

Kitaev Spin Models

Seminar: Topological states of matter

8. Juni 2012

1 Two-dimensional Toric Code

Spin-1/2 system on edges of a square lattice, described by

$$\begin{aligned}H_T &= -J_e \sum_s A_s - J_m \sum_p B_p, \\A_s &= \prod_{j \in \text{star}(s)} \sigma_j^z, \\B_p &= \prod_{j \in \partial p} \sigma_j^x.\end{aligned}$$

where

s ... runs over vertices

p ... runs over plaquettes

A_s ... star operator, acts on the 4 spins surrounding s

B_p ... plaquette operator, acts on the 4 spins surrounding a plaquette.

Results:

(1) Quasiparticles with different statistics:

- e and m with respect to their own kind are bosonic

- individual statistics of e vs. m are anyonic (more precisely: semionic)

\Rightarrow the mutual statistics of $e \times m$ are fermionic.

(2) Model exhibits a ground state degeneracy of 4^g when embedded on a surface of genus

g : On the torus, there are 4 topologically distinct ground states.

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2 Honeycomb Model

Spins sit on the vertices of a honeycomb lattice. The interaction between spins is of nearest neighbor type and link orientation dependent.

$$H = -J_x \sum_{x\text{-links}} \sigma_j^x \sigma_k^x - J_y \sum_{y\text{-links}} \sigma_j^y \sigma_k^y - J_z \sum_{z\text{-links}} \sigma_j^z \sigma_k^z.$$

Results:

- exact solution obtained by reduction to free fermions
- quasiparticles characterized as fermions and vortices
- spectrum with gapped and gapless phases depending on $|J_x|, |J_y|, |J_z|$.
- gapped phase in magnetic field acquires gap
- Vortices become non-Abelian anyons carrying an unpaired Majorana mode.

3 Main sources

- Alexei Kitaev, Anyons in an exactly solved model and beyond, Annals of Physics, Volume 321, Issue 1, January 2006
- A. Kitaev and C. Laumann, Topological phases and quantum computation, arXiv:0904.2771
- A. Yu. Kitaev, Fault-tolerant quantum computation by anyons, Annals of Physics, Volume 303, Issue 1, January 2003.