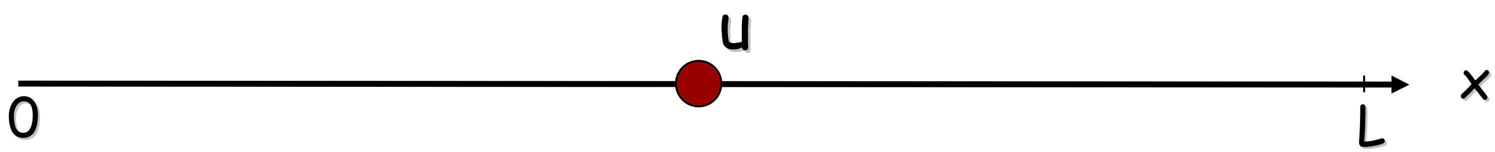


$\lambda_T \gtrless l \quad \leftrightarrow \quad T \lessgtr T_l \sim K/\kappa l$ — many / single impurity

Temperature scales



Single impurity (weak or strong), $l \to \infty$



$K < 1$ (repulsive interaction)

$\to$ impurity relevant

Kane & Fisher '92,
Furusaki & Nagaosa '93

$$G = \sigma/L \sim T^{2/K-2}$$

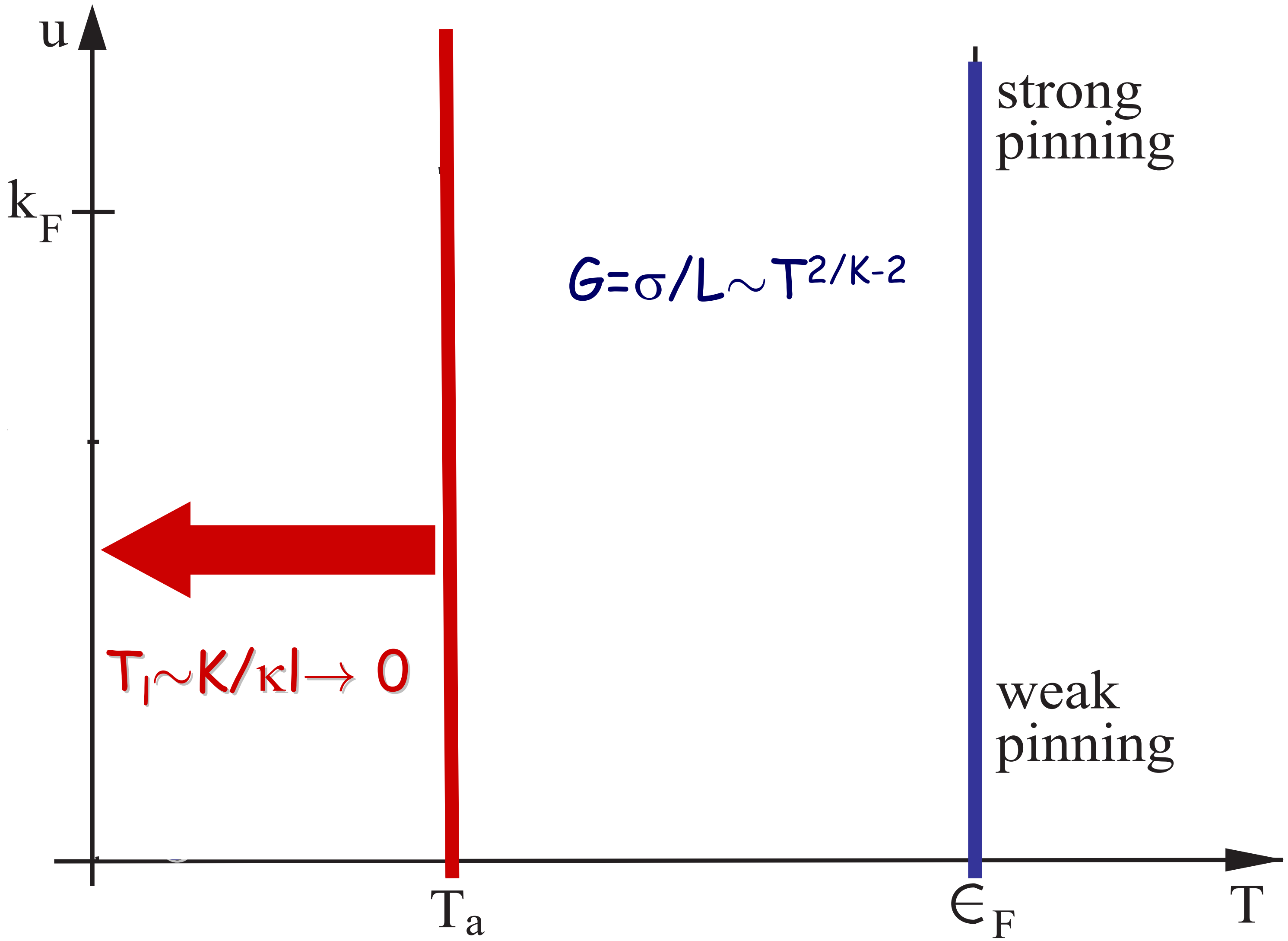
$K > 1$ (attractive interaction)

$\to$ impurity irrelevant

$$G \sim (K)\ e^2/h$$

Pinning strength

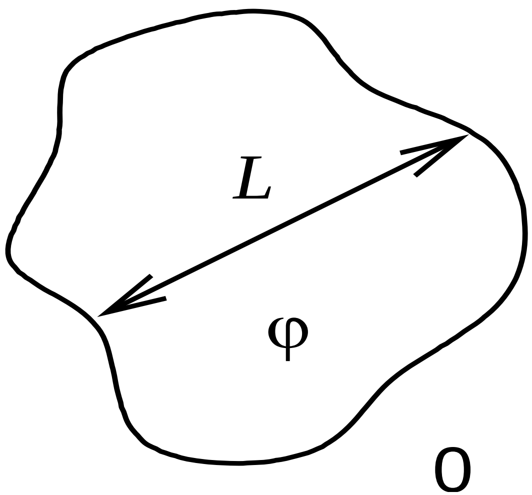
Single impurity



Disorder average

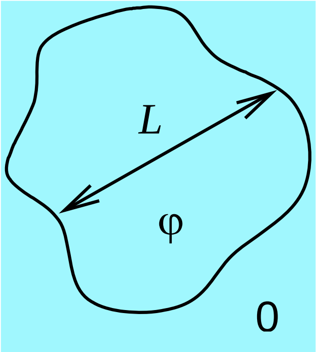
$$\langle v_R(\mathbf{x})\rangle_R = 0, \qquad \langle v_R(\mathbf{x}) v_R(\mathbf{x}')\rangle_R = v_R^2 \delta(\mathbf{x}-\mathbf{x}')$$

$$\left\langle \left(\int_0^L d^D x \, V_R(\mathbf{x},\varphi) \right)^2 \right\rangle_R \approx \left\langle \left(\int_0^L d^D x \, V_R'(\mathbf{x},0)\varphi + \dots \right)^2 \right\rangle_R \approx v_R^2 L^D \varphi^2$$



Relevance of weak disorder?

Free energy of domain of size *L*:



$$F \approx cL^D(\varphi/L)^2 - v_R L^{D/2}\varphi - T - c\left(\frac{K}{\varphi}\right)^2 L^{-D} - f\varphi L^D$$

$$= cL^{D-2}\left[\varphi^2 - \frac{v_R}{c}L^{\frac{4-D}{2}}\varphi - \frac{T}{c}L^{2-D} - \left(\frac{K}{\varphi}\right)^2 L^{2(1-D)} - \frac{f}{c}L^2\right]$$

$f = 0$: Min $\Rightarrow \varphi \approx (L/L_p)^\zeta$, $\zeta = \frac{4-D}{2}$ ''roughness exponent''

self similar ground state

Larkin-length $L_p \approx \left(\frac{c}{v_R}\right)^{\frac{2}{4-D}} \gg 1$, Larkin 1970, Imry & Ma 1975, Fukuyama & Lee 1978

Many weak impurities (Gaussian)

x

$u, l \to 0, \quad (l/u^2)^{1/3} \sim \xi_0 \gg k_F^{-1}, \quad$ fixed

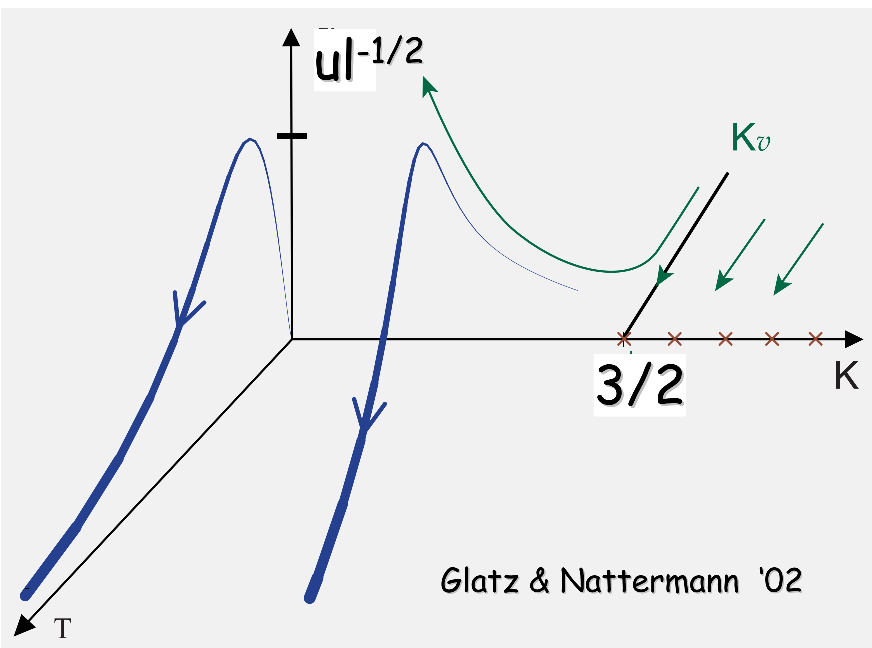
$K < 3/2$ impurities → relevant

$K > 3/2$ impurities → irrelevant

Suzumura & Fukuyama '83

Giamarchi & Schulz '88

$T_{loc} \sim K/\kappa\xi$

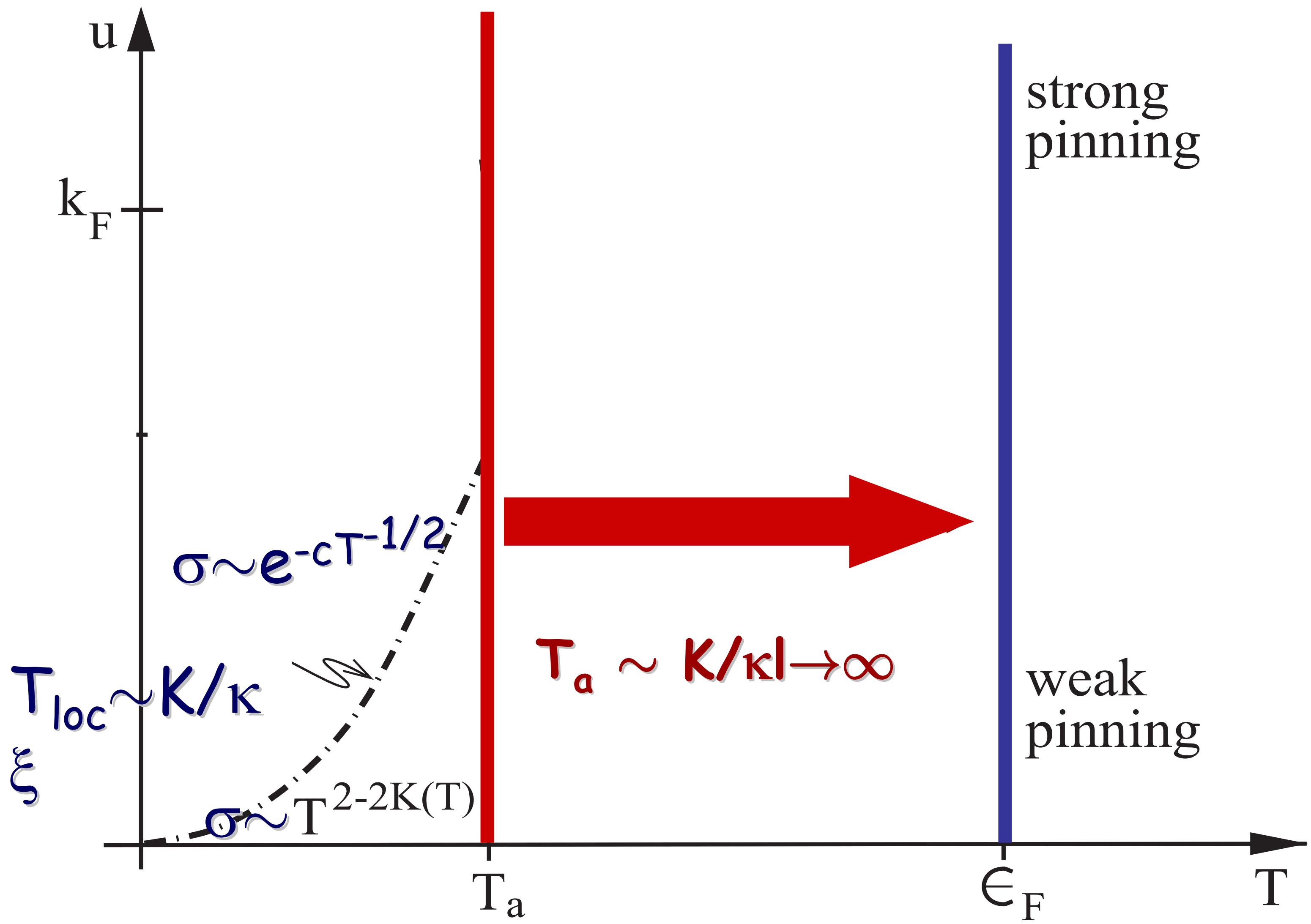


$\xi_0 \xrightarrow{\text{quantum fluctuations}} \xi \approx k_F^{-1} (\xi_0 k_F)^{3/(3-2K)}$

$\sigma \sim$ '88
$\begin{cases} T^{\,2-2K(T)} & T \gg T_{loc} \sim \hbar v/\xi \quad \text{Giamarchi \& Schulz} \\ e^{-cT^{-1/2}} & T \ll T_{loc} \sim \hbar v/\xi \end{cases}$

Pinning strength

Gaussian impurities



Here: Many strong Poissonian impurities

1. What is a strong impurity? $T<T_a$

no interaction (K=1): $u > u_c = k_F$

interaction (K<1):

$\rightarrow$ integrate out fluctuations with wave vector $\Lambda < |k| < k_F$

$u \rightarrow u_{eff} \approx u\,(k_F / \Lambda)^{-K}$ Glazman et al. '92

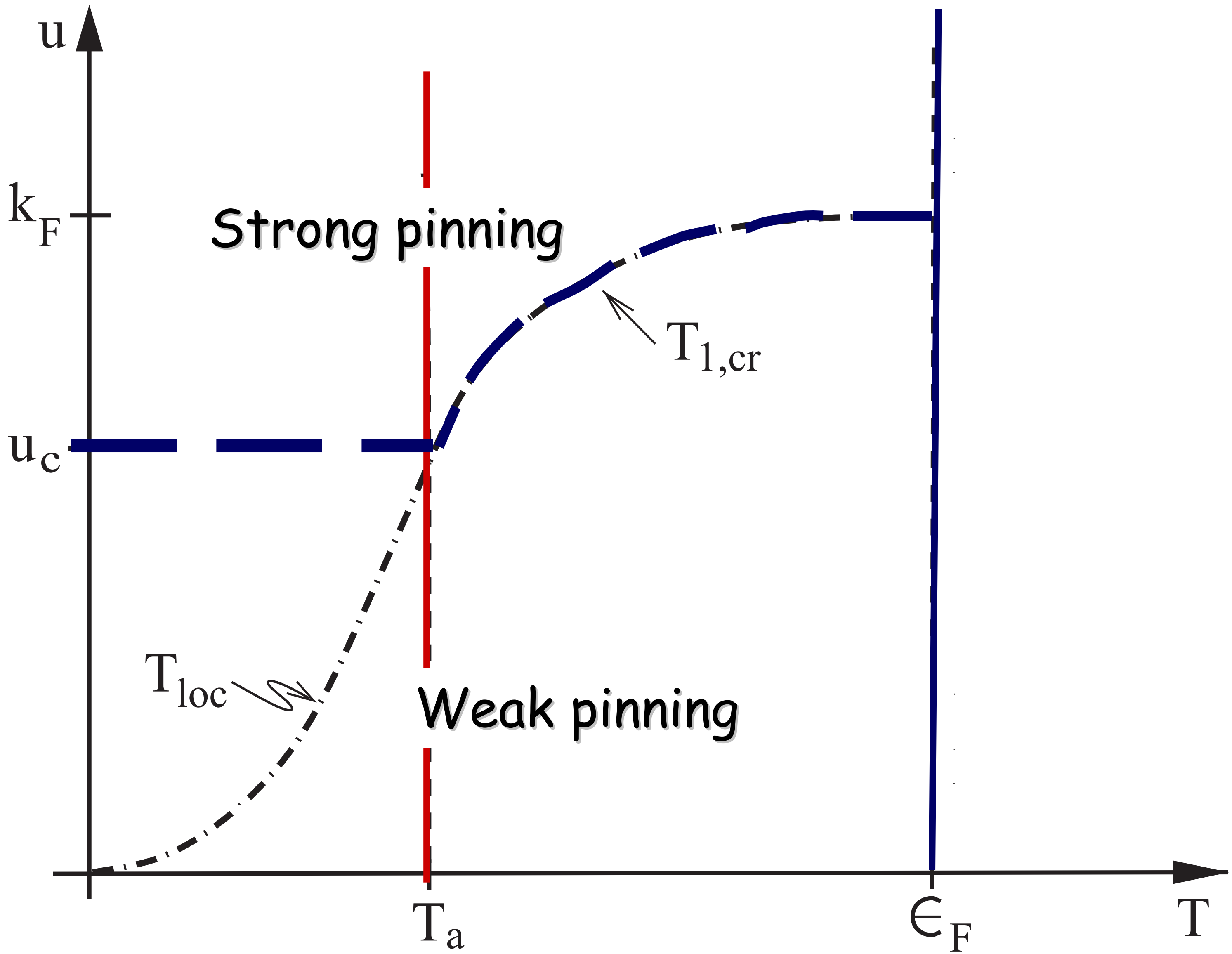
strong pinning : $l\, u_{eff} \gg 1$ Fukuyama & Lee '78

$\rightarrow \quad u > u_c = k_F\,(l k_F)^{K-1}$

2. What is a strong impurity? $T>T_a$

strong pinning: $\lambda_T\, u_{eff} \gg 1 \quad \rightarrow \quad u > u_1(T) = k_F\,(T/\varepsilon_F)^{1-K}$

$\rightarrow \quad T < T_{1,cr} = \varepsilon_F\,(u/k_F)^{1/(1-K)}$



u
k_F
u_c
Strong pinning
Weak pinning
$T_{1,cr}$
T_{loc}
T_a
ϵ_F
T

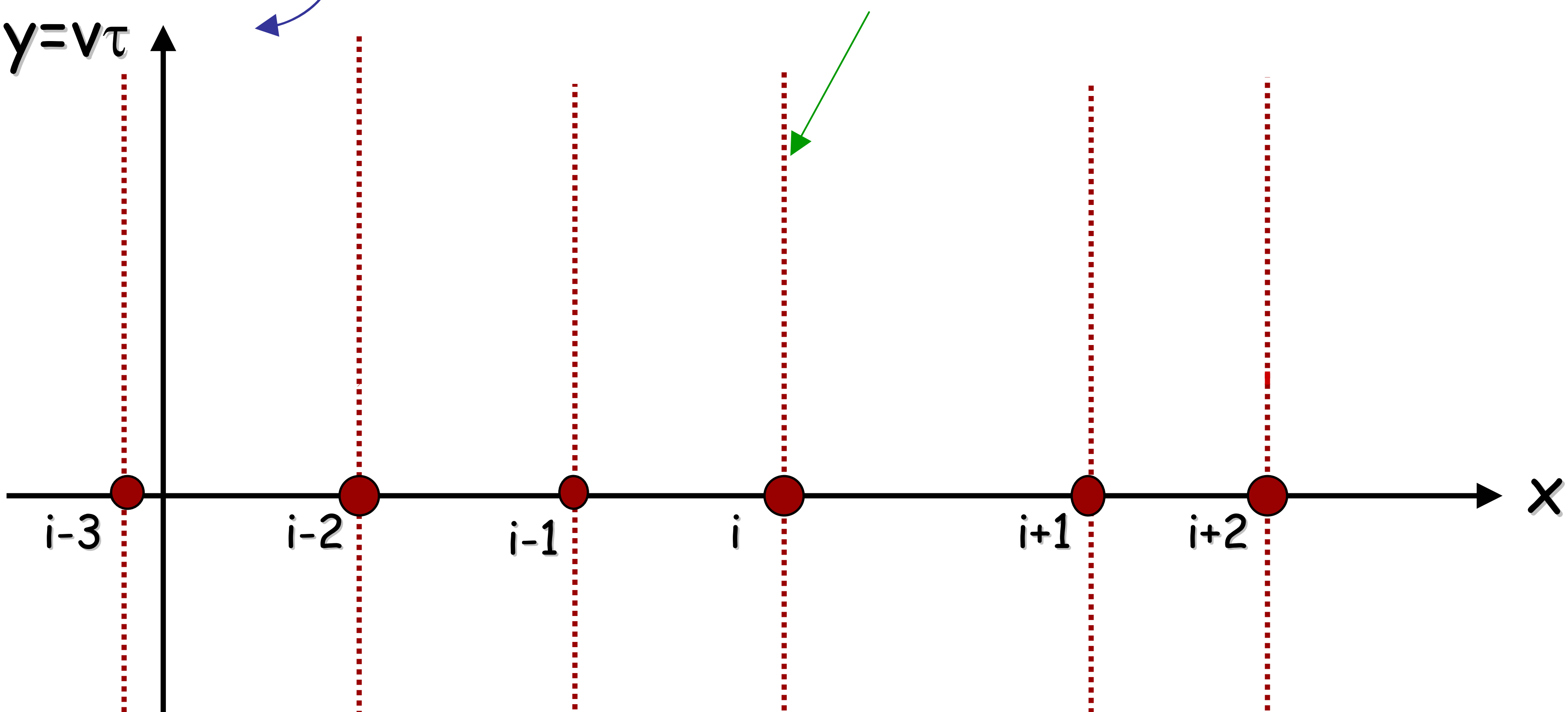
Many strong Poissonian impurities:

$$\frac{S}{\hbar} = \frac{1}{2\pi K}\int_0^L dx \int_0^{\lambda_T} d\tau \left\{ (\partial_x\phi + fx)^2 + (\partial_\tau\phi)^2 - u\sum_{i=1}^{N} \cos\left(2\phi - 2k_F x_i\right) \right\}$$

metastable states

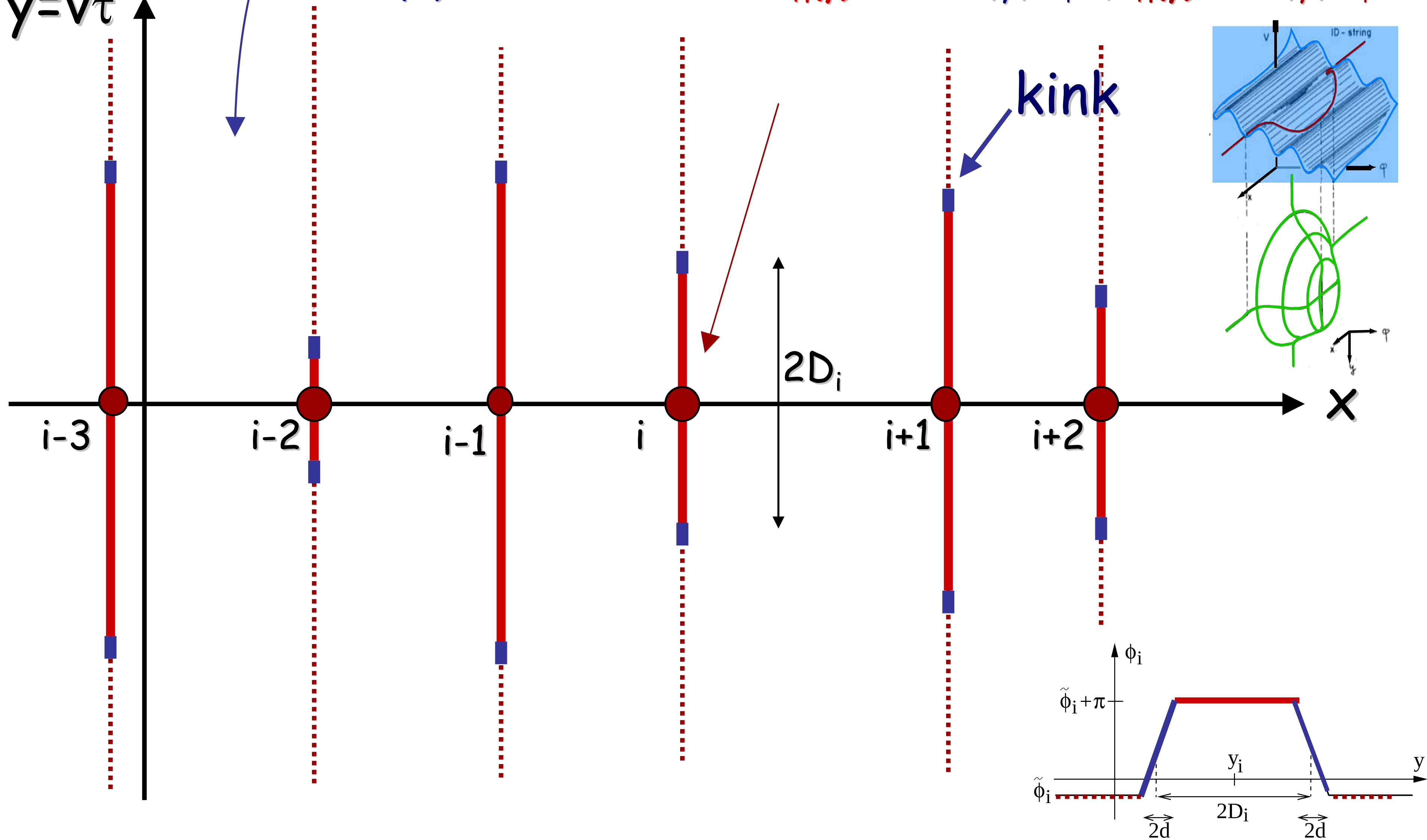
(i) integrate out $\phi(x,y)$ between impurities $\to$ $\phi(x_i,y) \equiv \phi_i(y)$

(ii) **metastable state:** $\Phi_i=\pi(n_i-\alpha_i)$, **ground state** $n_i=\sum_{j\geq i}[k_F(x_j-x_{j-1})]_G$,



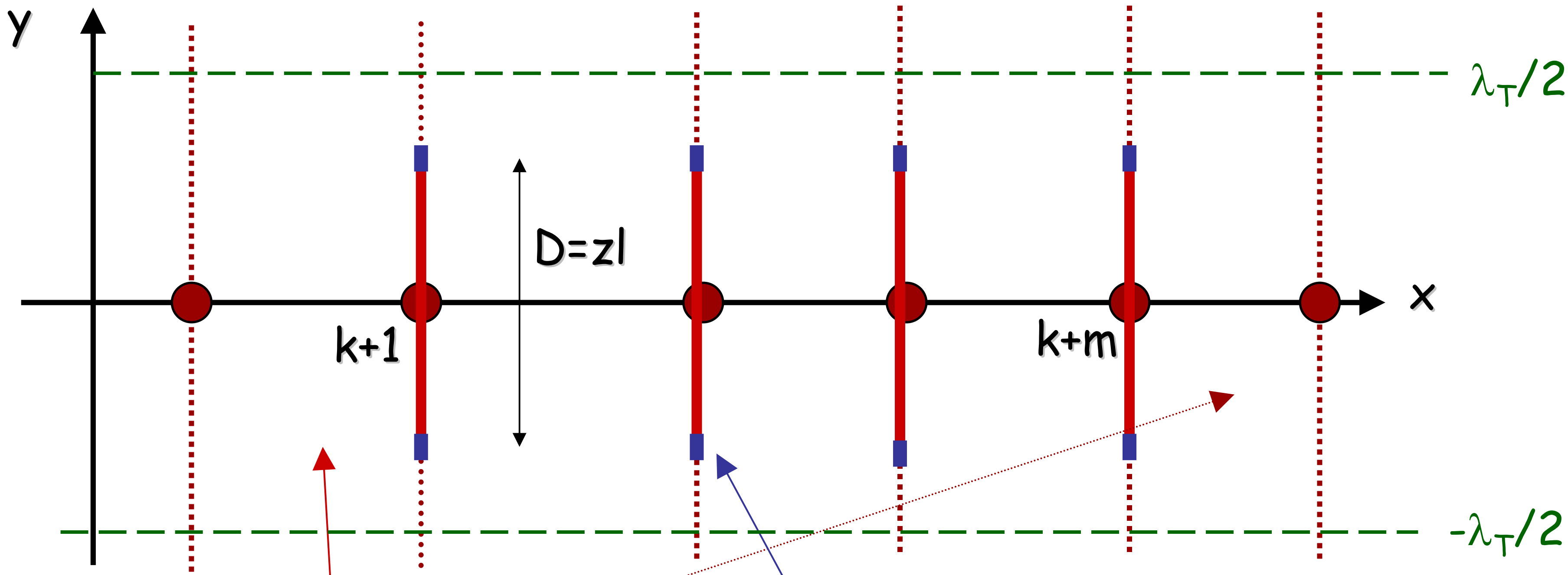
Tunneling under applied field: instantons
(=droplet of next metastable state)

(iii) consider **instantons** $\phi_i(y)=\Phi+\pi$ if $|y|<D_i-d$, $\phi_i(y)=\Phi$ if $|y|>D_i+d$



Tunneling under applied field: instantons

(iv) For simplicity: $D_i \equiv D$, $k < i \leq k+m$, D_i elsewhere



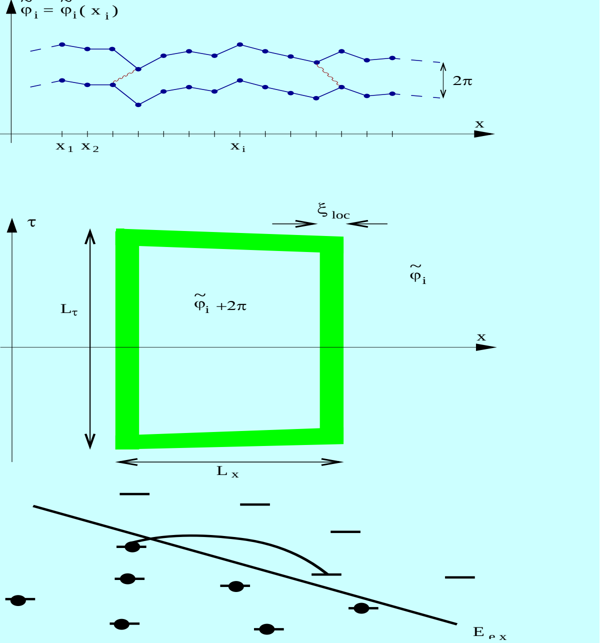
$$S/\hbar = 2/K \left\{ z\sigma_m(k) + \ln(1 + e^{-2z}) + m(s + \ln\tanh(z/2) - zfa^2) \right\}$$

$f \sim E$

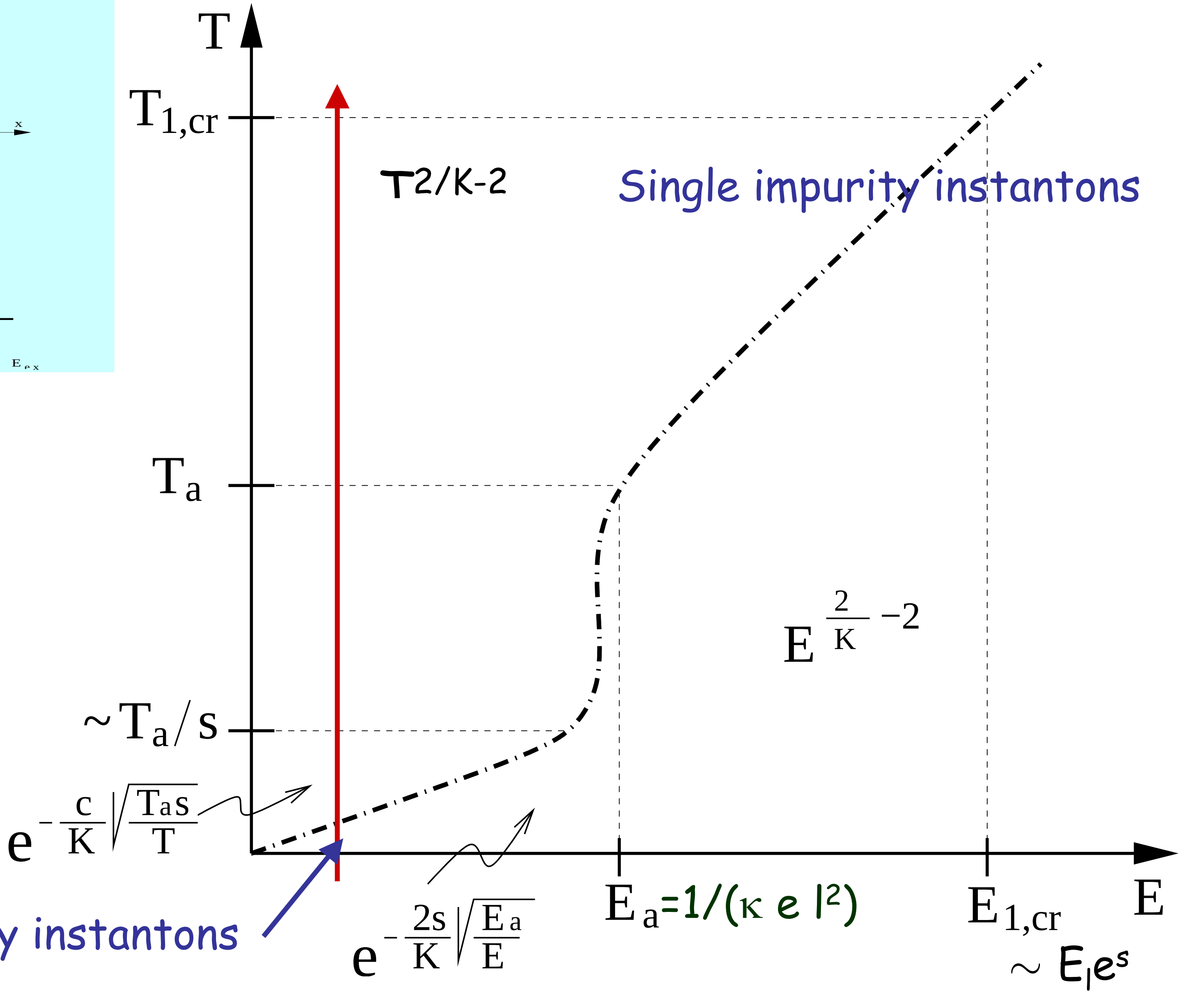
$s \sim \ln(Clu_{eff})$

Current $\sim$ tunneling probability $\sim$ $\exp\{- S_{\text{saddle point}}(E)/\hbar\}$

If instanton hits $\pm\lambda_T/2 \rightarrow$ Cross-over to linear response

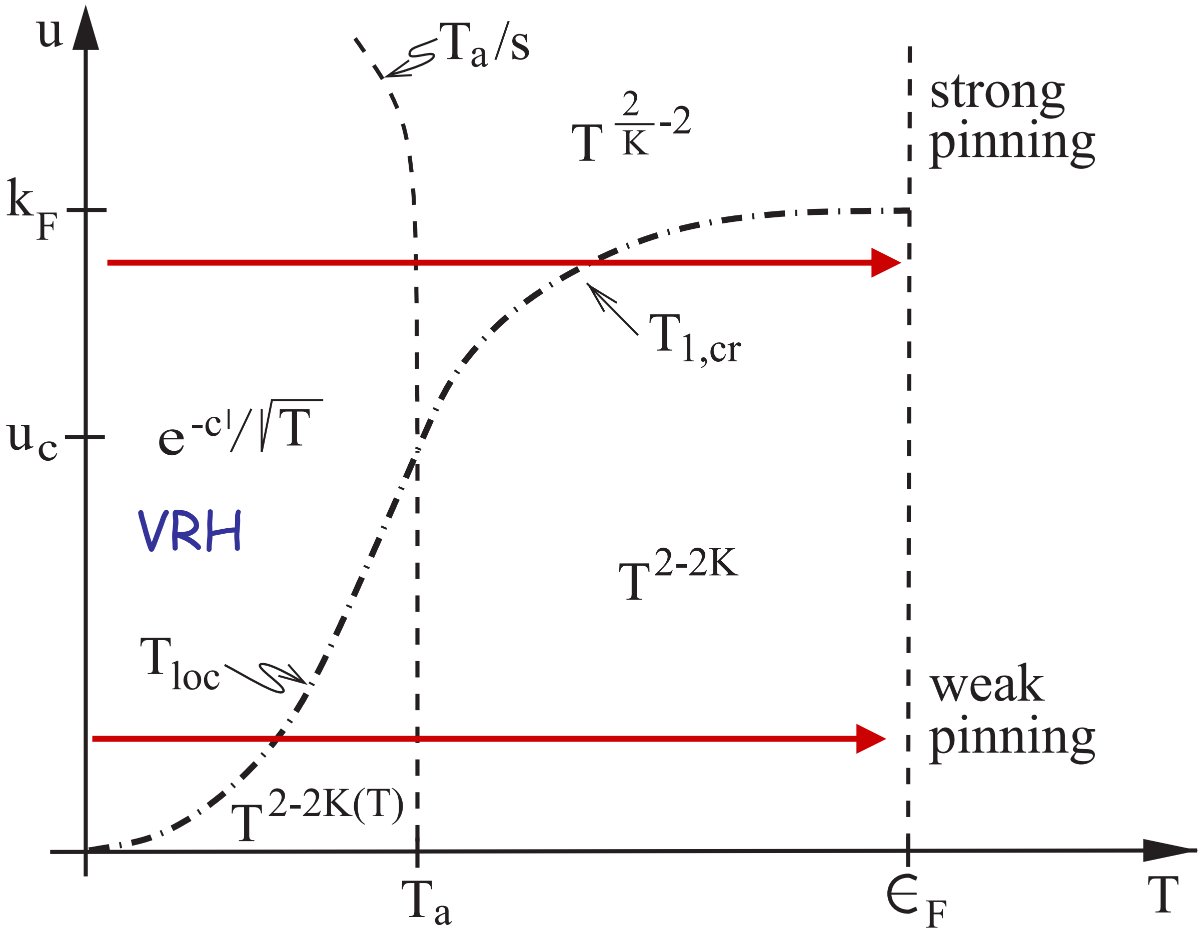


2π
x
x1 x2
xi
τ
ξ loc
Lτ
φ̃i +2π
φ̃i
Lx

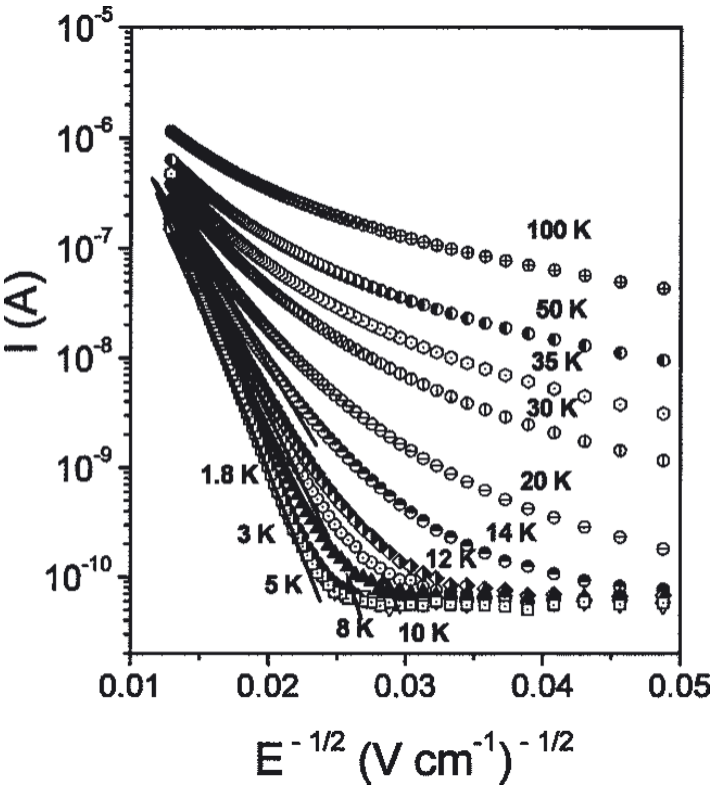


T
$T_{1,cr}$
$T^{2/K-2}$
Single impurity instantons
T_a
$E^{\frac{2}{K}-2}$
$\sim T_a/s$
$e^{-\frac{c}{K}\sqrt{\frac{T_a s}{T}}}$
$e^{-\frac{2s}{K}\sqrt{\frac{E_a}{E}}}$
$E_a=1/(\kappa e l^2)$
$E_{1,cr}$
$\sim E_l e^s$
E
Multi impurity instantons

Different regimes of conductivity



Polydiacetylen Aleshin et al. 2004



Carbon-nanotubes: Tang et al. 2000

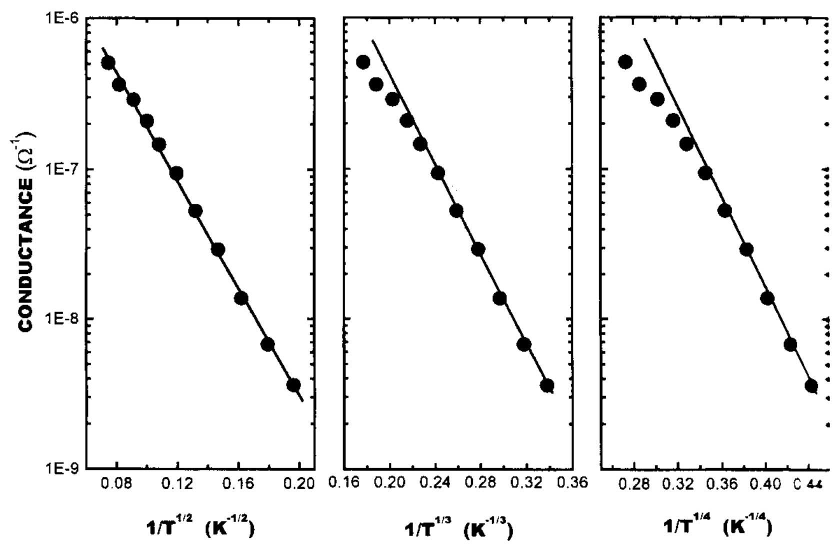


Fig. 4. The temperature dependence of the conductance measured at zero bias voltage.

Conclusions:

Considered 1D driven quantum model with periodic disorder:
CDWs and Luttinger liquids

Limit of strong disorder: instanton calculation

→ linear and non-linear conductivity
→ field and temperature cross-over between single and many impurity tunneling
→ small field and temperature: Mott-Shklovskii-VRH
→ larger E,T: Kane-Fisher-….. power law behavior
→ global weak/strong pinning regime diagram