STRONG DISORDER RENORMALIZATION GROUP ANALYSIS

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A renormalization method to probe the low energy properties of a system with strong disorder is described. We explain first in some detail a particular example and discuss afterwards some of the general features that are present in all models in which the procedure can be applied.

1 RTFIC

We start by considering a particular model: the Random Transverse Field Ising Chain (RTFIC). This means, we consider the model with hamiltonian

$$\mathcal{H} = -\sum_i J_i \sigma_i^z \sigma_{i+1}^z - \sum_i h_i \sigma_i^x + H \sum_i \sigma_i^z \tag{1}$$

where J_i and h_i are iid nonnegative random variables with distributions $\pi(J)$ and $\rho(h)$ respectively. A nonzero external field H is, in principle, allowed (and necessary to compute e.g. the magnetization) but since the renormalization group (RG) procedure to be described does not apply to it, we set it to zero for the purposes of this note. If the square root of the variance of J, h is much bigger than any other energy scale in the system, the system is said to have strong disorder and in this case, we expect the couplings and transverse fields to be widely distributed, this means in particular that $J_i \neq J_j$ for $i \neq j$ and similarly for h. An RG procedure to find a particular ground state is performed as follows [1]. Set Ω :=max$\{J_i, h_i\}$. If $\Omega = J_l$ for some l, we replace the spins σ_l and σ_{l+1} for a spin with twice the magnetic moment and with local transverse field

$$h_{l,l+1}^{eff} = \frac{h_l h_{l+1}}{\Omega}. \tag{2}$$

If, instead, $\Omega = h_l$ for some l, we remove the spin σ_l and couple the spins $\sigma_{l-1}, \sigma_{l+1}$ with strength

$$J_{l-1,l+1}^{eff} = \frac{J_{l-1} J_l}{\Omega}. \tag{3}$$

After the decimation, the highest energy scale is changed to some strictly smaller value, Ω', and the remaining couplings are still independent, since only the removed fields and couplings play any role in the rules (2),(3). This implies that 1) The procedure stops when the energy scale is lowered to the minimum value of $\Omega^* = 0$ (this is called the *fixed point* of the RG flow) and 2) When iterating this procedure, we need only consider the evolution of π and ρ at energy scale Ω. Straightforward analysis leads to flow equations for the distributions, i.e. differential equations for $\frac{\partial \pi}{\partial \Omega}$ and $\frac{\partial \rho}{\partial \Omega}$. The form of these equations is not particularly enlightening, but a solution for $\Omega \to 0$ is given by [1, 2]

$$\pi(J, \Omega) \sim \frac{\pi_0}{\Omega} \left(\frac{\Omega}{J}\right)^{1-\pi_0} \tag{4}$$

$$\rho(h, \Omega) \sim \frac{\rho_0}{\Omega} \left(\frac{\Omega}{h}\right)^{1-\rho_0} \tag{5}$$

with $\pi_0 = \pi(\Omega, \Omega)\Omega$ and $\rho_0 = \rho(\Omega, \Omega)\Omega$. From equations (4) (5) it can be seen that at the fixed point the randomness is infinite, i.e. the support of the distributions grows without bound. With the distributions at hand, the decimation procedure can be seen to become *asymptotically exact* approaching the fixed point. For example, the probability of decimating a field h when a bond $J \in (\alpha\Omega, \Omega)$ for $\alpha \in (0, 1)$ (a bad decimation) goes as [2]

$$\Pr(\alpha) \sim -\rho_0 \ln(\alpha) \to 0. \tag{6}$$

2 General Features

The RTFIC is just one example [3, 4] where a real space RG procedure for disordered systems can be performed. Rules of decimation like equations (2) and (3) are commonly referred to as *Ma-Dasgupta rules*. The rules depend on the particular system but they all share the properties [2]

1. They concern the maximum value of a random variable. This value evolves under RG and constitutes a “cutoff” scale.
2. The decimation is local in space, that is, at each stage only the immediate neighbors of the “cutoff” variable are affected.

Once the decimation rules are iterated, the interesting cases correspond to disorder growing without bound or to some finite value. These outcomes are referred to as the *Infite* and *Strong* randomness fixed points. Systems with an infinite randomness fixed point are characterized by strong dynamical anisotropy (i.e. the typical length scale ℓ goes as $\ln(\tau)$, where τ is the typical time scale) and a broad distribution of physical observables (typical and average observables differ significantly).

References

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[4] Fisher, D., Phys. Rev. B **50** 3799 (1994).