Majorana Fermions in 1D Nanowires

Majorana Fermions in solid state systems feature non-abelian statistics, i.e braiding not only changes the phase but leads to different quantum state. Therefor an application as qbits seems possible.

Kitaev chain

A toy tight binding Hamiltonian for a p-wave superconductor reads

$$H = \sum_{j=1}^{L} \left[-\mu a_j^\dagger a_j - t \left(a_{j+1}^\dagger a_j + \text{h.c} \right) + \left(\Delta a_{j+1}^\dagger a_j^\dagger + \text{h.c} \right) \right]$$

Introduce Majorana operators $\gamma_{2j-1} = a_j + i a_j^\dagger$ and $\gamma_{2j} = -i a_j + i a_j^\dagger$ with $\{\gamma_l, \gamma_m\} = 2\delta_{lm}$ and $\gamma_j = \gamma_j^\dagger$ Easy to solve in cases

1. $\Delta = t = 0$, $\mu < 0$
   - $\to$ $H = -\frac{i}{2} \sum \gamma_{2j-1}\gamma_{2j}$
   - $\to$ Majoranas on same site pair
   - $\to$ Unique unoccupied groundstate
2. $\Delta = t \neq 0$, $\mu = 0$
   - $\to$ $H = it \sum_{j=1}^{L-1} \gamma_{2j}\gamma_{2j+1}$
   - $\to$ Majoranas on neighbouring sites pair
   - $\to$ γ_1 and γ_{2L} are unpaired. Combine to highly non-local fermion $d_M = 1/2\left(\gamma_1 + i\gamma_{2L}\right)$
   - $\to$ groundstate *two-fold degenerate*: $|0\rangle$ and $|1\rangle = d_M^\dagger |0\rangle$

Associated with the transition is a topological invariant $\nu = s_0 s_\pi$ where $s_{0/\pi}$ is the sign of the kinetic energy at momentum $0/\pi$, i.e ν is -1 for an odd number of band crossings in half the Brioullin zone and can only changes when the gap closes. For parameters in the range $|\mu| < 2t$, the topological phase is realized with MFs no longer sharply localized to the ends, but decaying exponentially and overlapping, giving rise to an interaction

$$H_{int} = \frac{i}{2} t \gamma' \gamma''$$

with $t \propto \exp\left(-L/\xi\right)$ Therfor states split, but only exponentially $\to$ neglectable. Realisation is difficult, because no p-wave-SC is available and fermions must appear spin-less.

Physical Realization

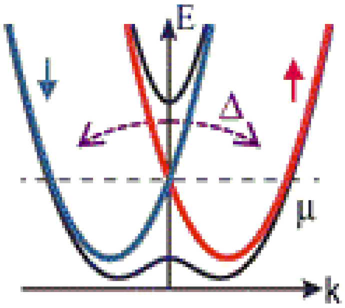
Solution: Semiconductor/s-wave-SC heterostructure can mimic p-wave-SC through Spin-Orbit coupling.

$$H = \int \mathrm{dx}\, \psi_\alpha^\dagger(x) \left\{ -\frac{\partial_x^2}{2m} - \mu - i\alpha\partial_x\sigma_y + V\sigma_x \right\} \psi_\beta(x) + H_{SC}$$

with

$$H_{SC} = \int \mathrm{dx} \left\{ \Delta \psi_\uparrow \psi_\downarrow + \text{h.c} \right\}$$

Spin-Orbit- and Zeemann-coupling split bands and give rise to an effective spinless regime for $|\mu| < V$. The Hamiltonian then maps to Kitaev's model and the topological phase occurs at $V > \sqrt{\Delta^2 + \mu^2}$.



Experiments

Two main effects should be visible. Measuring the DoS via tunnel spectroscopy should reveal a zero-energy-peak. A junction between two superconductors via a topological SC exhibits a fractional Josephson current.

References

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