Macroscopic entanglement, spin liquids & Kitaev models

Entanglement in Strongly Correlated Systems
Benasque, February 2020

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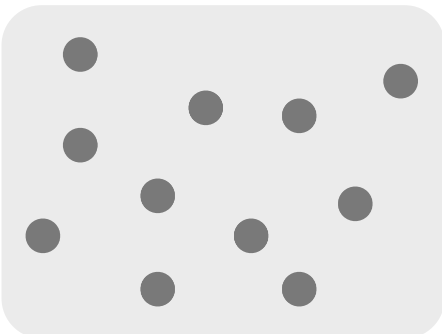
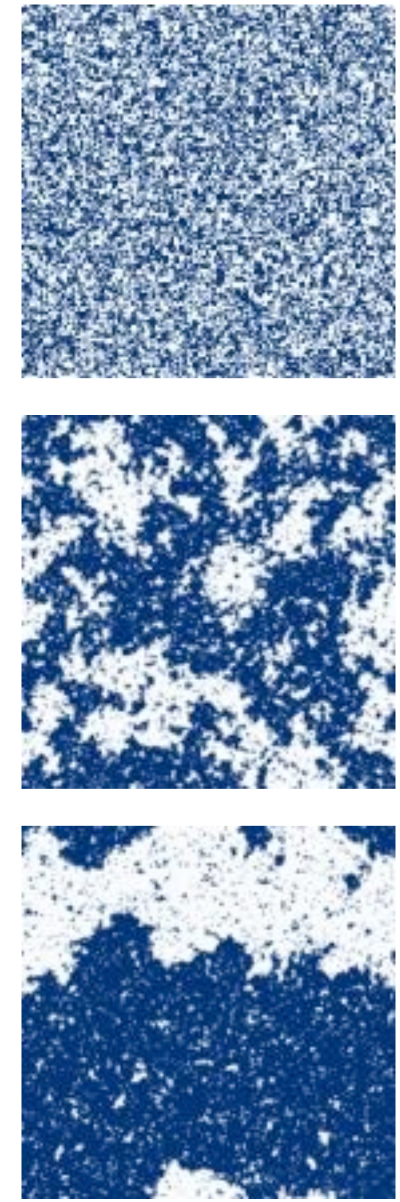
Motivation – a paradigm



interacting
many-body system

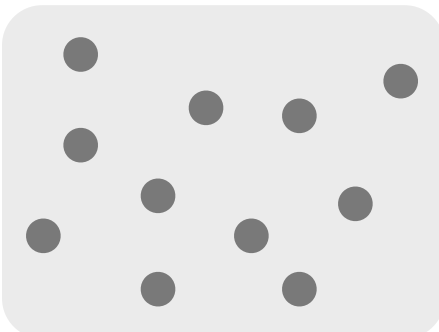
$$\mathcal{H} = -\sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z$$



Spontaneous symmetry breaking

- ground state has **less symmetry** than Hamiltonian
- **local** order parameter
- phase transition / **Landau-Ginzburg-Wilson** theory

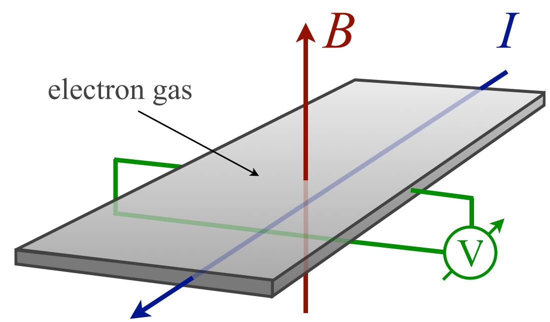
The quantum exception

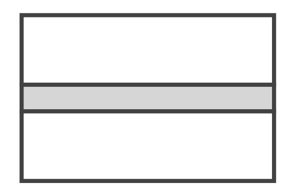


interacting
many-body system

$$\mathcal{H} = \sum_{j=1}^{N} \left(\frac{1}{2m} \left(\mathbf{p}_j - \frac{e}{c} \mathbf{A}(\mathbf{x}_j) \right)^2 + e\mathbf{A}_0(\mathbf{x}_j) \right) + \sum_{i<j} V(|\mathbf{x}_i - \mathbf{x}_j|)$$

$$\mathrm{tr} \left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right)$$





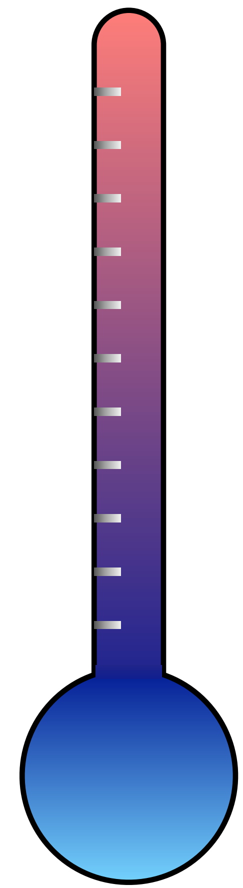
Sometimes, the exact opposite happens

- ground state has **more symmetry** than Hamiltonian
- **non-local order** parameter
- emergence of **long-range entanglement**, exotic statistics, …

Topological quantum matter

- **Spontaneous symmetry breaking**
  - ground state has **less symmetry** than Hamiltonian
  - **Landau-Ginzburg-Wilson** theory
  - **local** order parameter

- **Topological order**
  - ground state has **more symmetry** than Hamiltonian
  - topological quantum field theory (**TQFT**)
  - **non-local** order parameter
  - **macroscopic entanglement**

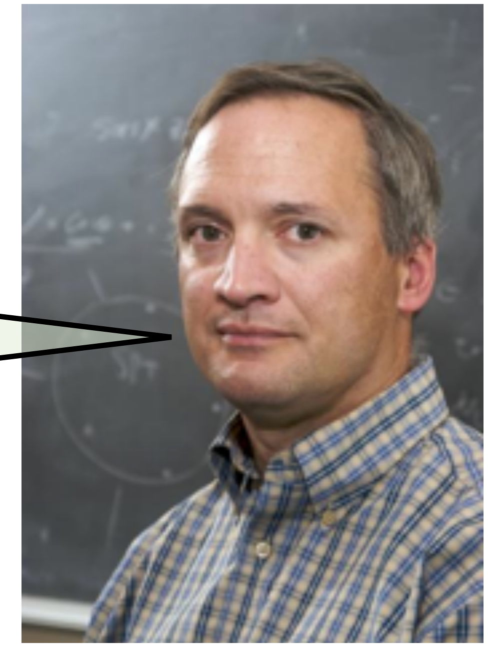


concepts

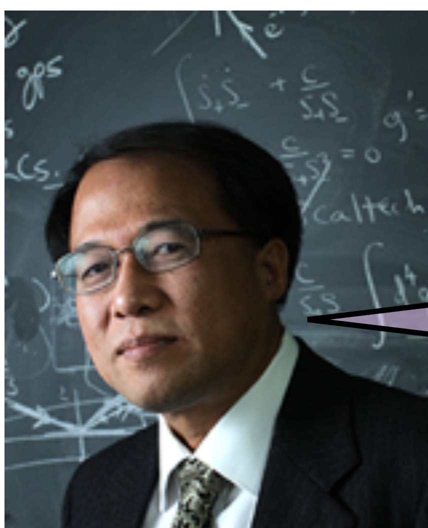
Quantum spin liquids

Quantum spin liquids are exotic ground states of frustrated quantum magnets, in which **local moments** are **highly correlated** but still **fluctuate strongly** down to zero temperature.

Nature 464, 199 (2010).



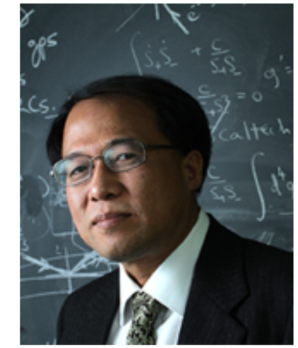
Leon Balents



Xiao-Gang Wen

Quantum spin liquids are **long-ranged entangled** states with **fractionalized excitations**. Some of them exhibit **intrinsic topological order** – very much like the fractional quantum Hall states.

How many are there?



Classification scheme

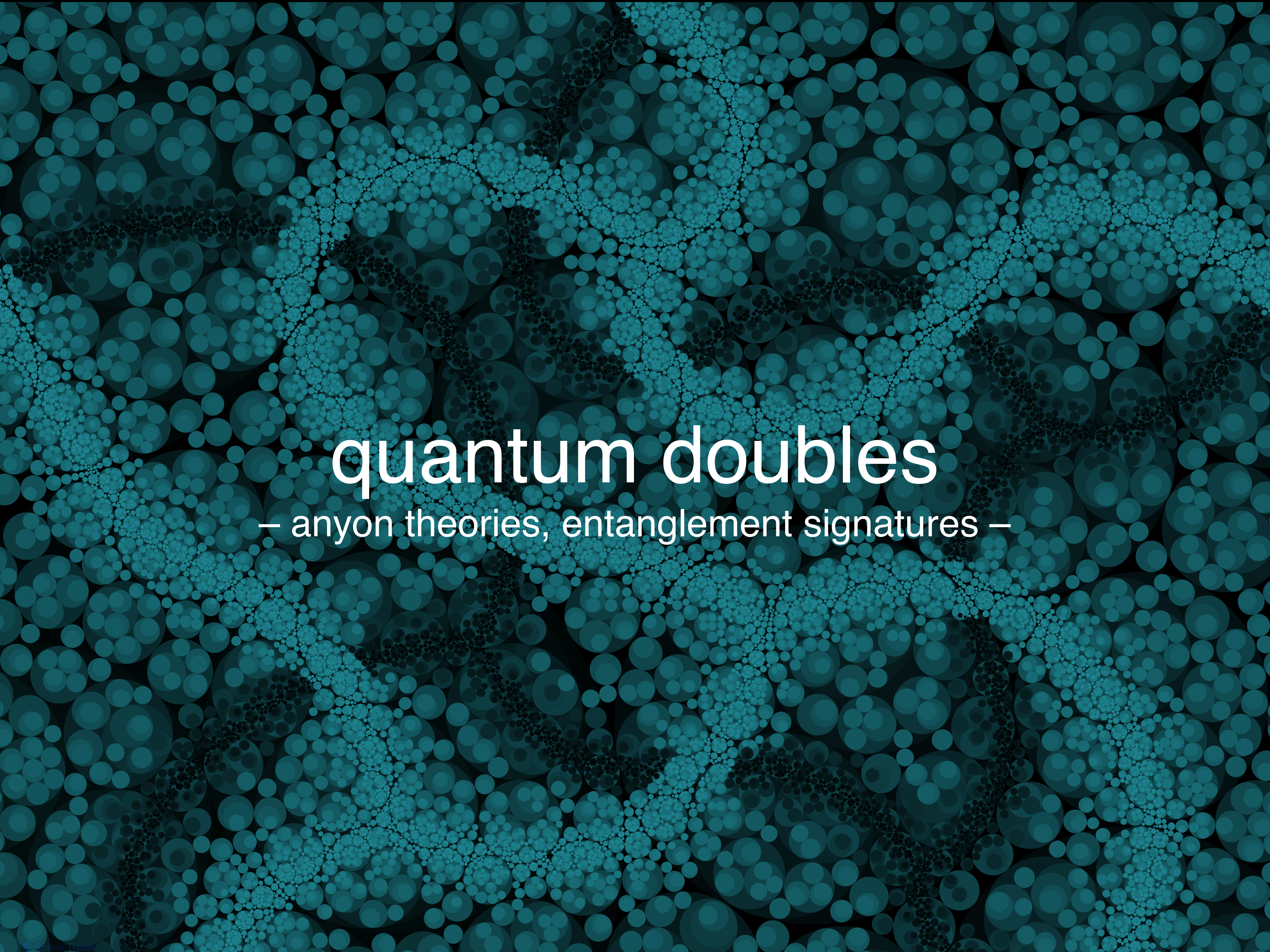
→ three spatial dimensions

spatial dimensions	two spatial dimensions
energy spectrum	gapped / gapless

gapped → **topological** spin liquids

gapless → **gapless** spin liquids → singular points, e.g. Dirac cones / spinon surface, Bose surface

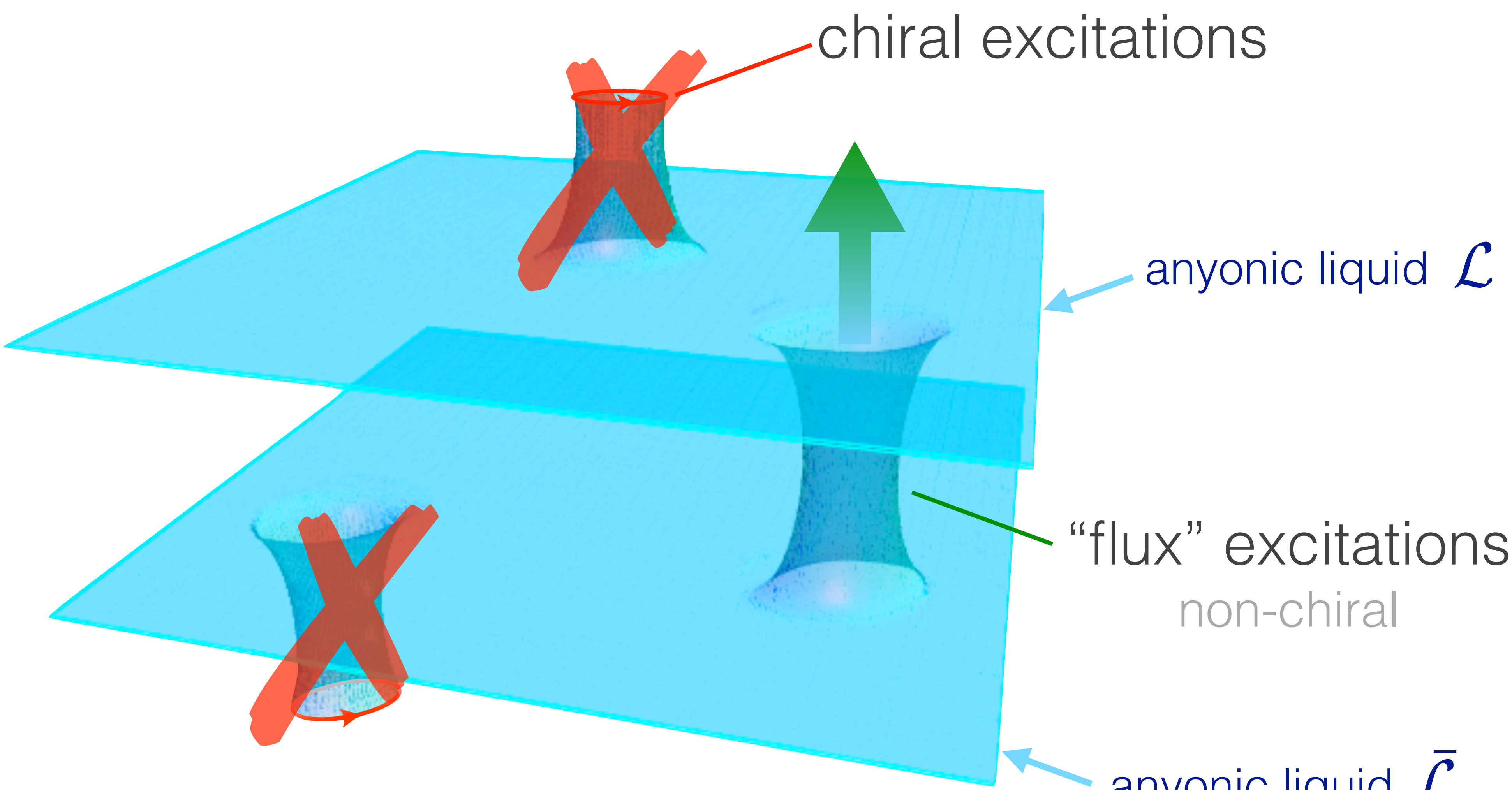
time-reversal symmetry				
	yes	quantum double models Levin-Wen / Kitaev models	**Kitaev model** (honeycomb)	**“Bose metals”** (Motrunich, Sheng & Fisher)
	no	**chiral spin liquids** Kalmeyer & Laughlin ✓	✗	✗ ✓



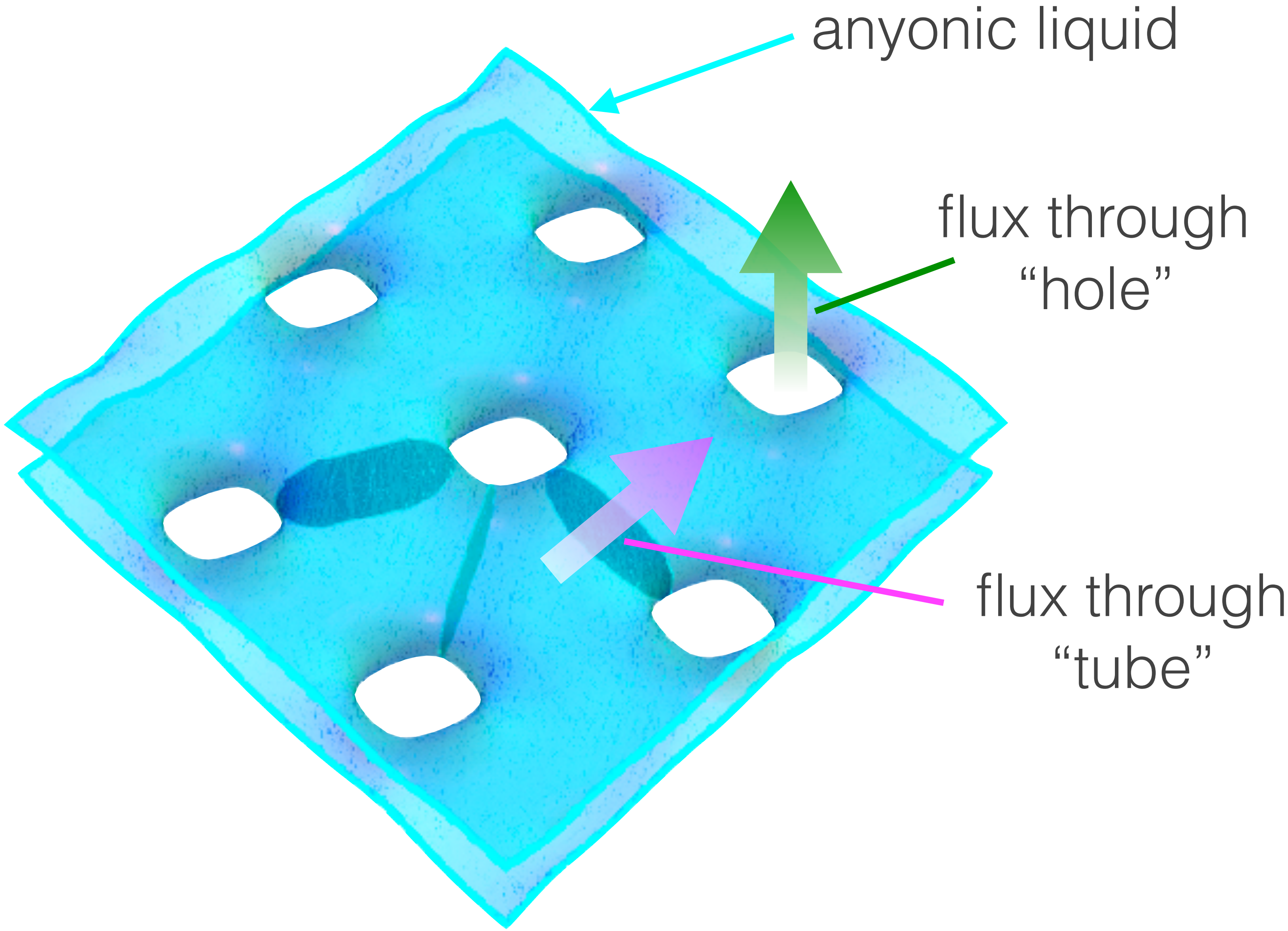
quantum doubles

– anyon theories, entanglement signatures –

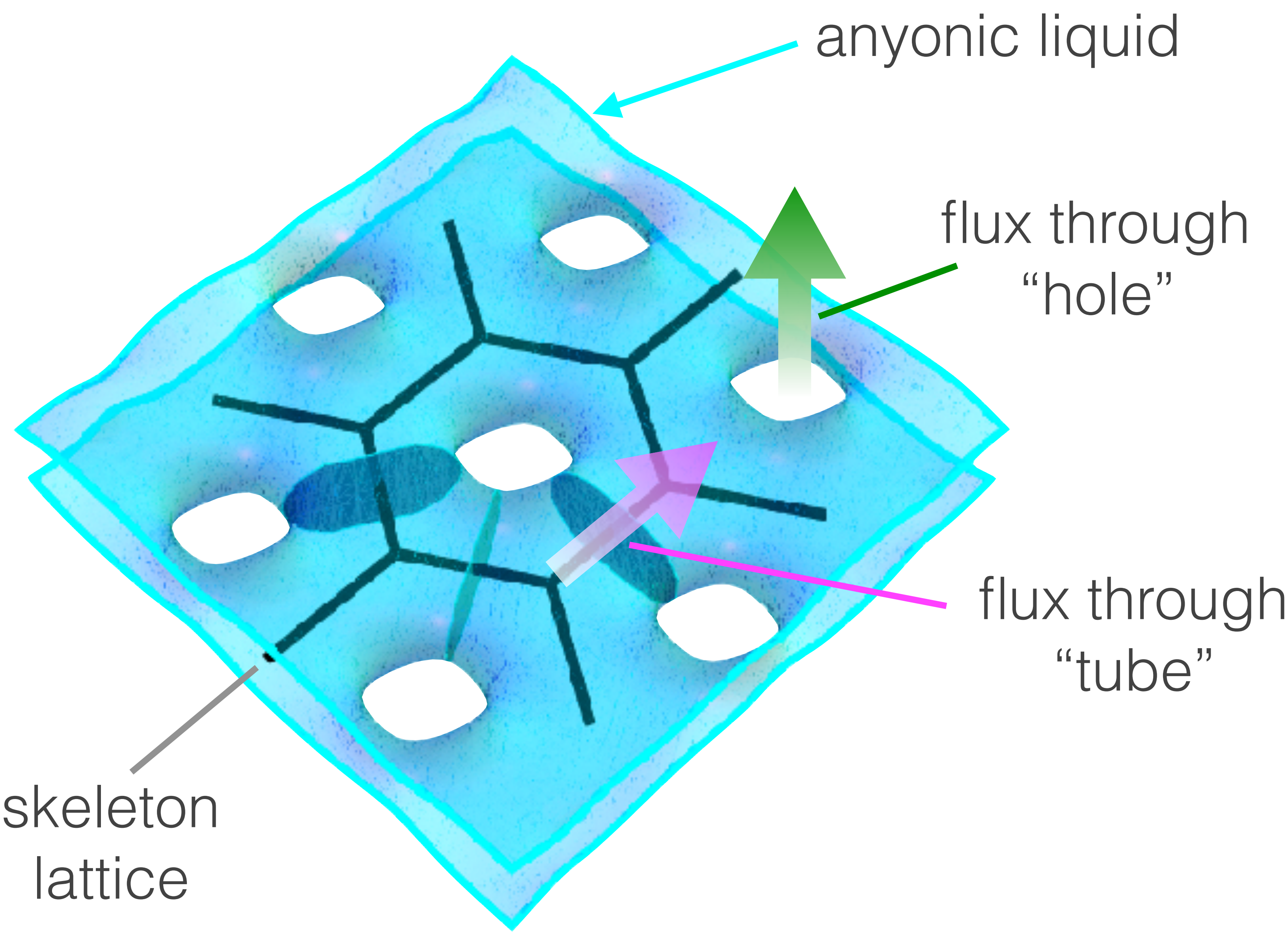
Time-reversal invariant liquids



Flux excitations

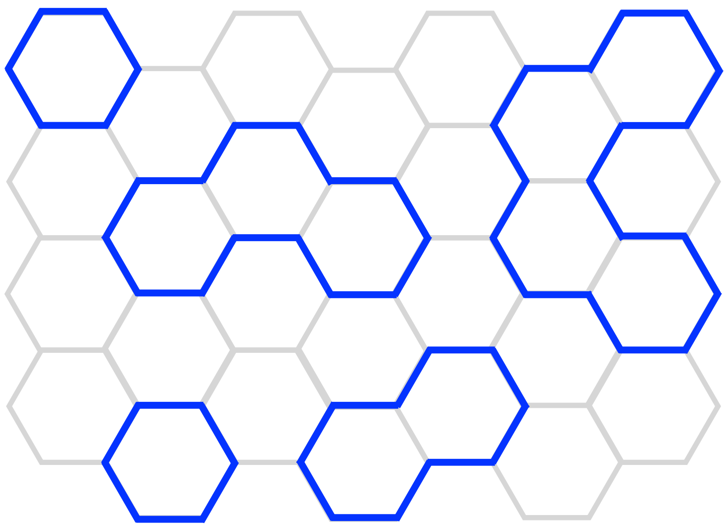


Flux excitations

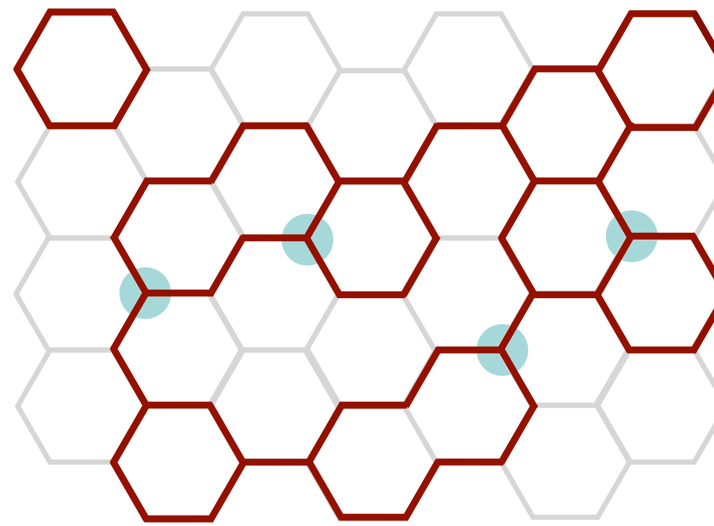


quantum double models

Quantum double models form a larger family of lattice models harboring non-trivial topological order, e.g. non-Abelian **string nets**.

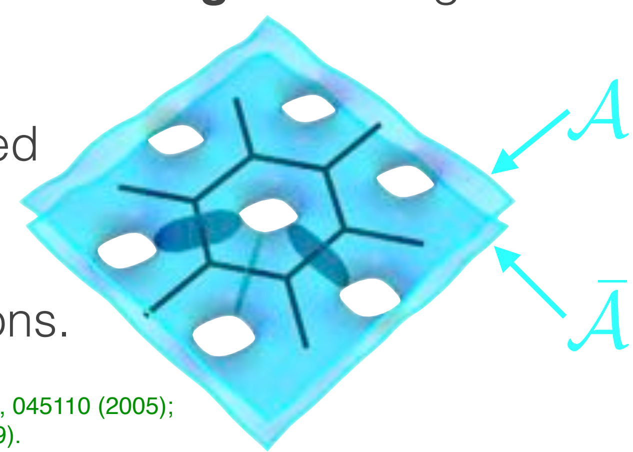


loop gas configuration
(toric code)



string net configuration

- Quantum double models are generally constructed from an underlying **anyon theory**.
- Key ingredient are so-called **fusion rules** of anyons.



anyon theories

M. Levin and X.-G. Wen, Phys. Rev. B **71**, 045110 (2005);
C. Gils et al., Nature Physics **5**, 834 (2009).

quantum double models

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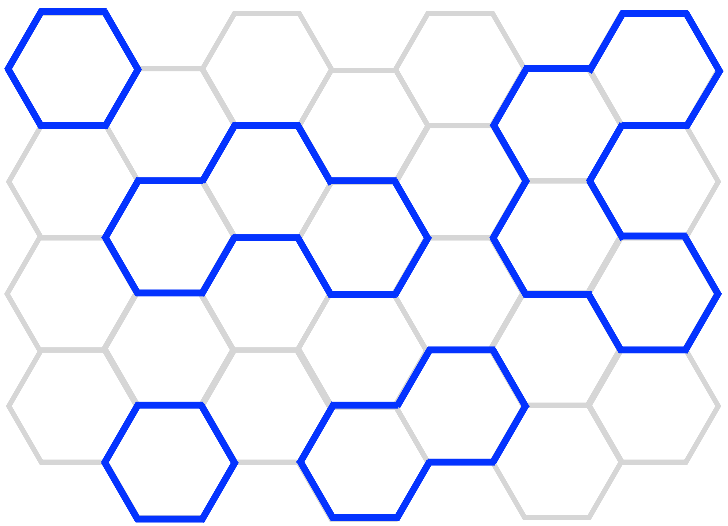
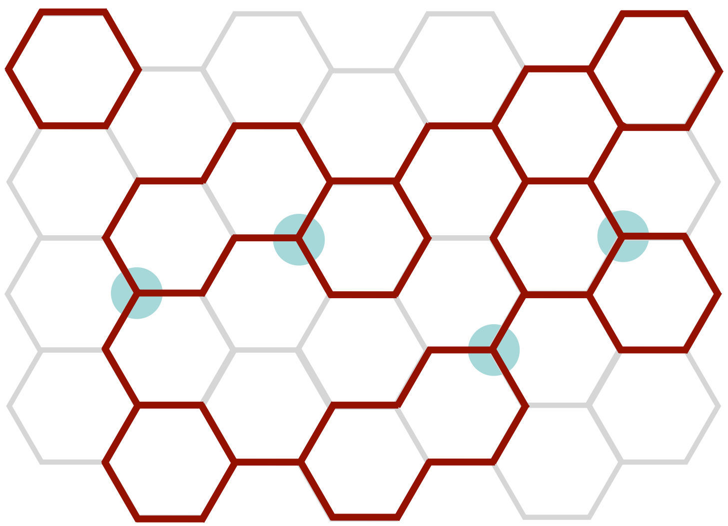


loop gas configuration



string net configuration

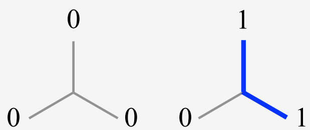
Z_2 anyon theory

$0 \times 0 = 0$

$0 \times 1 = 1$

$1 \times 1 = 0$

toric code



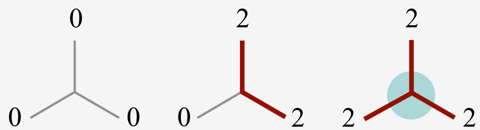
Fibonacci anyon theory

$0 \times 0 = 0$

$0 \times 2 = 2$

$2 \times 2 = 0 + 2$

non-Abelian



Entanglement

Corrections to the boundary law
can reveal the specific
character of the underlying quantum many-body state!

- **topological** spin liquids

$$S = aL - \gamma$$

quantum double models
Levin-Wen / Kitaev models

- **gapless** spin liquids
  - gapless modes at **singular point** in momentum space

$$S = aL + c\,\gamma(L_x, L_y)$$

Kitaev model
(honeycomb)

  - gapless modes on **surface** in momentum space

$$S = cL\ln(L)$$

"Bose metals"
(Motrunich, Sheng & Fisher)

- **critical points, conformal critical points, Goldstone modes,** …

$$S_{\mathrm{QCP}} = aL + c\,\gamma(L_x, L_y) \qquad S_{\mathrm{cQCP}} = \mu L + \gamma_{\mathrm{cQCP}} \qquad S_{\mathrm{G}} = aL + b\ln(L) + \gamma(L_x, L_y)$$

Topological entanglement entropy

The **topological correction** is **universal.**

$$\gamma = \ln \sqrt{\sum_{i=1}^{n} d_i^2}$$

← quantum dimension of excitation

Examples:

- toric code (loop gas)

	$\mathbf{1}$	e	m	em
d_i	1	1	1	1

$$\gamma = \ln \sqrt{1+1+1+1} = \ln 2 \approx 0.693$$

- Fibonacci theory (string net)

	$\mathbf{1}$	τ
d_i	1	$\phi = \frac{1+\sqrt{5}}{2}$

$$\gamma = \ln \sqrt{1+\phi^2} \approx 0.643$$

A. Kitaev and J. Preskill, Phys. Rev. Lett. **96**, 110404 (2006);
M. Levin and X.-G. Wen, Phys. Rev. Lett. **96**, 110405 (2006).

Let's try this on the toric code

Examples: • toric code (loop gas)

	$\mathbf{1}$	e	m	em
d_i	1	1	1	1

$$\gamma = \ln \sqrt{1+1+1+1} = \ln 2 \approx 0.693$$

ARTICLES
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nature physics

Identifying topological order by entanglement entropy

Hong-Chen Jiang[1], Zhenghan Wang[2] and Leon Balents[1]*

Nature Physics **8**, 902 (2012).

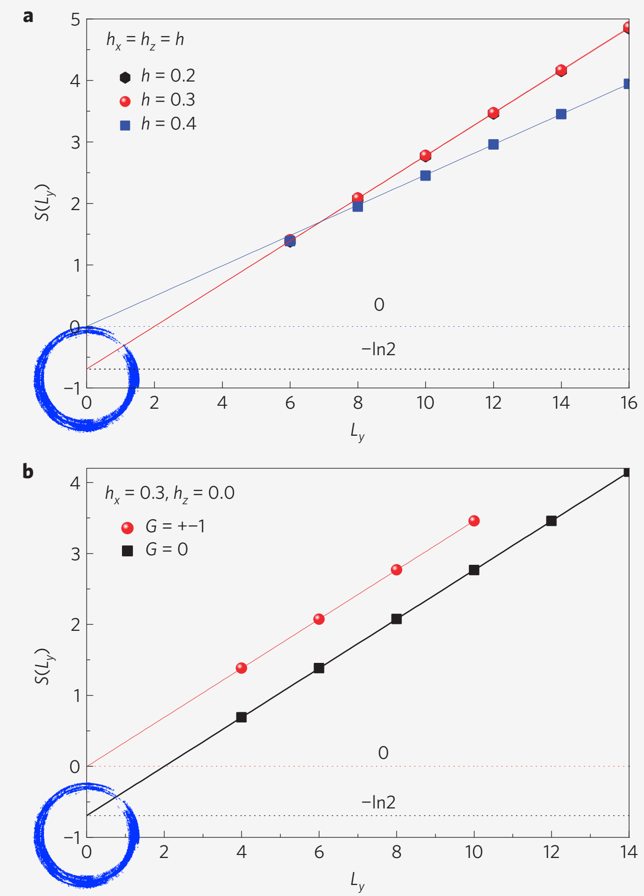
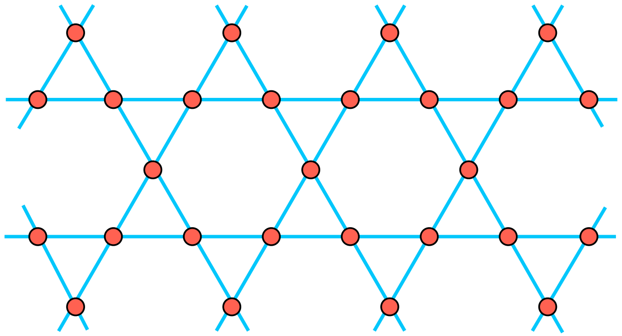


Figure 2 | The von Neumann entropy $S(L_y)$ for the toric-code model in magnetic fields. a, $S(L_y)$ with $L_y = 4$–16 at $L_x = \infty$ for symmetric magnetic fields at $h_x = h_z = h = 0.2$, 0.3 and 0.4. By fitting $S(L_y) = aL_y - \gamma$, we get $\gamma = 0.693(1)$, 0.691(4) and 0.001(5), respectively. **b**, The pure electric case, $h_x = 0.3, h_z = 0$, and comparison of $S(L_y)$ in the MES obtained in the large L_x limit (black squares) with that of the absolute ground state from systems of dimensions $L_x \times L_y = 20 \times 4, 24 \times 6, 24 \times 8, 24 \times 10$ (red circles). Extrapolation shows that the MES has the universal TEE, whereas the absolute ground state has zero TEE.

Kagomé antiferromagnet



Heisenberg model

$$H = \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j$$

on kagomé lattice

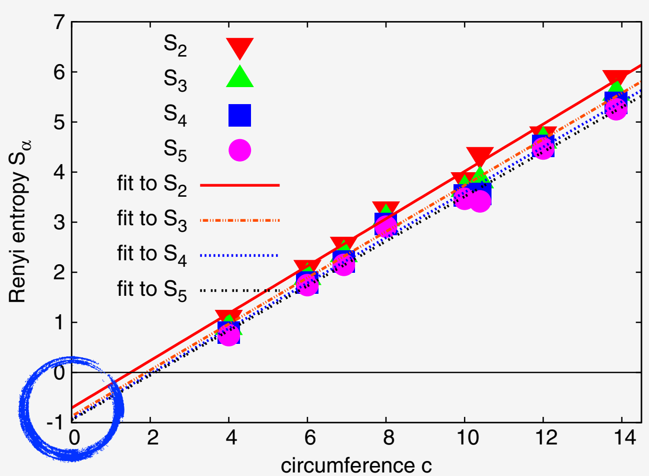


FIG. 6 (color online). Renyi entropies $\mathcal{S}_\alpha$ of infinitely long cylinders for various α versus circumference c, extrapolated to $c = 0$. The negative intercept is the topological entanglement entropy γ.

S. Depenbrock, I.P. McCulloch, and U. Schollwöck, Phys. Rev. Lett. **109**, 067201 (2012).

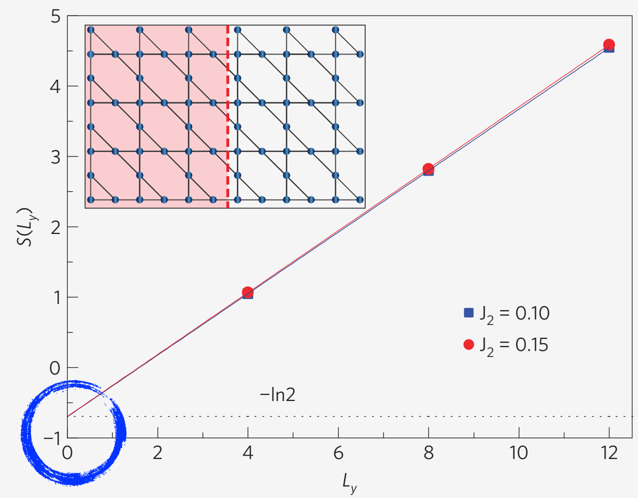
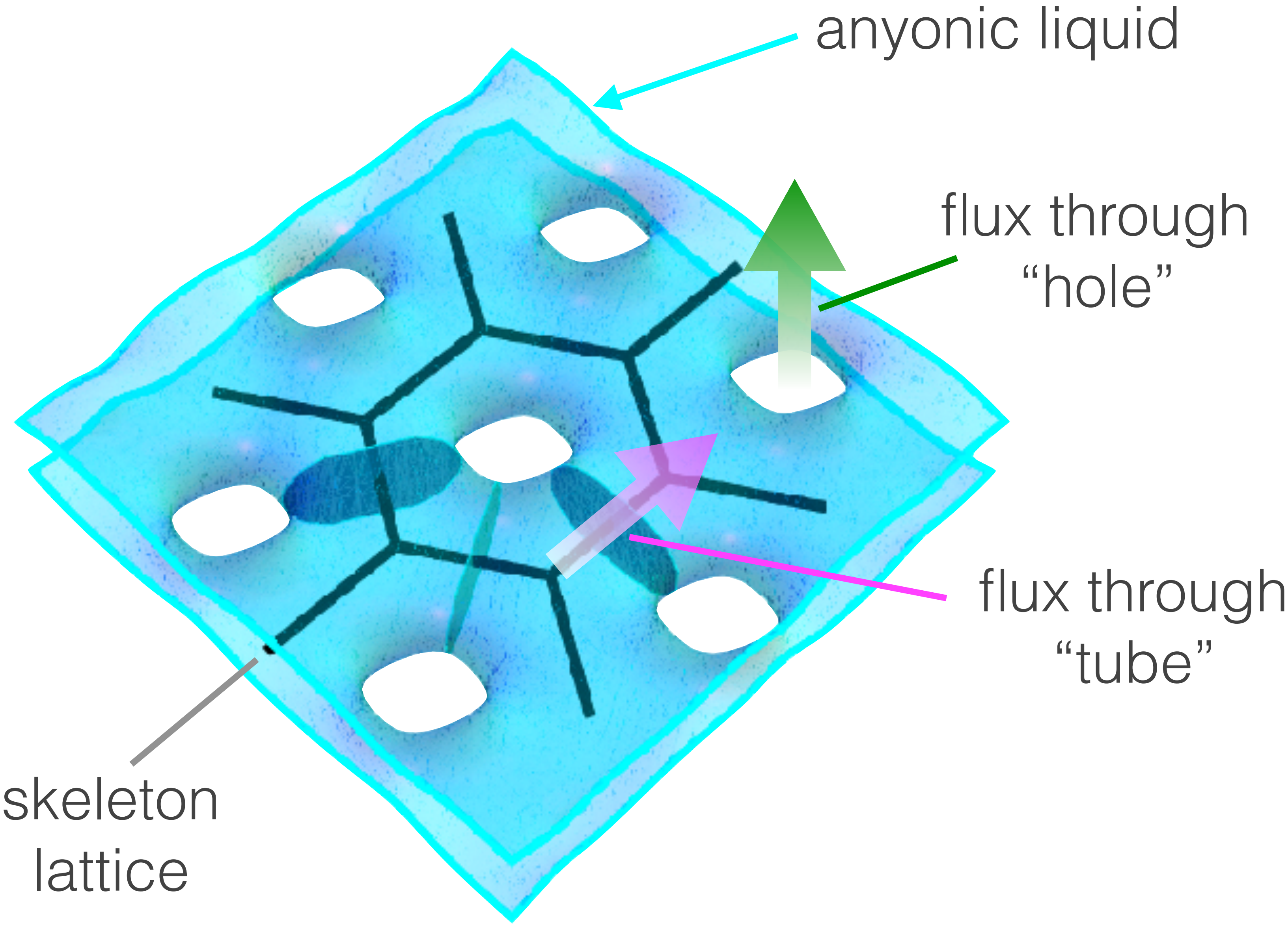


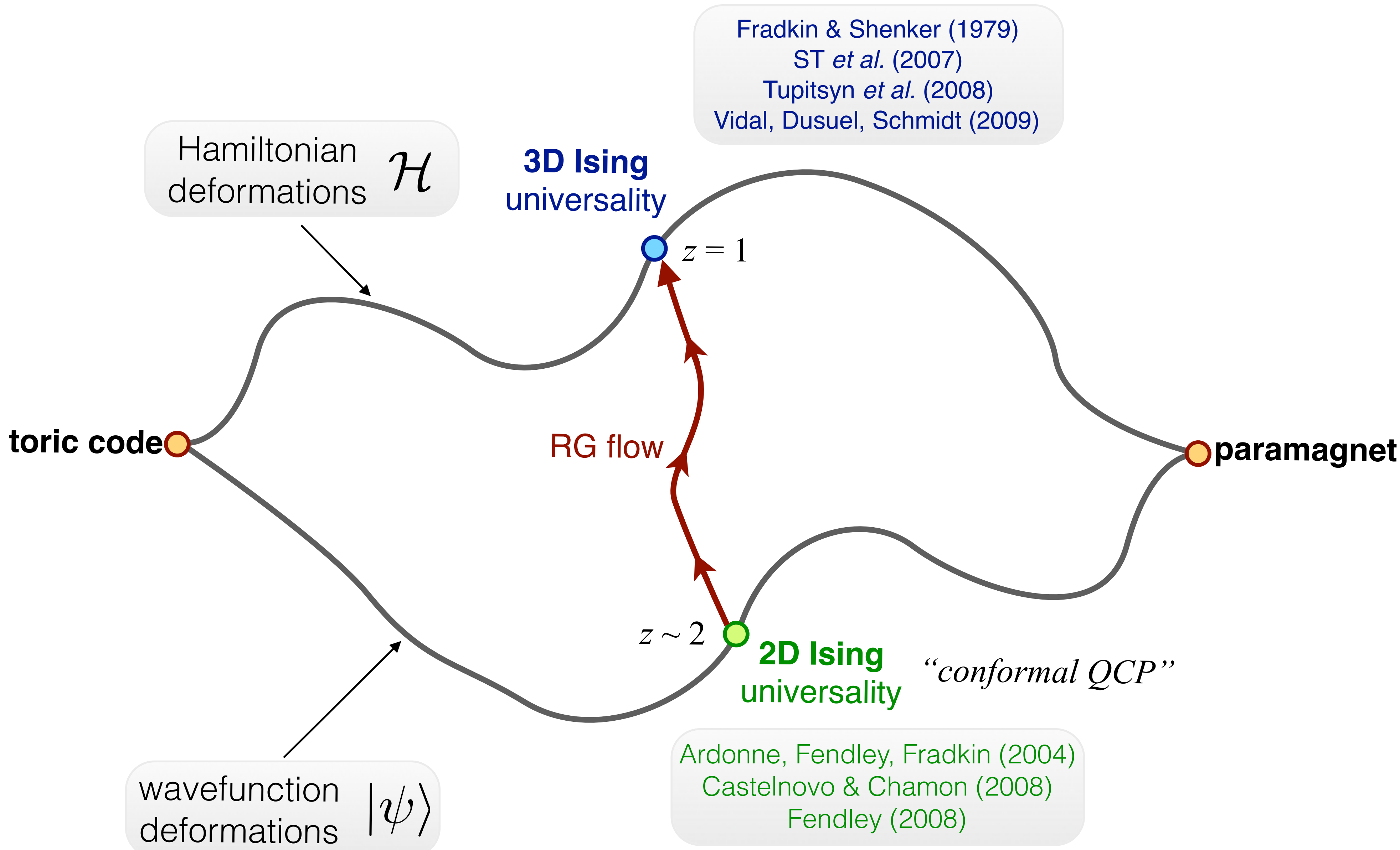
Figure 3 | The entanglement entropy $S(L_y)$ of the kagome J_1–J_2 model in equation (2), with $L_y = 4$–12 at $L_x = \infty$. By fitting $S(L_y) = aL_y - \gamma$, we get $\gamma = 0.698(8)$ at $J_2 = 0.10$ and $\gamma = 0.694(6)$ at $J_2 = 0.15$. Inset: kagome lattice with $L_x = 12$ and $L_y = 8$.

H.-C. Jiang, Z. Wang, and L. Balents, Nature Physics **8**, 902 (2012).

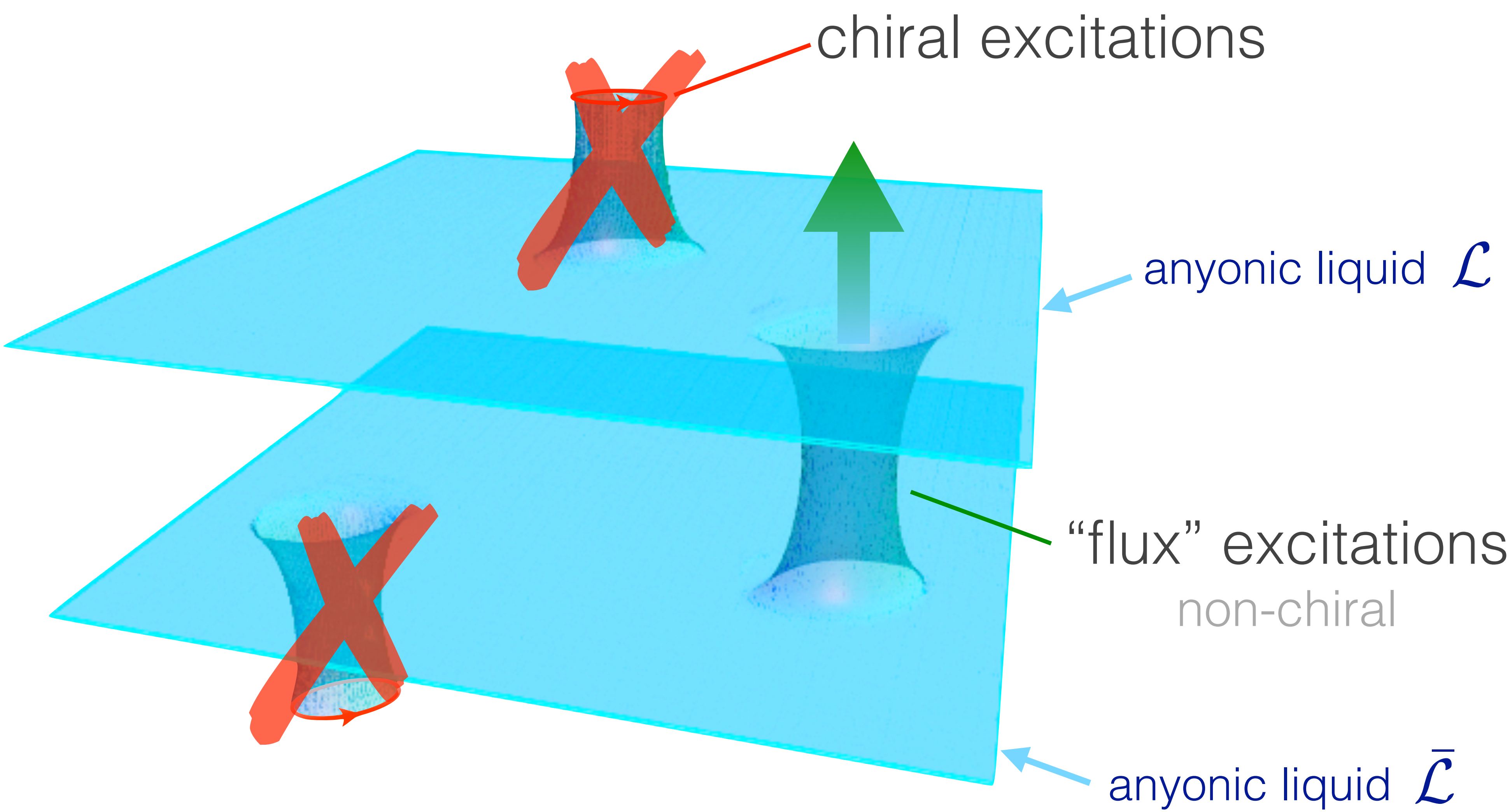
Flux excitations



string tension & phase transitions



Time-reversal invariant liquids



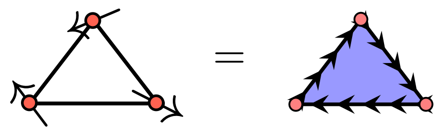
concepts

How do we talk about spin liquids?

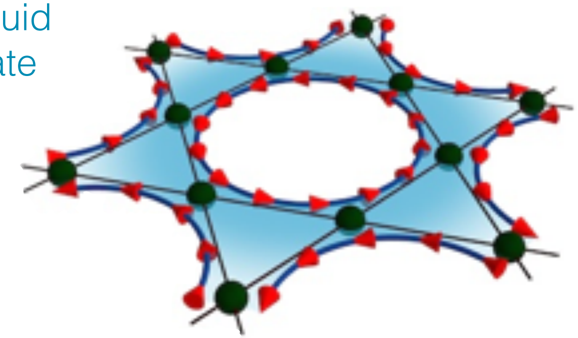
anyon models

quantum doubles

chiral spin liquids



topological liquid
with edge state



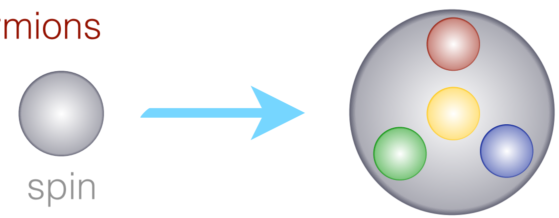
parton constructions

Majorana fermions

Z_2 spin liquids

Majorana fermions

$$S^\gamma = i a^\gamma c$$



+
Z_2 gauge field

Schwinger representation

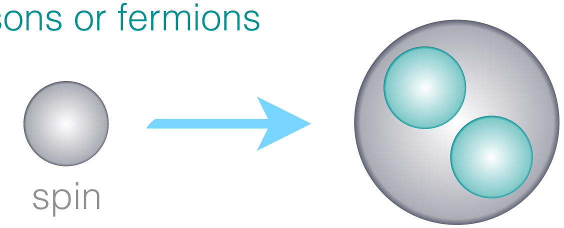
U(1) spin liquids

$c_1^\dagger c_1 + c_2^\dagger c_2 = 2S$

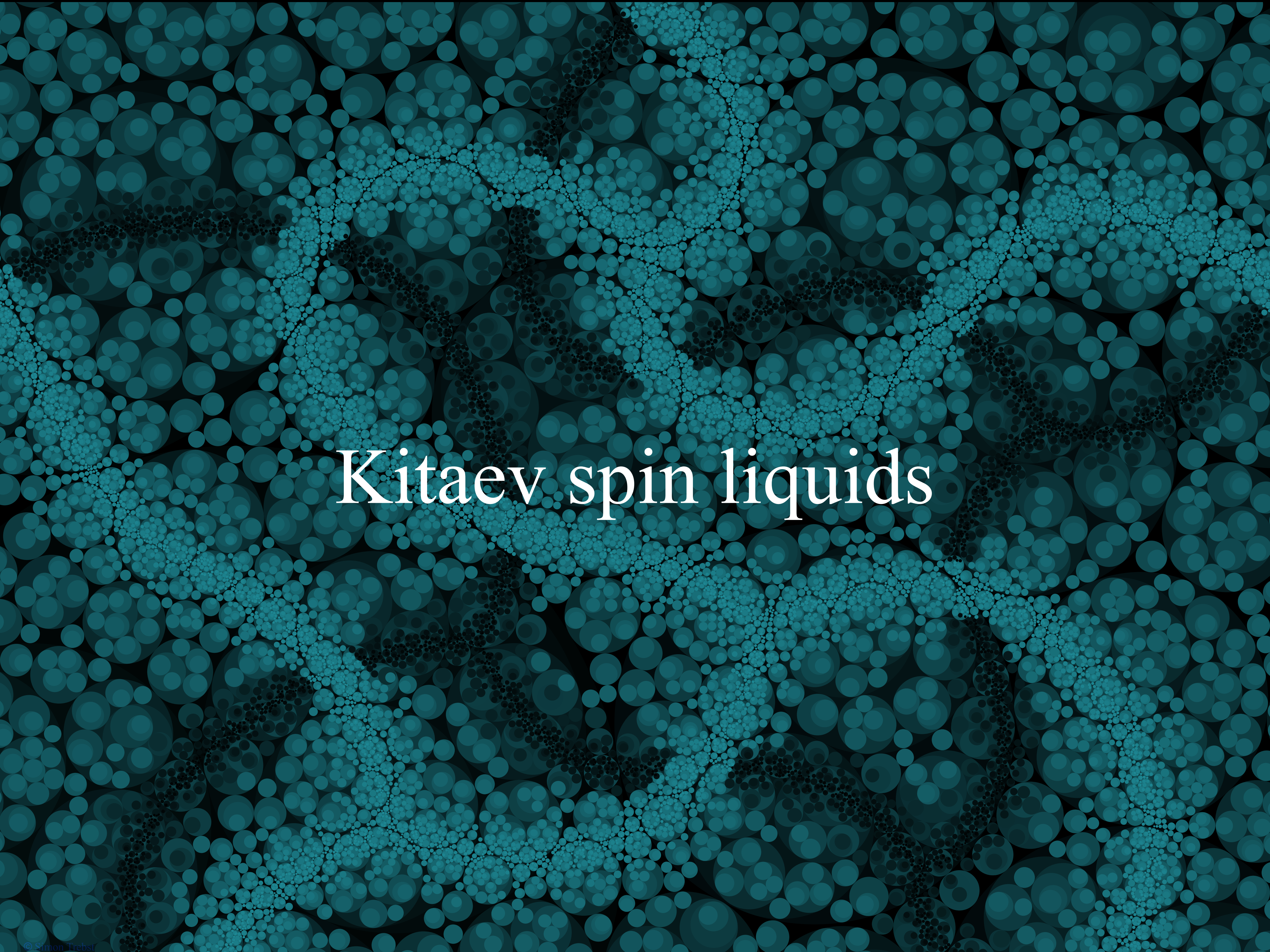
bosons or fermions

$$S^\gamma = \frac{1}{2} c_\alpha^\dagger \sigma_{\alpha,\beta}^\gamma c_\beta$$

Pauli matrices



+
U(1) gauge field



Kitaev spin liquids

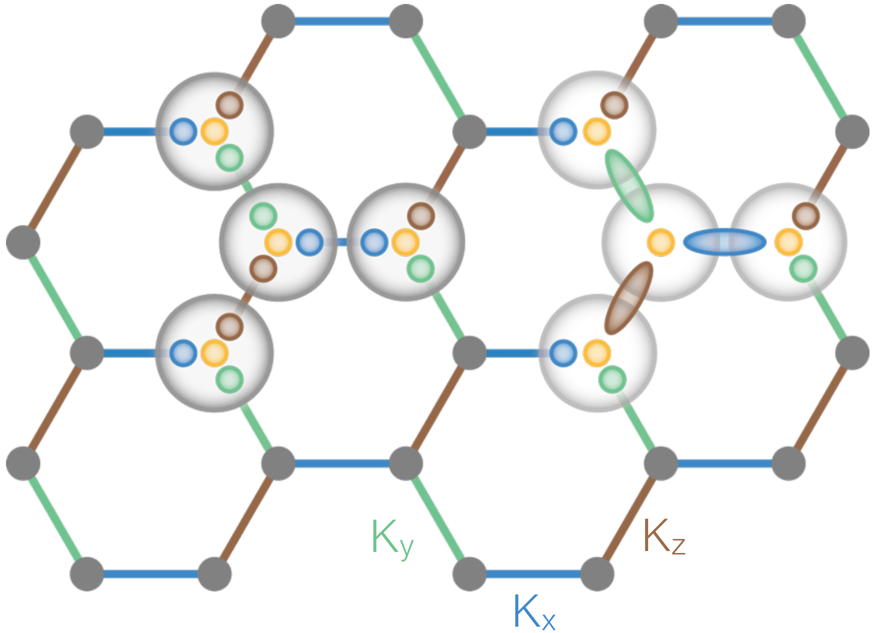
Kitaev model

$$\mathcal{H} = - \sum_{\gamma-\text{bonds}} K_\gamma \, S_i^\gamma S_j^\gamma$$

Represent spins in terms of four **Majorana fermions**

$$S_i^\gamma = i \; c_i \; c_i^\gamma$$

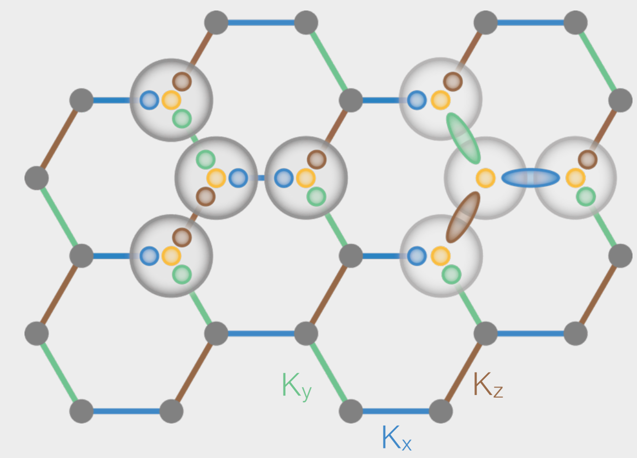
$$S_i^\alpha \to S_i^\alpha \qquad c_i^\alpha \to e^{-i\phi_i} c_i^\alpha \qquad c_i \to e^{i\phi_i} c_i$$



$$(c_i)^2 = 1 \to (e^{i\phi_i} c_i)^2 = 1 \qquad e^{i\phi_i} = \pm 1$$

Z$_2$ redundancy

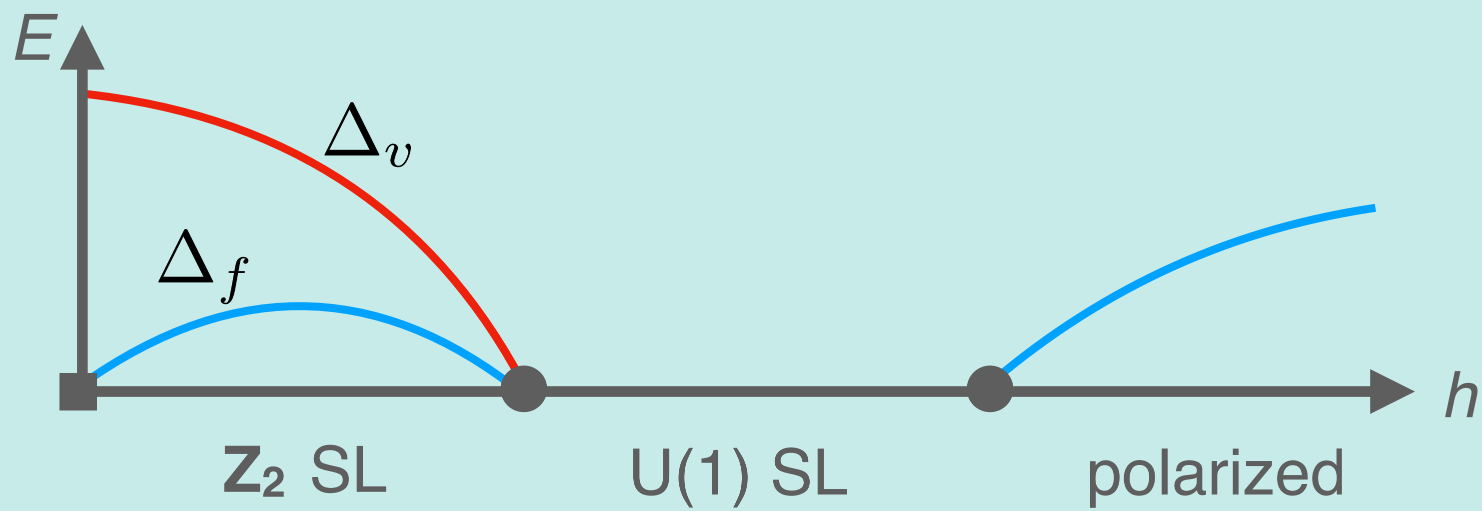
Kitaev spin liquids



$$\mathcal{H} = - \sum_{\gamma-\text{bonds}} K_\gamma \, S_i^\gamma S_j^\gamma$$

Kitaev spin liquids are textbook examples of **Z_2 spin liquids.**

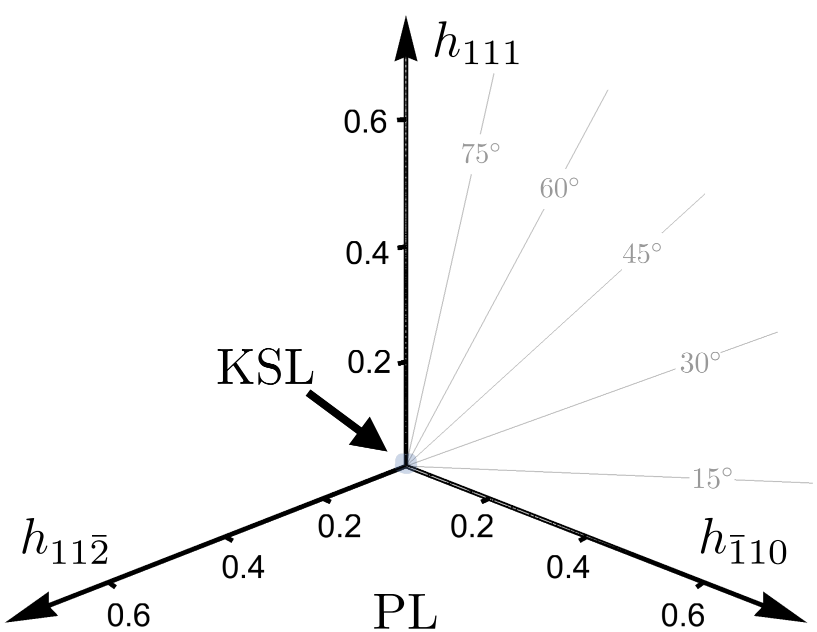
For strong magnetic fields, this picture no longer holds.
The Kitaev model exhibits a **gauge transition** to a **U(1) spin liquid.**



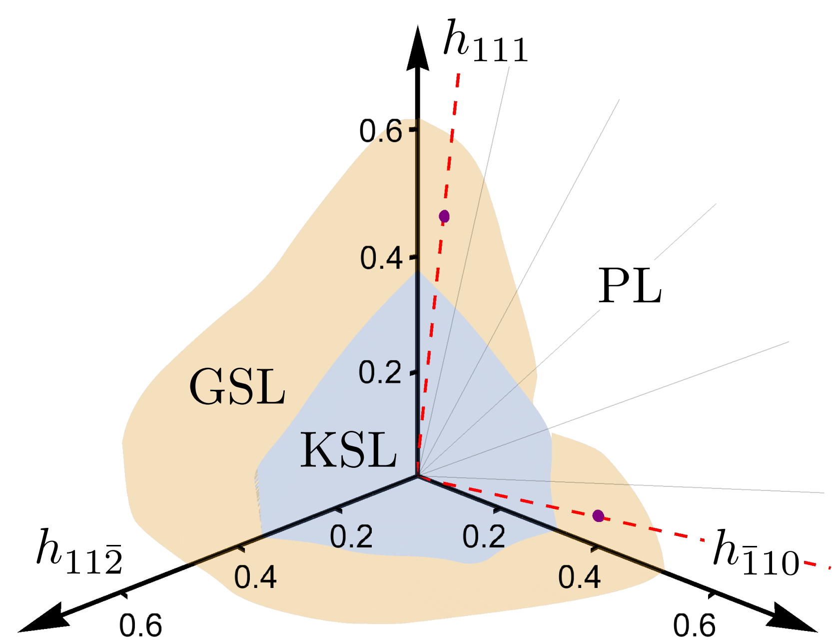
Kitaev model – magnetic field effects

$$\mathcal{H} = -\sum_{\gamma-\text{bonds}} K_\gamma S_i^\gamma S_j^\gamma - \sum_i \mathbf{h}\cdot\mathbf{S}_i$$

FM Kitaev coupling



AFM Kitaev coupling



Kitaev model – magnetic field effects

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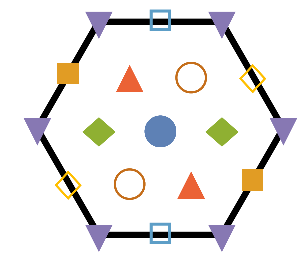


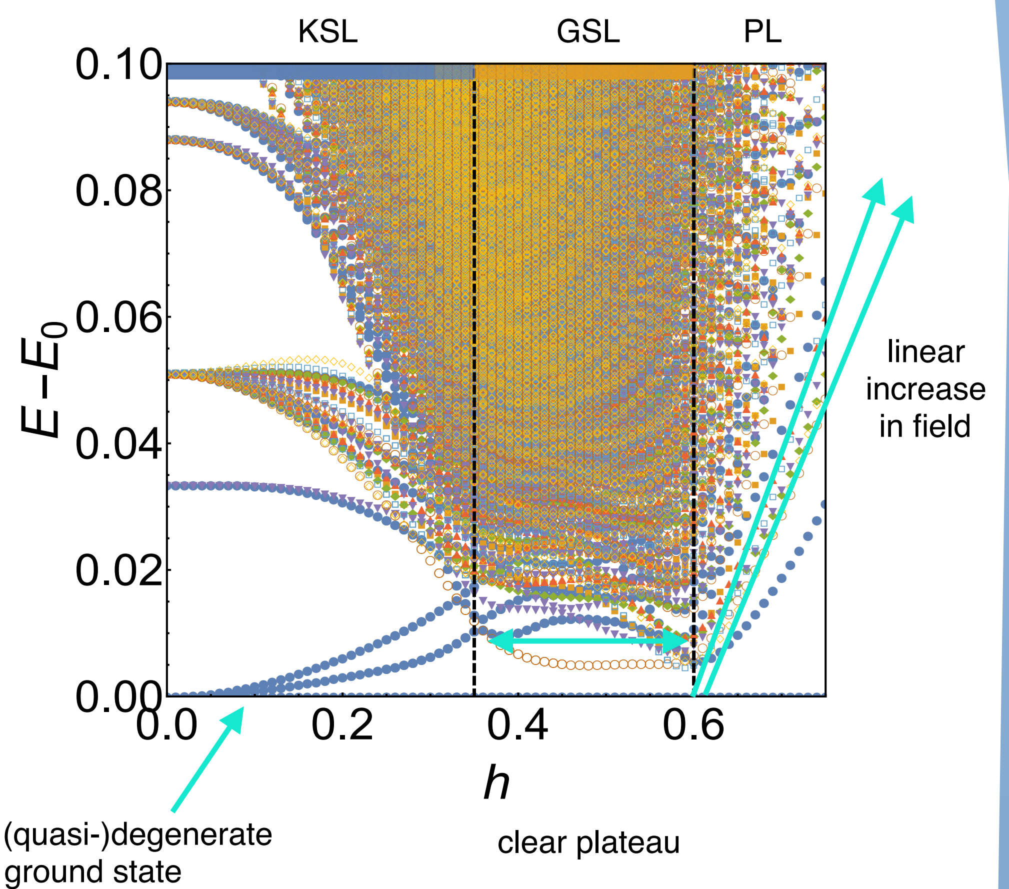
modular matrices

$$S = \begin{pmatrix} 0.50 & 0.71 & 0.50 \\ 0.71 & 0.00 & -0.71 \\ 0.50 & -0.71 & 0.50 \end{pmatrix}$$

Ising TQFT

$$S = \begin{pmatrix} 0.46 & 0.74 & 0.47 \\ 0.71 & 0.04e^{-0.91i} & -0.70 \\ 0.49 & -0.67e^{0.02i} & 0.58e^{-0.13i} \end{pmatrix}$$

numerical result



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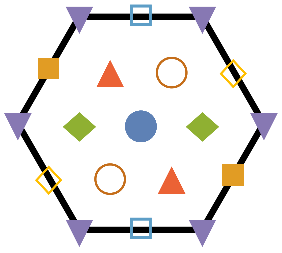
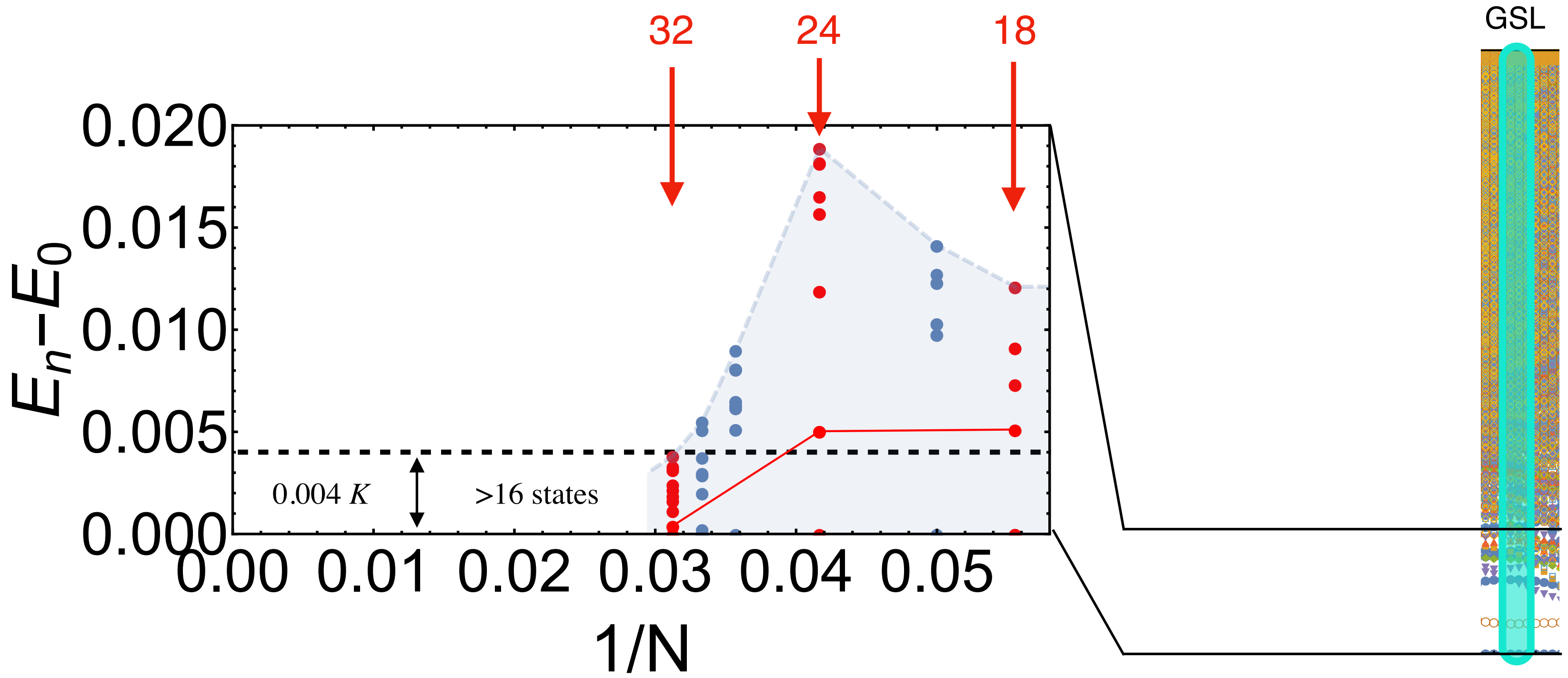


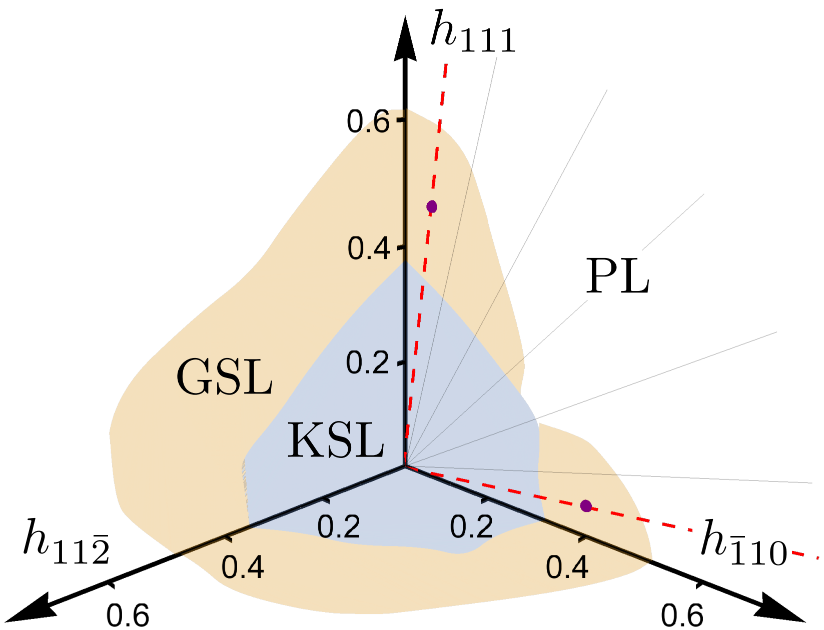


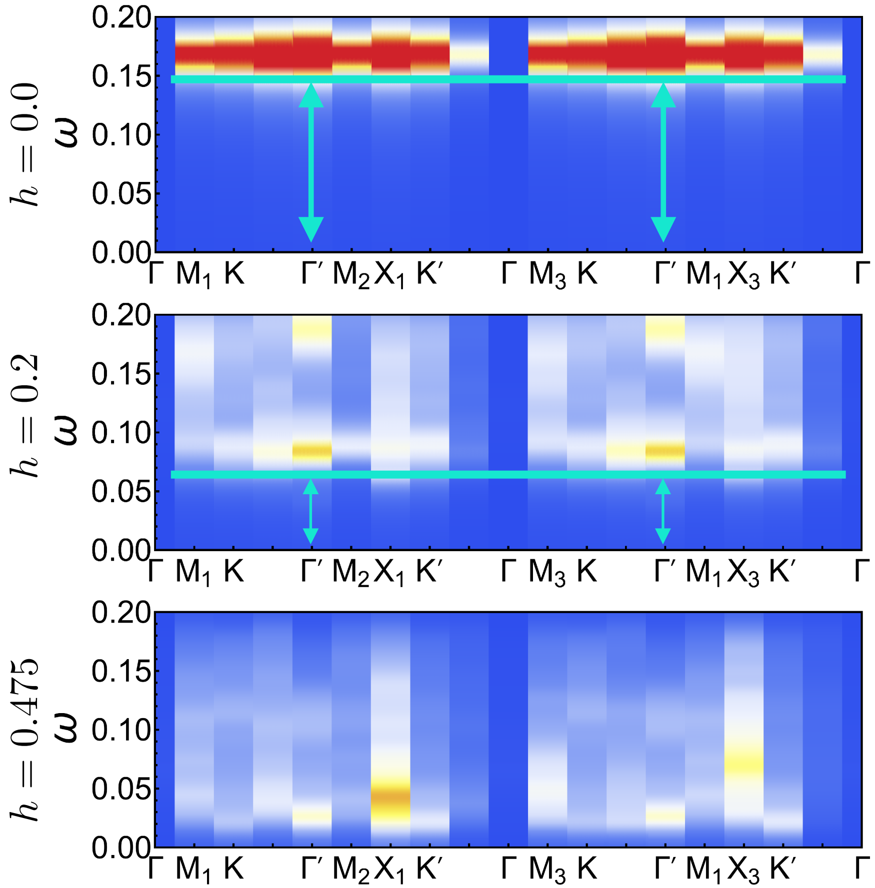
The intermediate phase has the **smallest finite-size gap** ever seen!

dynamical structure factor

$$\mathcal{H} = -\sum_{\gamma-\mathrm{bonds}} K_\gamma S_i^\gamma S_j^\gamma - \sum_i \mathbf{h} \cdot \mathbf{S}_i$$

AFM Kitaev coupling





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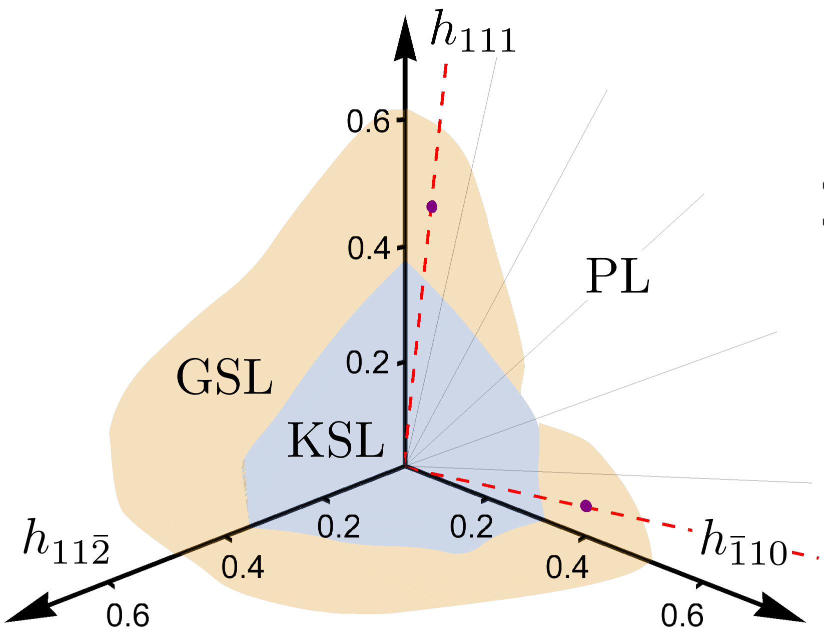
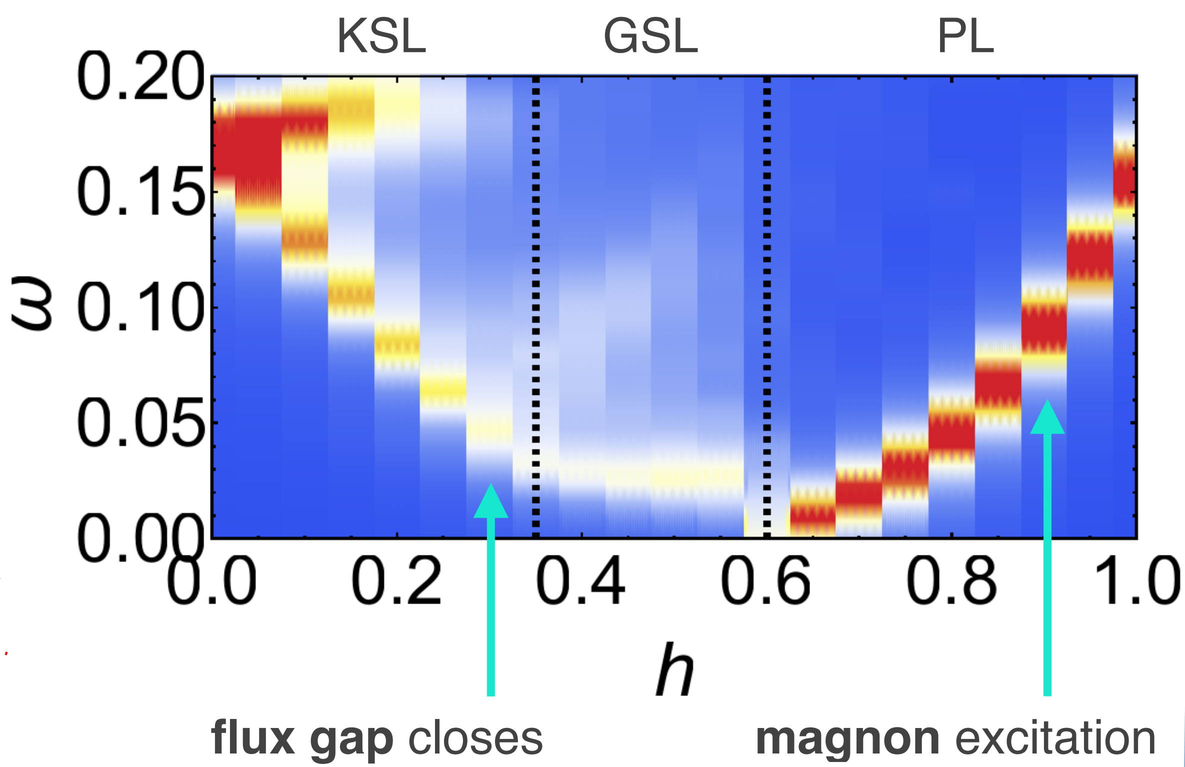
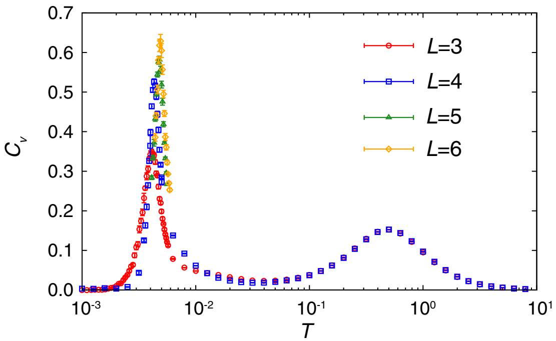




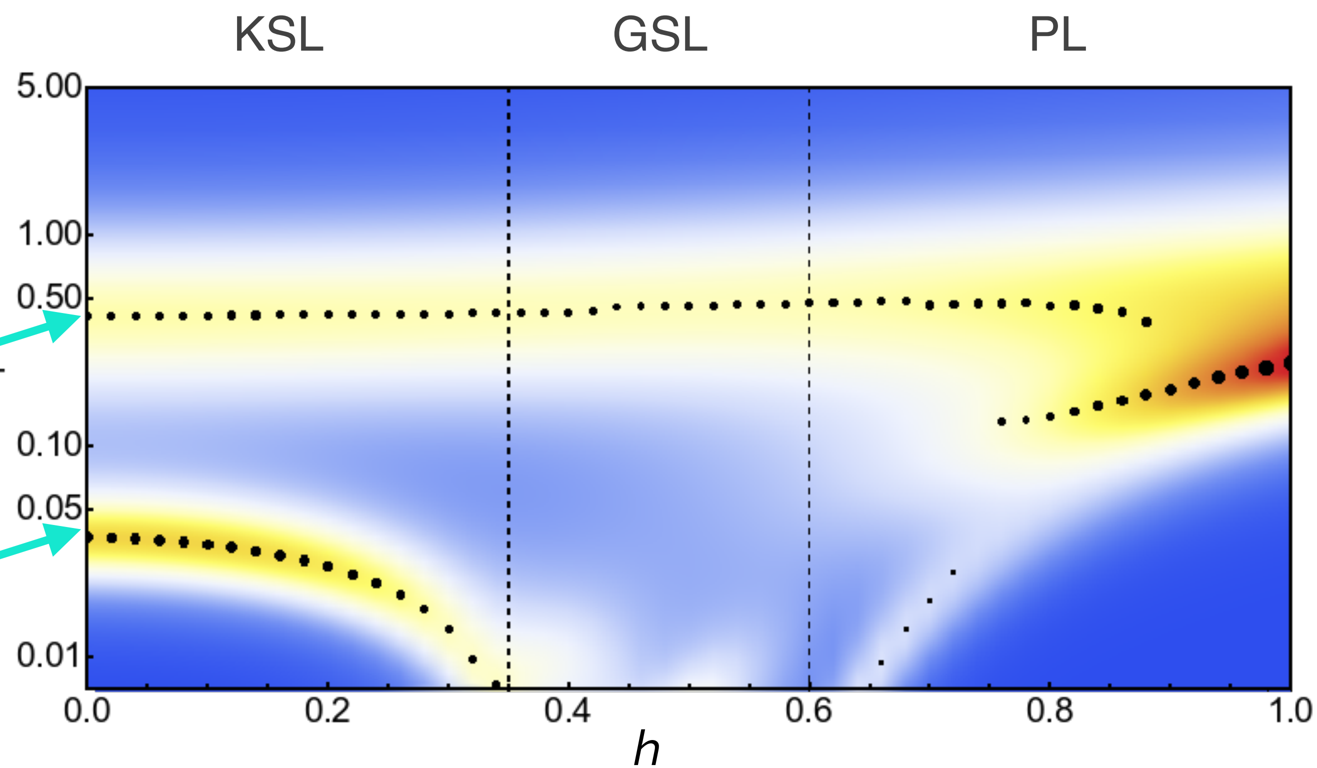
specific heat

$$\mathcal{H} = -\sum_{\gamma-\text{bonds}} K_\gamma S_i^\gamma S_j^\gamma - \sum_i \mathbf{h}\cdot\mathbf{S}_i$$



fractionalization crossover

Z_2 flux ordering



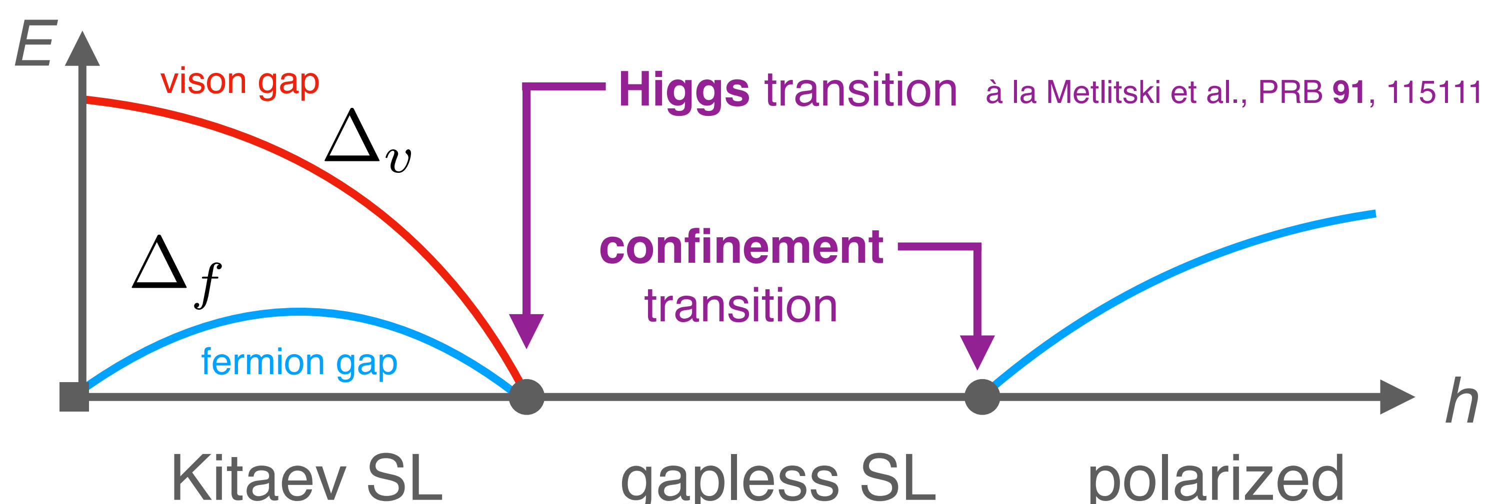
Kitaev unHiggsed!

Synopsis: for strong magnetic fields, the Kitaev model exhibits a **Higgs transition** to a gapless U(1) spin liquid.

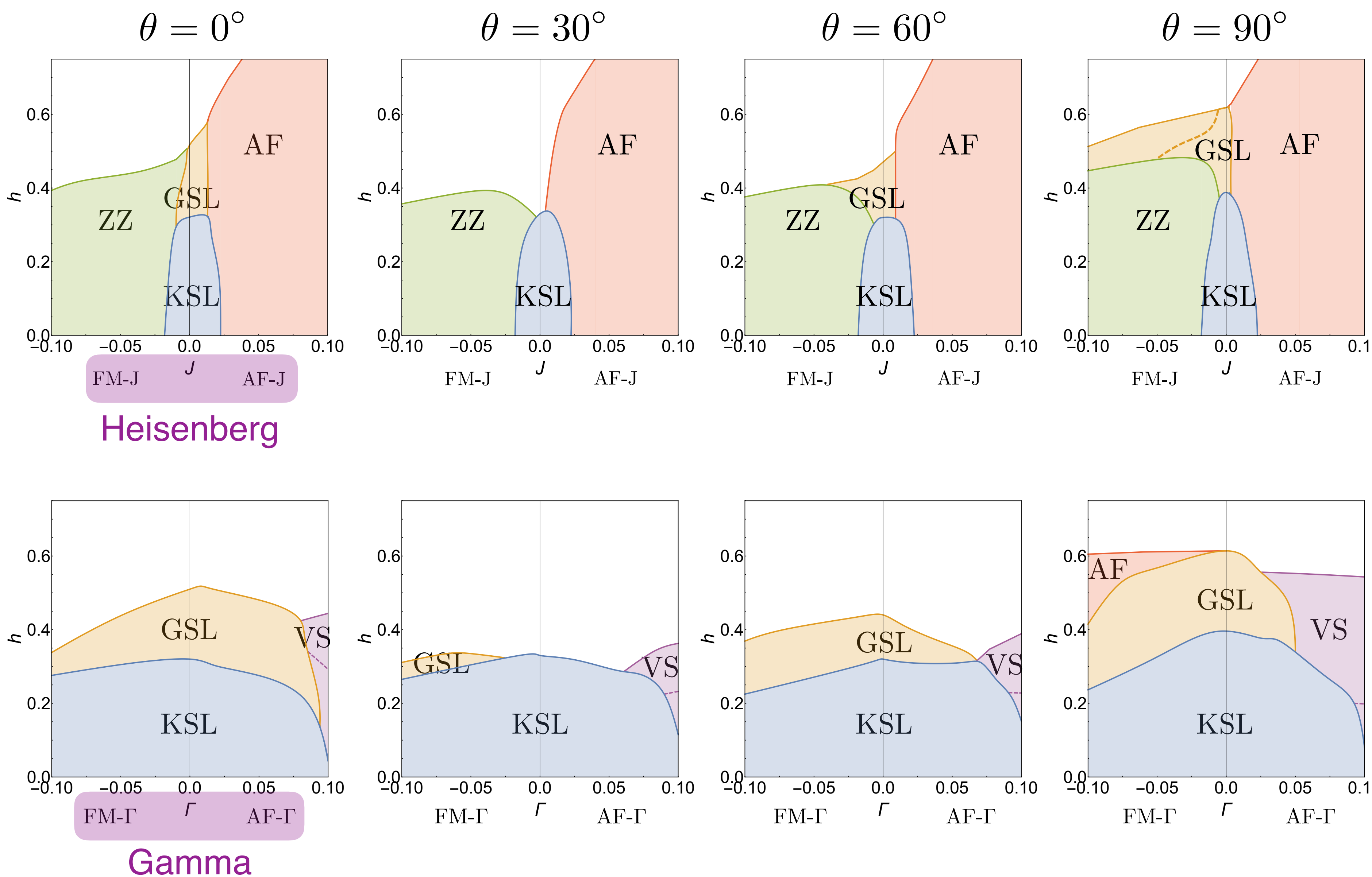
Represent spins in terms of **complex fermions** $S_i^\alpha = f_{i,\mu}^\dagger \sigma_{\mu\nu}^\alpha f_{i,\nu}$

F. J. Burnell and C. Nayak, PRB **84**, 125125 (2011)

fermions	gapped topological SC	gapless Fermi surface	gapped trivial insulator
gauge field	Z_2 (Higgsed)	U(1)	U(1) [confined]



Stability of U(1) spin liquid



Summary

C. Hickey and ST
Nature Comm. **10**, 530 (2019)

$$\mathcal{H} = -\sum_{\gamma-\mathrm{bonds}} K_\gamma \, S_i^\gamma S_j^\gamma - \sum_i \mathbf{h} \cdot \mathbf{S}_i$$

Kitaev spin liquids are textbook examples of **Z_2 spin liquids.**

For AFM Kitaev couplings and strong magnetic fields,
a **Higgs transition** to a gapless **U(1) spin liquid** occurs.

The U(1) spin liquid is probably the generic high-field phase, and **parent phase** to the KSLs, but also all kinds of magnetic order.

classification
of quantum spin liquids

Quantum spin liquids

Quantum spin liquids are exotic ground states of frustrated quantum magnets, in which **local moments** are **highly correlated** but still **fluctuate strongly** down to zero temperature.

Classification scheme

→ three spatial dimensions

spatial dimensions	two spatial dimensions	
energy spectrum	gapped	gapless
	topological spin liquids	**gapless** spin liquids
		singular points e.g. Dirac cones / spinon surface, Bose surface
entanglement	$S = aL - \gamma$	$S = aL + c\,\gamma(L_x, L_y)$ / $S = cL\ln(L)$

Summary

- The formation of **quantum spin liquids** in an interacting quantum many-body system is one of the **most fascinating** phenomena in condensed matter physics
- The identification of **topological order or gapless spin liquids** builds on concepts from
  - statistical physics — van Neumann & Renyi entropies
  - quantum information theory — entanglement
  - mathematical physics — boundary laws, anyon theories
- The exploration of **macroscopic entanglement** is a rich and quickly evolving research field – a great field to work in.

All slides of this presentation will become available on our group webpage at **www.thp.uni-koeln.de/trebst**