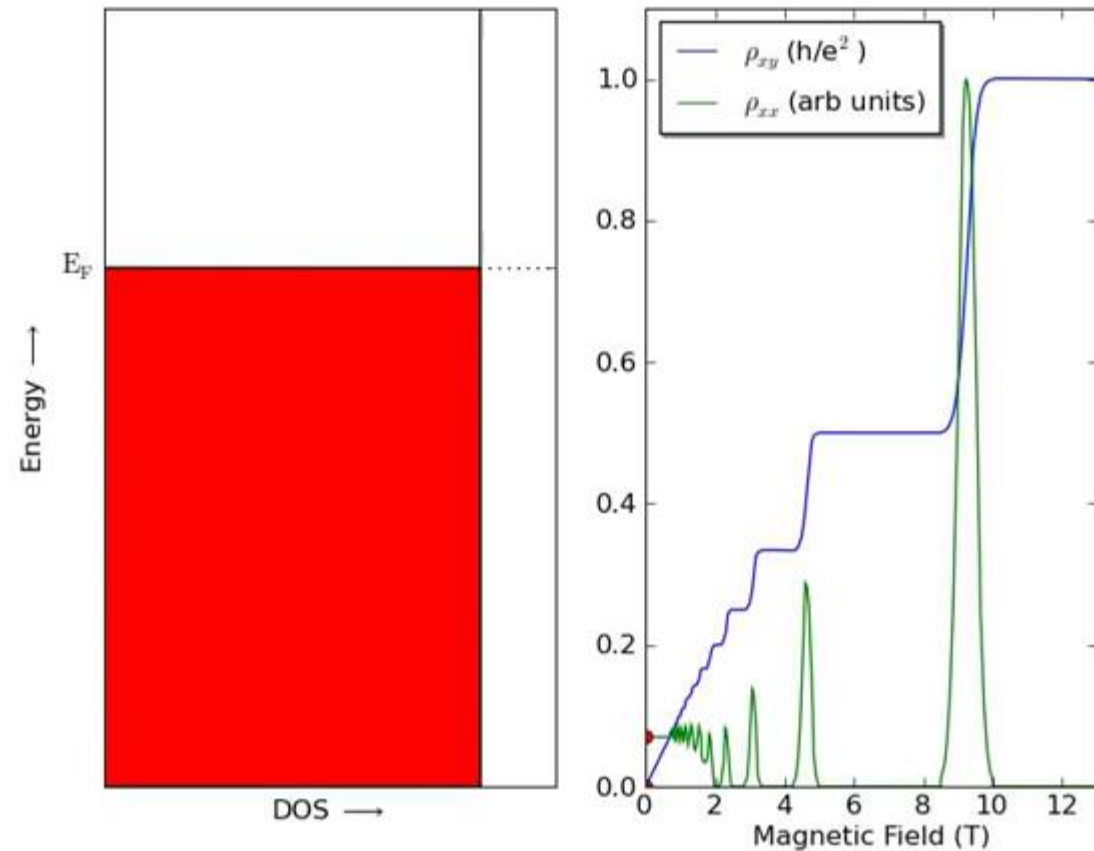# Thermal Quantum Hall Effect
Seminar on Majorana Fermions

Marcel Gievers

Jannis Brodherr

29.06.2018

Landau Levels in Quantum Hall Effect

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Theoretical Physics
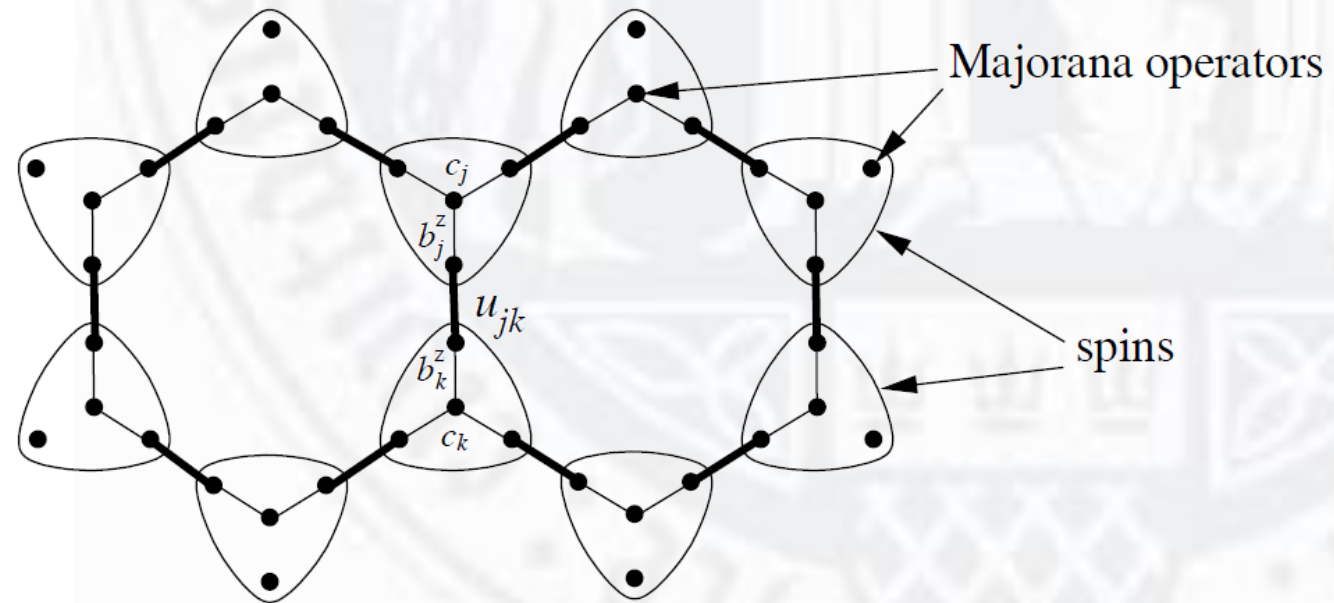University of Cologne

Kitaev Honeycomb Model

$$\mathcal{H} = -J_x \sum_{x-\text{bonds}} \sigma_j^x \sigma_k^x - J_y \sum_{y-\text{bonds}} \sigma_j^y \sigma_k^y - J_z \sum_{z-\text{bonds}} \sigma_j^z \sigma_k^z$$

$$\sigma_j^\alpha = \mathrm{i} b_j^\alpha c_j, \quad \alpha = x, y, z$$

$$\Rightarrow \mathcal{H} = \frac{\mathrm{i}}{4} \sum_{j,k} A_{jk} c_j c_k$$

$$A_{jk} = \begin{cases} 2 J_\alpha \hat{u}_{jk} & j, k \text{ connected} \\ 0 & \text{else} \end{cases}$$

$$\hat{u}_{jk} = \mathrm{i} b_j^\alpha b_k^\alpha$$

Kitaev, Alexei. "Anyons in an exactly solved model and beyond."
Annals of Physics 321.1 (2006): 2-111.

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Fourier transform and spectrum

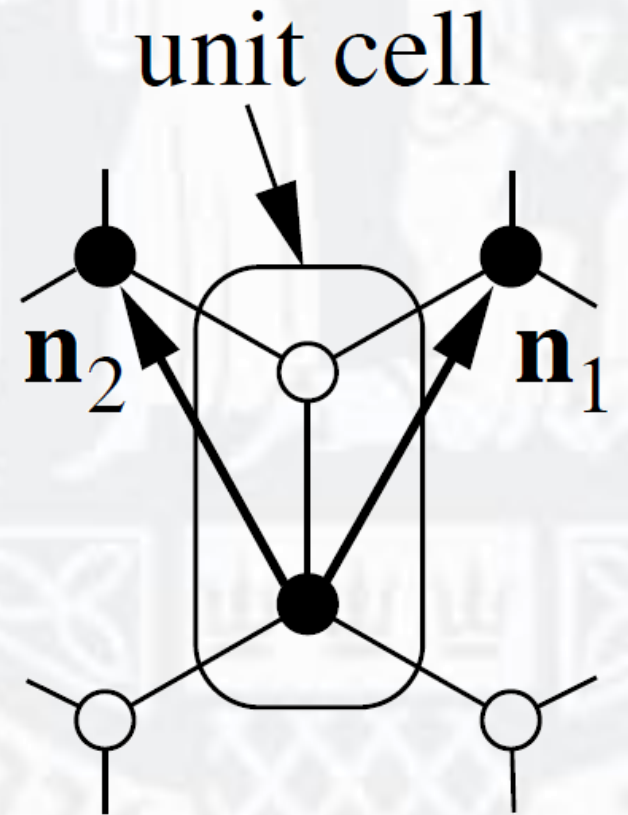$$\mathcal{H} = \frac{i}{4} \sum_{\lambda,\mu=1,2} \sum_{t,s} A_{s\lambda,t\mu} c_{s\lambda} c_{t\mu}$$

$$= \frac{1}{4} \sum_{k} \left(c_1^\dagger(\boldsymbol{k}), c_2^\dagger(\boldsymbol{k}) \right) \begin{pmatrix} 0 & if(\boldsymbol{k}) \\ -if^*(\boldsymbol{k}) & 0 \end{pmatrix} \begin{pmatrix} c_1(\boldsymbol{k}) \\ c_2(\boldsymbol{k}) \end{pmatrix}$$

$$c_\lambda(\boldsymbol{r}) = \frac{1}{\sqrt{N}} \sum_{k} e^{i\boldsymbol{k}\cdot\boldsymbol{r}} c_\lambda(\boldsymbol{k})$$

$$f(\boldsymbol{k}) = 2 \left(J_x e^{i\boldsymbol{k}\cdot\boldsymbol{n_1}} + J_y e^{i\boldsymbol{k}\cdot\boldsymbol{n_2}} + J_z \right)$$

$$\Rightarrow \epsilon(\boldsymbol{k}) = \pm|f(\boldsymbol{k})|$$

unit cell

$\boldsymbol{n}_2$ $\boldsymbol{n}_1$

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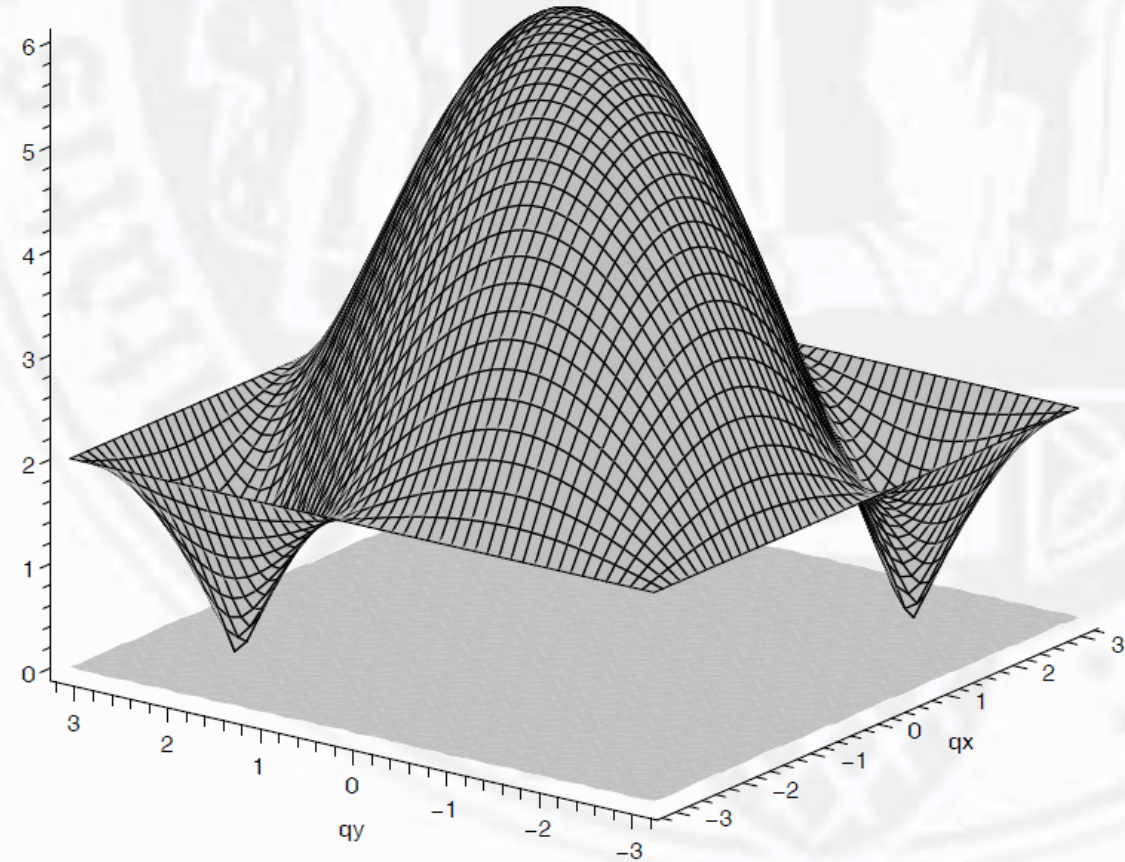Institute for
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Phase diagram

gapless phase B for:

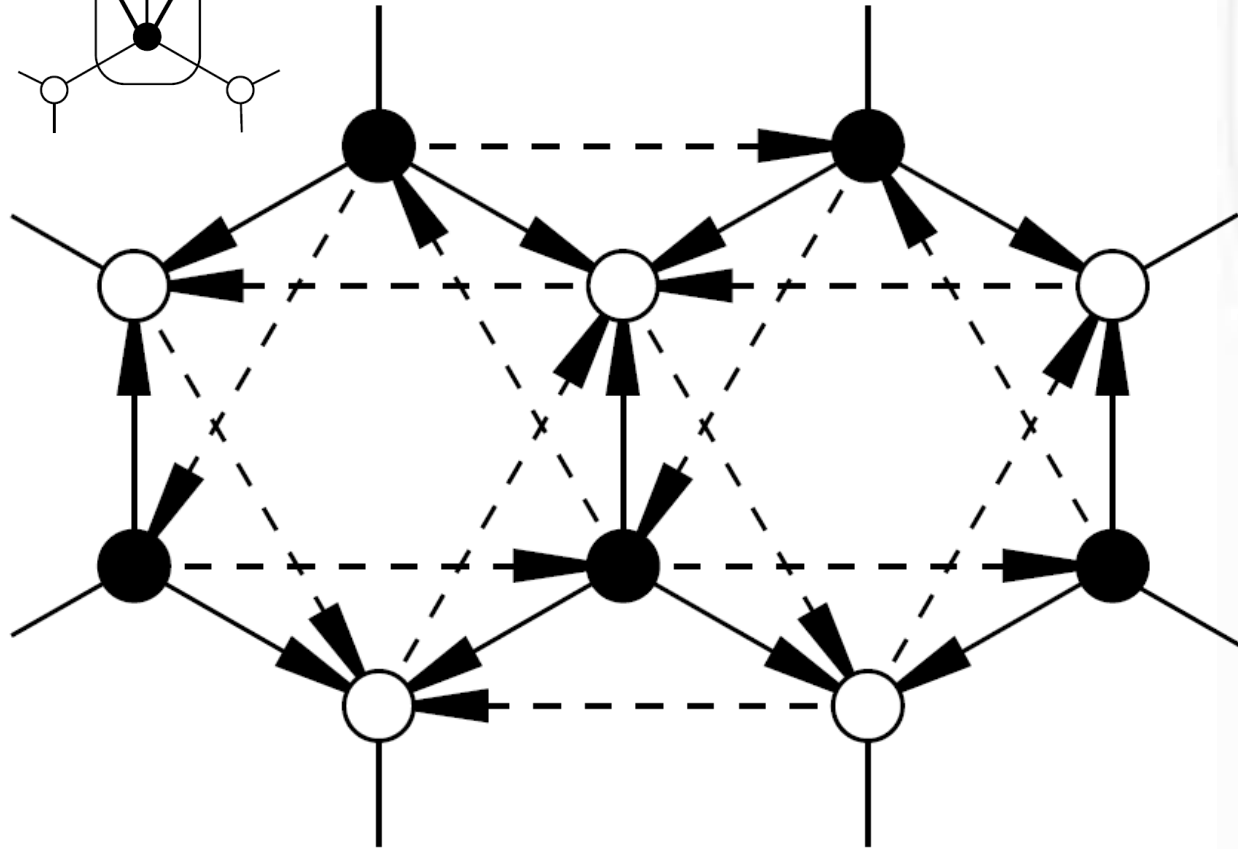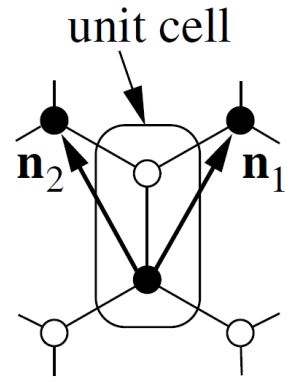$$|J_\alpha| \leq |J_\beta| + |J_\gamma|, \quad \alpha, \beta, \gamma = x, y, z \text{ cyclic}$$
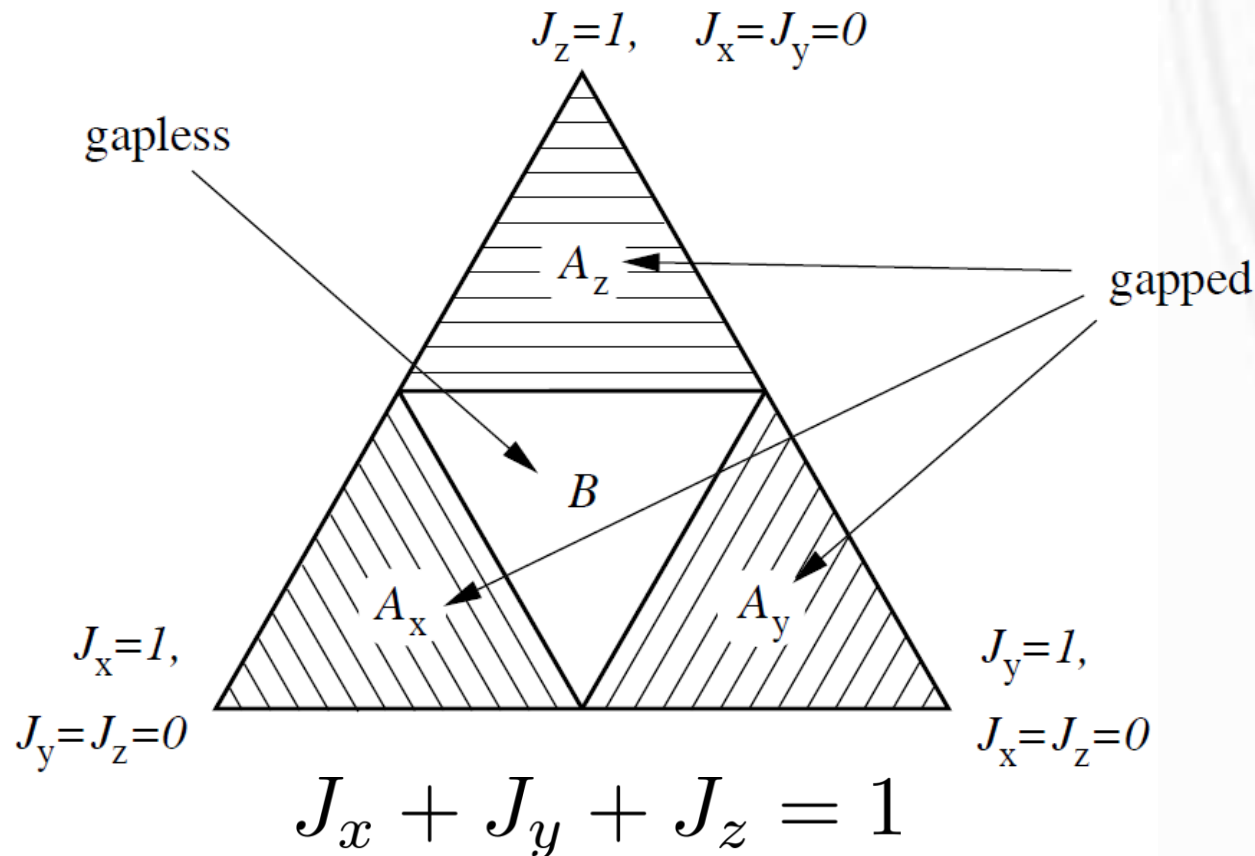
Mancini, Matteo. "Fermionization of Spin Systems." *Università degli Studi di Perugia* (2008).

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Perturbative Magnetic Field

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Perturbative Magnetic Field

$$\mathcal{H}_{\text{eff}} = \frac{i}{2} \sum_{(j,k)\text{bonds}} A_{jk} c_j c_k$$
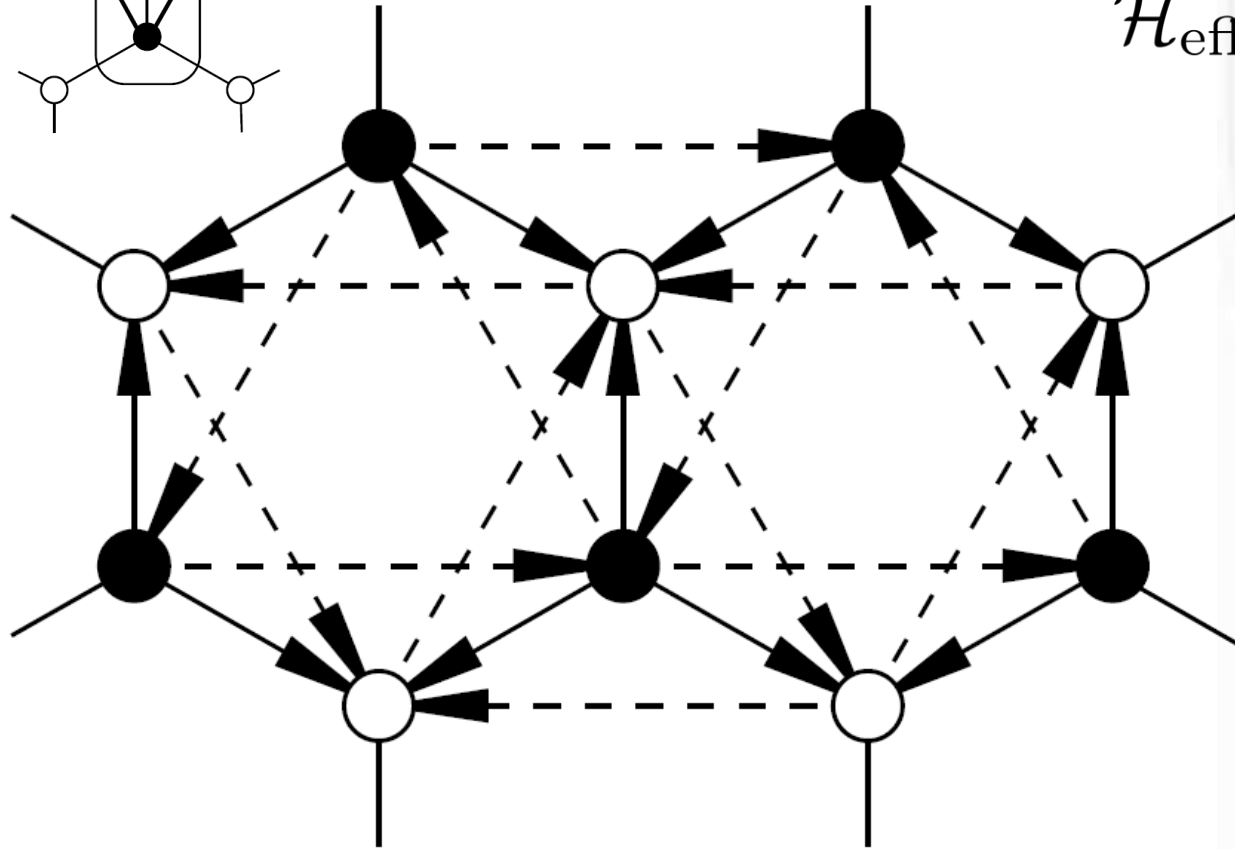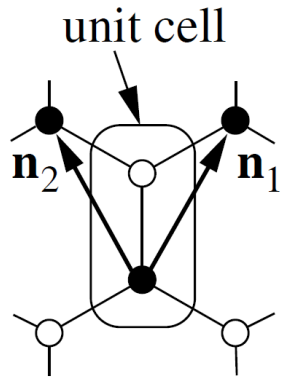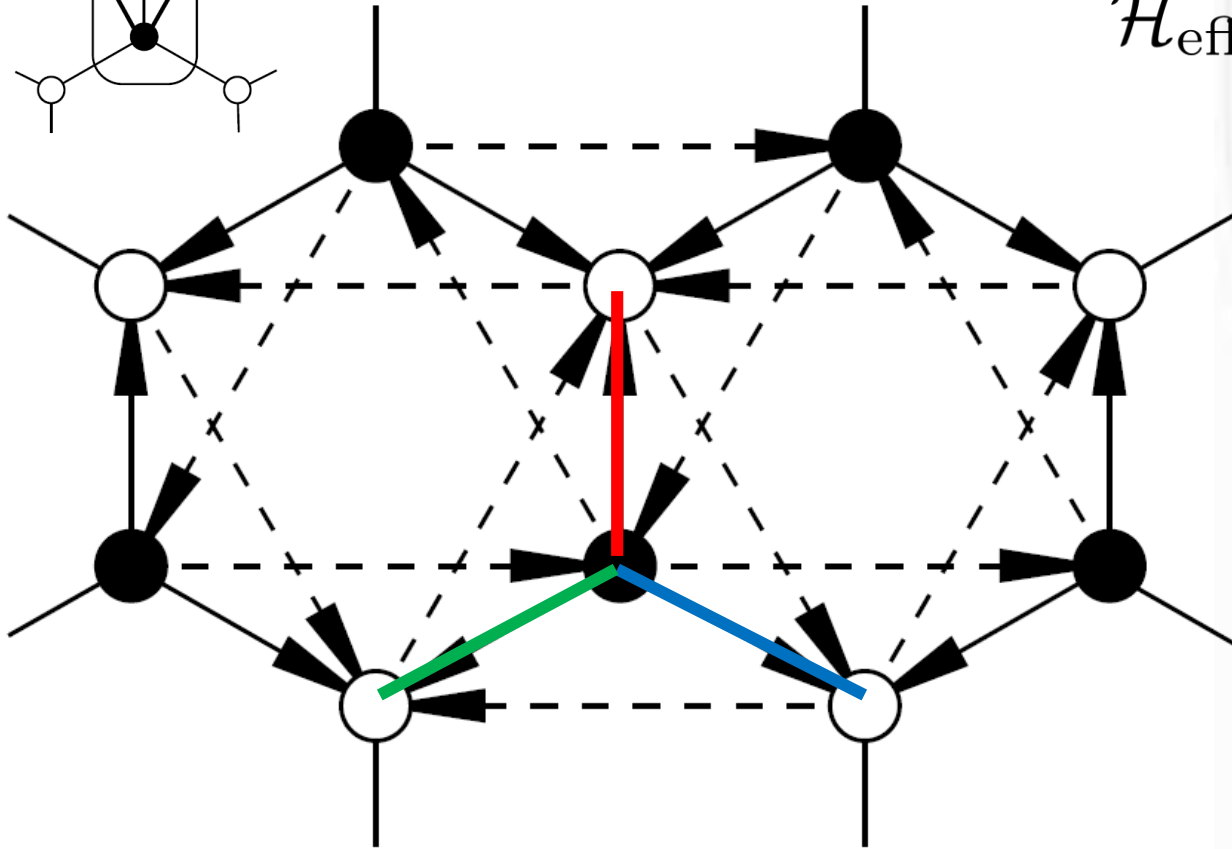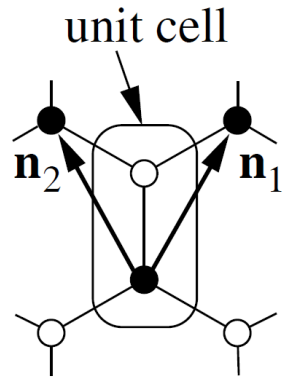
Kitaev, Alexei. "Anyons in an exactly solved model and beyond."
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Perturbative Magnetic Field

$$\mathcal{H}_{\text{eff}} = \frac{i}{2} \sum_{(j,k)\text{bonds}} A_{jk} c_j c_k$$

$$= \frac{i}{2} \sum_r \left[2J \left(c_{1,r} c_{2,(r-n_1)} + c_{1,r} c_{2,(r-n_2)} + c_{1,r} c_{2,r} \right) \right.$$

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Perturbative Magnetic Field

$$\mathcal{H}_{\mathrm{eff}} = \frac{\mathrm{i}}{2} \sum_{(j,k)\mathrm{bonds}} A_{jk} c_j c_k$$

$$= \frac{\mathrm{i}}{2} \sum_{r} \left[2J \left(c_{1,r} c_{2,(r-n_1)} \right. \right.$$
$$\left. + c_{1,r} c_{2,(r-n_2)} + c_{1,r} c_{2,r} \right)$$
$$+ 2K \left(c_{1,r} c_{1,(r-n_1)} \right.$$
$$- c_{1,r} c_{1,(r-n_2)}$$
$$\left. + c_{1,r} c_{1,(r+n_1-n_2)} \right)$$

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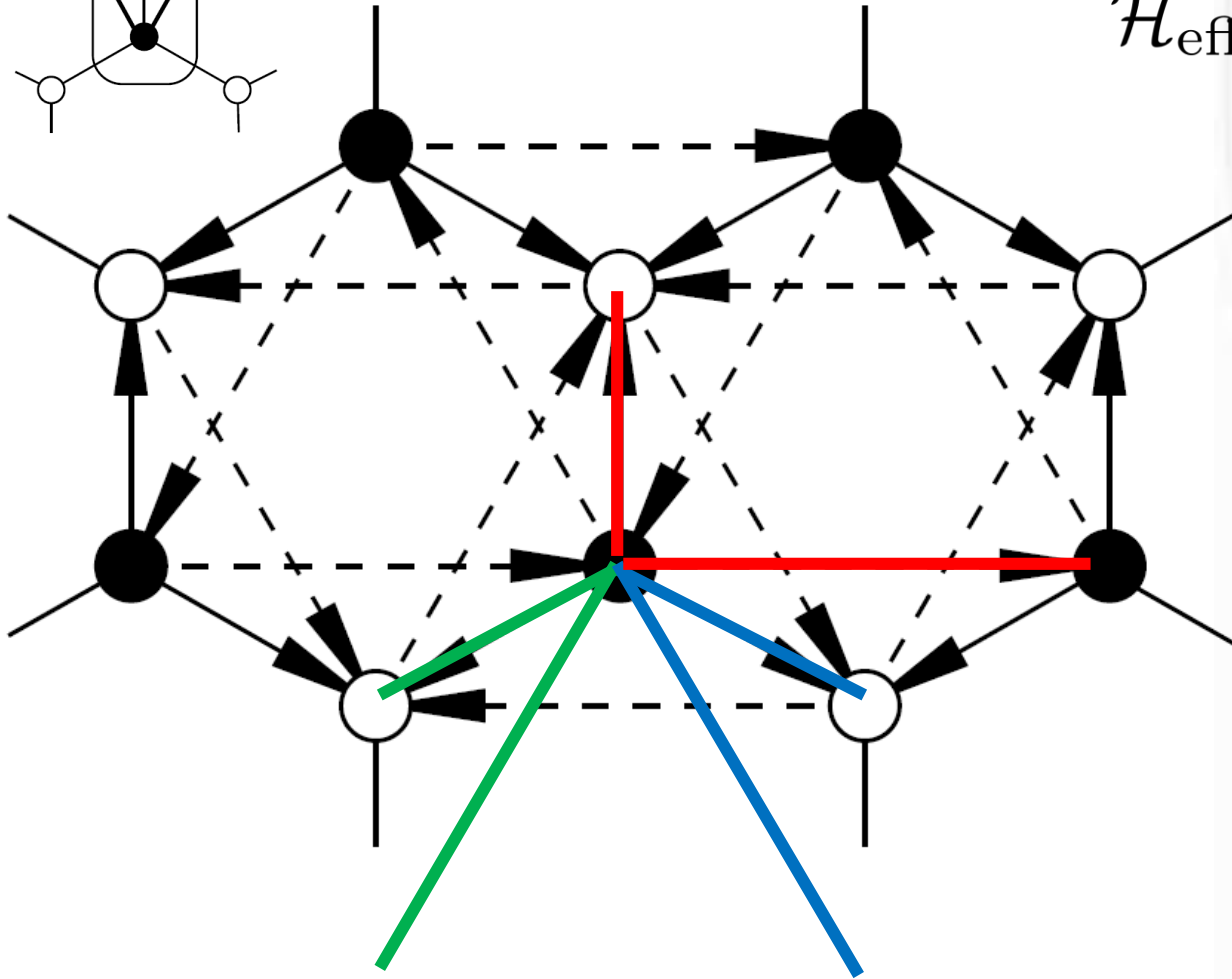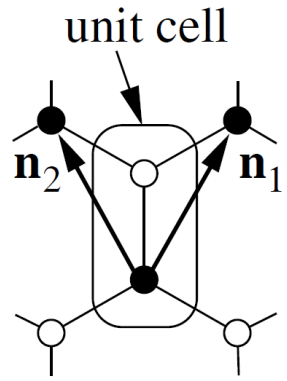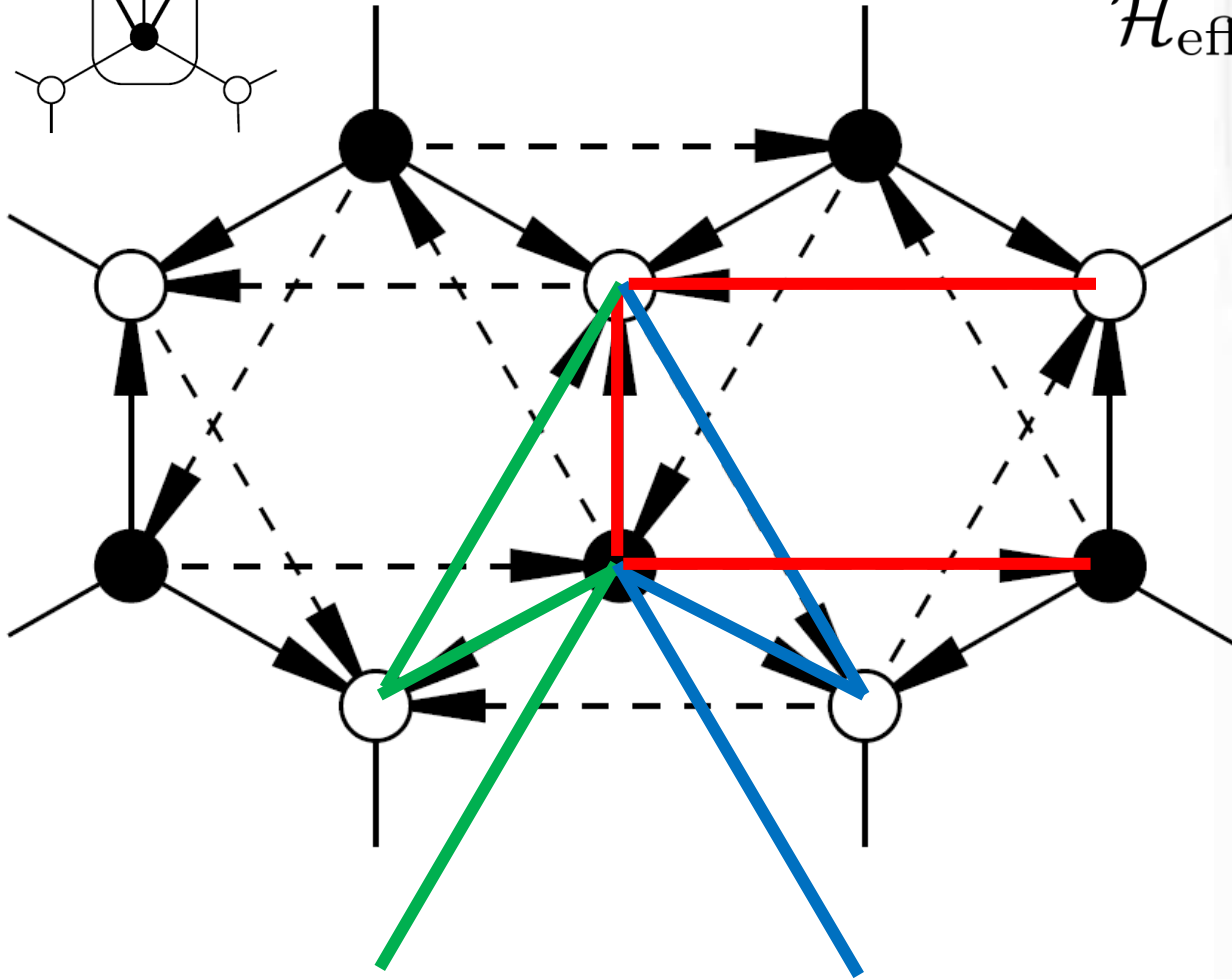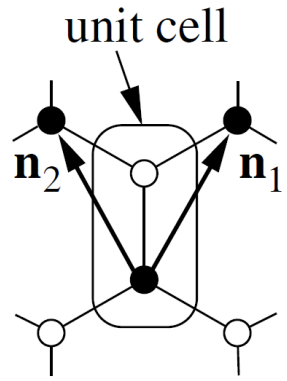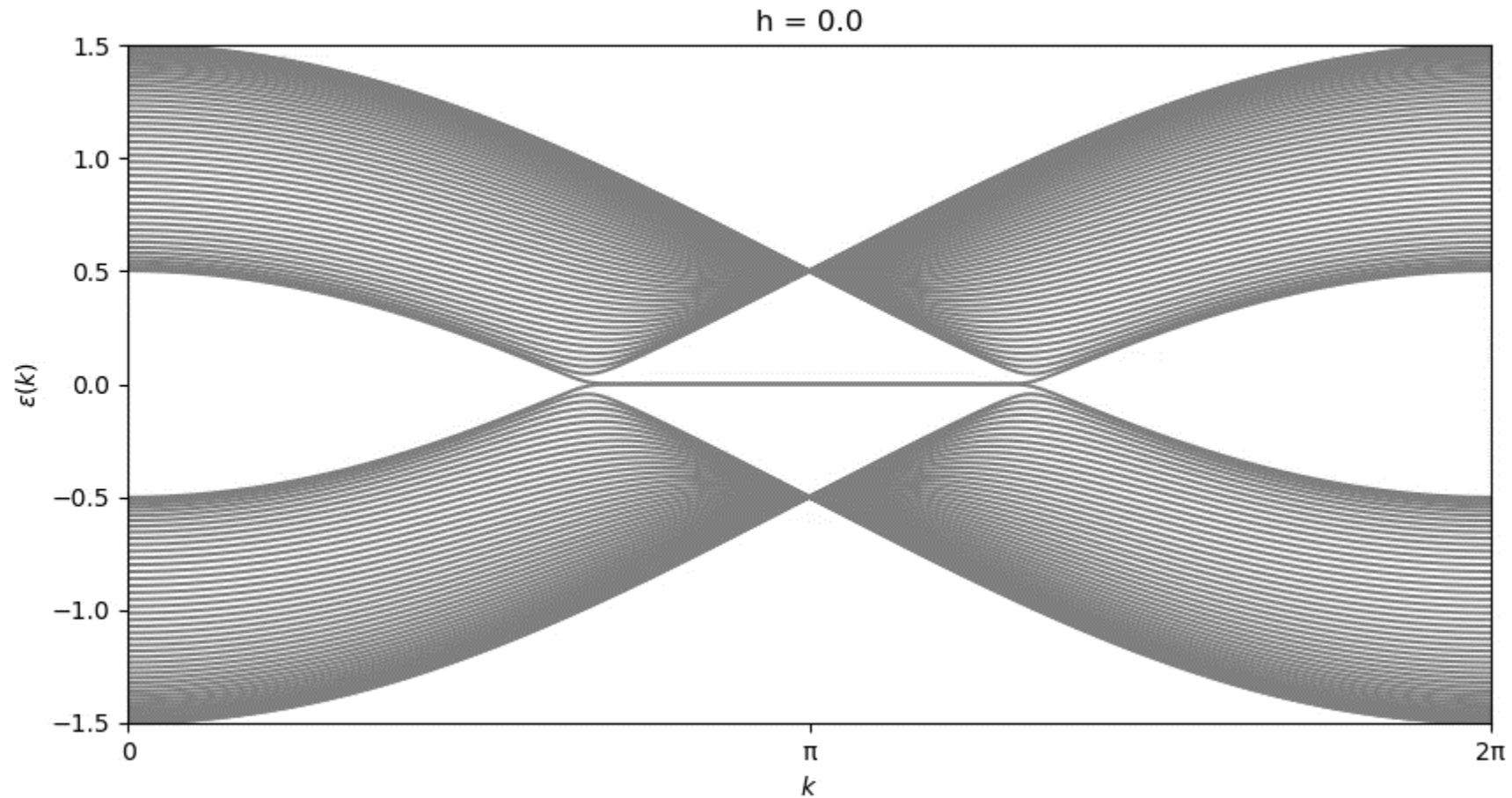*Annals of Physics* 321.1 (2006): 2-111.

Perturbative Magnetic Field

unit cell

$$\mathcal{H}_{\text{eff}} = \frac{i}{2} \sum_{(j,k)\text{bonds}} A_{jk} c_j c_k$$

$$= \frac{i}{2} \sum_{r} [2J\left(c_{1,r}c_{2,(r-n_1)}\right.$$
$$+ c_{1,r}c_{2,(r-n_2)} + c_{1,r}c_{2,r}\right)$$

$$+2K\left(c_{1,r}c_{1,(r-n_1)}\right.$$
$$- c_{1,r}c_{1,(r-n_2)}$$
$$+ c_{1,r}c_{1,(r+n_1-n_2)}$$
$$- c_{2,r}c_{2,(r-n_1)}$$
$$+ c_{2,r}c_{2,(r-n_2)}$$
$$\left.\left.- c_{2,r}c_{2,(r+n_1-n_2)}\right)\right]$$

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Majorana Edge Modes in Kitaev Model

Majorana edge currents & phase diagram of αRuCl$_3$

Kasahara, Y., Ohnishi, T., Mizukami, Y., Tanaka, O., Ma, S., Sugii, K., ... & Shibauchi, T. (2018). Majorana quantization and half-integer thermal quantum Hall effect in a Kitaev spin liquid. *arXiv preprint arXiv:1805.05022*.

Experimental Setup

Kasahara, Y., Ohnishi, T., Mizukami, Y., Tanaka, O., Ma, S., Sugii, K., ... & Shibauchi, T. (2018). Majorana quantization and half-integer thermal quantum Hall effect in a Kitaev spin liquid. *arXiv preprint arXiv:1805.05022*.

Experimental Results

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