Pinning by planar defects and the planar glass phase*

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* T. Emig and T.N., Phys. Rev. Lett. **97**, 177002 (2006)
A. Petkovic and T.N., Phys. Rev. Lett. **101**, 267005 (2008)
A. Petkovic, T. Emig and T.N., Phys.Rev. B **79**, 224512 (2009)

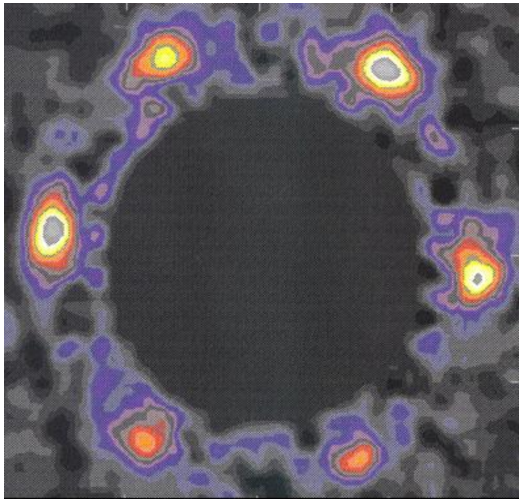
1. Bragg glass : A reminder
2. Planar defects
3. A single planar defect
4. Many defects
5. Relation to other problems

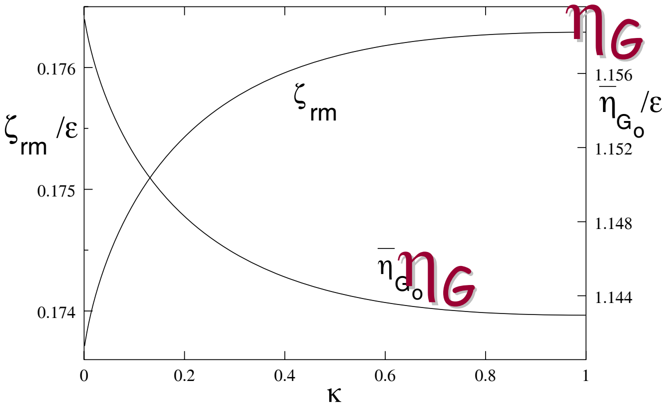
Bragg Glass - A Reminder

Bragg glass: a reminder

Elastic vortex lattice + point disorder $\rightarrow$ quasi-long range order

$$S(\mathbf{k}+\mathbf{G}) \sim (k_\perp^2 + (c_{44}/c_{66}) k_z^2)^{-(3-\eta_G)/2}$$





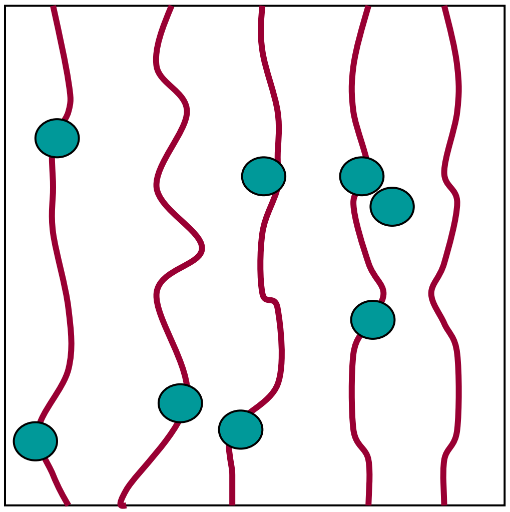
$1.14 < \eta_{G_0} < 1.16$

$G = m b_1 + n b_2$

Elastic constants c_{11}, c_{44}, c_{66} remain finite!

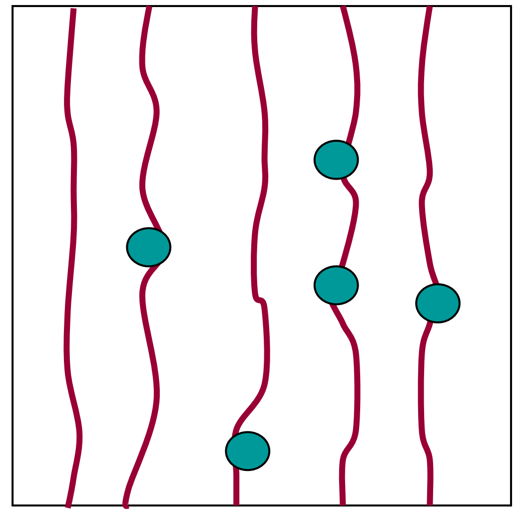
$$\mathcal{H}_{\rm el} = \frac{1}{2}\int d^2x dz \left\{ c_{11}(\boldsymbol{\nabla}_\perp \mathbf{u})^2 + c_{66}(\boldsymbol{\nabla}_\perp \times \mathbf{u})^2 + c_{44}(\nabla_\parallel \mathbf{u})^2 \right\}$$

Sample to sample fluctuations of free energy



$F \sim L^3$

$\Delta F \sim L^{\chi}$



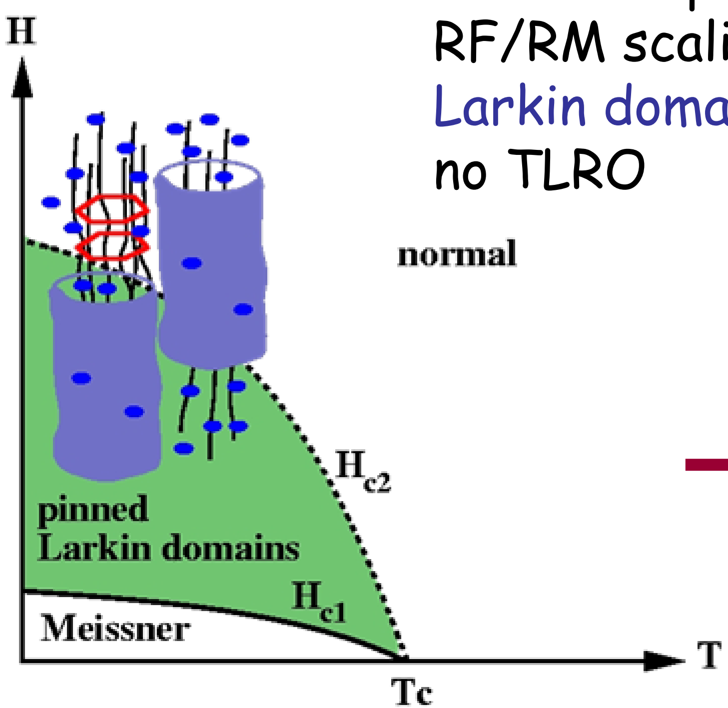
$F \sim L^3$

non-linear resistivity $\rho(j) \sim \exp -[j(H,T)/j]^{\mu}$

Bragg glass $\chi=1$, $\mu=\chi/(2-\zeta)=1/2$

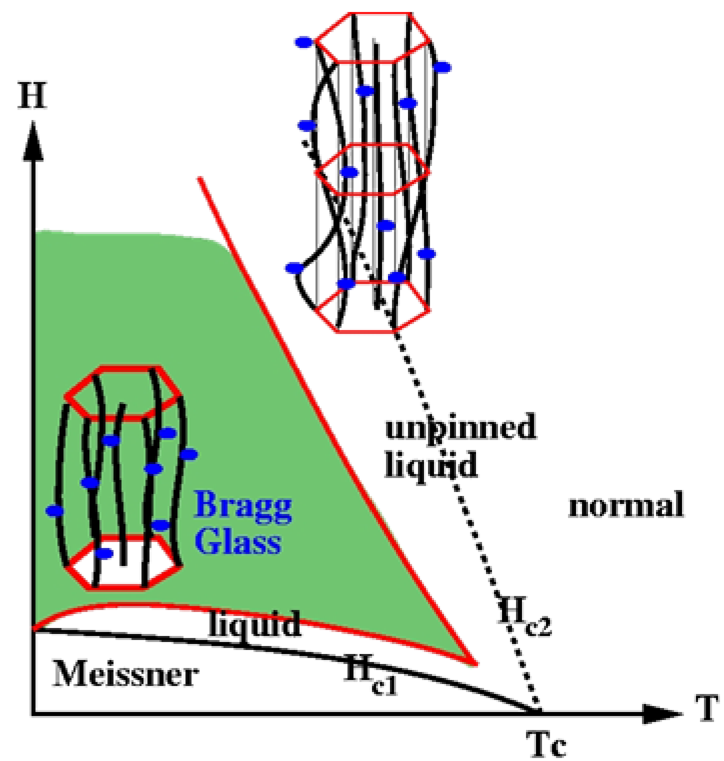
Vortex phase diagram, elastic theory

collective pinning
RF/RM scaling
Larkin domains
no TLRO



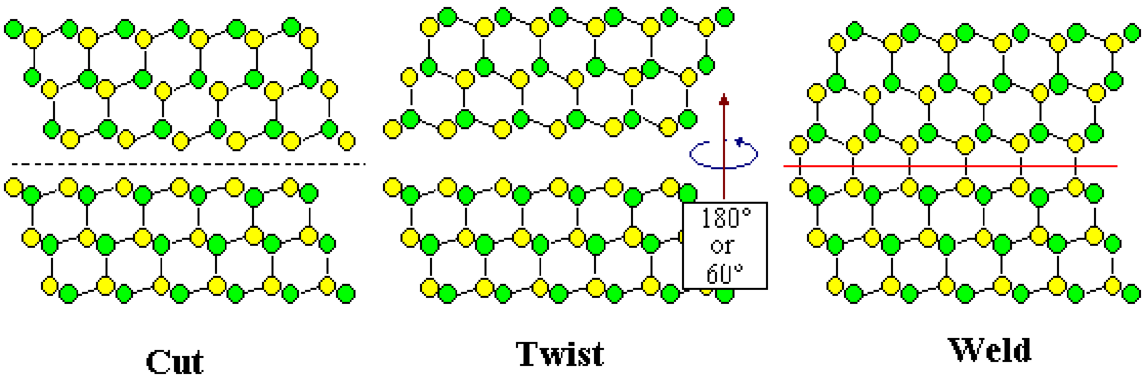
→

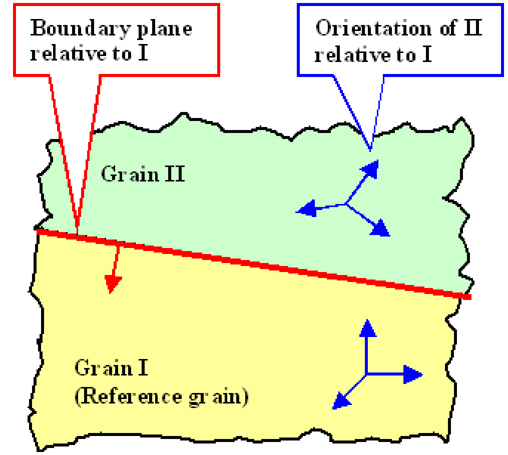
thermal
lattice melting

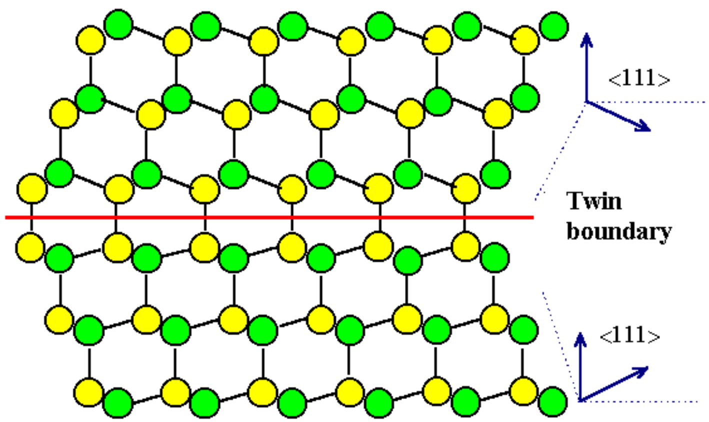


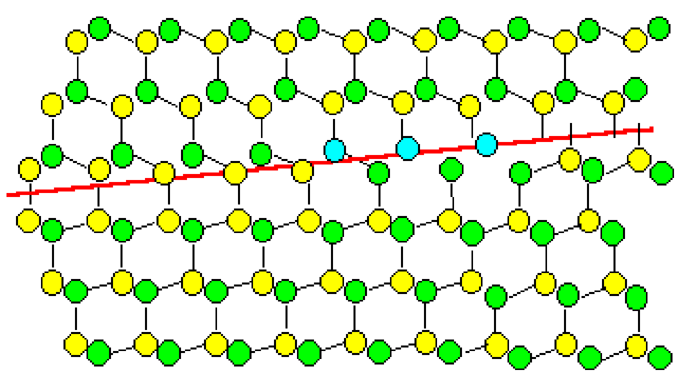
Bragg glass scaling
quasi-TLRO

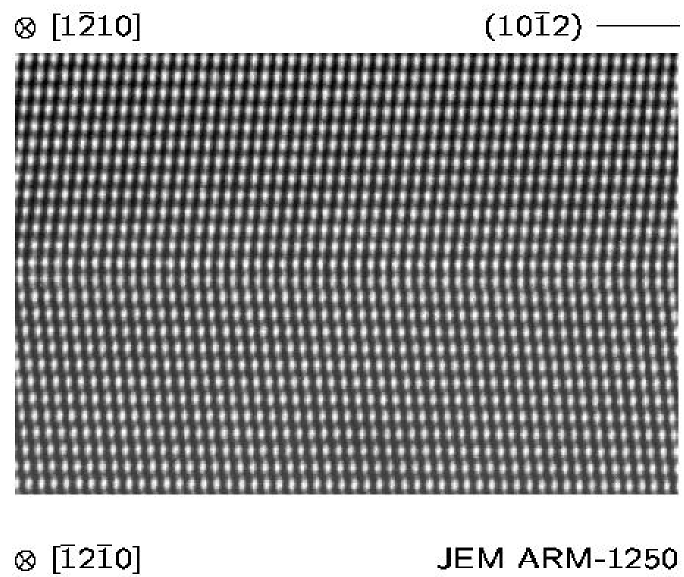
Planar Defects


180°
or
60°
Cut
Twist
Weld


Boundary plane
relative to I
Orientation of II
relative to I
Grain II
Grain I
(Reference grain)


<111>
Twin
boundary
<111>




⊗ [1$\bar{2}$10]
(10$\bar{1}$2)
⊗ [$\bar{1}$2$\bar{1}$0]
JEM ARM-1250

Letters to Nature

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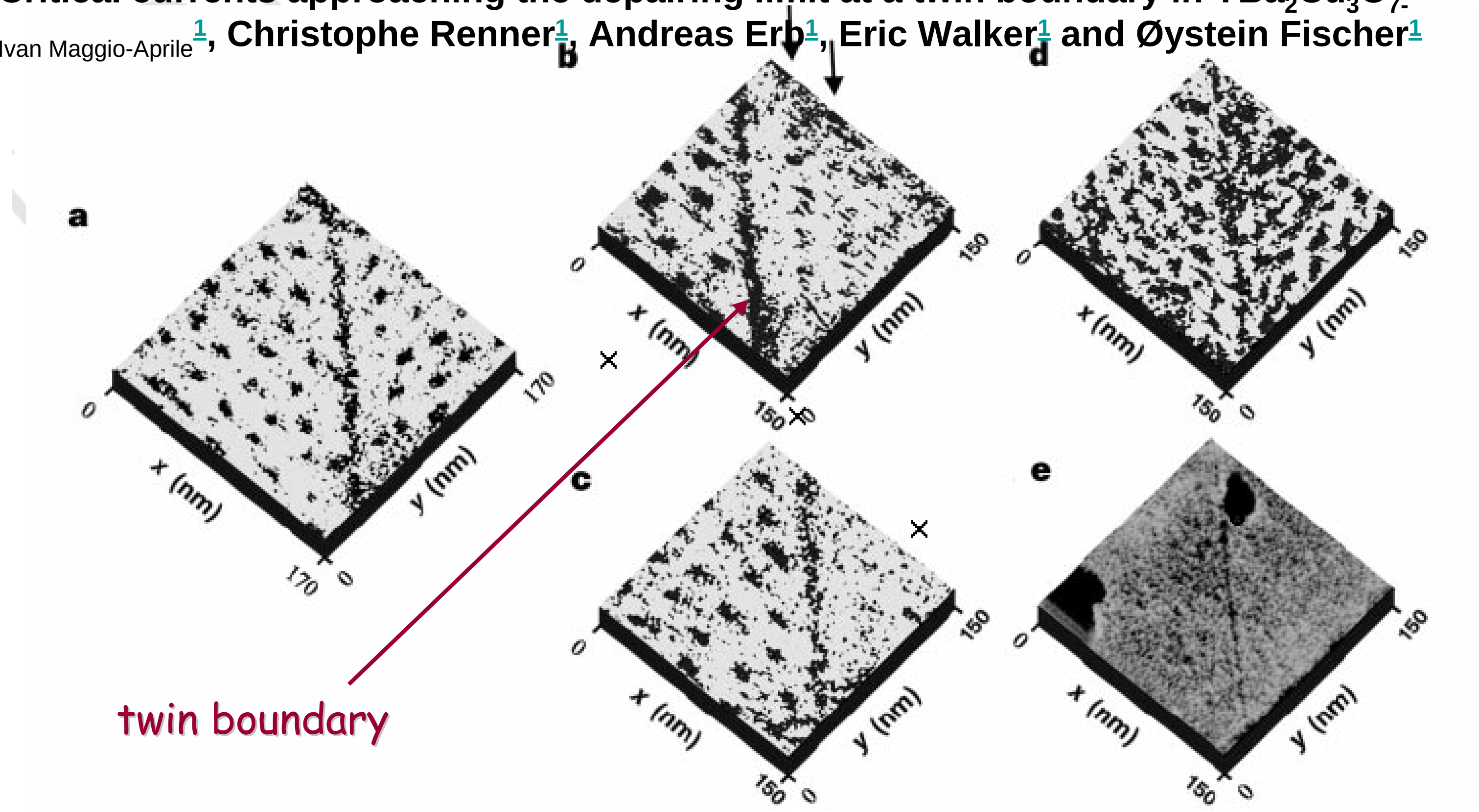
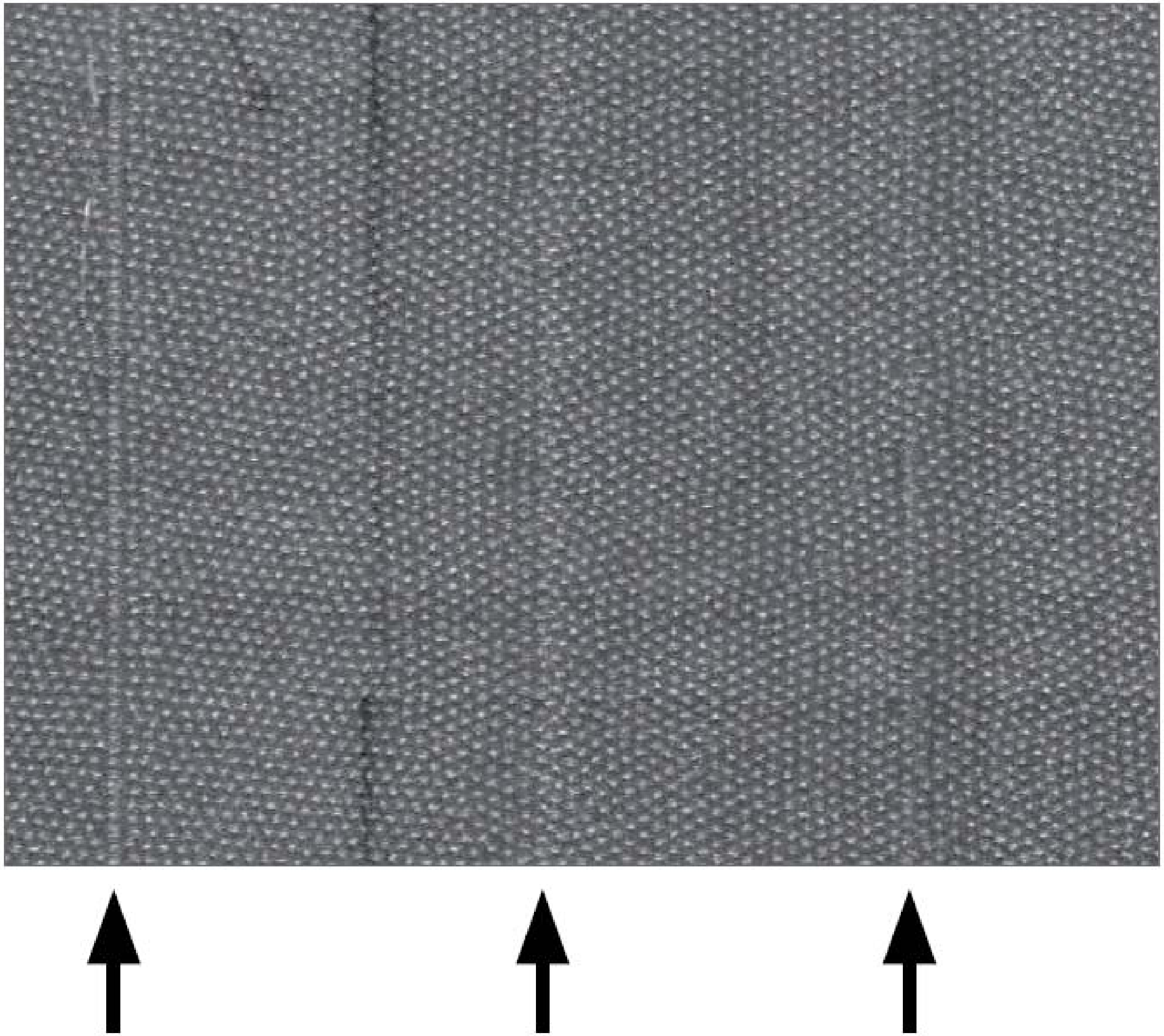
Critical currents approaching the depairing limit at a twin boundary in $YBa_2Cu_3O_{7-}$

Ivan Maggio-Aprile[1], **Christophe Renner[1], Andreas Erb[1], Eric Walker[1] and Øystein Fischer[1]**

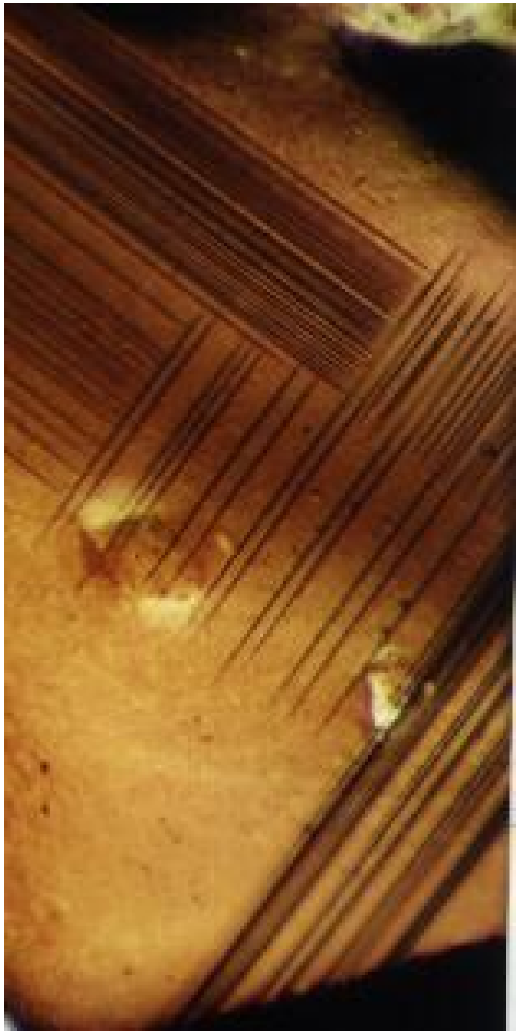


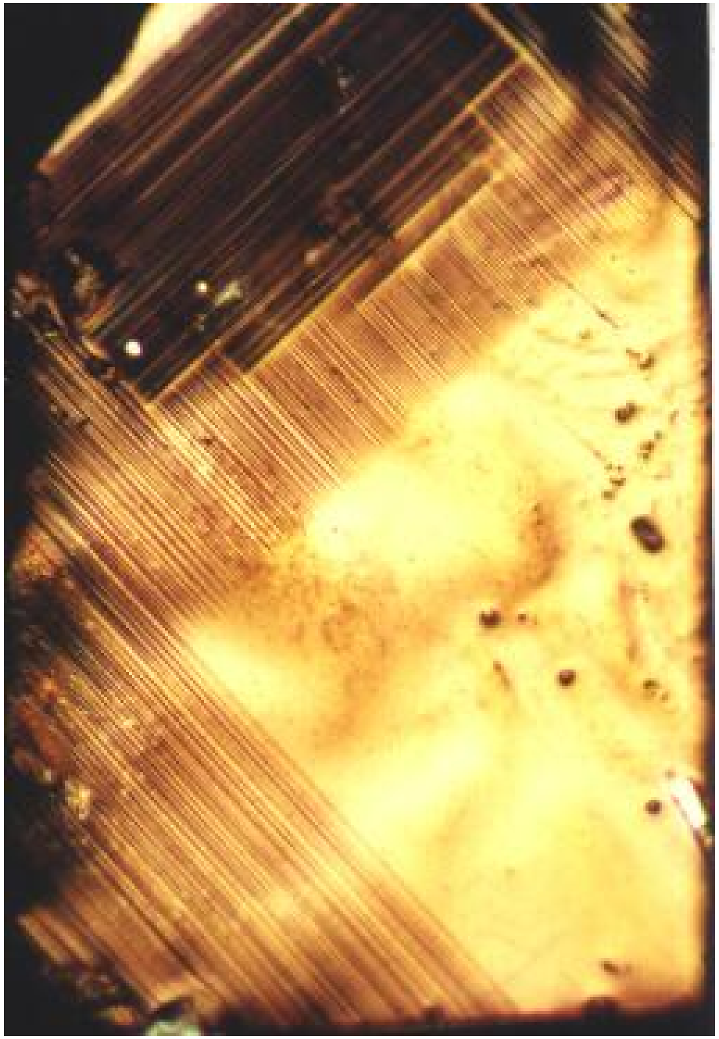
The grey scale is defined by the ratio of the conductance measured at 20 meV to the conductance measured at zero bias. Grey tones varying from clear to dark correspond to ratios ranging from high to low. Note that for experimental reasons, the centre of the different images presented here are not exactly at the same position; this puts the twin boundary (TB) at slightly different positions relative to the centre in the various images. **a**, 170 170 nm^2 image taken at 3 T (field-cooled). Both domains are filled with a nearly equal density of flux lines, and the 90° rotation of the *ab* -plane anisotropy is observed across the TB. **b**, 150 150 nm^2 image taken 12 hours after the field was reduced from 3 to 1.5 T. The arrows indicate the vortex movements observed in the domain to the right, forming non-continuous lines extending in a direction parallel to the TB. **c**, Three days after field reduction, no more flux lines can be detected throughout the domain to the right over at least 80 nm. **d**, 150 150 nm^2 image taken after a 3 T–1.5 T–0 T–6 T field cycle. Both domains show a high density of flux lines, and a flux gradient is measured across the TB. **e**, Topographic image of the YBCO surface taken simultaneously with **d**. The TB appears as a narrow structure about 0.1 nm deep. Note that the width of this line is much smaller than the width of the dark line in the spectroscopic images, giving further support to the interpretation that the dark line comes from a high density of vortices along the TB.



Planar defects in BSCCO (M. Menghini, Y. Fasano, F. De la Cruz, and E. Zeldov)

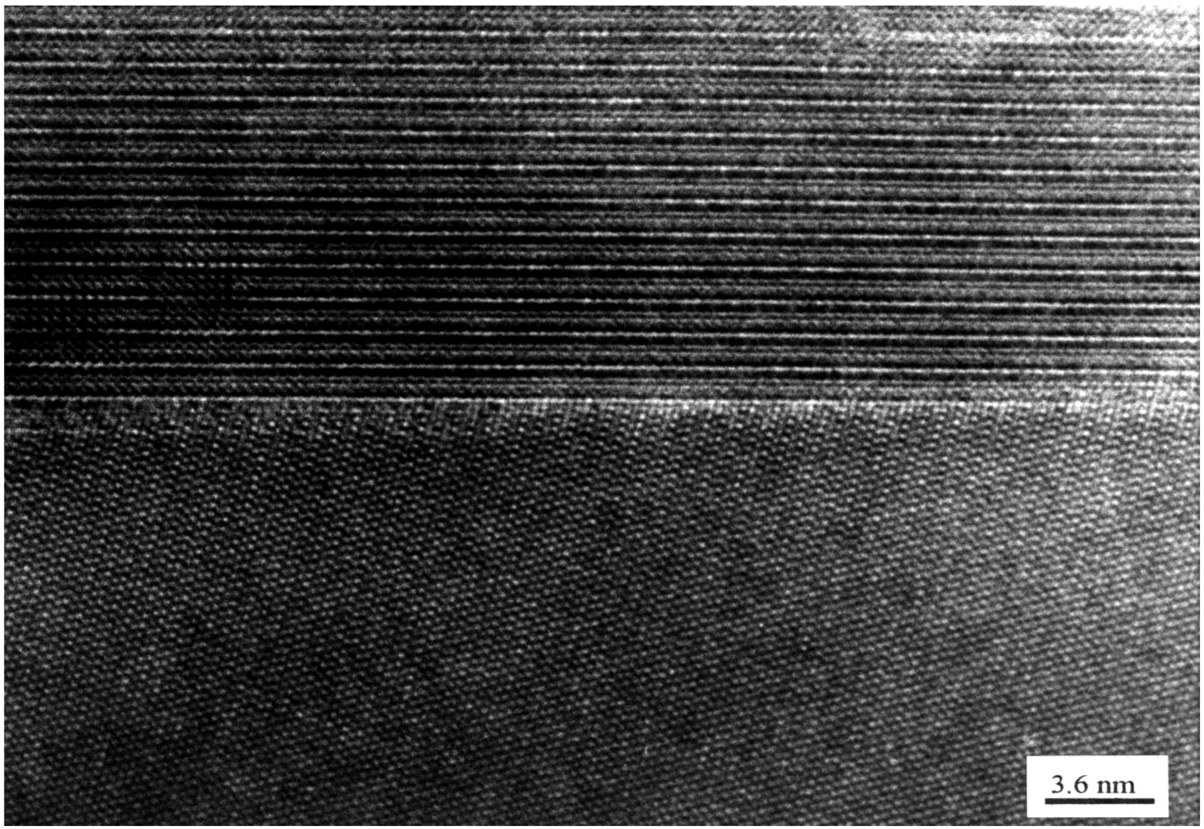
- twin boundaries in YBCO crystals
- width ~ 2nm distance, ~ 1μ m





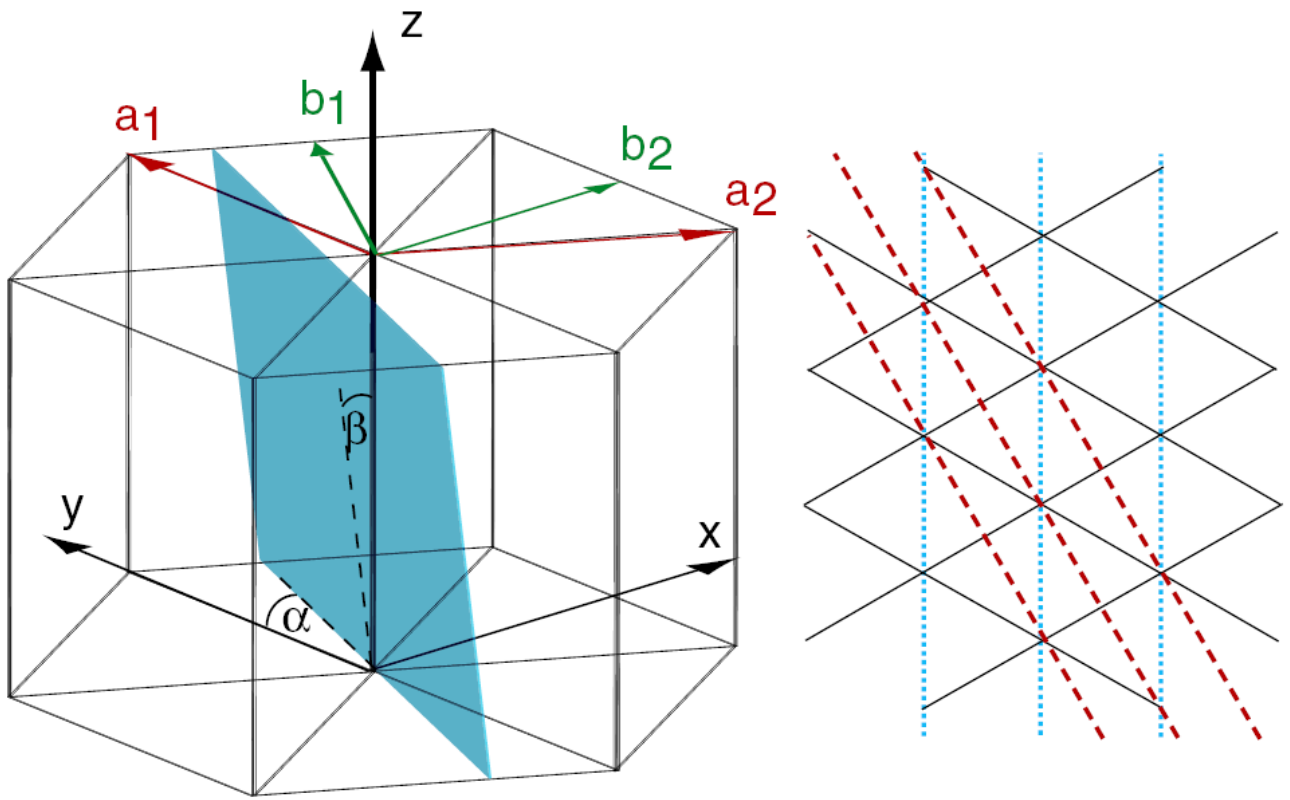
These images were taken by Martín Irigoyen under the direction of Eduardo Rodríguez R

- TEM micrography showing the presence of twin boundaries in the YBCO melt-textured samples.
[sample by IFW, Dresden; image by CNRSM-PASTIS]



A Single Planar Defect

Parametrization of the defect plane



Position vector on the defect plane

unit vector normal to the defect plane

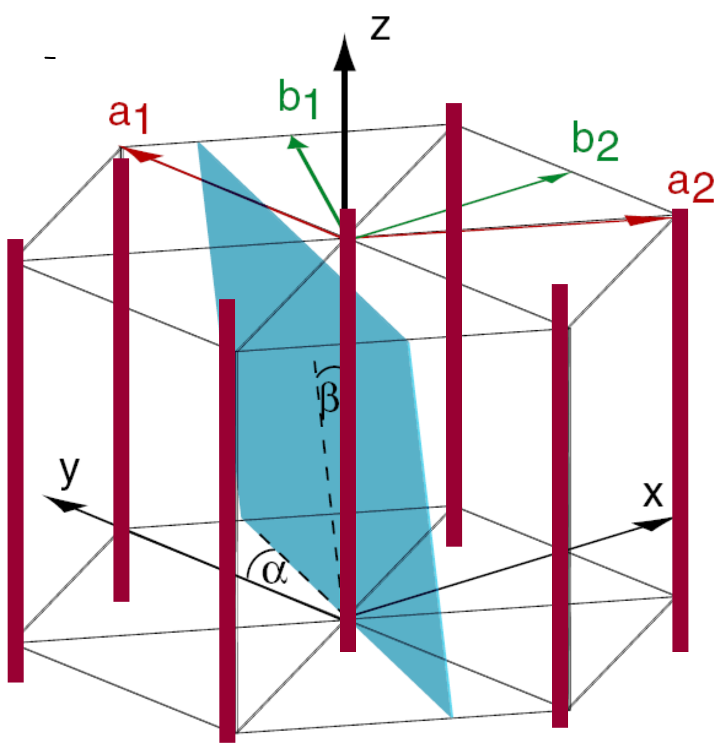
$$\mathbf{r}_D = (\mathbf{x}_D, z_D) + \Delta\,\mathbf{n}_D, \qquad z_D = t\cos\beta,$$

$$\mathbf{x}_D = (s\sin\alpha - t\cos\alpha\sin\beta,\ s\cos\alpha + t\sin\alpha\sin\beta)$$

Defect planes Hamiltonian

$$\rho_{\text{vortex}}(\mathbf{u},\mathbf{r}) \approx \frac{1}{a_B^2}\frac{1}{|1+\partial_\alpha u_\alpha|}\sum_{\mathbf{G}} e^{i\mathbf{G}\left(\mathbf{x}-\mathbf{u}(\mathbf{r})\right)}$$

vortex density



Defect energy

$$H_D = \int d^3r\, V_D(\mathbf{r})\, \rho(\mathbf{r})$$

Defect potential

$$V_D(\mathbf{r}) \approx -v\, \delta(\mathbf{r}-\mathbf{r}_D)$$

$$\mathcal{H}_D = v\rho_0 \int dt ds \left\{ \nabla_{\mathbf{x}} \mathbf{u}(\mathbf{r}_D) - \sum_{\mathbf{G}\neq\mathbf{0}} e^{i\mathbf{G}[\Delta\mathbf{n}_D + \mathbf{x}_D - \mathbf{u}(\mathbf{r}_D)]} \right\}$$

Is a defect a relevant perturbation?

$$< \mathcal{H}_D = v\rho_0 \int dtds \left\{ \nabla_{\mathbf{x}} \mathbf{u}(\mathbf{r}_D) - \sum_{\mathbf{G}\neq 0} e^{i\mathbf{G}[\Delta \mathbf{n}_D + \mathbf{x}_D - \mathbf{u}(\mathbf{r}_D)]} \right\} > \sim L^{\Psi}$$

➡ $\exp\{G x_D(s,t)\} = 1$ ➡ defect plane parallel **B**

➡ $G x_D(s,t) = 0$ defect plane characterized by (integer) Miller indices

$\langle H_D \rangle \sim v \cos(2\pi\Delta/\delta)\, (L/L_a)^{2-g}$

δ : distance between equivalent defect planes

$g \equiv (3/8)\, \eta_G\, a^2/\delta^2$ ➡ $\psi - \chi = 1 - g > 0$

Thus defect relevant if $g \leq 1$ and

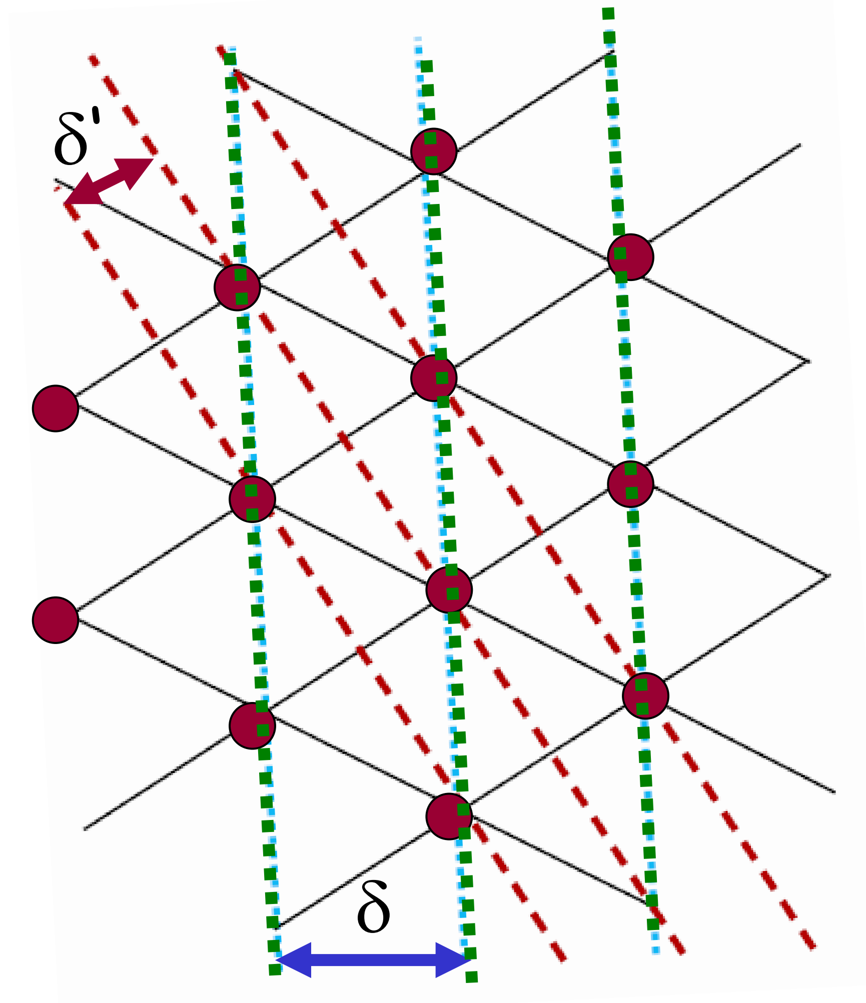
$G = m b_1 + n b_2$

$$L > L_D \equiv L_a (c a^2 L_a / v)^{1/(1-g)}$$

$$g \equiv (3/8)\, \eta_G\, a^2/\delta^2$$

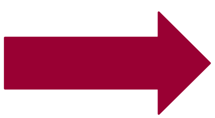
$\psi - \chi = 1 - g > 0$

$1.14 < \eta_G < 1.16$



$$a^2/\delta^2 = 4/3(m^2 + mn + n^2)$$

m, n Miller indices of defect plane



$$g = \eta_G/2,\ 3\eta_G/2,\ 7\eta_G/2, \ldots, < 1\ !$$

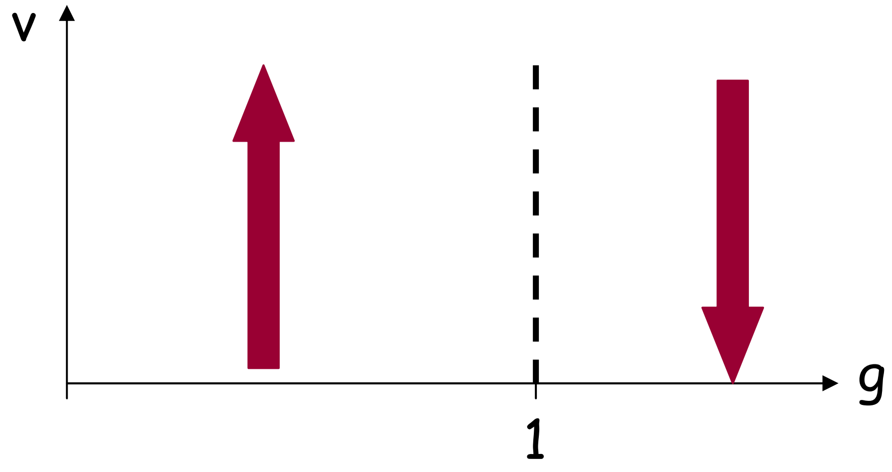
Weak tilted defects always irrelevant

So far: weak defects
Now: Critical coupling for strong defects?

Inegrate out all displacements out of defect plane

➡ effective model for $\varphi = 2\pi\, u_D(\mathbf{r}_D)/\delta$

$$\mathcal{H}_{2D} = \frac{K}{2}\int d^2\mathbf{q}|\mathbf{q}||\varphi_{\mathbf{q}}|^2 + \int d^2 r_D \left\{ \frac{2\sqrt{\pi g}K}{\xi}\cos(\varphi-\alpha) + \frac{v_1}{L_a^2}\cos(\varphi) \right\}$$



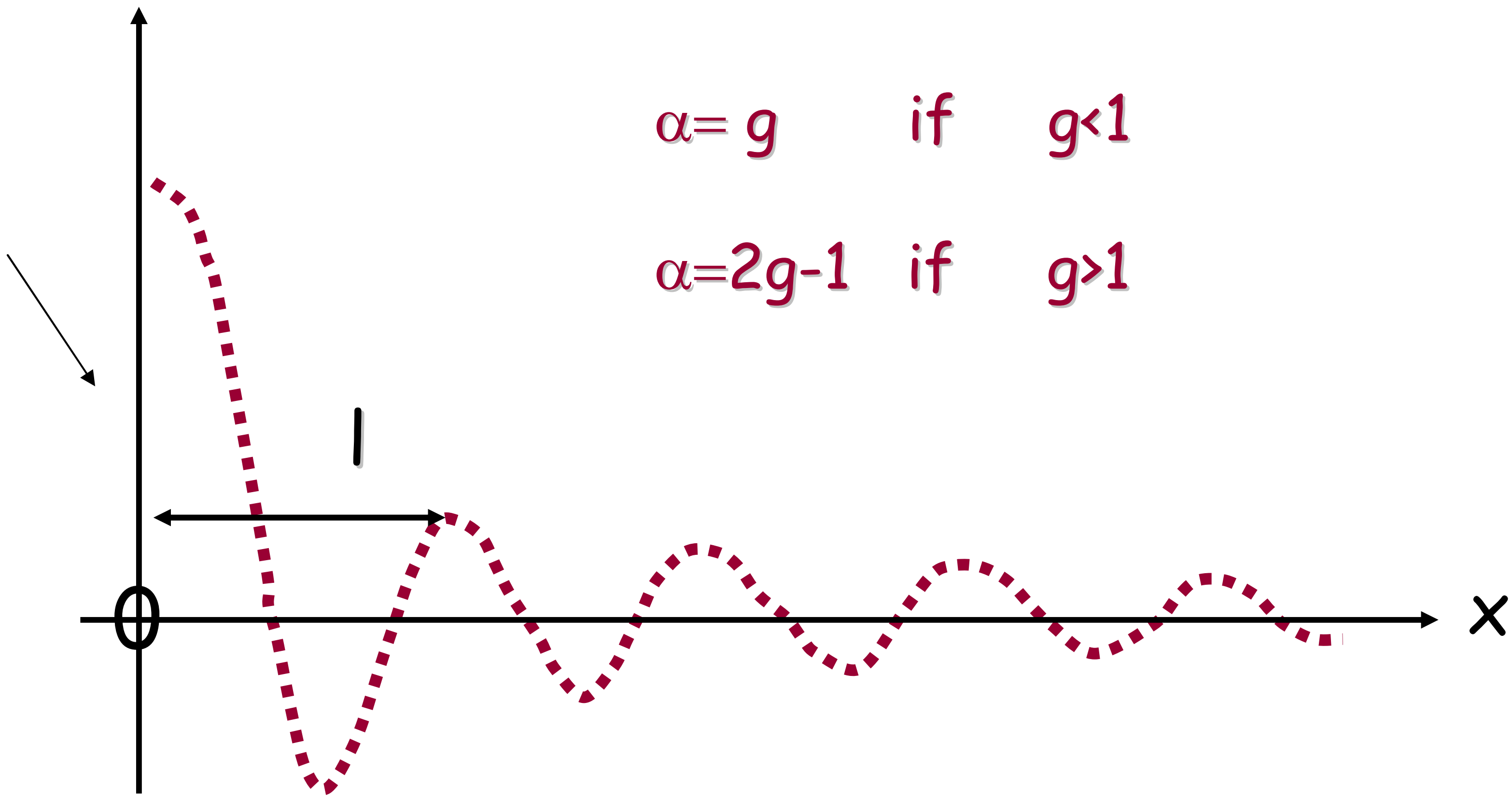
Density oscillations close to the defect

$$\langle \rho(x)-\rho_0 \rangle \sim (L_a/x)^{\alpha} \cos(2\pi x/\delta)$$

$\alpha = g$ if $g<1$

$\alpha = 2g-1$ if $g>1$

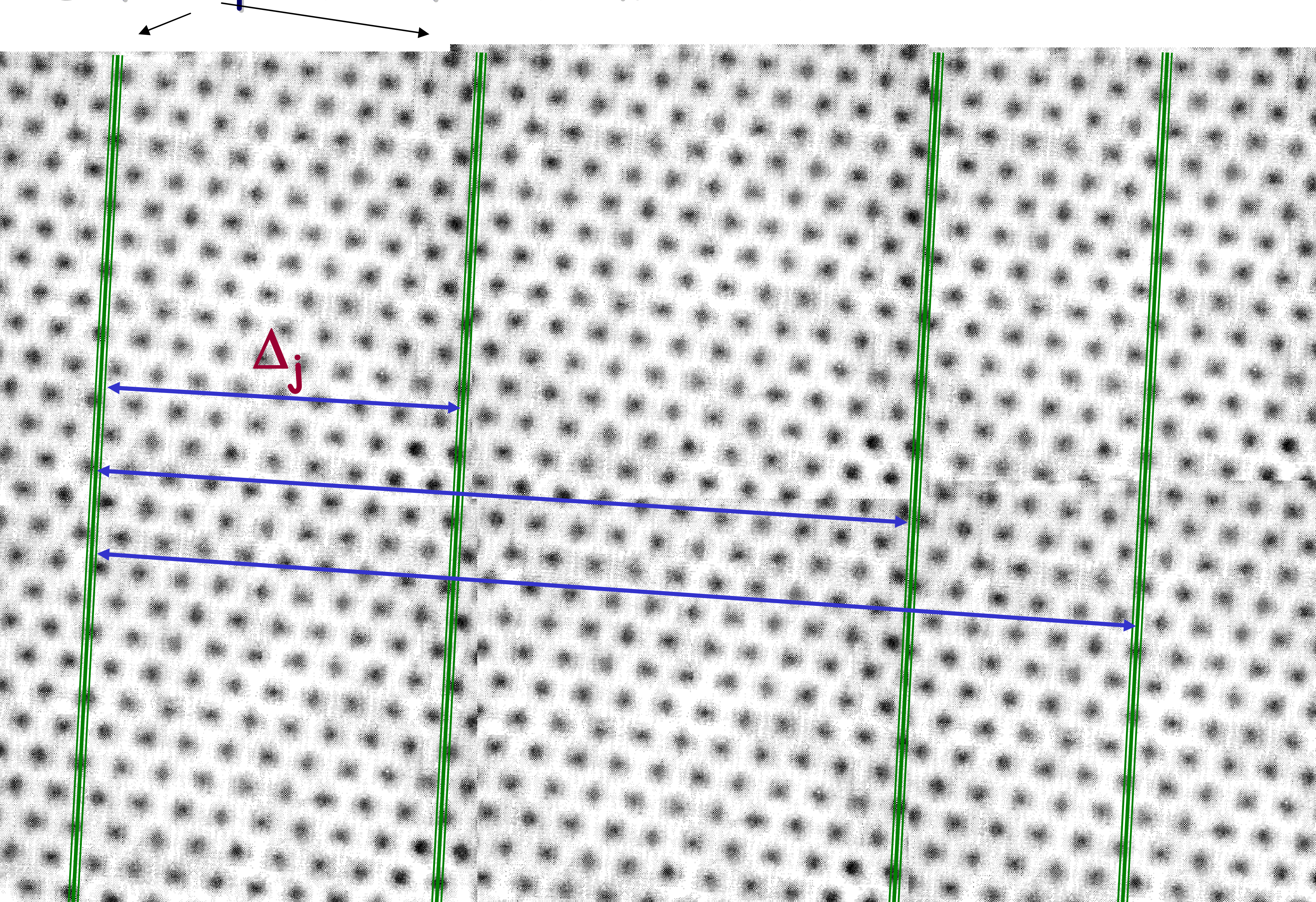
Defect plane x=0



If defect not parallel B: additional factor e^{-x/x_B}, $x_B = 1/(G_D\,|\sin\beta|)$

Many Defects

Defect planes of random distance



Relevance of many defects (no point disorder)

Ignore displacement parallel to defects:

dimensionality shift D ➡ D-2

➡ D=3: exponential decay of correlations in direction perpendicular to defects

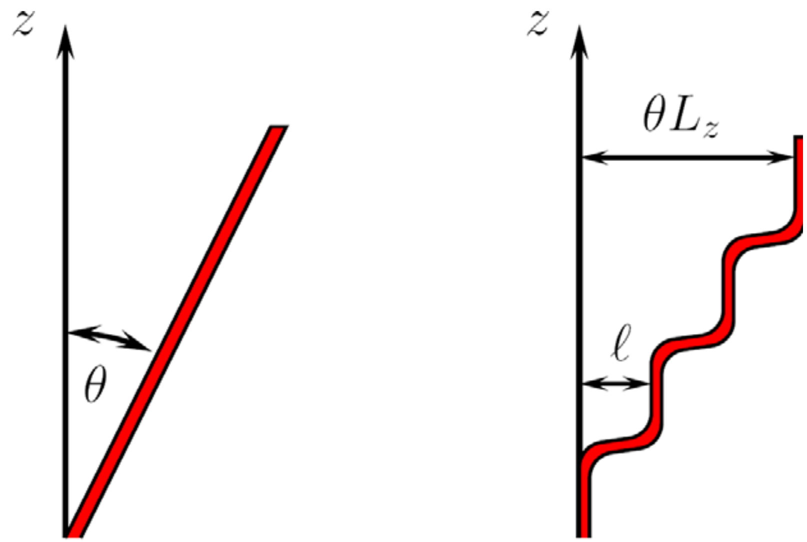
➡ upper critical dimension D=6

Functional renormalization group calculation in $D=6-\varepsilon$

Elastic constants $c_{44}, c_{66} \to \infty$

Elastic constants

$c_{44}, c_{66} \to \infty$



without defects | with defects

$\theta \ll 1$

$$\frac{\Delta E}{L_x L_u} = \frac{c_{44}}{2} L_z \theta^2 \qquad \frac{\Delta E}{L_x L_u} \approx \Sigma_z \theta$$

Transverse Meissner effect $H_{x,c} \sim \Sigma_z \delta / \phi_0$

Resistance agains shear strees $\sigma_{xy,c} \sim \Sigma_y \delta$

Diverging sample to sample fluctuation of magnetic susceptibility

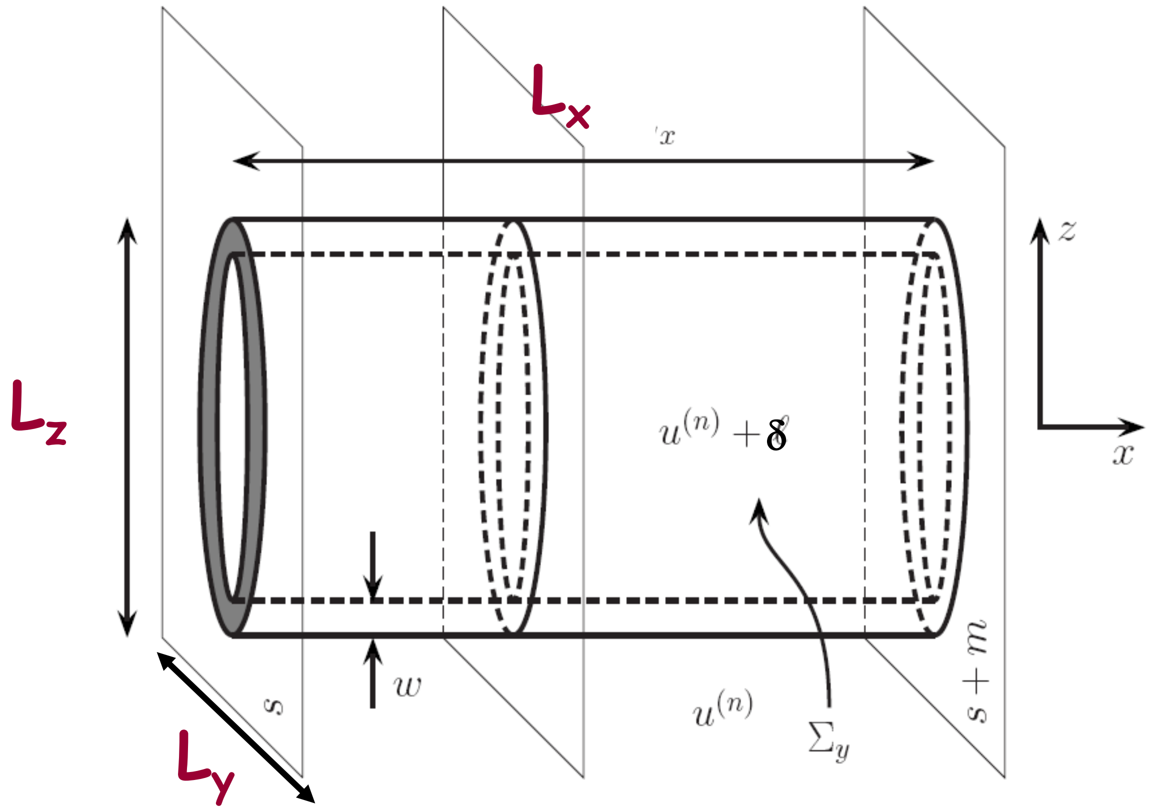
- longitudinal susceptibility
  $\chi = \rho_0 \phi_0 \frac{\langle \partial_x u_x \rangle}{\partial H_z}$
  $\overline{\chi} = \frac{(\rho_0 \phi_0)^2}{4\pi c_{11}}$
  No dependence on disorder due to statistical tilt symmetry.

- Perturbation theory gives
  $(\overline{\chi^2} - \overline{\chi}^2)/\overline{\chi}^2 = R_D''''(0) L^\epsilon / (5 c_{11}^2) \sim (L/L_D)^\epsilon$, for $L < L_D$.
  Signature of a glassy phase!

Flux creep

$$E_{drop} = L_x L_y L_z \left(c_{11}\delta^2/L_x^2 + \Sigma_z/L_z + \Sigma_y/L_y - f\delta\right)$$

$L_z \sim L_y \sim L_x^2$



non-linear resistivity

$$\rho(j) \sim \exp -[j(H,T)/j]^{3/2}$$

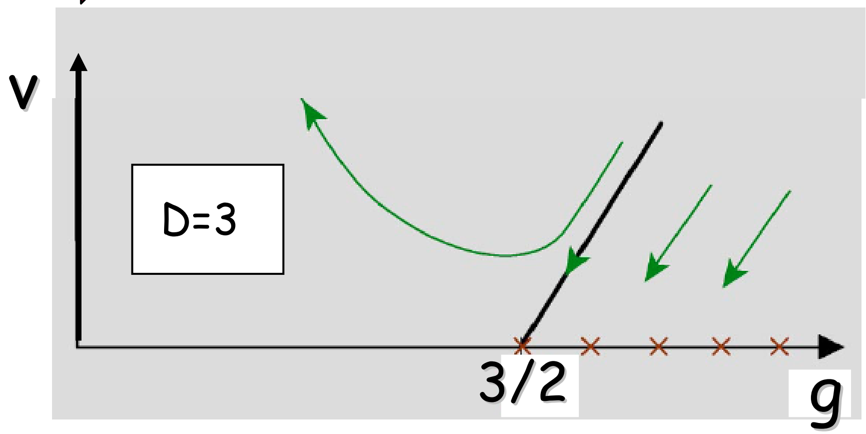
Relevance of many defects in the presence of point disorder

Assume defects of random distance but identical orientation

$$< \sum_j H_D(\Delta_j) > = 0$$

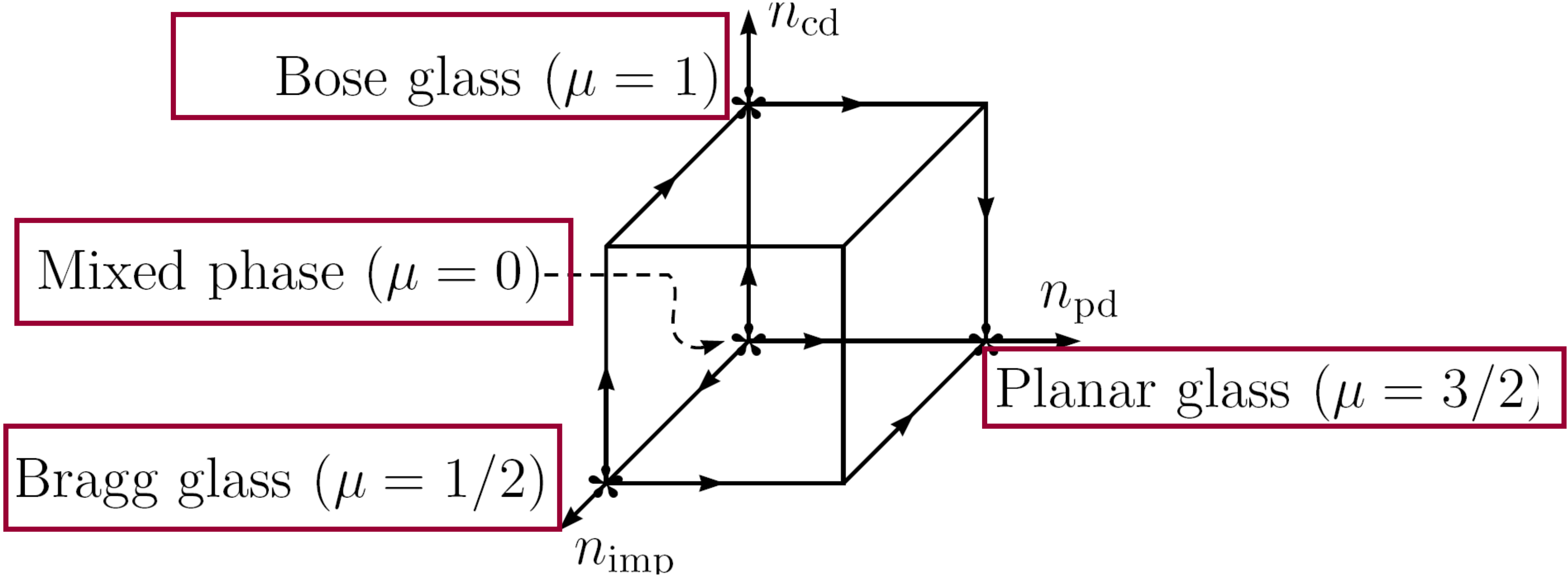
$$< (\sum_j H_D(\Delta_j))^2 > \sim n_D L^{5-2g}$$

Defect array relevant if $g < g_c = 3/2$

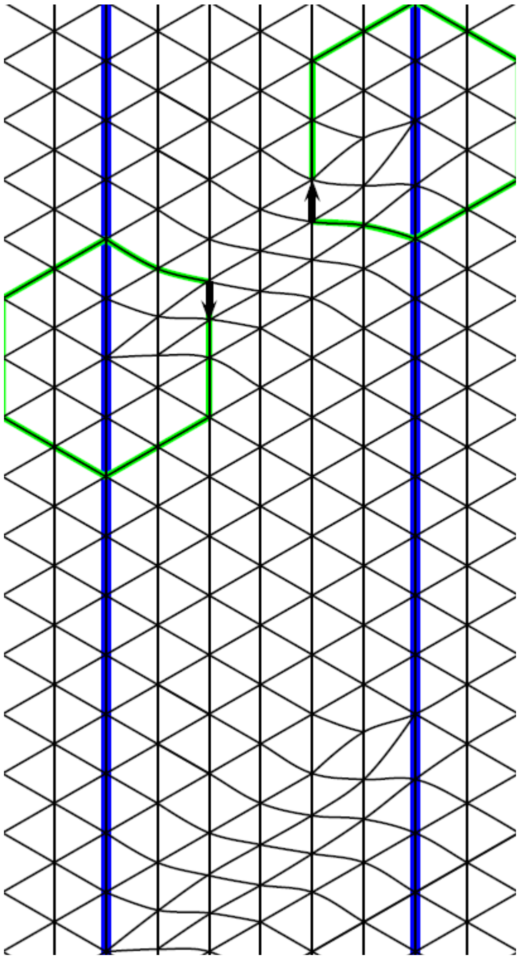


non-linear resistivity

$\rho(j) \sim \exp -[j(H,T)/j]^{\mu}$

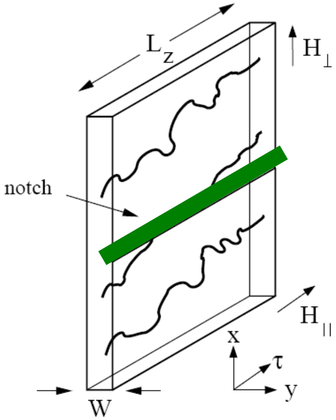


Transverse displacements: dislocations



Relations to other problems

Closely related problems:



• Superconducting plane with line defect

EUROPHYSICS LETTERS 15 April 2004

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Non-Hermitian Luttinger liquids and vortex physics

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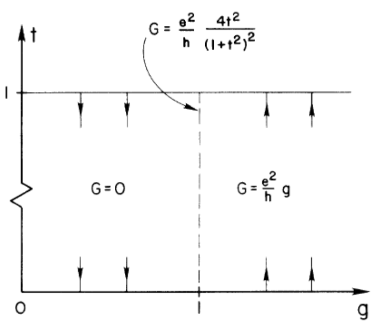
[3] *Sektion Physik, Universität München - Theresienstr. 37 D-80333 München, Germany*

d=2+0

g~T/κ

• 1-d electron liquid (Luttinger liquid) with point defect(s)



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Transport in a One-Channel Luttinger Liquid

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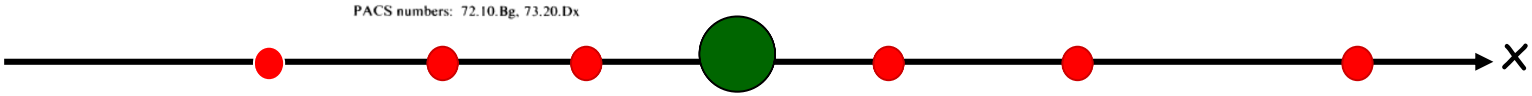
(Received 15 November 1991)

We study theoretically the transport of a one-channel Luttinger liquid through a weak link. For repulsive electron interactions, the electrons are completely reflected by even the smallest scatterer, leading to a truly insulating weak link, in striking contrast to that for noninteracting electrons. At finite temperature (T) the conductance is nonzero, and is predicted to vanish as a power of T. At $T=0$ power-law current-voltage characteristics are predicted. For attractive interactions, a Luttinger liquid is argued to be perfectly transmitted through even the largest of barriers. The role of Fermi-liquid leads is also explored.

PACS numbers: 72.10.Bg, 73.20.Dx

d=1+1

g conductance



Bragg glass Luttinger liquid with defect(s)

- Single defect relevant if $g < 1$ for all defect strength
- Many defects relevant if $g < 3/2$
- Density (Friedel) oscillations close to the defect with power g and $2g-1$ for relevant and irrelevant defects, respectively.
- Coupling constant g tunable

Summary

- The Bragg glass phase shows quasi-LRO with non-universal decay exponent $\eta_{BG}(c_{66}/c_{11})$.
- Single planar defect in the Bragg glass phase is a relevant perturbation provided the defect is parallel to B and
- $g=(3/8)\,\eta_G(a/\delta)^2 < 1$.
- Close to the defect the vortex density shows Friedel like density oscillations with decay exponent $g<1$.
- Randomly arranged weak defects become relevant in D<6 dimensions. In D=3 and in the presence of point disorder defects are relevant for $g<3/2$.
- Planar defects lead to a transverse Meissner effect and a threshold against shear deformation.
- The creep exponent is $\mu=3/2$.