# Many-Body-Localisation

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## I. Single and Many Particle Anderson Localisation are different

Disorder changes the nature of the wavefunction significantly. For non-interacting particles, localisation occurs in real space and wavefunctions are characterised by an exponentially decaying envelope function $\psi(x) \sim \exp(-x/\xi)$, with localisation length $\xi$. The origin of localisation is a quantum interference enhanced probability for particles in a disordered potential to form closed loops. Localisation is much more than the absence of diffusion. It is possible e.g. to make statements on the Hamiltonian level using Random Matrix-Theory to describe the distribution of energy eigenvalues for localised systems.

Many-Body Localisation (MBL) is an extension of single-particle Anderson localisation to interacting systems. Theoretical investigation and experimental treatment of many interacting particles in a disordered potential, isolated from its environment is an extremely challenging and modern-day research problem. For this reason, the following statements are limited to the state of knowledge as of July, 2015.

## II. Many-body localisation is closely connected to thermalisation

Classical systems thermalise chaotically, through interactions. The thermal state contains no information about the initial state. Quantum thermalisation is very different. Quantum systems obey a linear, unitary time evolution where no information can get lost. What states can the interacting, random system achieve under its own dynamics? Put simply: Localisation or thermalisation. Isolated systems thermalise by acting as their own heat bath that thermalises all of its parts. But all information about the initial state has to remain in the system forever (unitarity). Thermalisation hides this information in the entire system. Many-body localised systems fail to thermalise, they fail to transport conserved quantities across the system. Information about the initial state remains visible in some local region (e.g. in real space). Thus, localisation in real space refers to localisation of information.

## III. In isolated quantum systems, eigenstates can be thermal states

The eigenstate thermalisation hypothesis was put forward in the '90s [10] to propose how quantum systems thermalise. The hypothesis states that in the thermodynamic limit the expectation value of an operator will be determined by its diagonal elements only, i.e. by the true many-body eigenstates of the system, if the distribution of energies over the states is sufficiently narrow and if energy is the only conserved quantity [8]. More profoundly, a single eigenstate correctly predicts the thermal expectation value of this operator.

$$\lim_{t\to\infty} \langle A(t)\rangle = \sum_{\alpha\alpha} |c_\alpha|^2 A_{\alpha\alpha} = \langle A\rangle_{\text{microcan}}(E_0) \qquad (1)$$

This defines a new statistical ensemble: the single-eigenstate microcanonical ensemble. The MBL transition is characterised not as a thermodynamical transition but as an eigenstate phase transition in the sense that the ETH is false in the MBL phase and true in the thermal phase. Investigation of this property of the eigenstates is restricted to numerical methods of exact diagonalisation to detect the transition.

## IV. Numerical investigation of finite size systems predict an MBL transition in 1D

The MBL phase transition is supported by the Hamiltonian for a disordered Heisenberg $s = 1/2$ chain in 1D [6],[7].

$$H = \sum_{i=1}^{L} \left[ J\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_{i+1} + h_i \sigma_i^z \right] = \delta H(J) + H(J=0) \qquad (2)$$

that can also be cast in the form of a particle model with the Jordan-Wigner transformation. Here, $h_i$ is a random magnetic field drawn from a box distribution of width $h$. The main aspects of the MBL transition are listed in the table which is reproduced from [5]

| Thermal phase | $(h/J) \sim 3 \pm 1$ Many-body localised phase $h/J$ |
|---|---|
| • System is a reservoir for itself | • System is not a reservoir for itself |
| • All local info $\rho(t=0)$ spreads through entire system for $t \to \infty$ thus hidden | • Some local info in $\rho(t=0)$ remains localised (quantum memory) |
| • Diffusive transport of energy and $\sigma_z$ | • Zero thermal and spin conductivity |
| • ETH is true | • ETH is false |
| • Eigenstates extended in Fock space | • Eigenstates localised in Fock space |

As of today, nothing about the nature of the phase transition is set in stone. There is no finite-size scaling or (dynamical)mean-field approach available in the literature. Naively, interactions destroy phase coherences necessary for single-particle Anderson localisation. In 2006 [1] it was predicted however, that electron-electron interactions for weak and short-range Hamiltonians do not lead to a finite conductivity unless the temperature exceeds a critical value. Astonishingly, the many-body localisation/delocalisation transition is a phase transition at elevated energy densities where one might expect the system to be just classical, but we have to take its quantum character very serious. In the thermodynamic limit, the localised system is characterised by a strictly zero conductivity, which sets it apart from conventional metal-insulator transitions.

## V. Localisation has a meaning in real space and in Fock space

Localisation in real space means that information about the initial state remains accessible at all times by a local measurement.

If we start from the limit of $J = 0$ in Eq. 2 the many-body eigenstates will be product states of localised single particle eigenstates that are any pattern of spin up and down precessing at local Larmor Frequencies.

Fock space localisation is the statement that weak $(J \ll h)$ and short-ranged interactions, will not mix the old eigenstates (for $J = 0$) strongly with states that are close (states connected by a few applications of $\delta H(J)$) in Fock space. It can be motivated by an argument based on perturbation theory. Mixing with states close in Fock space, is suppressed with $O(J/h)$. States that are close in energy to our old eigenstates require multiple powers of $\delta H(J)^n$ (far apart in Fock space) such that the smallness of the energy mismatch is suppressed by the smallness of $(J/h)^n$. A thorough investigation was done recently [1].

## References

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