Kitaev Chain

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Introduction

We will introduce a 1D toy model proposed by Kitaev [1] In the specific setup we are able to theoretically observe the emergence of Majorana modes. Furthermore, we will discuss the possibilities of the experimental realization of the system and some experimental results. If Majorana fermions were observed in the experiments is still the topic of a controversial discussion.

1D Kitaev Chain - Model

Kitaev proposed a simple, one dimensional model containing a tight-binding chain of spinless electrons and a superconducting term. The corresponding Hamiltonian reads

$$H = -\mu \sum_j c_j^\dagger c_j + \sum_{j=0}^{N-1} \Big[-t \left(c_{j+1}^\dagger c_j + c_j^\dagger c_{j+1} \right) - |\Delta| \left(c_j c_{j+1} + c_{j+1}^\dagger c_j^\dagger \right) \Big]. \tag{1}$$

Now one introduces Majorana operators, where two Majorana fermions (MF) describe one fermionic state,

$$\gamma_{j,1} = c_j + c_j^\dagger, \quad \gamma_{j,2} = i \left(c_j^\dagger + c_j \right), \tag{2}$$

with the properties

$$\gamma_{j,\alpha} = \gamma_{j,\alpha}^\dagger, \quad \{\gamma_{i,\alpha}, \gamma_{j,\beta}\} = 2\delta_{ij}\delta_{\alpha\beta} \tag{3}$$

After rewriting the Hamiltonian in eq.1 in terms of Majorana operators one considers two cases of a specific choice of the parameters:

- for $\mu < 0$, $t = |\Delta| = 0$ the Hamiltonian reads

  $$H = -i\frac{\mu}{2} = \sum_j \gamma_{j,1}\gamma_{j,2}. \tag{4}$$

  MFs on the same lattice site j are coupled.

- for $\mu = 0$, $t = |\Delta| \neq 0$ the Hamiltonian reads

  $$H = -it \sum_{j=0}^{N-1} \gamma_{j,1}\gamma_{j+1,2}. \tag{5}$$

  Here MFs of different lattice sites are coupled. At both ends of the chain one finds unpaired MFs $\gamma_{0,2}$, $\gamma_{N,1}$ and the ground state shows a two-fold degeneracy.

These two different couplings of MF correspond to two distinct topological phases. Here, the first case refers to a trivial, the second to a non-trivial phase. The Majorana edge modes are also stable under less fine-tuned parameters [2, 3].

Physical Realization

Two basic realizations of the Kitaev chain exist. One is based on quantum wires made of a semiconductor with strong spin–orbit coupling such as InSb or InAs, and the other employs a 3D topological insulator. In both cases, superconductivity in the wire is induced by the proximity effect of the heterostructure. In the first kind of realizations, the semiconductor / s-wave-SC heterostructure, four ingredients, namely 1) 1 D semiconductor wire, 2) Spin-orbit interaction, 3) Zeemann-coupling and 4) Superconductivity (proximity effect by s-wave SC), are basic to mimic the theoretical model of the Kitaev chain and further the chemical potential μ is controlled to be in the effective spinless regime $|\mu| < V$ (V is the Zeemann field). In the regime $V > \sqrt{\mu^2 + \Delta^2}$, the semiconductor wire realizes the Kitaev chain topological paradigm associated with Majorana edge modes [4].

Observing Unpaired Majoranas

Two main experimental methods are available for observing unpaired Majoranas. One is observing the Majorana zero modes via tunneling spectroscopy. A Zero bias peak is present in tunneling conductance plot corresponding to a Majorana mode when the wire is in the topological phase. Another kind of experiment tests the 4π-periodic Josephson effect occuring between two SC wires in the topological phase. The experimental result weighs in favour of Majorana interpretation but does not have a definitive proof. Some parts of experiments are missing and there are many alternative explanations which do not involve Majorana modes. These need to be ruled out for further identification [5].

References

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