Semiclassical approach to disorder and weak localization

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1 Introduction

In systems of mesoscopic length scales, quantum interference effects of conduction electrons give a negative correction to the classical electric conductivity. The transport of an electron through a medium can be determined by using the semiclassical approach of R. Feynman, Feynman Path Integral. Since the number of electrons is very large, the **average** of the sum over the probability amplitude of each possible trajectory for all electrons is an important quantity. Due to dephasing of paired trajectories is large an random, the term of quantum corrections vanishes under averaging process, and solution is given by the classical diffusion equation.

2 Quantum corrections

Only under special conditions quantum corrections survive the averaging process. One of these special conditions is the regime of mesoscopic length scales, making objects like the Cooperon possible.

2.1 Weak localization

A Cooperon is an object describes two trajectories $(i \neq j)$ with $\vec{r} \simeq \vec{r}\ '$, on which the particles passing the same scattering impurities, but in time reversed order. As long as time reversal symmetry is not broken, the classical actions and therefore the phases for both trajectories are equal and dephasing becomes zero and we remain quantum corrections of interference effects. To determine the magnitude of these interference effect one has to calculate the probability for the occurrence of a Cooperon. The time of traveling through a Cooperon is restricted by a lower and an upper time limit $dt \in [\tau_e, \tau_{\Phi/D}]$. In $d = 3$ dimension the probability of return to initial point is given by solving of classical diffusion equation. $\left(\frac{1}{4\pi Dt}\right)^{\frac{3}{2}}$ For an induced dephasing smaller then 2π during dt the final point of the path of the electron has to enter the volume $v_F \lambda_F^2 \cdot dt$, so that the **total probability** is given by the integral, which is proportional to the relative correction of the average conductance:

$$\frac{\triangle\sigma}{\sigma} \sim - \int_{\tau_e}^{\tau_{\Phi/D}} \frac{v_F \lambda_F^2 \cdot dt}{(4\pi Dt)^{\frac{3}{2}}}$$

The minus sign has its origin in the reduction of conductance (electron remains longer in region of Cooperon). Since the magnitude of correction is very weak, we call it **weak localization**.

By solving the former integral over time one get for the correction $\triangle\sigma \sim -\frac{e^2}{\hbar l_e} + \frac{e^2}{\hbar L_{\Phi/D}}$. Due to the upper time boundary strongly depends on temperature, the correction to conductivity depends on temperature. Such that only bellow a certain temperature negative quantum corrections become important and therefore the conductivity begins to decrease with decreasing temperature.

3 Magnetic fields

In a loop in an external magnetic field the vector potential $\vec{A}$ changes its sign, while the momentum $\frac{d\vec{x}}{dt}$ does not, so that for the trajectories of a Cooperon the resulting phase difference of the phases $\varphi_{j/i} = S_{j/i}/\hbar$ is given by

$\triangle\varphi_H = \varphi_j - \varphi_i = \frac{2e}{c\hbar} \int \vec{A}\ d\vec{x} = 4\pi \frac{\Phi}{\Phi_0}$ with the flux quantum Φ_0 and the magnetic field flux Φ across the loop. For a rapid change in phase $(\Phi >> \Phi_0)$, the phase is randomized and correction disappears. For $\Phi \simeq \Phi_0$ and for times $t < \tau_H$ interference exists and for times $t > \tau_H$ it is broken (withmagnetic phase breaking timeτ_H).

3.1 Influence to conductivity

With a replacement of the upper time boundary $\tau_\Phi \rightarrow \tau_H$ if $\tau_H << \tau_\Phi$, the resulting correction (in $d = 3$) is:

$$\sigma_d(H) - \sigma_d(0) \sim \frac{e^2}{\hbar} \left(\frac{eH}{\hbar c}\right)^{\frac{1}{2}}$$

So as long as the magnetic field strength is not to strong, the magnitude of correction of interference effects depends on the strength of the applied magnetic field.

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