Disordered Quantum Wires*

Thomas Nattermann	Universität zu Köln
Thierry Giamarchi	Geneva
Pierre Le Doussal	Paris
Bernd Rosenow	Köln/Harvard
Sergey Malinin	Köln/Detroit
Michel Fogler	UCSD

*$e=h=k_B=1$

- Electrons in 3 dimensions
- Electrons in 1 dimension: what is different?
- Electrons in 1 dimension: experiments
- Quantum wire as a chain of quantum dots
- Long wires: Rare events
- Conclusions

1. Electrons in three dimensions

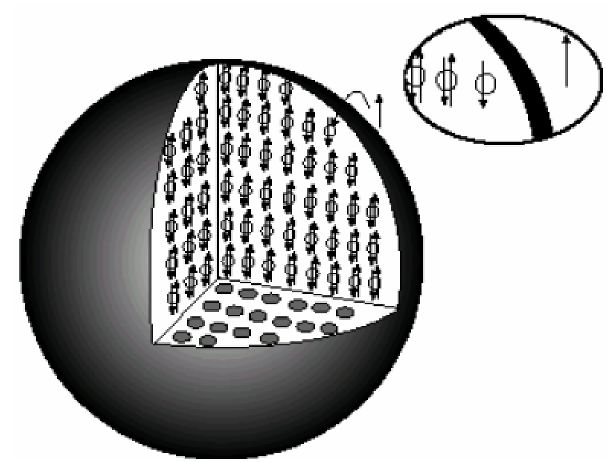
Non-interacting electrons: mass m, mean spacing a, Planck's constant $\hbar$

$\rightarrow$ energy scale $\hbar^2/ma^2 \sim E_F$

$T \ll E_F$: Pauli principle determines pressure $\rightarrow$ $p \sim \hbar^2/ma^{d+2}$

Compressibility*: $\kappa = - a^{-2d}\, \partial \ln V/\partial p \sim 1/(E_F a^d)$

Specific heat : $C \sim T/E_F$



1. Electrons in three dimensions

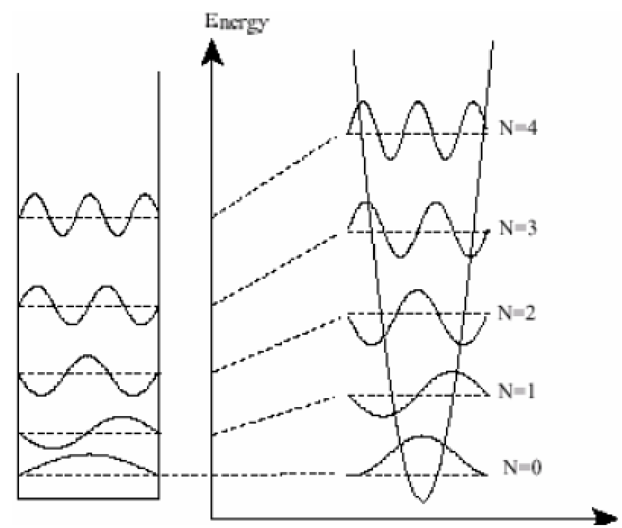
Interacting spinless electrons:

charge e → strength of interaction $\sim (e^2/a) / E_F \sim a/a_B$!

but energy and momentum conservation reduce phase space

⇒ Landau's Fermi fluid of quasi-particles

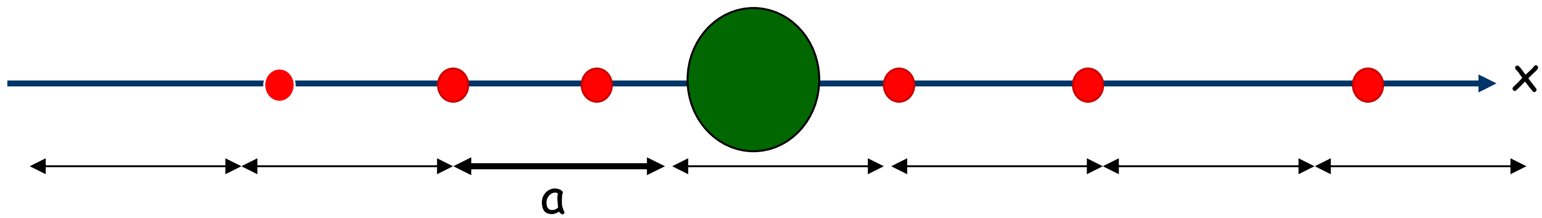
with finite lifetime $\hbar/\tau \sim E_F(T/E_F)^2$



Disordered spinless electrons:

Metal insulator transition

2. Electrons in one dimension - what is different?



Electrons cannot avoid each other $\rightarrow$ Landau's picture breaks down

$\Rightarrow$ density wave excitations : plasmons

(a) 1D clean wire: Luttinger liquid $G = dJ/dV = \sigma/L = (K)\, e^2/h$

K

(b) + single impurity : K<1: impurity relevant, $\rightarrow G \sim (\max(T, eV))^{2/K-2}$

Kane & Fisher '92, Furusaki & Nagaosa

K>1: impurity irrelevant

3. Electrons in one dimension: Experiments

- **MoSe Nanowires**

Venkataraman, PRL (2006)

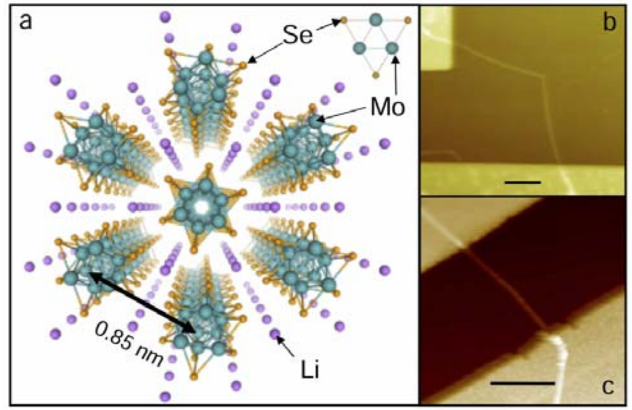
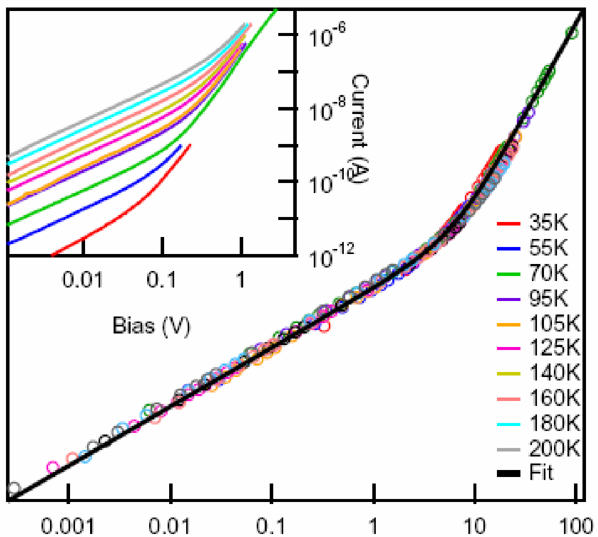


FIG. 1 (color online). (a) Structural model of a 7-chain MoSe nanowire along with the triangular Mo_3Se_3 unit cell. (b) and (c) AFM height images of MoSe nanowires between two Au electrodes. The wire heights are 7.2 nm and 12.0 nm, respectively. Scale bar = 500 nm.

$$J / T^{\alpha+1} \sim \max \left(V/T,\ V^{\beta+1}/T^{\alpha+1} \right)$$

$$\frac{J}{T^{a+1}}$$



$$\frac{\text{Voltage}}{\text{Temperature}}$$

short wires (L ~ 1 μm):

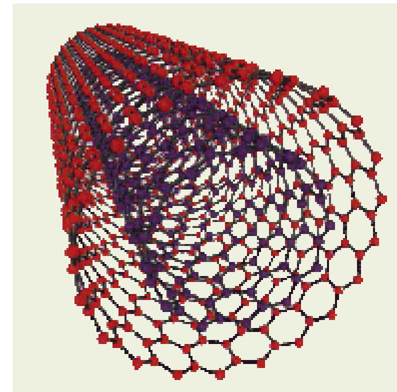
"Temperature" Exponent (a) is close to "Voltage" Exponent (b)

Agrees with the conventional "Luttinger-liquid" picture with

$$\alpha = \beta = 2/K - 2$$

3. Electrons in one dimension: Experiments

• Multiwall carbon nanotubes



conductance

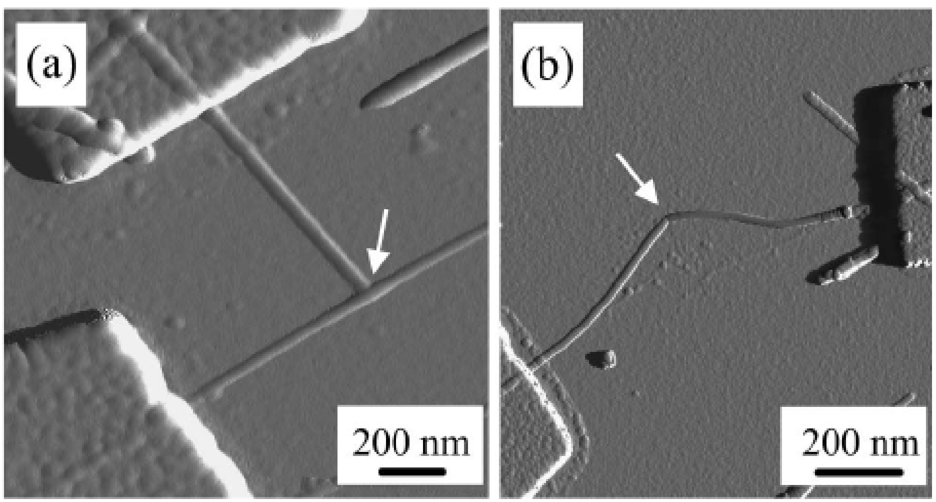
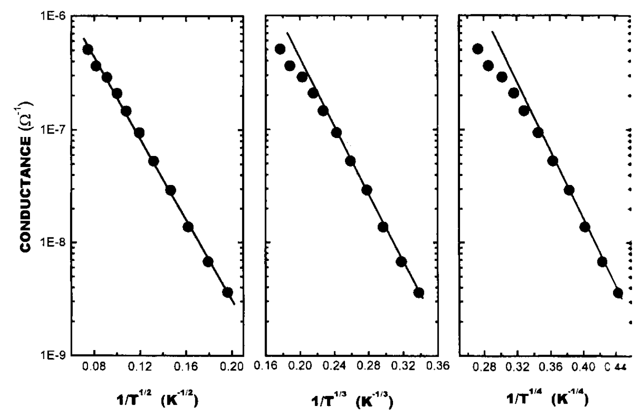


FIG. 3. AFM images of junctions formed between two MWNTs: (a) end-bulk junction and (b) end-end junction. The arrows indicate the position of the junctions.



$1/T^{1/2}$

$J \sim e^{-(T_0/T)^{1/2}}$

3. Electrons in one dimension: Experiments

Variable Range Hopping conduction in polydiacetylene single crystals

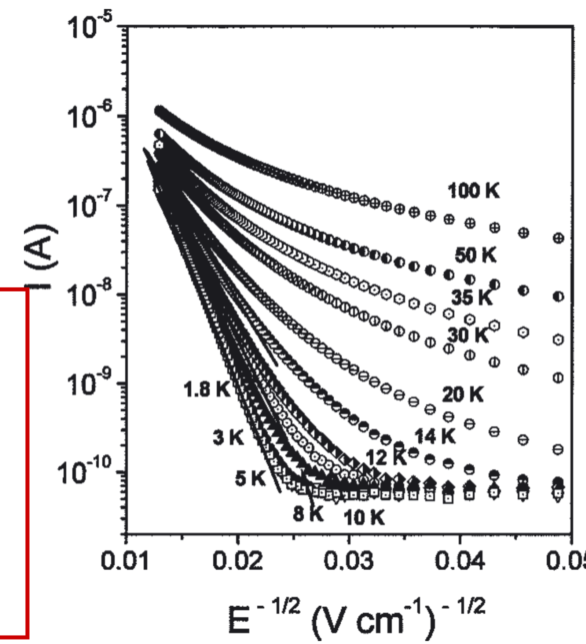
A. N. Aleshin*
School of Physics and Condensed Matter Research Institute, Seoul National University, Seoul 151-747, Korea
and A. F. Ioffe Physical-Technical Institute, Russian Academy of Sciences, St. Petersburg 194021, Russia

J. Y. Lee, S. W. Chu, S. W. Lee, B. Kim, S. J. Ahn, and Y. W. Park
School of Physics and Condensed Matter Research Institute, Seoul National University, Seoul 151-747, Korea

T_0(K)	E_0(V/cm)
2.57×10^3	4.9×10^5
2.47×10^3	3.2×10^5
4.72×10^3	1.1×10^6

$$J \sim e^{-(E_0/E)^{1/2}}$$

The charge transport in polydiacetylene quasi-1D single crystals (PDA-PTS) has been studied as a function of temperature, electric and magnetic fields. In the Ohmic regime the temperature dependence of the resistivity, $\rho(T)$, is characteristic of hopping conduction with a crossover at $T<50$ K from activated $\rho(T) = \rho_0 \exp[(E_A/k_BT)]$, with $E_A \sim 13-19$ meV to variable-range hopping transport $\rho(T)=\rho_0 \exp[(T_0/T)^p]$, with $p\sim 0.65-0.70$. At modest electric fields the resistivity depends as $\rho(E,T)=\rho(0,T)\exp(-eEL/k_BT)$, where the characteristic hopping length changes as $L\sim T^{-m}$ with $m\sim 0.5$ at $T>50$ K and $m\sim 0.75$ at $T<50$ K. At high electric fields the low temperature current becomes temperature independent and follows: $I(E) = I_0 \exp[-(E_0/E)^{0.5}]$, which corresponds to the regime of activation-free phonon-emission-assisted hopping



3. Electrons in one dimension: Experiments

- **Polymer nanofibers**

long wires: 10μm

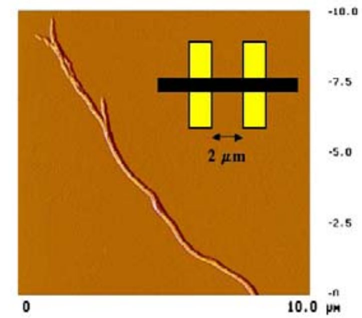
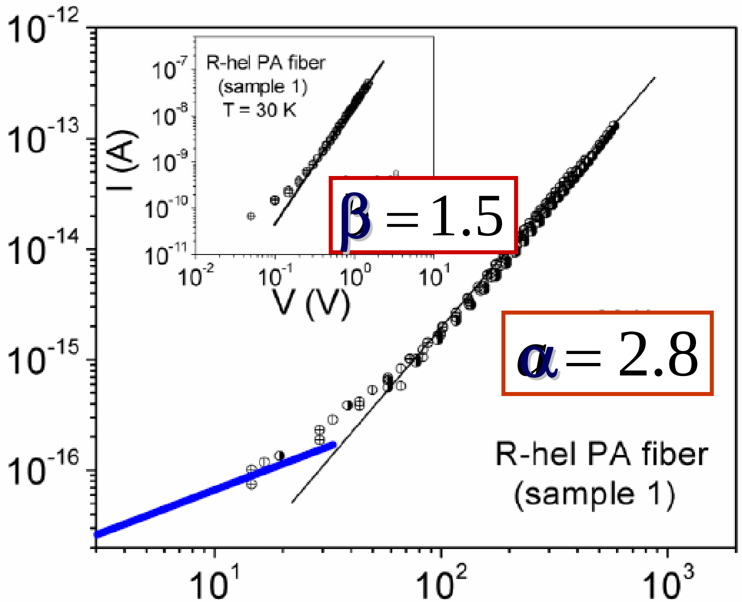


FIG. 1 (color online). AFM image of a *R*-hel PA fiber; the inset shows the schematic of a two-probe device based on such a *R*-hel PA fiber on top of Pt electrodes.

$$\frac{I}{T^{\alpha+1}}$$



$$\frac{\text{Voltage}}{\text{Temperature}}$$

"temperature" exponent **exceeds** "voltage" exponent!

$$J / T^{\alpha+1} \sim \max\left(V/T,\ V^{\beta+1}/T^{\alpha+1} \right)$$

Aleshin et al., PRL (2004)

Disagrees with the conventional Luttinger-liquid picture

3. Electrons in one dimension: Experiments

Observed Power-Law Exponents

L ~10 μm polymers
Aleshin et al.,
PRL (2004)

Sample	1	2	3	4	5	6
α	2.8	5.5	7.2	5.6	5.0	4.1
β	1.5	3.8	4.7	1.0	1.1	1.8

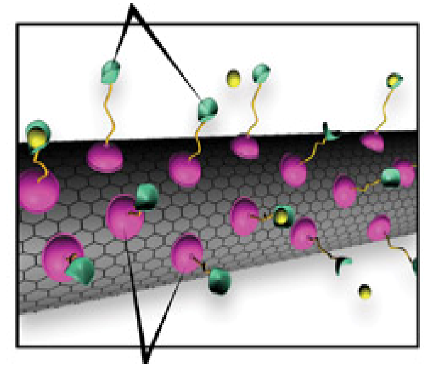
L ~ 100 μm InSb wires
Zaitsev-Zotov et al.,
JPCM (2000)

Sample	1	2	3	4
α	2.3	3.4	4.5	4.6
β	1.3	3.4	2.8	2.0

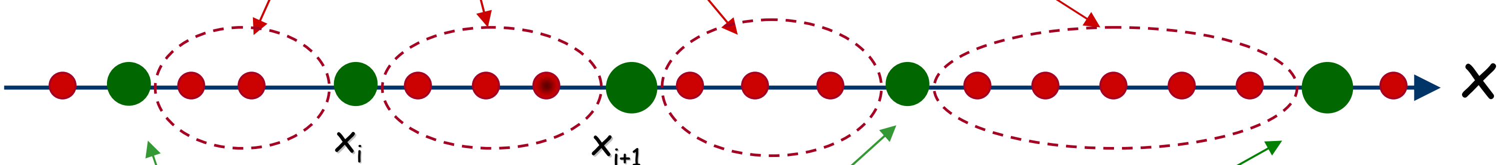
- In long wires T-exponent exceeds V-exponent: $\alpha > \beta$
- Exponents are sample-dependent

4. Quantum wire as a chain of quantum dots

"quantum dots" with integer number of electrons



Model:



Adding or removing a charge costs energy!

strong impurities, randomly (Poissonian) distributed

V=0 : classical ground state (K<<1)

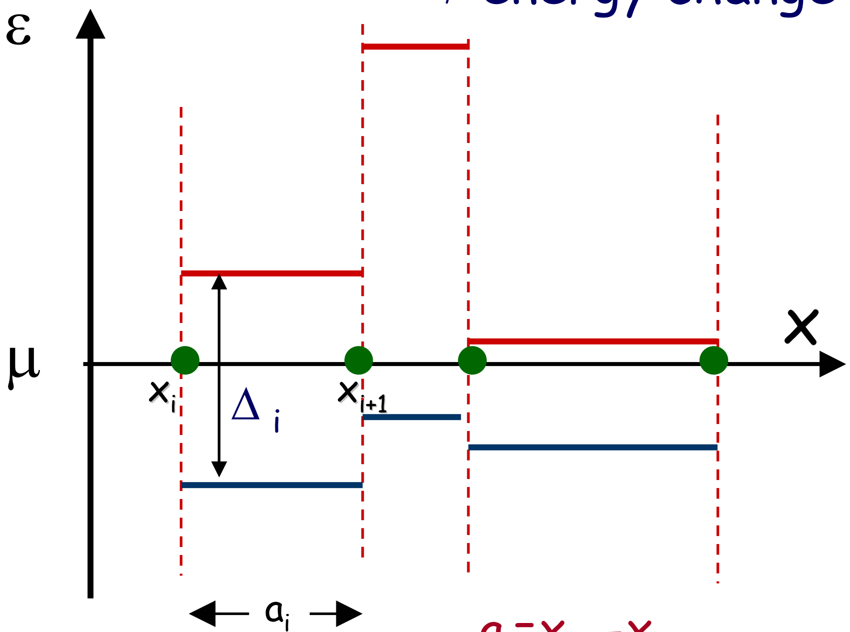
integer number q_i of electrons between impurities i and i+1

$$q_i = [Q_i]_G \quad , \qquad Q_i = a_i\,(2\pi k_F + \mu\, C) \quad \text{background charge}$$

excited states: change number of electrons by n_i

→ energy change

$$E_i(n_i) = \Delta_i \,\{(1/2)n_i^2 + n_i(q_i - Q_i)\}$$



$a_i = x_{i+1} - x_i$

$\Delta_i = 1/(Ca_i)$ charging energy of quantum dot

C: capacitance/unit length

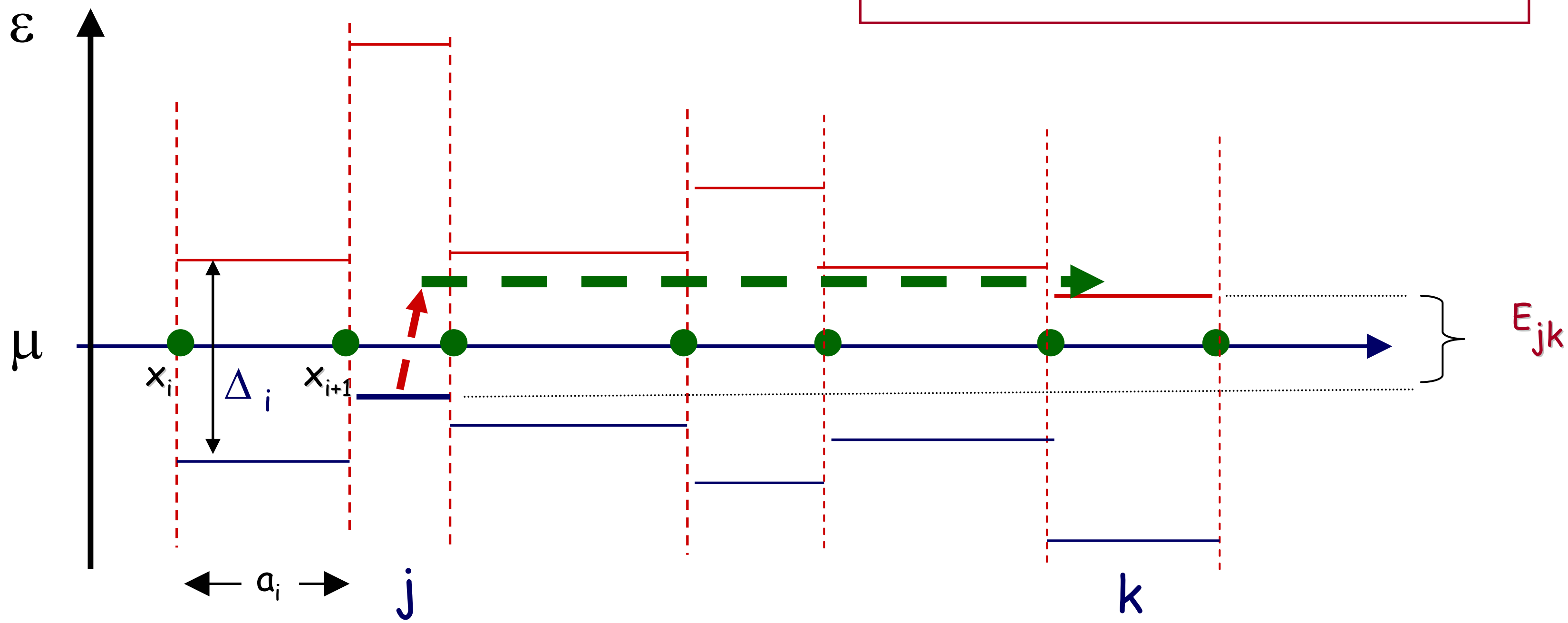
$\Delta = 1/Ca$

Transport from dot j to dot k by activation and co-tunneling

$$J_{jk} \sim e^{-E_{jk}/T} \quad e^{-s|k-j|} \ \sinh\left((\xi_k - \xi_j)/T \right)$$

local electro-chemical potential $\xi_j = Fx_j - \mu_j$

tunneling transparency $s = 2(K^{-1} - 1)\ \ln(ak_F)$ Larkin, Lee '78



Transport from dot j to dot k by activation and co-tunneling

$$J_{jk} \sim e^{-E_{jk}/T} \quad e^{-s|k-j|} \; \sinh\left((\xi_k - \xi_j)/T \right)$$

Ohmic regime

typical energy mismatch $E_{jk} \sim \Delta / |k-j| \;\rightarrow\; |k-j| \sim (\Delta/Ts)^{1/2} \equiv x_{VRH}/a$

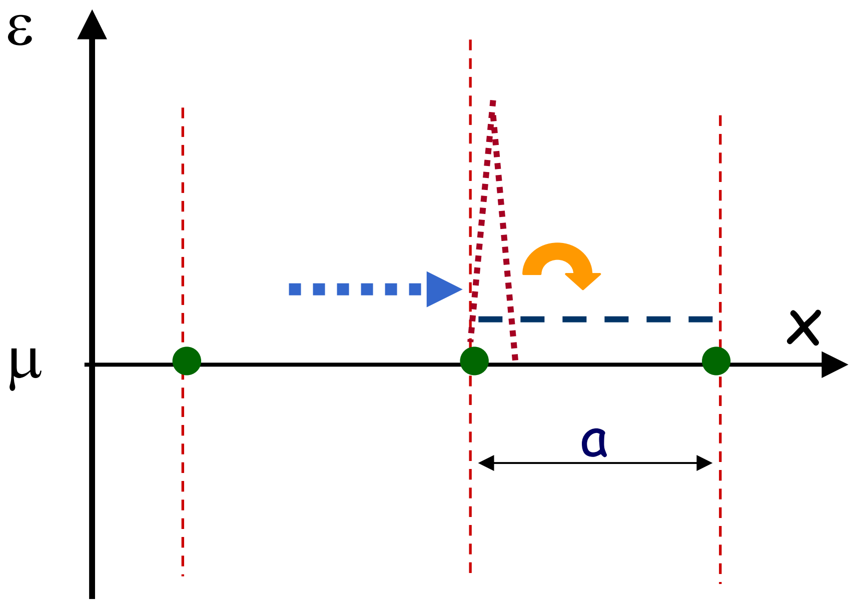
$$\Rightarrow \quad J \sim e^{(s\Delta/T)^{1/2}} F, \qquad Ts \ll \Delta \qquad \text{VRH}$$

Non-ohmic regime

typical $|k-j| aF \sim \Delta / |k-j| \;\rightarrow\; |k-j| \sim (\Delta/aF)^{1/2}$

$$\Rightarrow \quad J \sim e^{s(\Delta/aF)^{1/2}}, \qquad aF \ll \Delta \qquad \text{VRH}$$

Large Voltage/temperature: power laws



Larkin and Lee '78

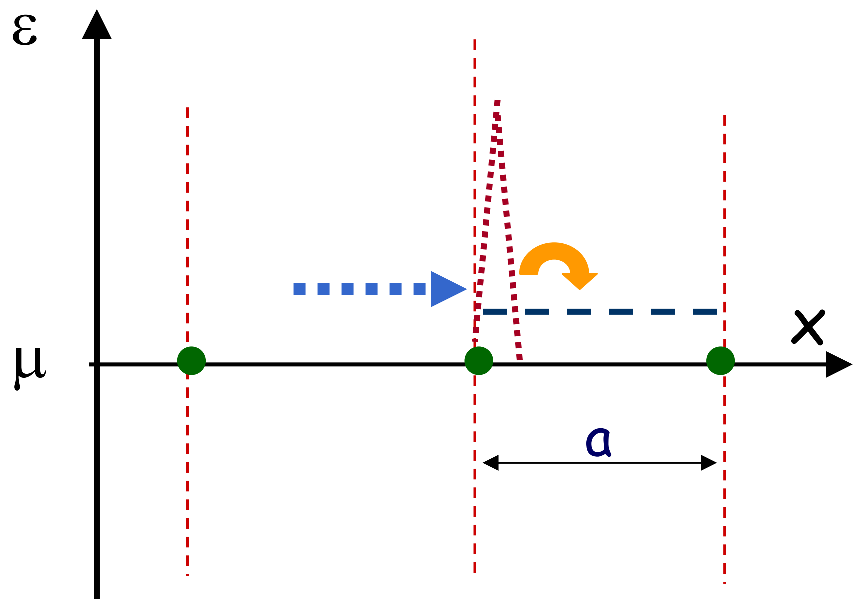
Tunneling action dominated by spreading of charge:

$S_{tun} \sim \int d\tau\, E(\tau) \sim \int d\tau\, (C\, x(\tau))^{-1} \sim K^{-1} \ln (E_{initial}/E_{final})$

$E_{initial} = k_F/C$

$E_{final} = \max(\Delta, T, V)$

Large Voltage/temperature: power laws



Larkin and Lee '78

Tunneling action dominated by spreading of charge:

$S_{tun} \sim \int d\tau\, E(\tau) \sim \int d\tau\, (C\, x(\tau))^{-1} \sim -K^{-1} \ln (E_{final}/E_{initial})$

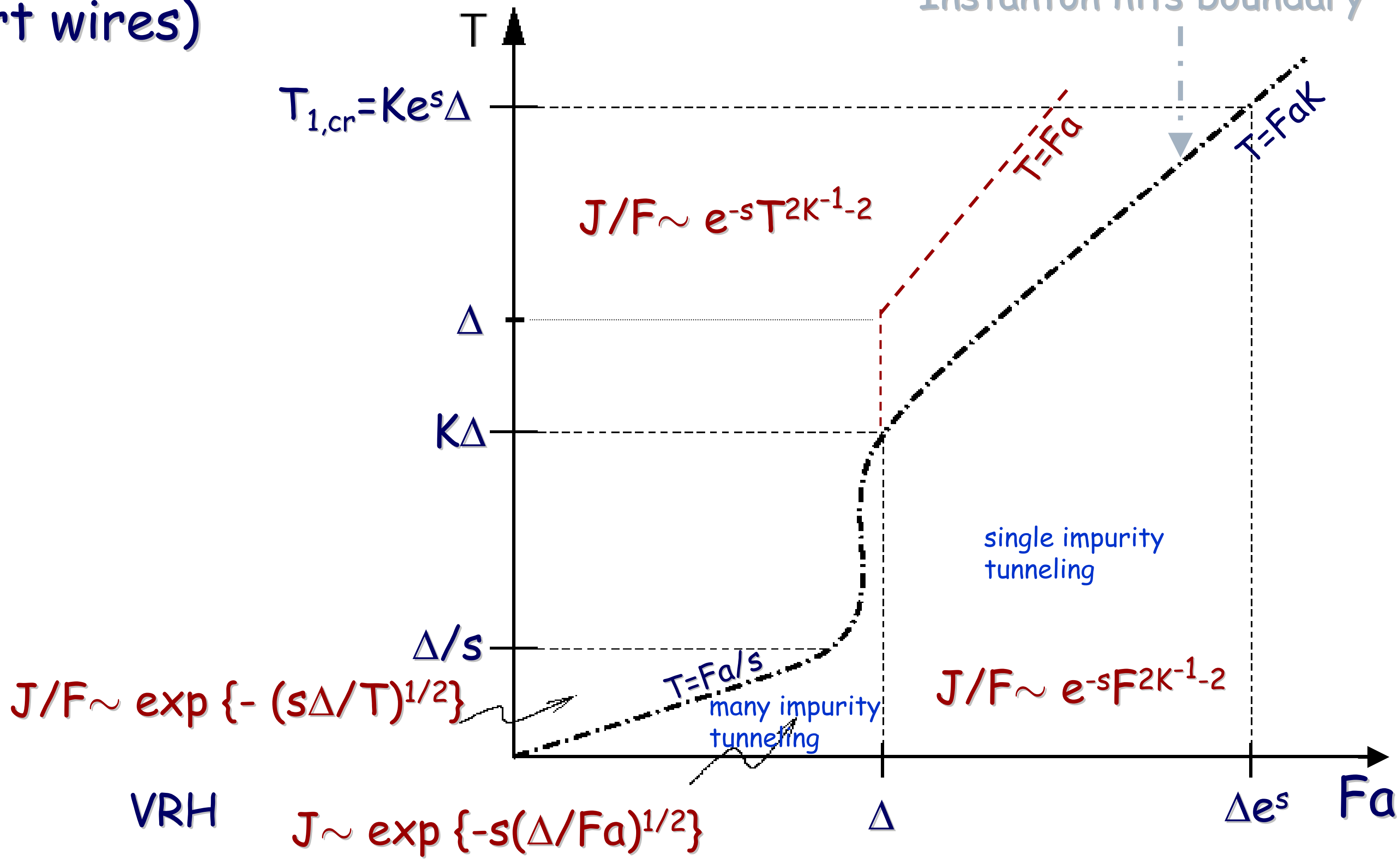
$E_{initial}=k_F/C$

$E_{final}=\max(\Delta,T,V)$

$s_{eff} \approx 2K^{-1} \ln [ak_F / \max(1,T/\Delta,V/\Delta)] \approx s - 2K^{-1} \ln(\max(T,V))$ if $\max(T,V) > \Delta$

$\rightarrow T > \Delta \;:\; J_{k,k+1} \sim (\max(T,V)/\Delta)^{2/K}$ Kane-Fisher 1992

Results from typical dots: (short wires)



$\xi_{loc} \approx a/s$

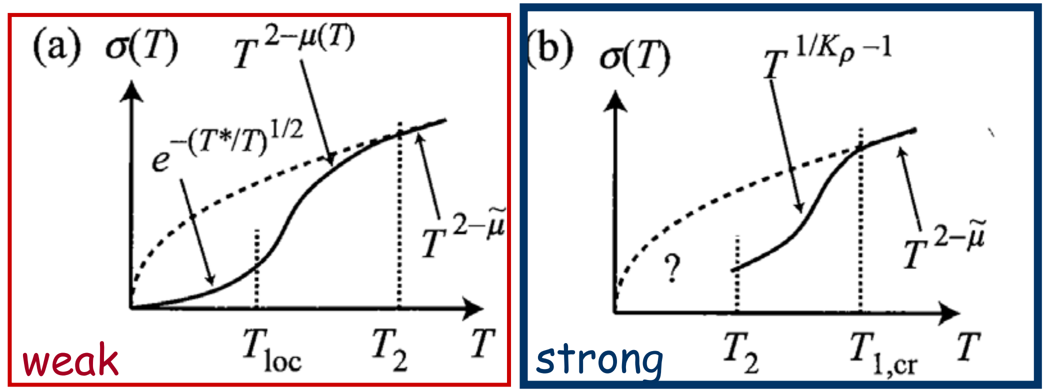
Strong and weak pinning

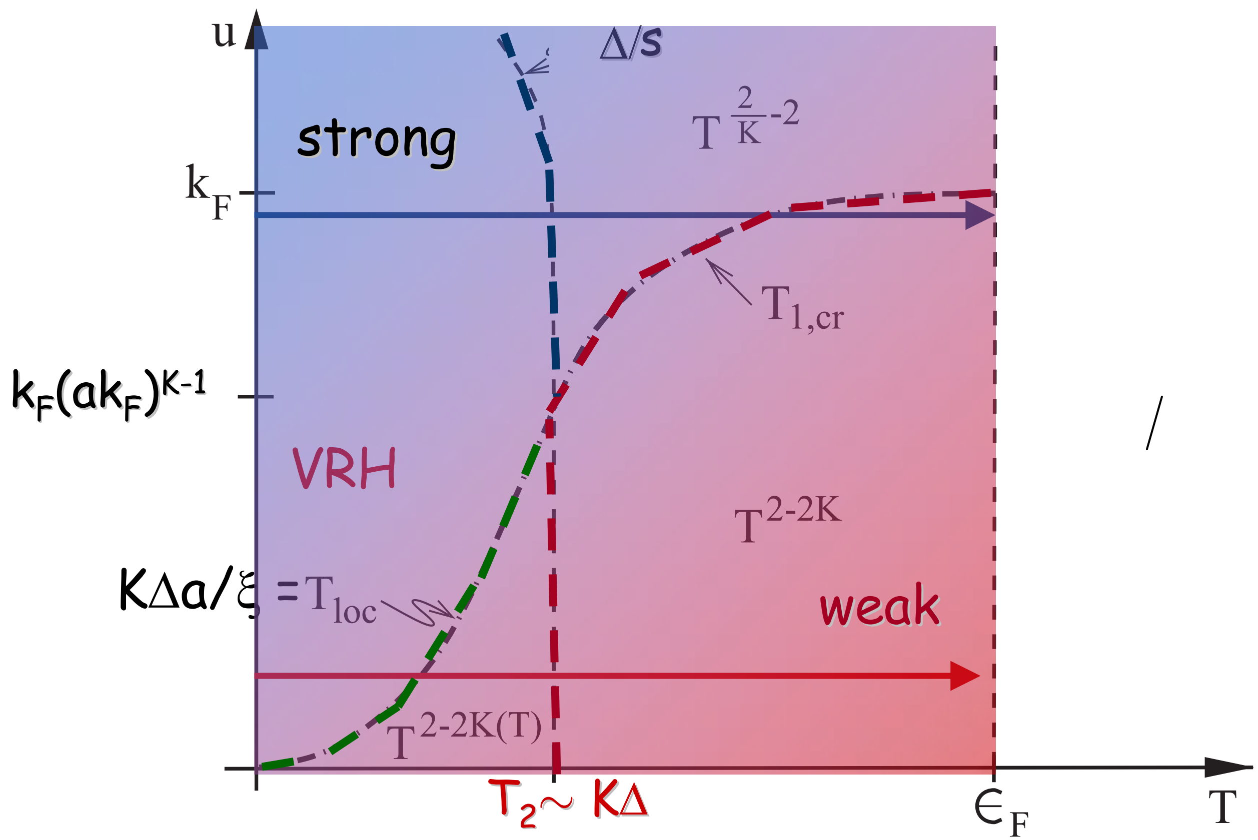
$T > T_{1,cr}$: single impurity weak

$T < T_2$: collective effects

$u < k_F$: SCHA $u \to u_{eff}$

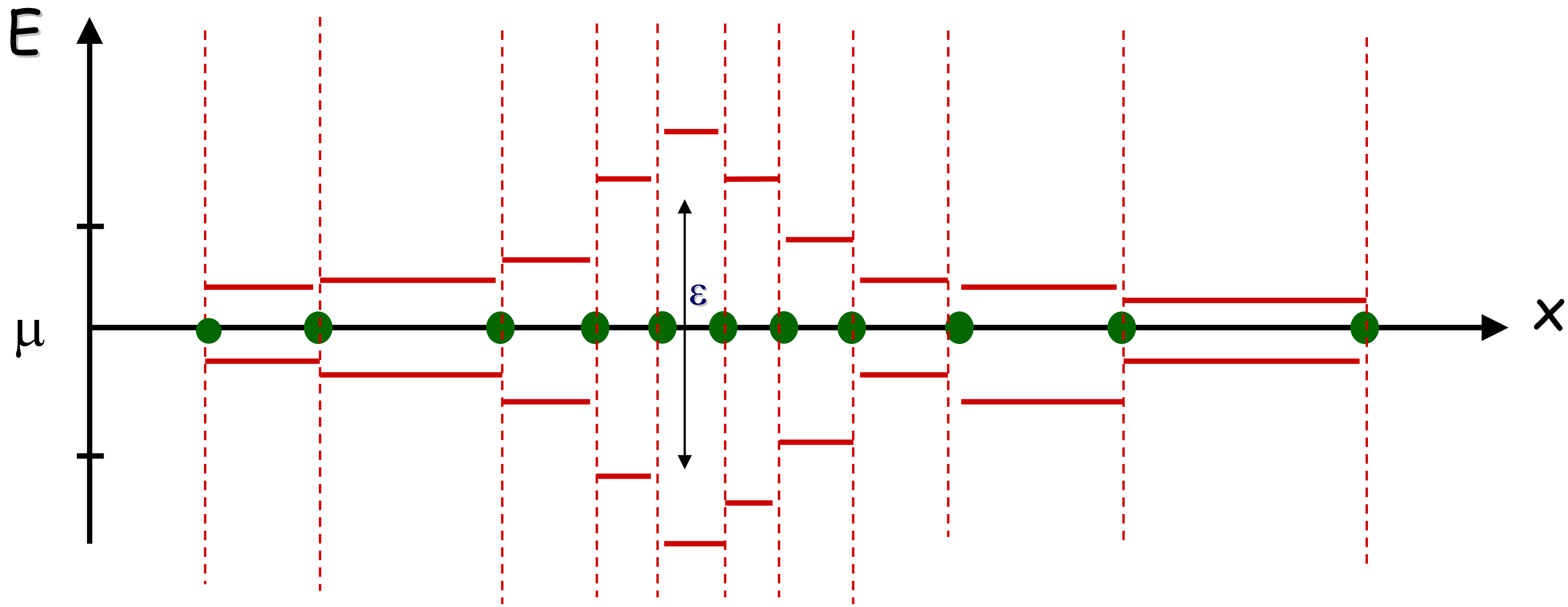
Giamarchi, 2004: "Quantum Physics in One Dimension"





4. Long wires: rare events

- So far considered: typical quantum dots with $a_i \approx a$
- Now: consider regions with many narrow dots with $a_i \ll a$



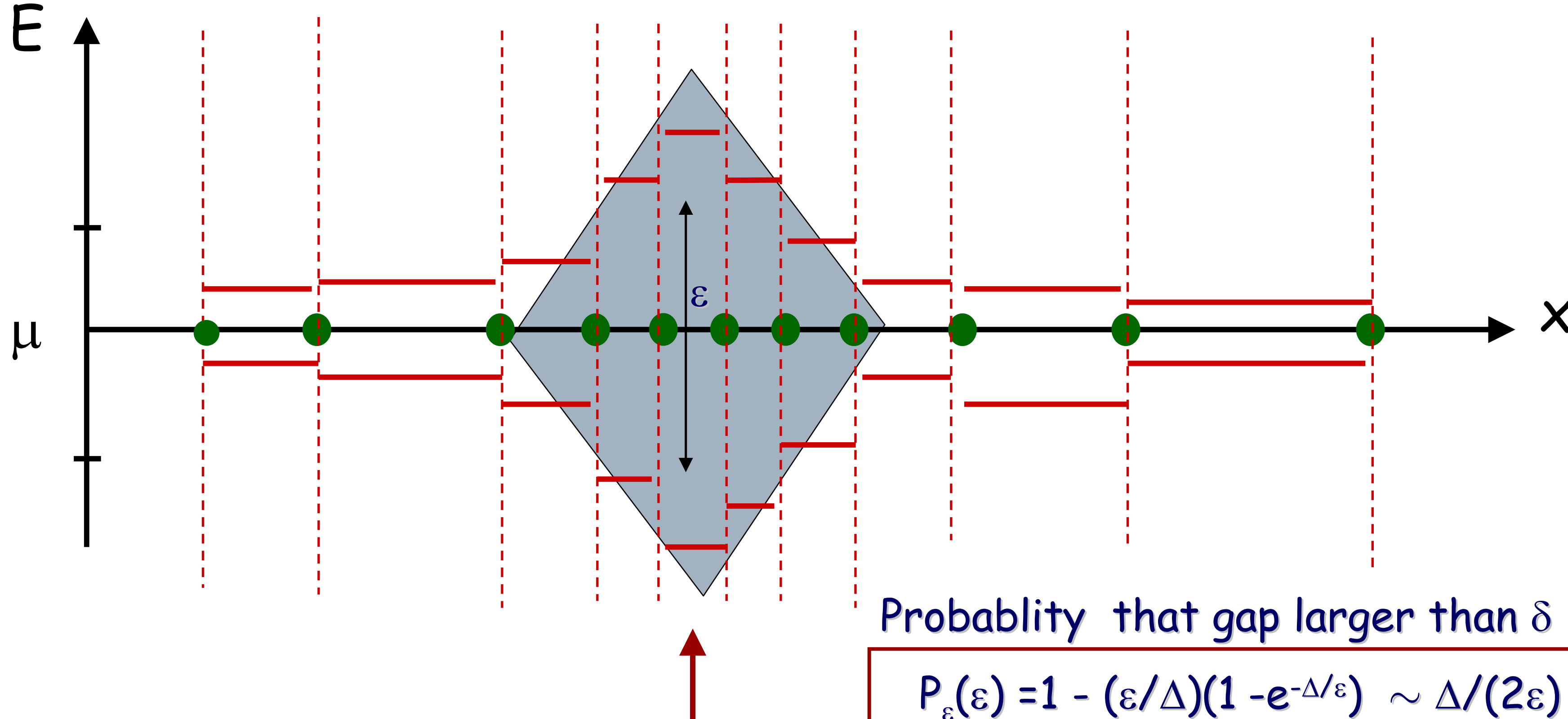
Probablity that gap larger than ε

$\rightarrow \langle \Delta_i \rangle \rightarrow \infty$

$$P(\varepsilon) = 1 - (\varepsilon/\Delta)(1 - e^{-\Delta/\varepsilon}) \underset{\varepsilon \to \infty}{\sim} \Delta/(2\varepsilon)$$

4. Long wires: rare events

- So far considered: typical quantum dots with $a_i \approx a$
- Now: consider regions with many narrow dots with $a_i \ll a$



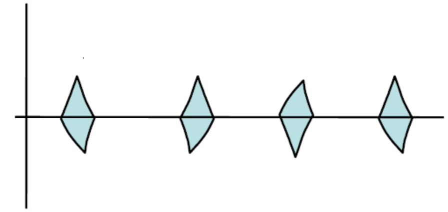
"Break" : sequence of narrow dots

Probablity that gap larger than δ

$$P_\varepsilon(\varepsilon) = 1 - (\varepsilon/\Delta)(1 - e^{-\Delta/\varepsilon}) \sim \Delta/(2\varepsilon)$$
$$\varepsilon \to \infty$$

Wire: non-overlapping breaks + low resistance connecting pieces

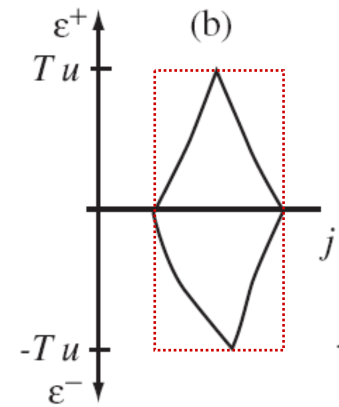
break resistance $R_0 e^u \rightarrow R = L \int P_u(u)\, R_0 e^u\, du$



$P_u(u)$: prob. /unit length that break resistance at least e^u

$I_{jk} \sim e^{-s|j-k|}\, e^{-E_{jk}/T} \sinh\left((\xi_j - \xi_k)/T \right)$ $\qquad \xi_j = F x_j - \mu_j$

(a) Ohmic break of |j-k| dots



envelope: $E_i(\pm 1) \geq \varepsilon_i^{\pm} \rightarrow s|j-k| + \max(\varepsilon_j^{\pm}, \varepsilon_k^{\pm})/T \geq u$

simplification: rectangular break: $s|j-k| \sim u$, $\varepsilon_j^{\pm} \sim uT$

$$P_u(u) \sim x_{VRH}^{-1} P_\varepsilon(\varepsilon)^{|j-k|} \sim x_{VRH}^{-1} P_\varepsilon(uT)^{u/s} \begin{cases} \ln P_u(u) \sim -u^2 T/\Delta s & \text{if } uT \ll \Delta \\ \ln P_u(u) \approx -u/s \ln(2uT/\Delta) & \text{if } uT \gg \Delta \end{cases}$$

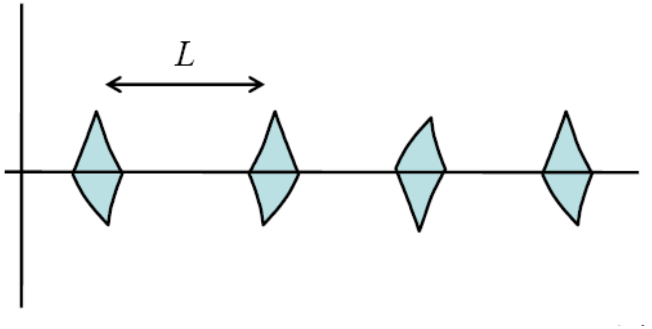
$\ln P_u(u) \sim - u^2 T/\Delta\, s$ if $uT \ll \Delta$

$\ln P_u(u) \approx - u/s \ln (2uT/\Delta)$ if $uT \gg \Delta$

→ infinite wire:

$s \ll 1$: $\rho \sim \exp(s\Delta/4T)$ Raikh Ruzin '89

$s \gg 1$: $\rho \sim \exp(s\Delta\, e^{(s-1)}/2T)$ new



provided $L > a \exp(e^s \Delta(s-1)/Ts)$

→ infinite wire:

Low T , $s \ll 1$: $\rho \sim \exp(s\Delta/4T)$ Raikh Ruzin '89

High T, $s \gg 1$: $\rho \sim \exp(s\Delta\, e^{(s-1)}/2T)$ new

finite wire: $P_u(u)(L/x_{VRH}) \approx 1$ → u_{max}

$sT \ln(L/x_{VRH}) \ll \Delta$: $\rho \sim \exp[\, s\Delta \ln(L/a) / T\,]^{1/2}$ VRH

Raikh Ruzin '89

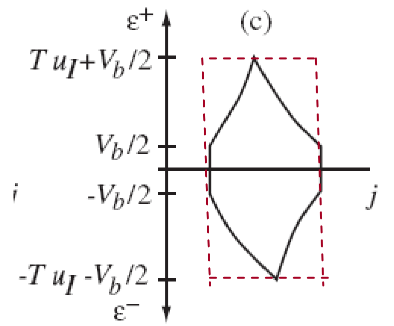
$sT \ln(L/x_{VRH}) \gg \Delta$:

$$\rho \sim \exp[s \ln(L/x_{VRH}) / \ln(Ts/\Delta)] \sim T^{\alpha}$$

$$\alpha = s \ln(L/x_{VRH}) / \ln^2(Ts/\Delta)]$$

(b) Non-Ohmic breaks of m dots $V_b=\xi_j-\xi_k \gg T$

(main voltage drop across the breaks,
but constant $I = I_0 e^{-u}$ current everywhere)



$I_{jk} \sim e^{-s|j-k|} e^{-E_{jk}/T} \sinh((\xi_j-\xi_k)/T)$ $\rightarrow$ $s|j-k| + \max(\varepsilon_j^{\pm}, \varepsilon_k^{\pm})/T - V_b/2T \geq u$

average electric field: $V/L = F = \int dV_b\, V_b P_V(V_b) = x_{VRH}^{-1} 2Ts\, (Tu/\Delta)^{-u/s}$

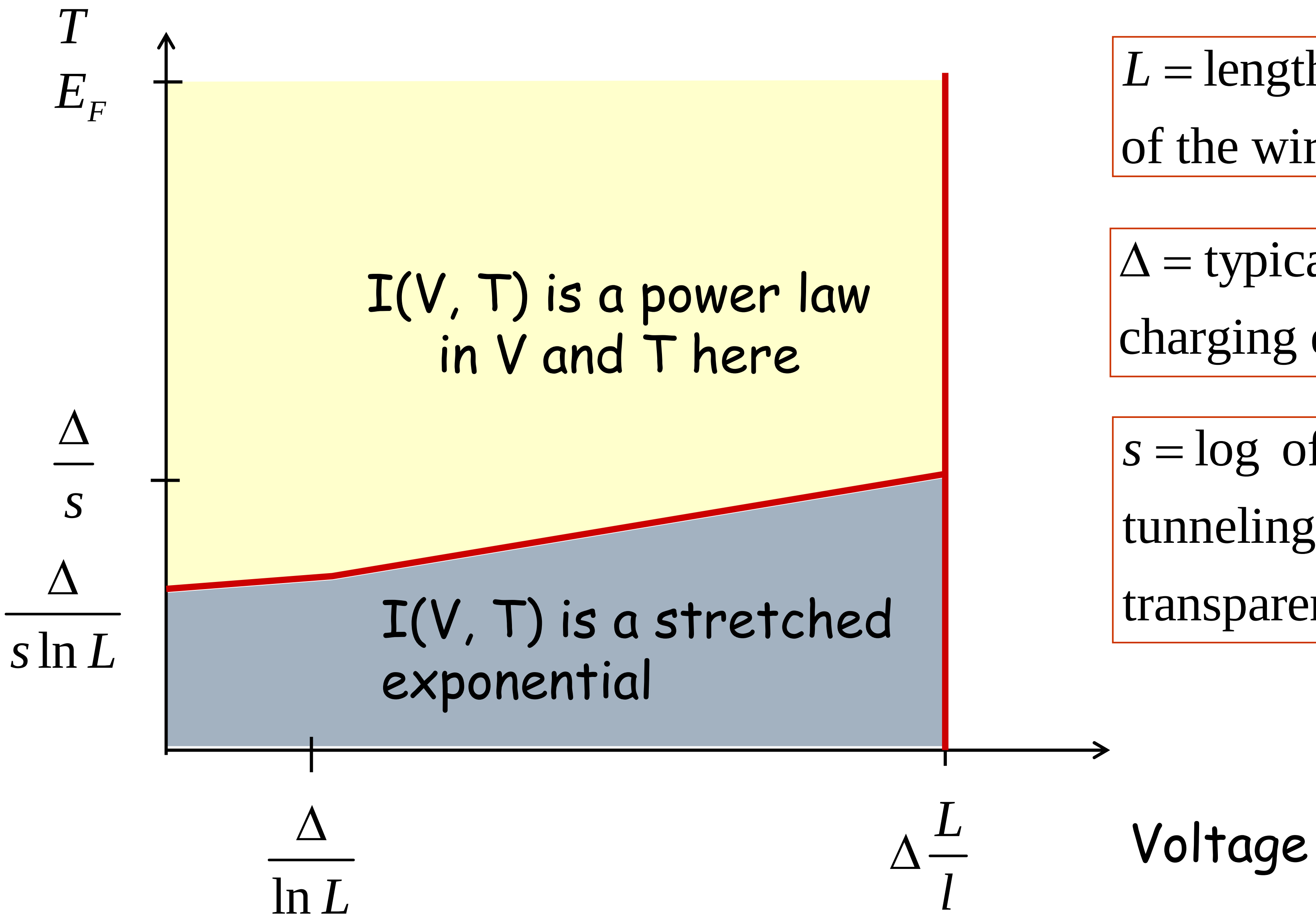
$P_V(V_b) = x_{VRH}^{-1} [P_\varepsilon(uT+V_b/2)]^{u/s} = x_{VRH}^{-1} (Tu/\Delta)^{-u/s} e^{-V_b/2Ts}$

$V_b \ll u_I T$

$u = -\ln(I/I_0) = s \ln(2Ts/Fx_{VRH}) / \ln(Ts/\Delta)$

$I \sim V^\beta$, $\beta = s/\ln(Ts/\Delta)$, $\alpha/\beta = \ln(L/x_{VRH}) / \ln(Ts/\Delta) > 1$

Regime diagram



$L =$ length of the wire

$\Delta =$ typical charging energy

$s = \log$ of tunneling transparency

Conclusions:

- linear and non-linear conductivity
- field and temperature cross-over between single and many impurity tunneling
- low field and temperature: Mott-Shklovskii-VRH
- larger E,T: Kane-Fisher - power law behavior
- Kane-Fisher “single-dominant-barrier” theory is not valid in long wires that contains many (> 100 ?) impurities
- true power-law exponents exceed the single-barrier ones by a “large” log-factor (perhaps, by 2 or 3 in practice)!
- resistance is controlled by “difficult spots” - dense clusters of impurities
- global weak/strong pinning regime diagram