Monitored Criticality

Learning topology from quantum randomness.

Simon Trebst
University of Cologne

CRC183 ENTANGLED STATES OF MATTER

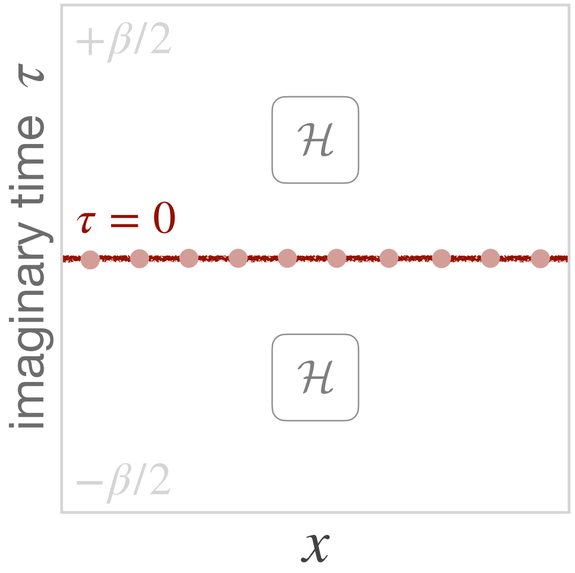
2026 November workshop CRC “Entangled States of Matter”

Berlin, November 2026

ML4Q MATTER AND LIGHT FOR QUANTUM COMPUTING

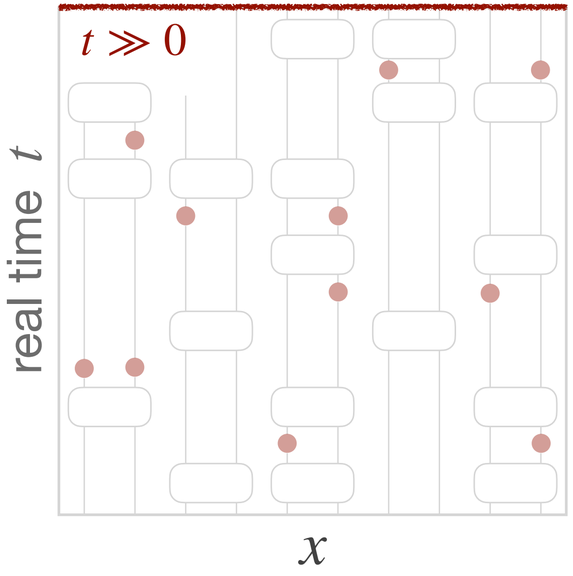
measurements & criticality

(1+1)d



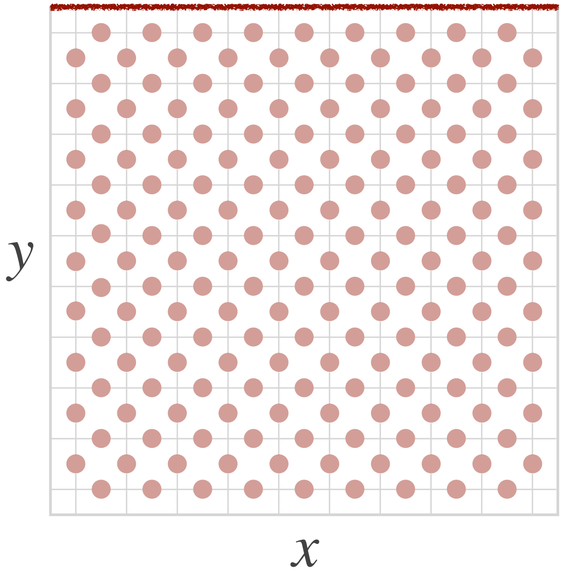
measurement-altered criticality

(1+1)d



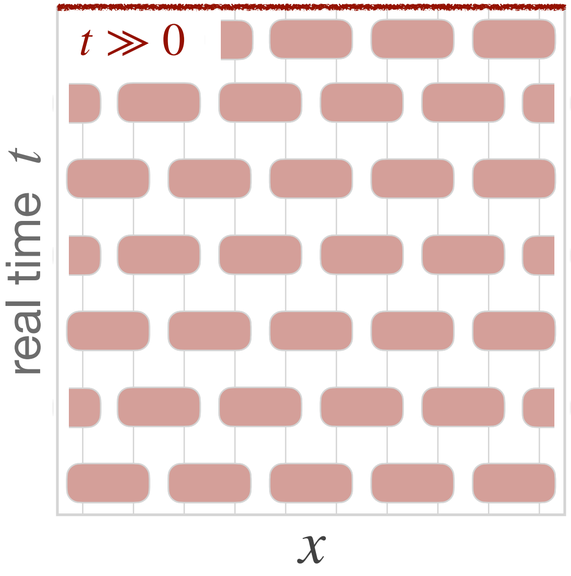
measurement-induced phase transition

(2+0)d



monitored criticality

(1+1)d



measurement-only dynamics

All of these settings lead to hitherto unexplored (non-unitary) conformal field theories.

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monitored toric codes

arXiv:2608.20268
arXiv:2604.06324
arXiv:2502.14034
PRX Quantum (2026)
PRL **136**, 190402 (2026)
PRX Quantum **5**, 040313 (2024)

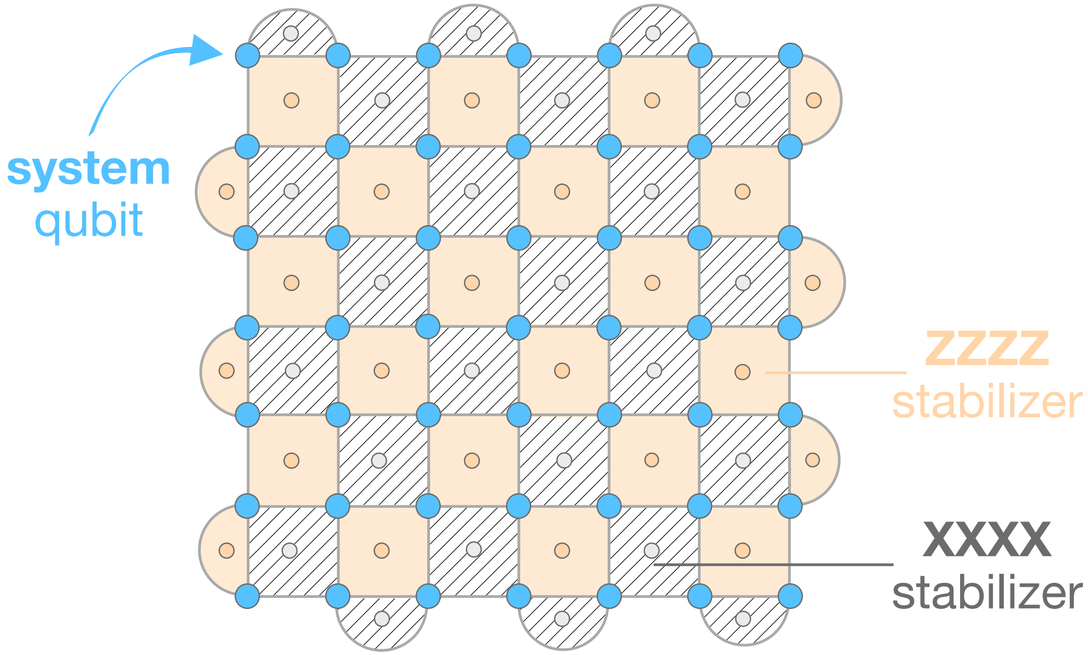
F. Eckstein, B. Han, Q. Wang, R. Vasseur, A. Ludwig, G-Y. Zhu



entanglement by measurement: **surface code**



Kitaev (1997)



d=7 surface code

The toric/surface code
is an ingenious
measurement protocol.

The toric/surface code
is an exactly solvable
Hamiltonian model.

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phase transitions & deformations

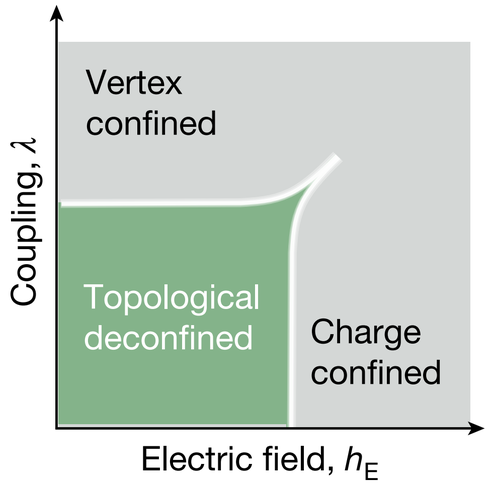


3D Ising*
universality

Castelnovo, ST, Troyer (2010)

Hamiltonian deformation $\mathcal{H}$

toric code



product state
(paramagnet)

Article

Visualizing dynamics of charges and strings in (2+1)D lattice gauge theories

Tyler A. Cochran *et al.*, Nature **642**, 315 (2025)

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phase transitions & deformations



3D Ising*
universality

Castelnovo, ST, Troyer (2010)

Hamiltonian
deformation $\mathcal{H}$

z = 1

RG flow
RK-Hamiltonians

toric
code

product state
(paramagnet)

z > 2

wavefunction
deformation $|\psi\rangle$

2D Ising
universality

conformal QCP

2D Nishimori
universality

measurement
deformation

Ardonne, Fendley, Fradkin (2004),
Castelnovo & Chamon (2008), Fendley (2008),
Isakov, Fendley, Ludwig, ST, Troyer (2011)

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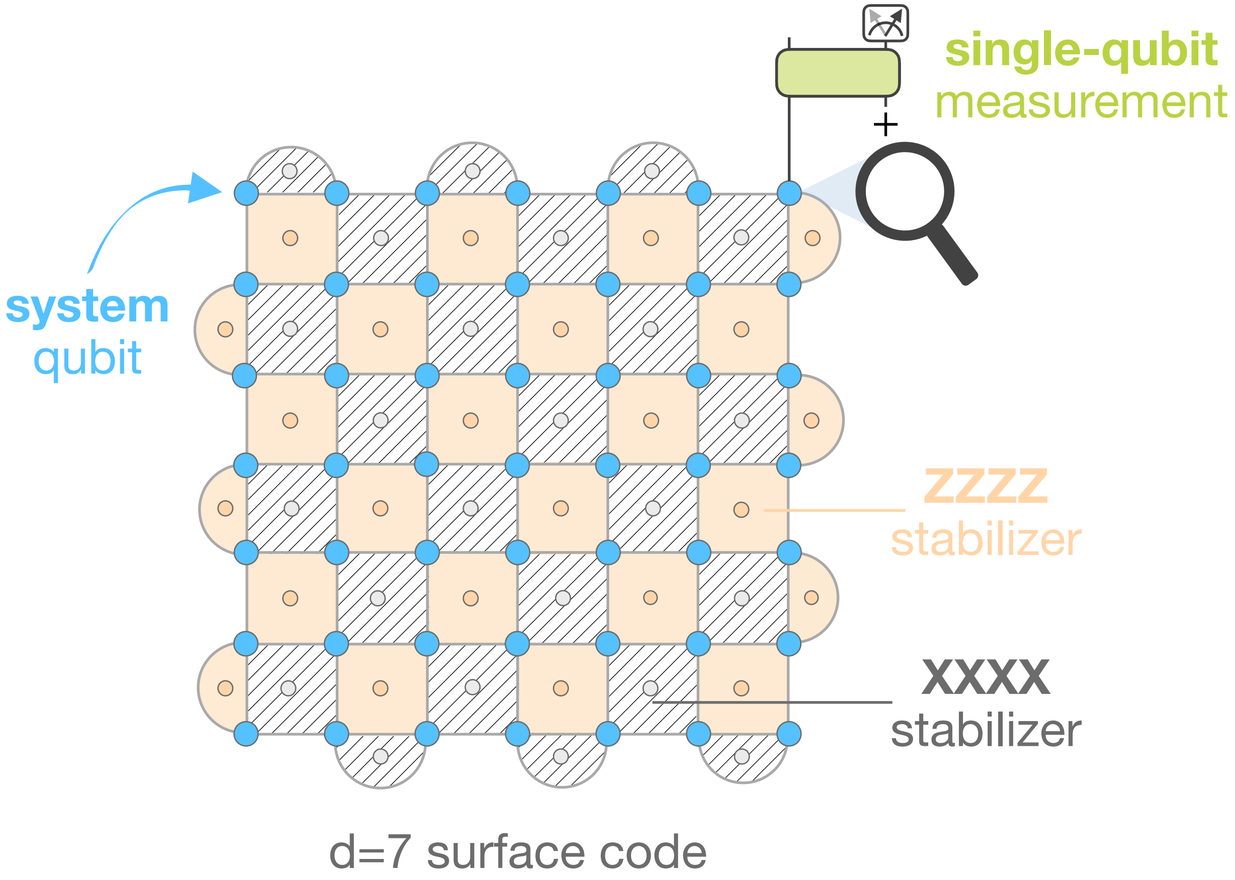
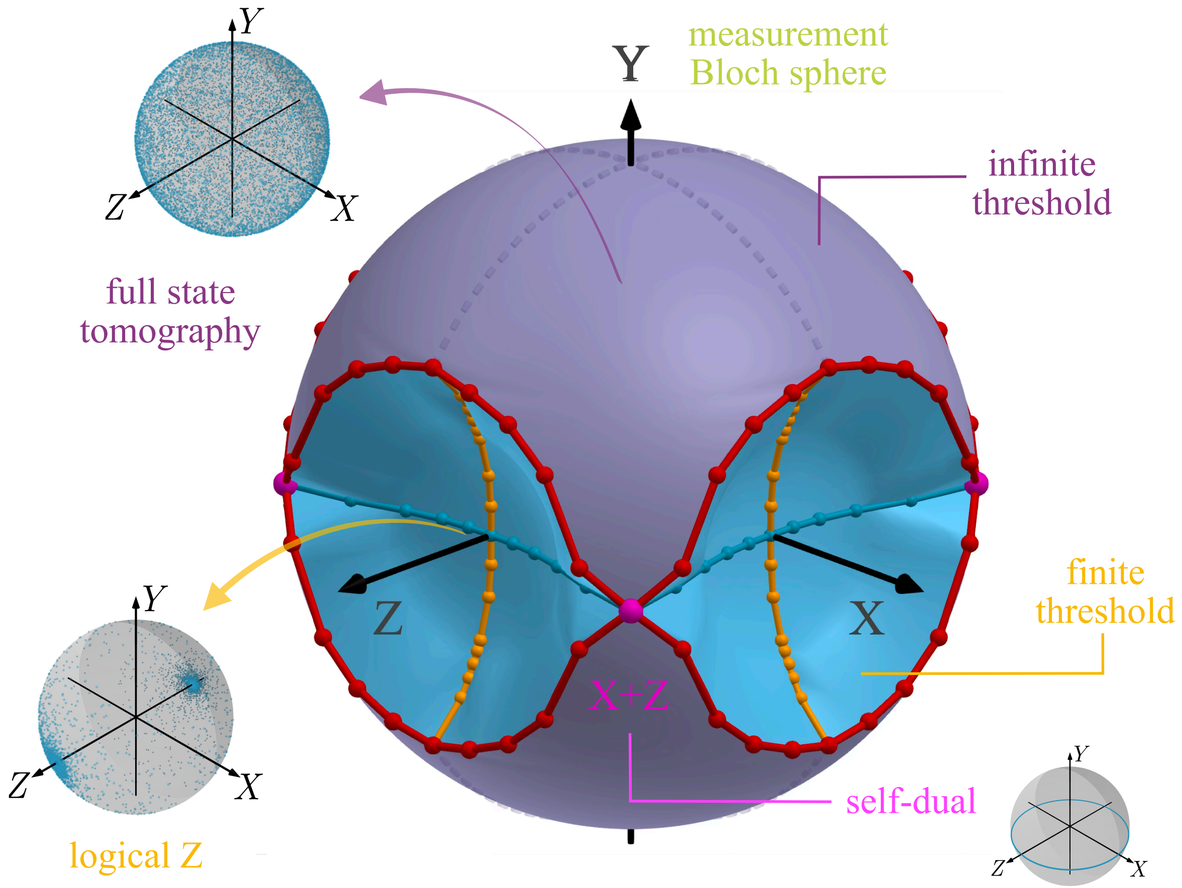
learning by measurement

Infer the logical qubit of a surface code from single-qubits measurements.

Eckstein, Han, **ST** & Zhu, PRX Quantum 2024, arXiv:2512.19786



d=7 surface code



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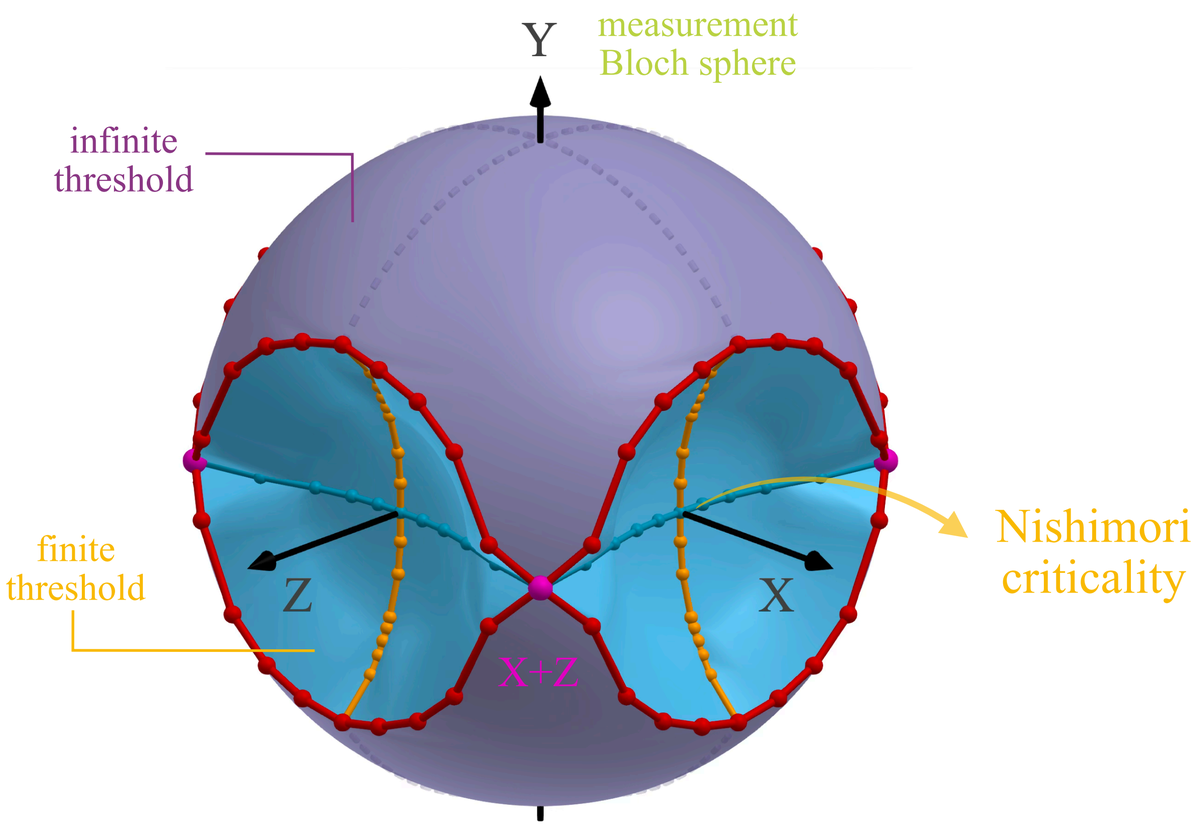
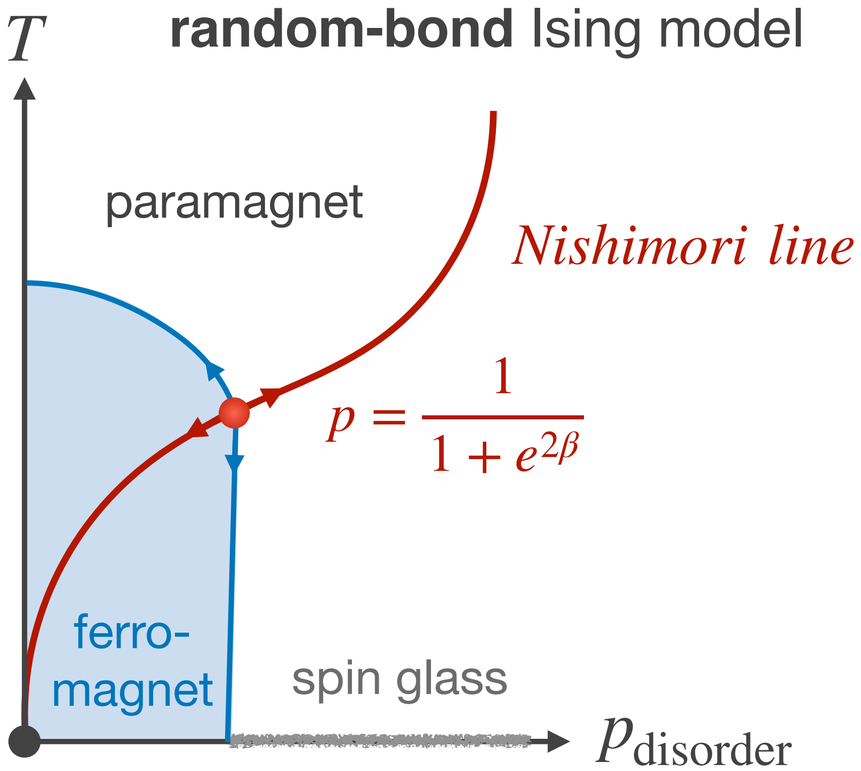
learning by measurement

The learning thresholds exhibit monitored quantum criticality.

Eckstein, Han, **ST** & Zhu, PRX Quantum 2024, arXiv:2512.19786





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learning by measurement

The learning thresholds exhibit monitored quantum criticality.

Eckstein, Han, **ST** & Zhu, PRX Quantum 2024, arXiv:2512.19786

$$|\psi\{s\}\rangle = e^{-\frac{\beta}{2}\sum_{ij} J_{s_{ij}} \sigma_i^z \sigma_j^z} |+\rangle^{\otimes N}$$

Born's rule

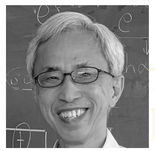
thermal **fluctuations** and **disorder** are **locked**

interpret as **classical stat mech model**

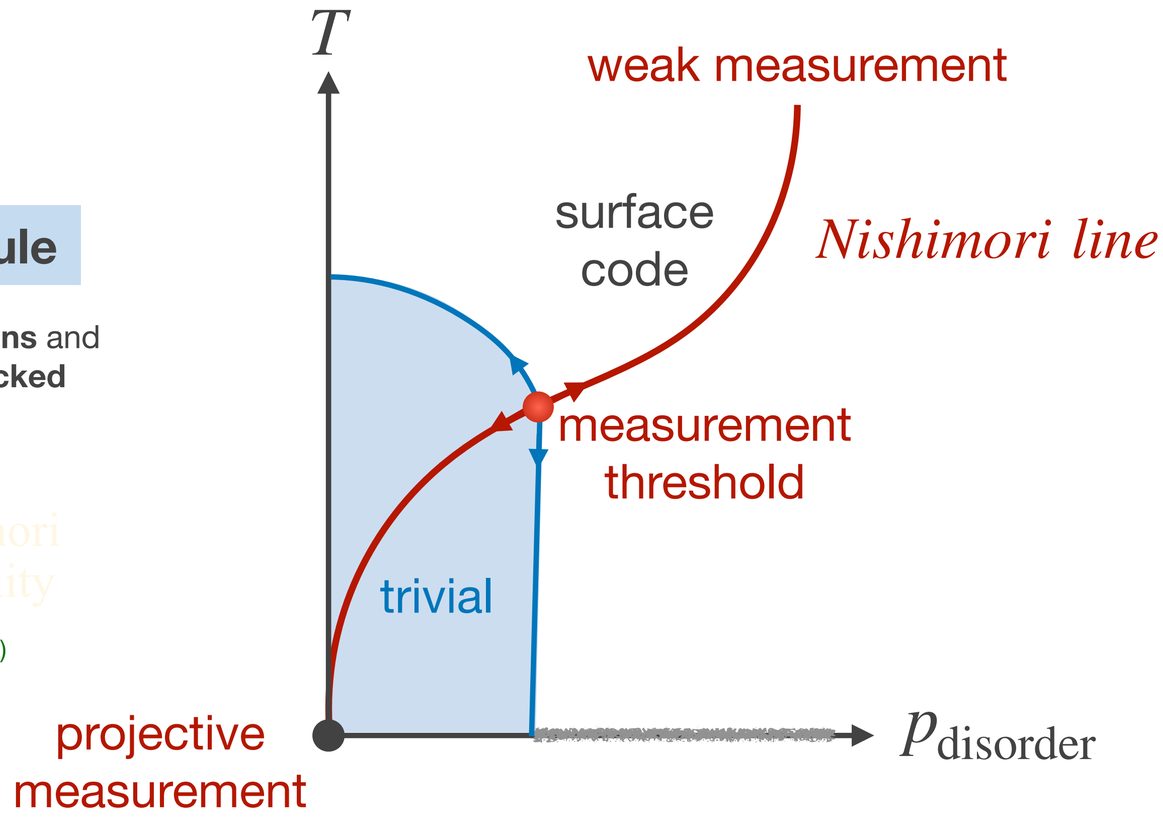
$$\tanh \frac{\beta J_\pm}{2} = \pm \tan t$$

$$Z_{\{s\}} = \sum_{\{\sigma\}} e^{-\beta \sum_{ij} J_{s_{ij}} \sigma_i \sigma_j}$$

random bond Ising model



Nishimori (1981)



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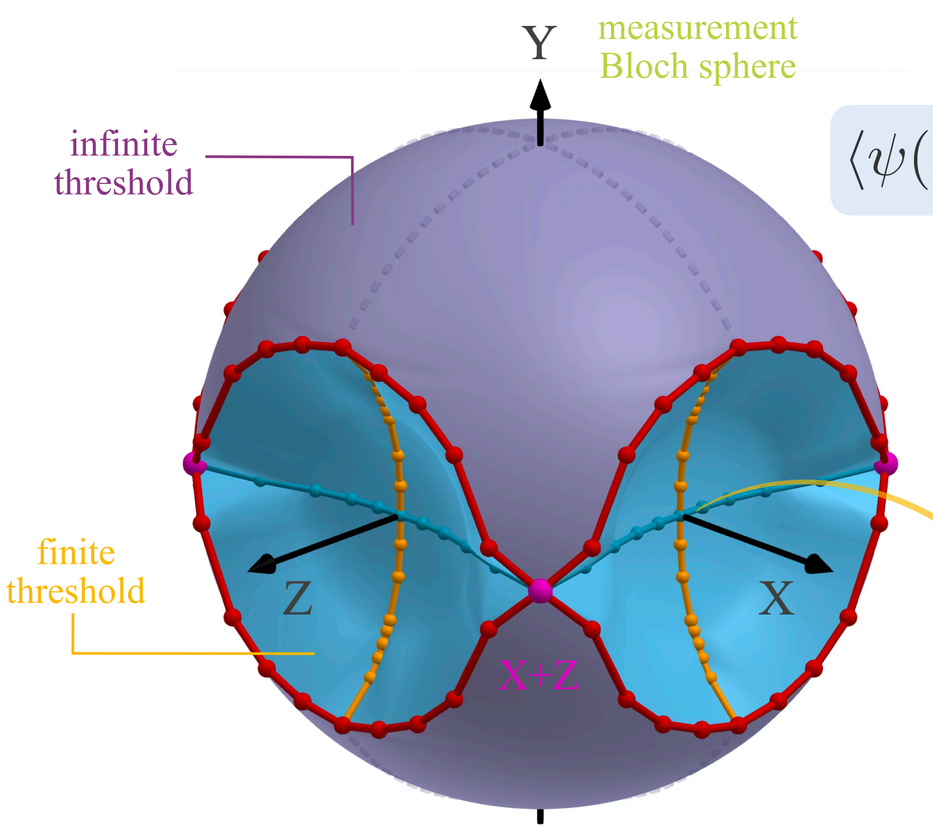
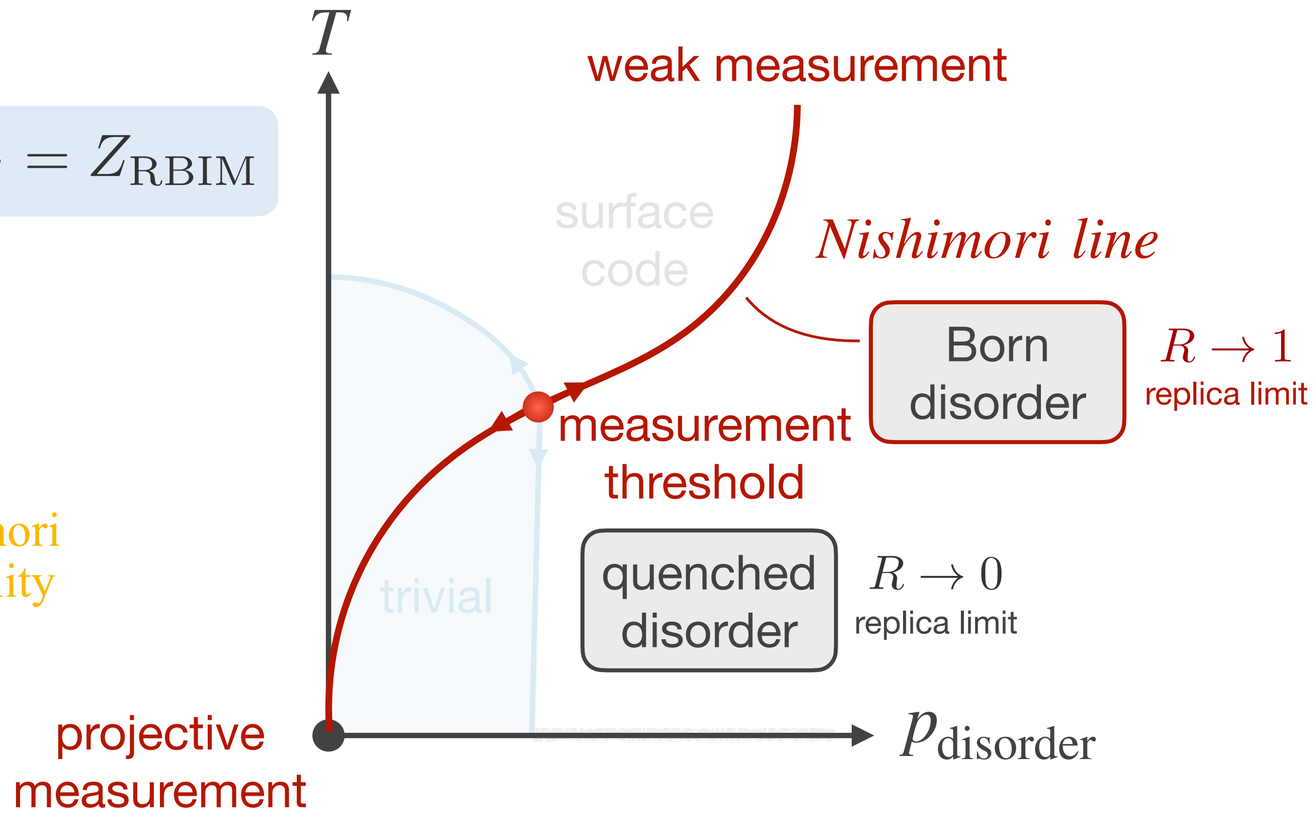
conditional randomness by measurement

The learning thresholds exhibit monitored quantum criticality.

Eckstein, Han, **ST** & Zhu, PRX Quantum 2024, arXiv:2512.19786



$$\langle\psi(s)|\psi(s)\rangle = Z_{\mathrm{RBIM}}$$



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conditional randomness by measurement

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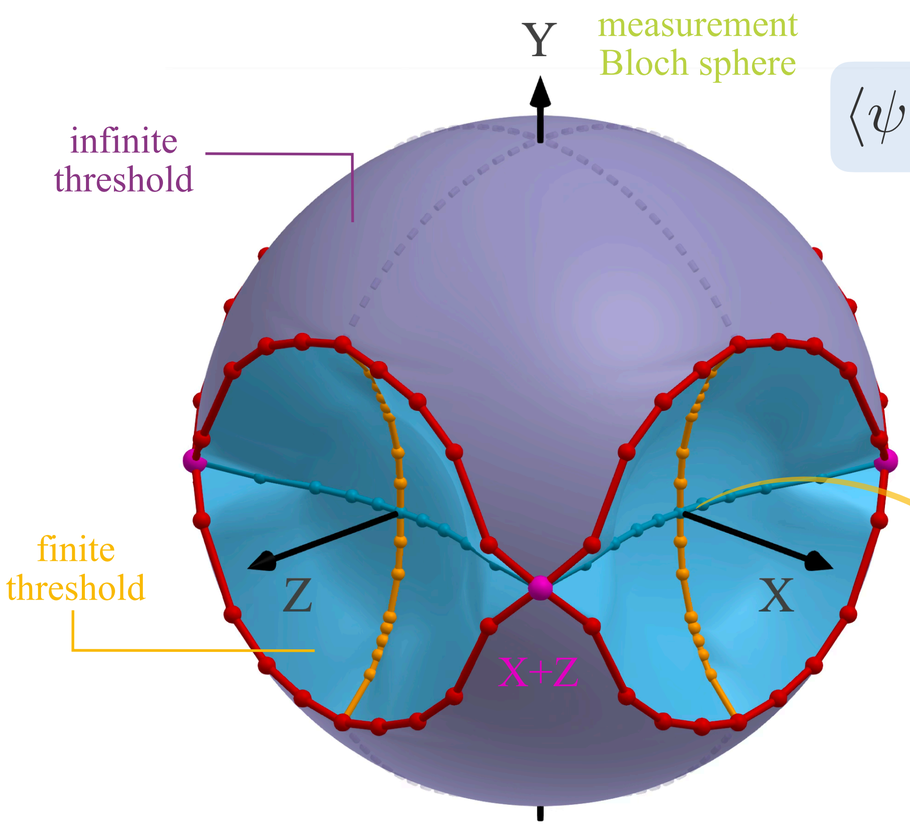


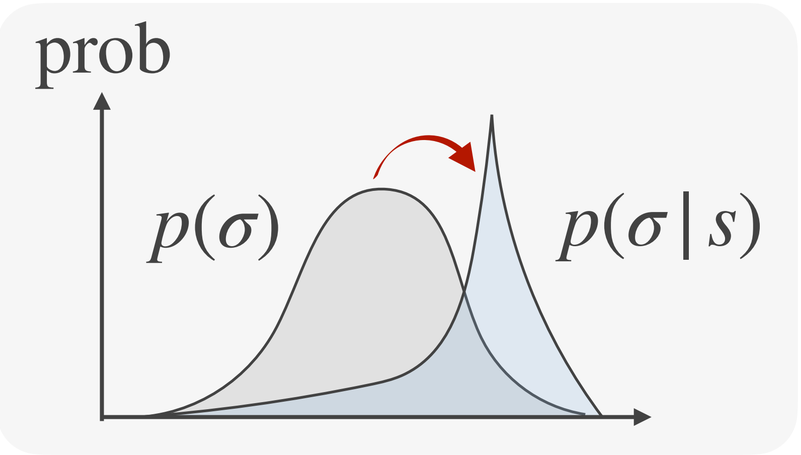
$\langle\psi(s)|\psi(s)\rangle = Z_{\mathrm{RBIM}}$

Born disorder

$R \to 1$
replica limit

Nishimori criticality

conditional randomness



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conditional randomness by measurement

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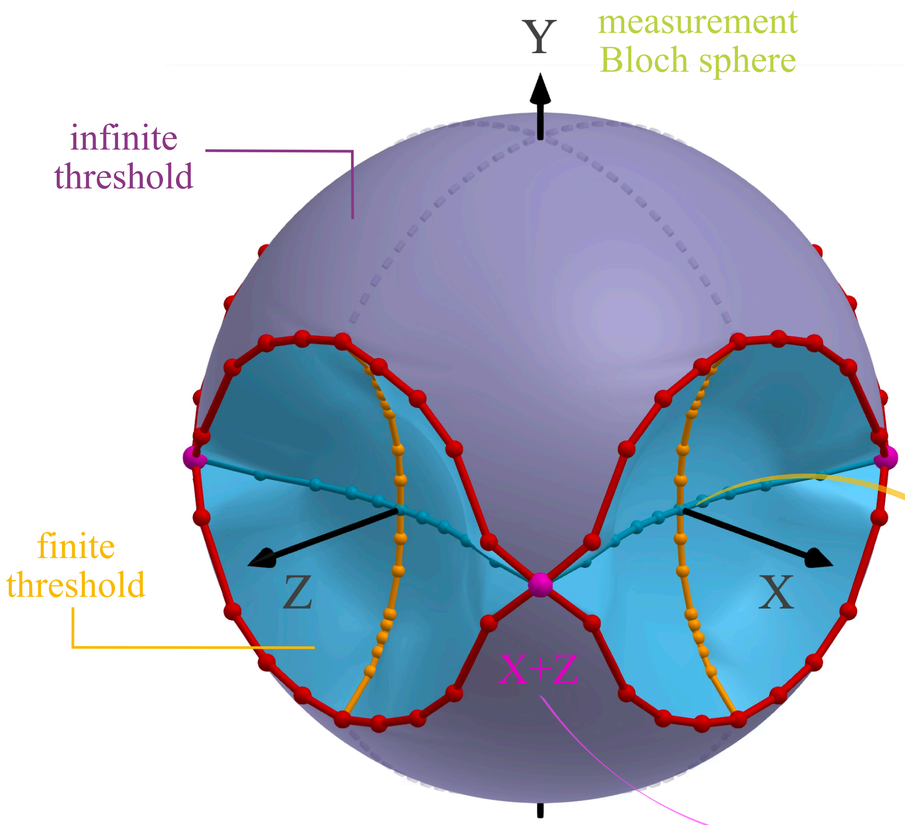
Eckstein, Han, **ST** & Zhu, PRX Quantum 2024, arXiv:2512.19786



Born disorder

$R \to 1$
replica limit

- **conditional** randomness
- **monitored** criticality
- **non-unitary** CFTs
- **sampling** in circuits

Nishimori criticality

Pütz, Vasseur, Ludwig, **ST** & Zhu, PRX Quantum 2025

self-dual criticality

arXiv:2502.14034

A gigantic playground to explore **quantum critical phenomena**.

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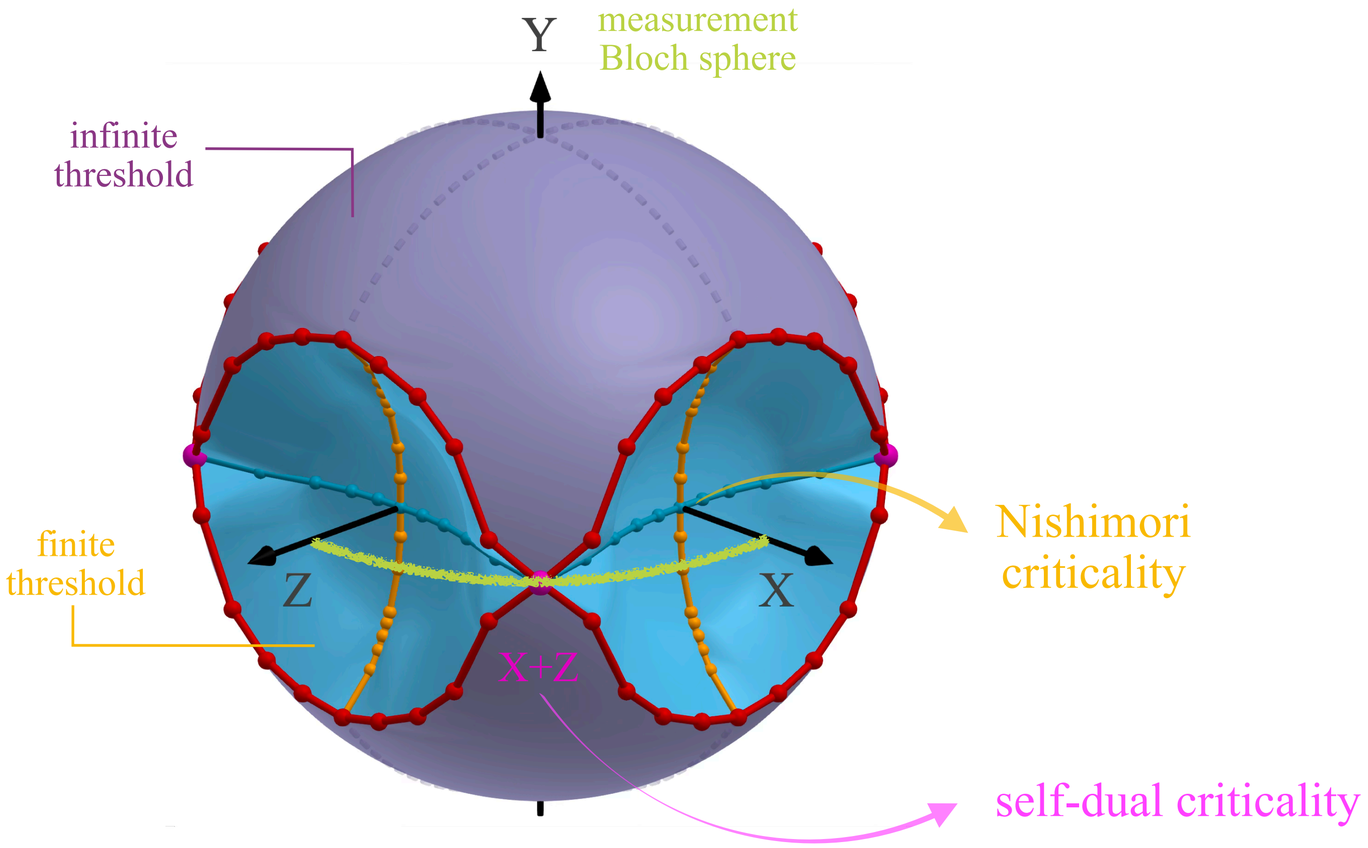
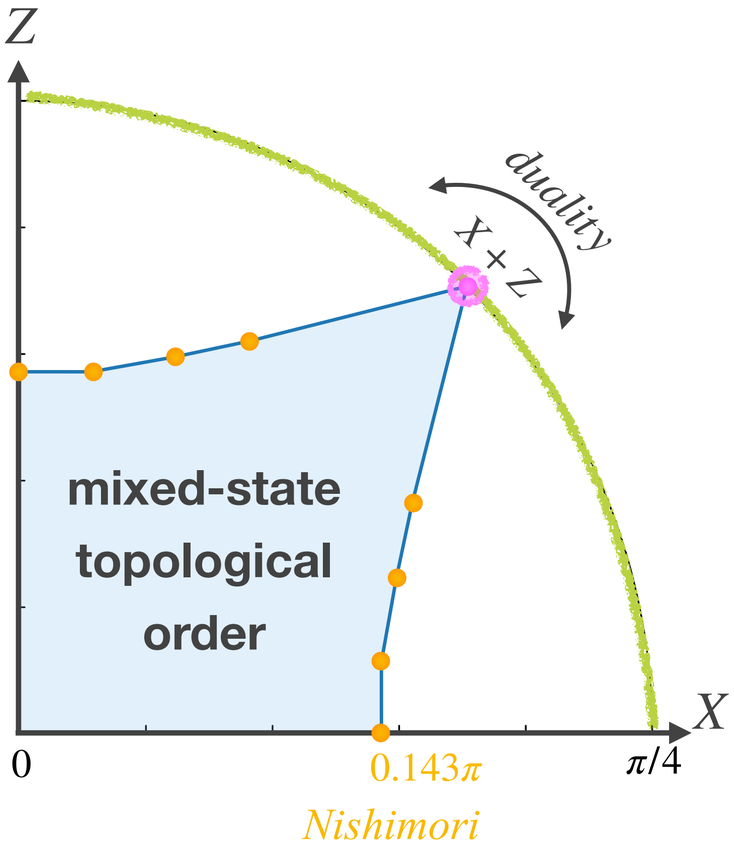
learning by measurement

The learning thresholds exhibit monitored quantum criticality.

Eckstein, Han, **ST** & Zhu, PRX Quantum 2024, arXiv:2512.19786





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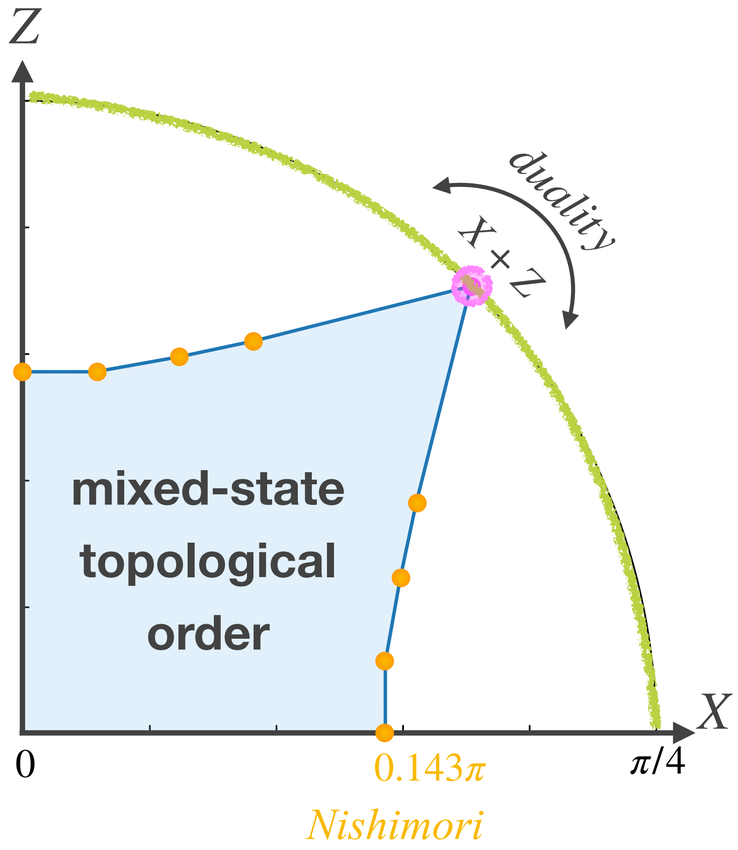
learning by measurement

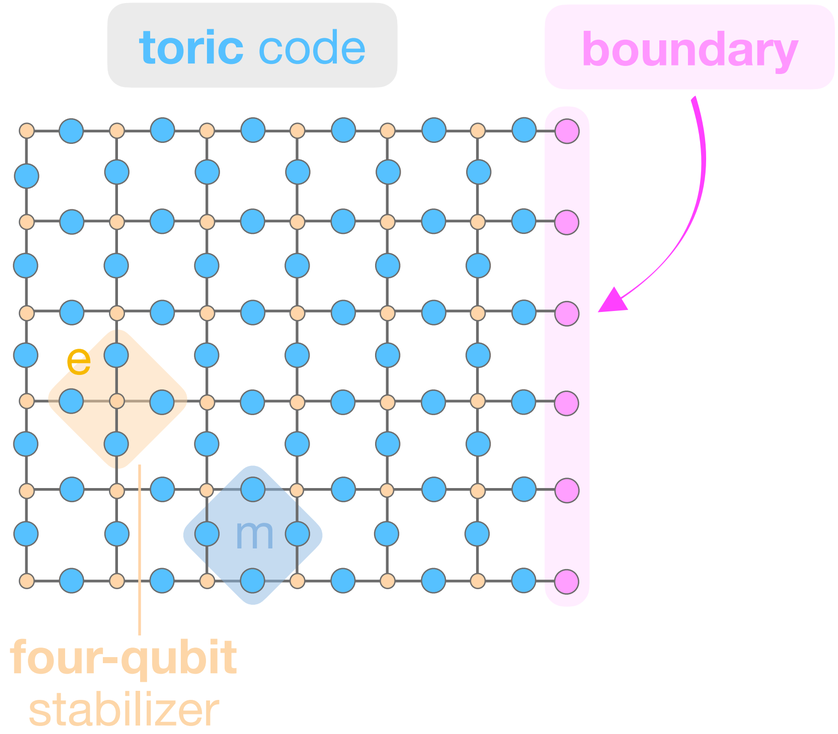
arXiv:2502.14034

$$\Pi_{ij}[1 \pm \tanh \frac{\beta}{2} (\cos\theta\, Z_{ij} + \sin\theta\, X_{ij})]|\psi\rangle$$

measurement **angle**



Why is there a **critical point** when all the bulk qubits **collapse**?



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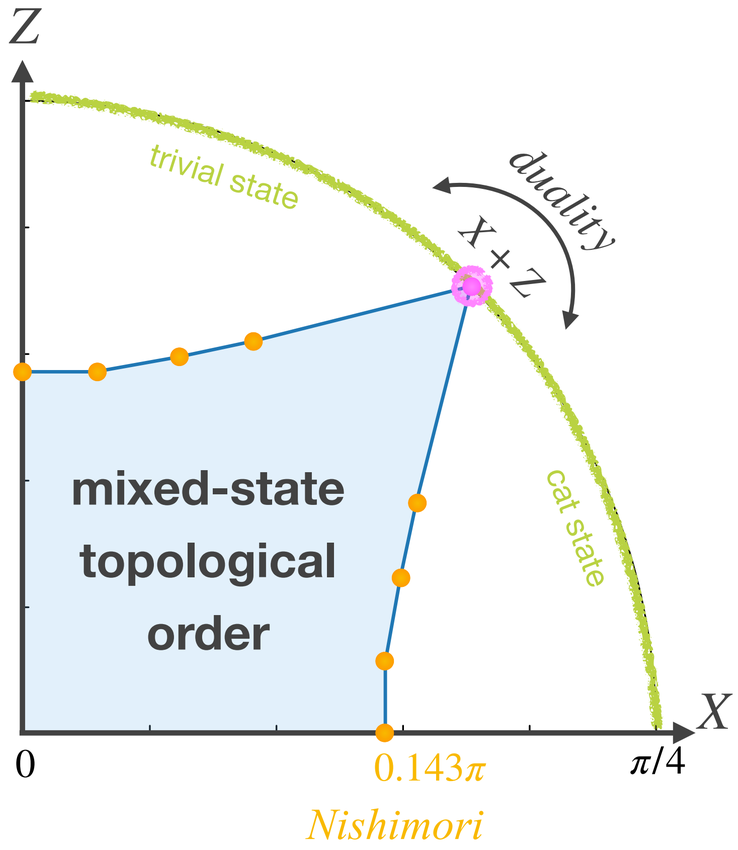
bulk-boundary correspondence

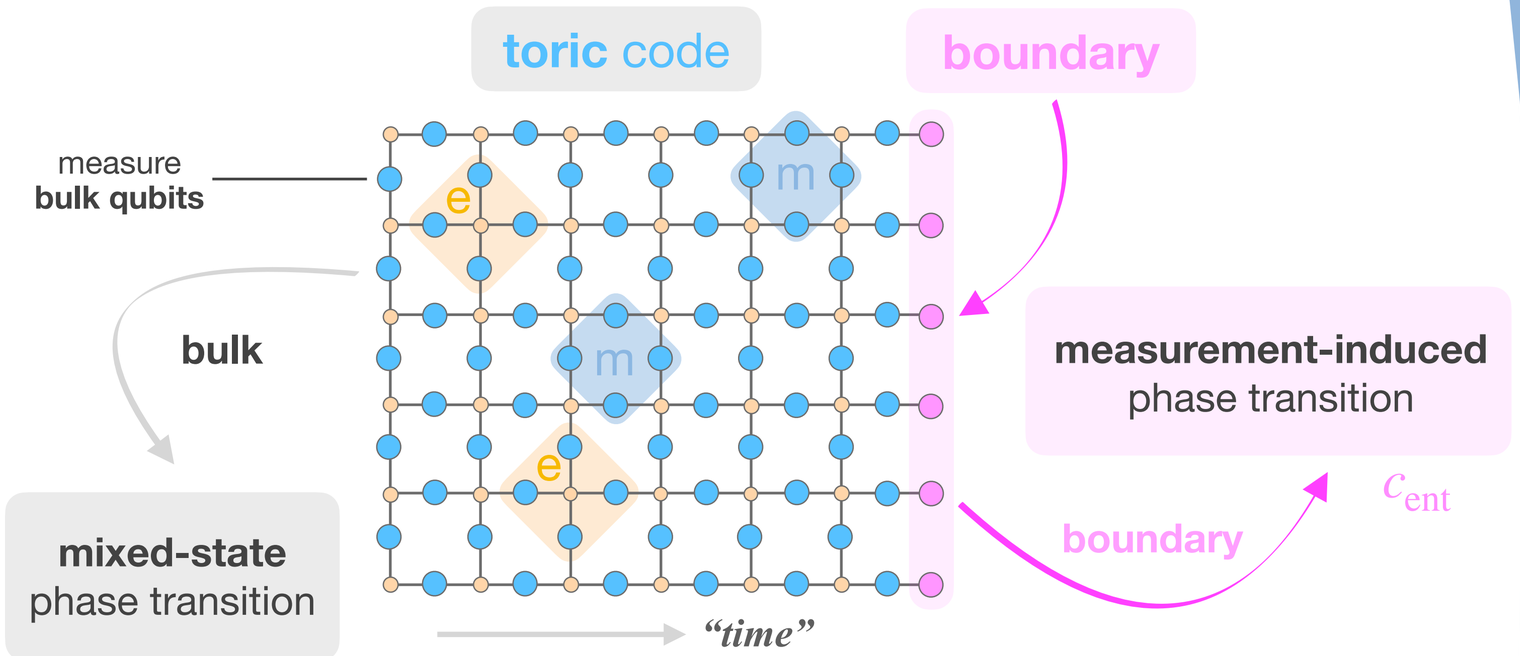
arXiv:2502.14034

$$\Pi_{ij}[1 \pm \tanh \frac{\beta}{2} (\cos\theta\, Z_{ij} + \sin\theta\, X_{ij})]|\psi\rangle$$

measurement **angle**

Why is there a **critical point** when all the bulk qubits **collapse**?





classical quantum

$$\rho = \sum_{em} |em\rangle\langle em|_C \otimes |\psi(em)\rangle\langle\psi(em)|_Q$$

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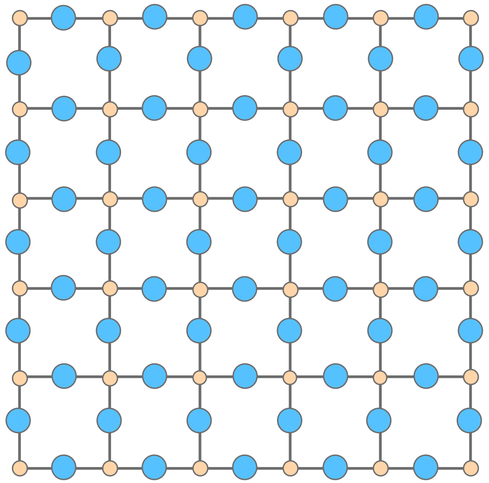
(1+1)d representations

arXiv:2502.14034

toric code

(2+0)d representation



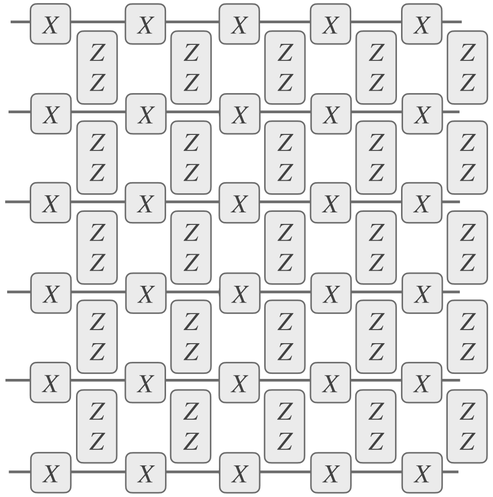
"time"

self-dual

$X + Z$ measurement

transverse field Ising

(1+1)d representation



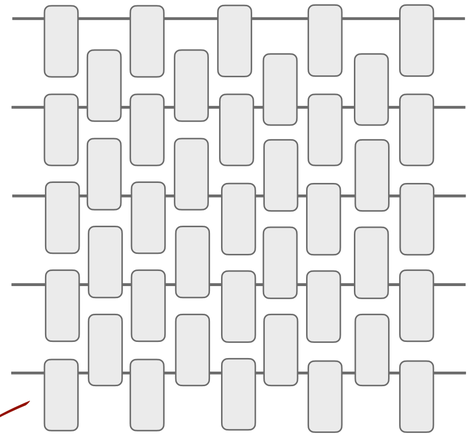
time

equal-weight

X and ZZ checks

monitored Majoranas

(1+1)d representation



time

"parity" check

$\sigma \times \sigma \to 1 + \psi$

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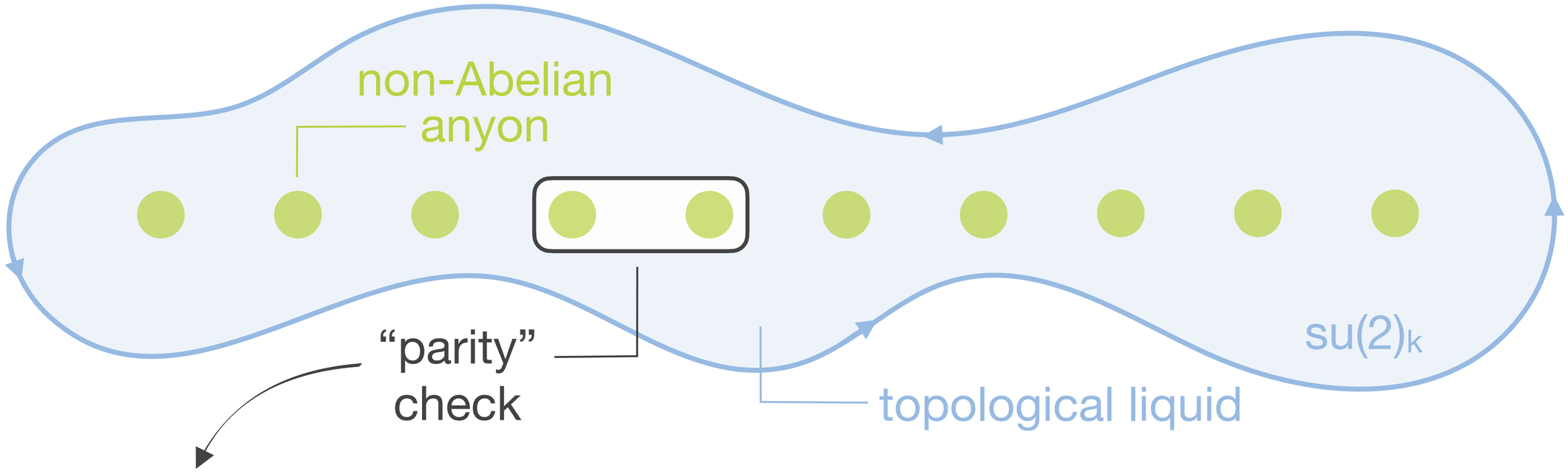
The monitored golden chain

arXiv:2610.18965
PRL **98**, 160409 (2007)

Z. Zhi, B. Han,R. Vasseur, A. Ludwig, G-Y. Zhu



monitored topology



Fibonacci anyons

$\tau \times \tau \to 1 + \tau$

Hamiltonian dynamics

“golden chain”

CFT minimal models

$c = 1/2, 7/10, 4/5, \ldots$ ✔

quenched disorder

infinite randomness fixed points ✔

measurement randomness ?

family of CFTs w/ non-invertible symmetry

Feiguin, **ST**, Ludwig, Troyer, Kitaev, Wang, Freedman (2007), Yang & Bonesteel (2007)

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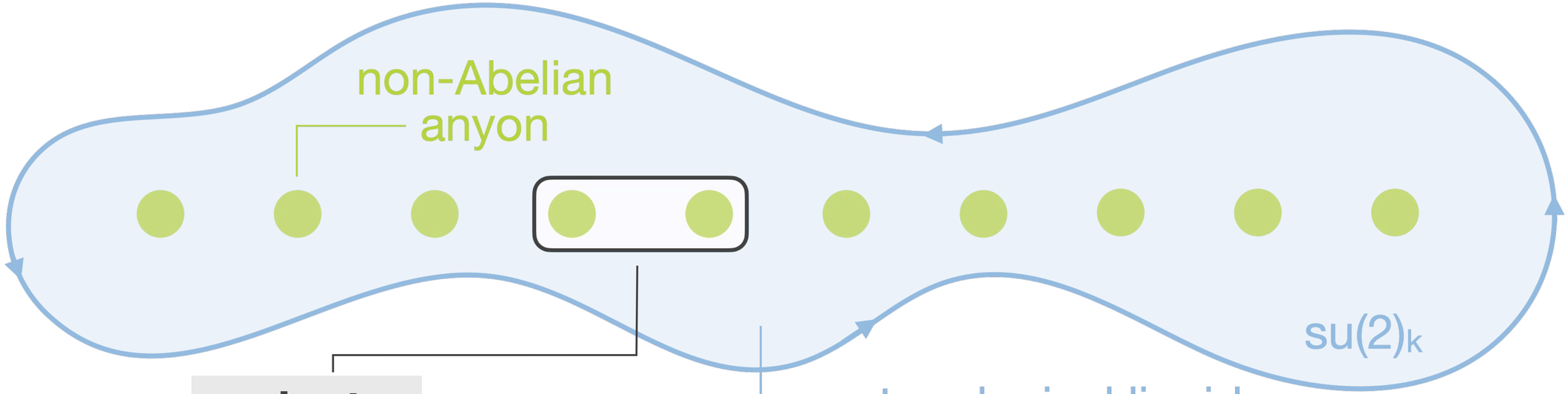
The golden chain

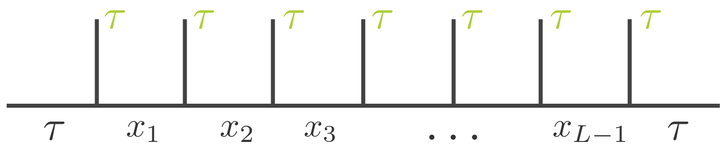
Phys. Rev. Lett. **98**, 160409 (2007)



projector
onto fusion channel

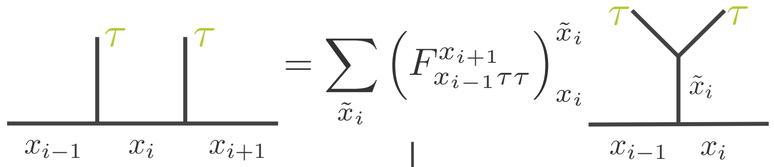
Fibonacci anyons

$\tau \times \tau \to \mathbb{1} + \tau$



Hilbert space

fusion basis



Hamiltonian

F-matrix basis change

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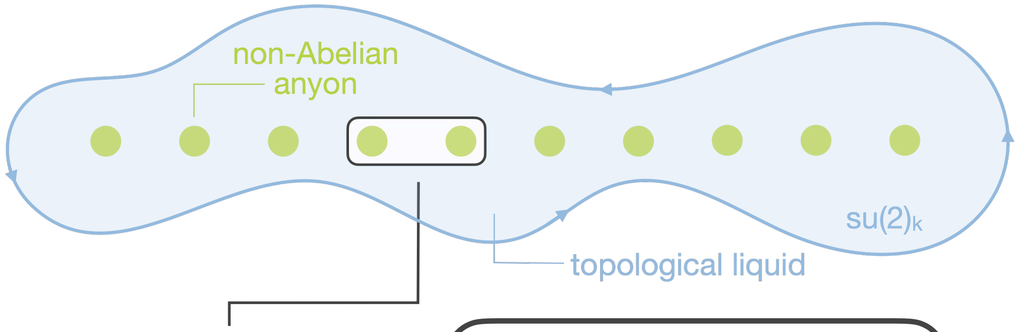
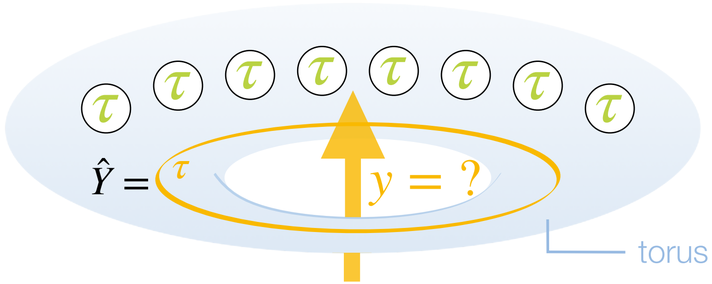
The golden chain



projector
onto fusion channel

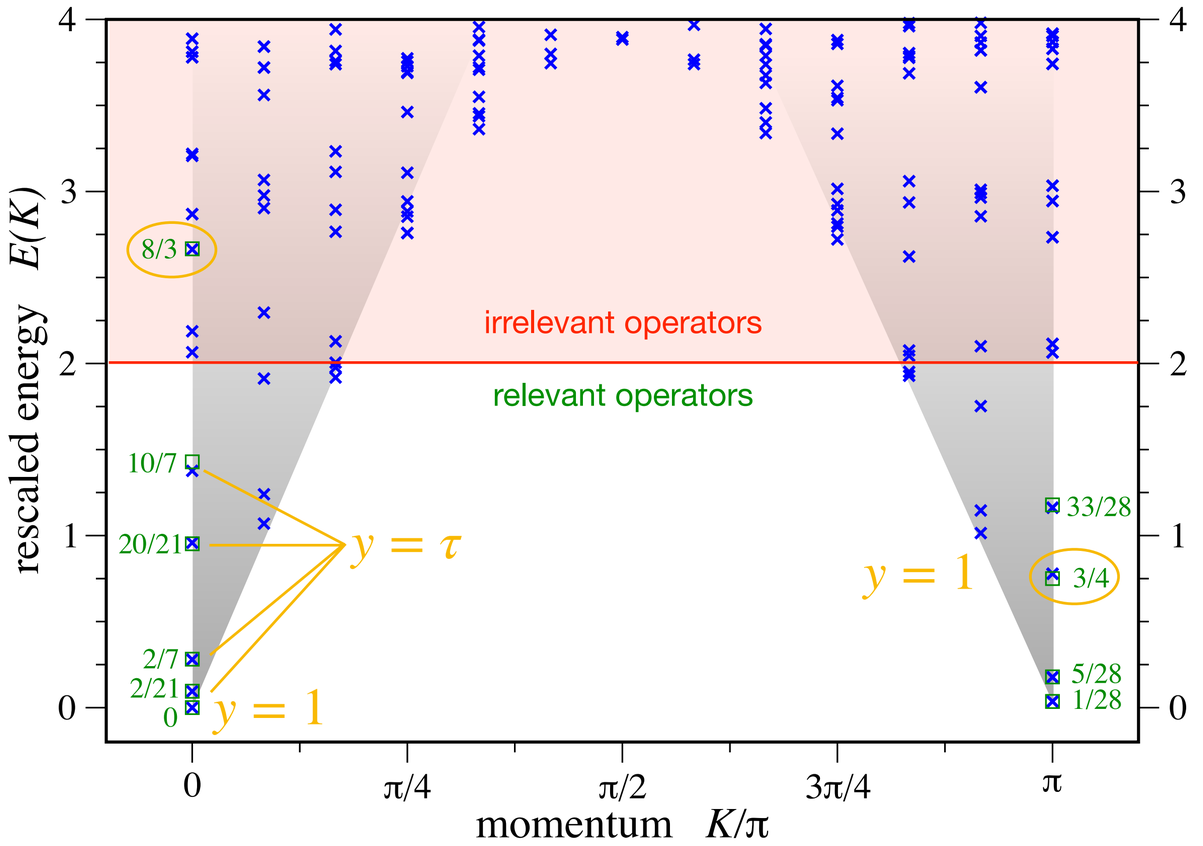
Fibonacci anyons

$$\tau \times \tau \to 1 + \tau$$



Topological symmetry
protects criticality.

Phys. Rev. Lett. **98**, 160409 (2007)



$$E = E_1 L + \frac{2\pi v}{L}\left(-\frac{c}{12} + h_L + h_R\right)$$

conformal tower

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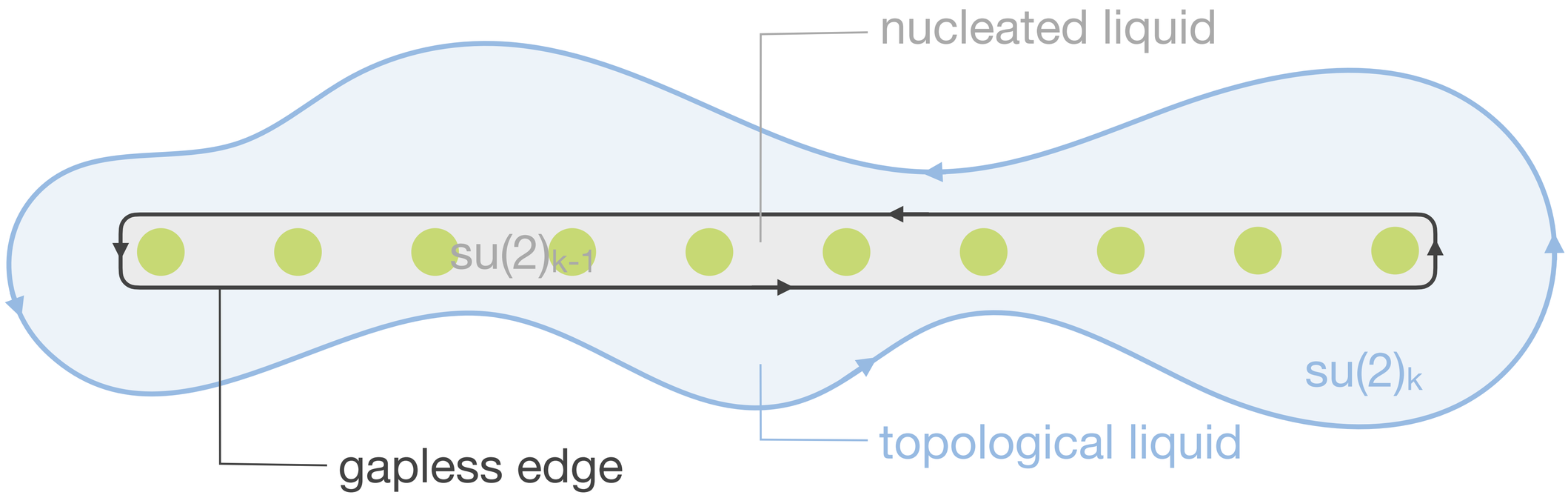
The golden chain

Phys. Rev. Lett. **101**, 050401 (2008)



$$\frac{su(2)_{k-1} \times u(1)}{su(2)_k}$$

Topological symmetry is an early example of a **generalized, non-invertible symmetry**.

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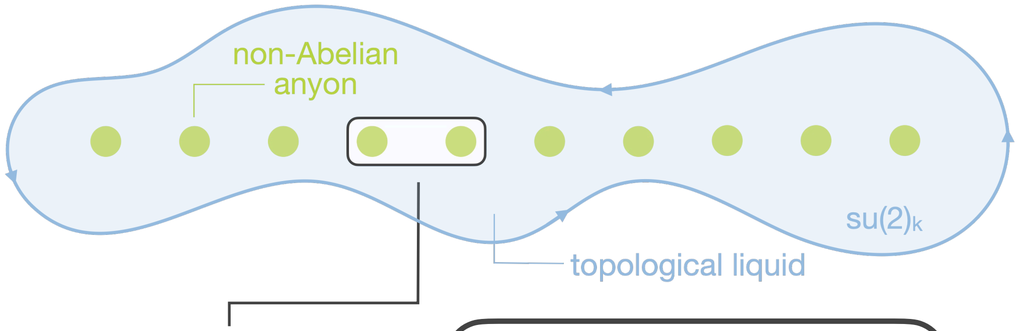
The **monitored** golden chain

arXiv:2610.18965

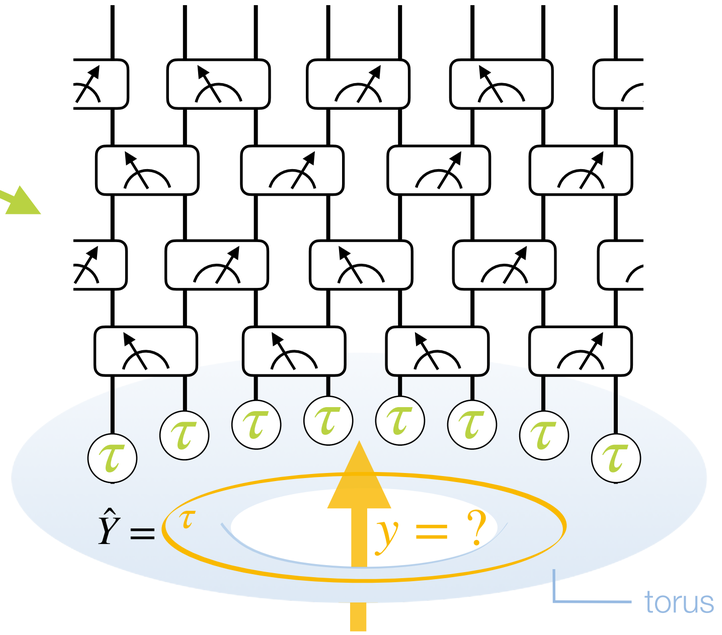


monitor
fusion channel

Fibonacci anyons

$$\tau \times \tau \to 1 + \tau$$

brick-wall circuit



- “simplest” non-Abelian anyons
- quantum dimension $\phi = (1+\sqrt{5})/2$
- non-trivial fusion outcome

- trajectories according to Born probabilities
- topological symmetry is preserved

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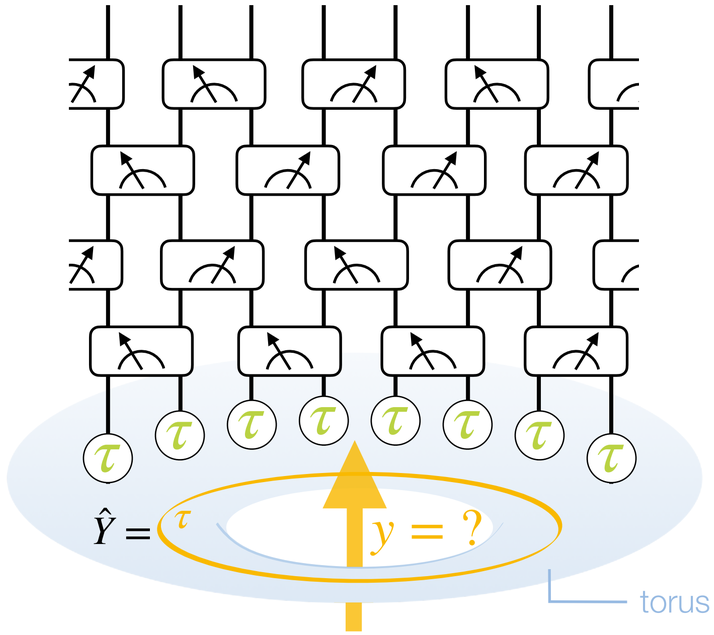
The monitored golden chain

arXiv:2610.18965

$|\psi(\mathbf{s})\rangle$

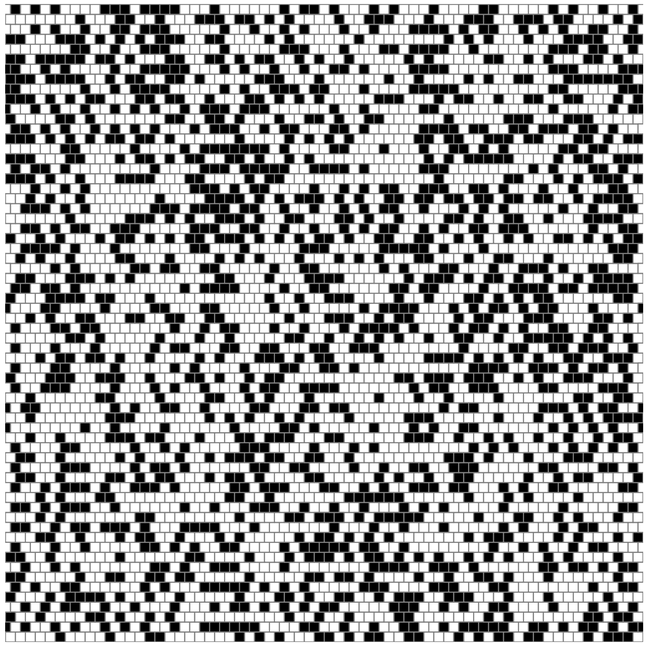


quantum system



measure

$\mathbf{s}$



time

classical observer

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The monitored golden chain

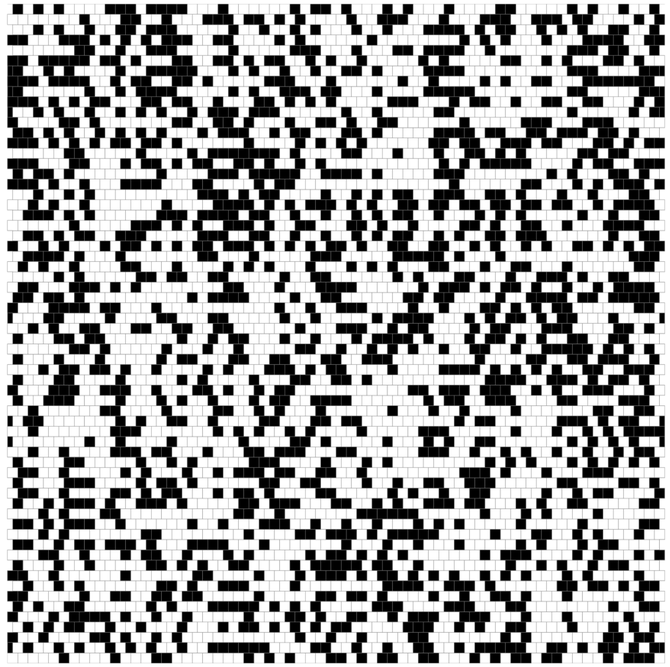
arXiv:2610.18965

Can you **learn the** **topological charge** from the measurement record?

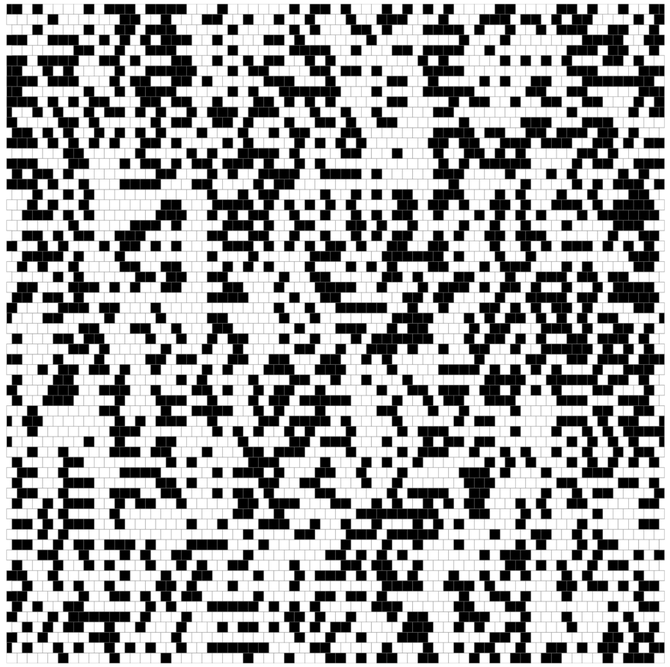
global

local

$P(\tau) = 1/\phi$ $P(1) = 1/\phi^2$



$y = 1$



$y = \tau$



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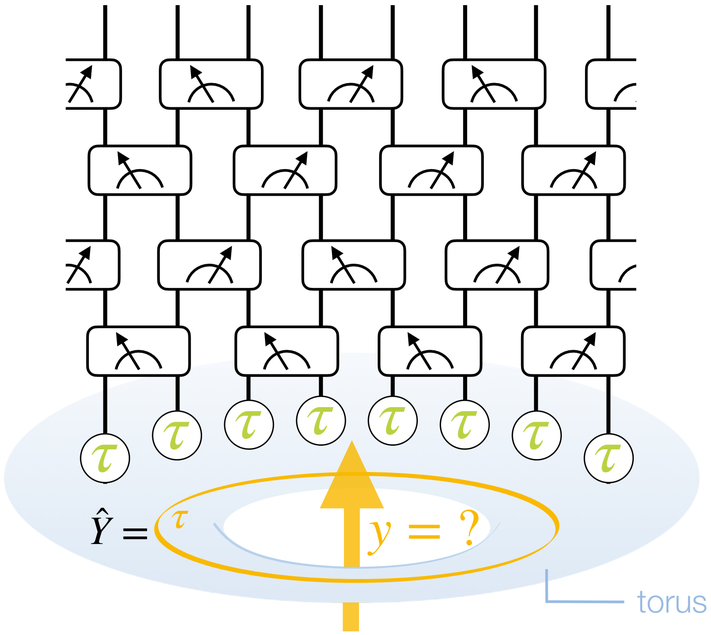
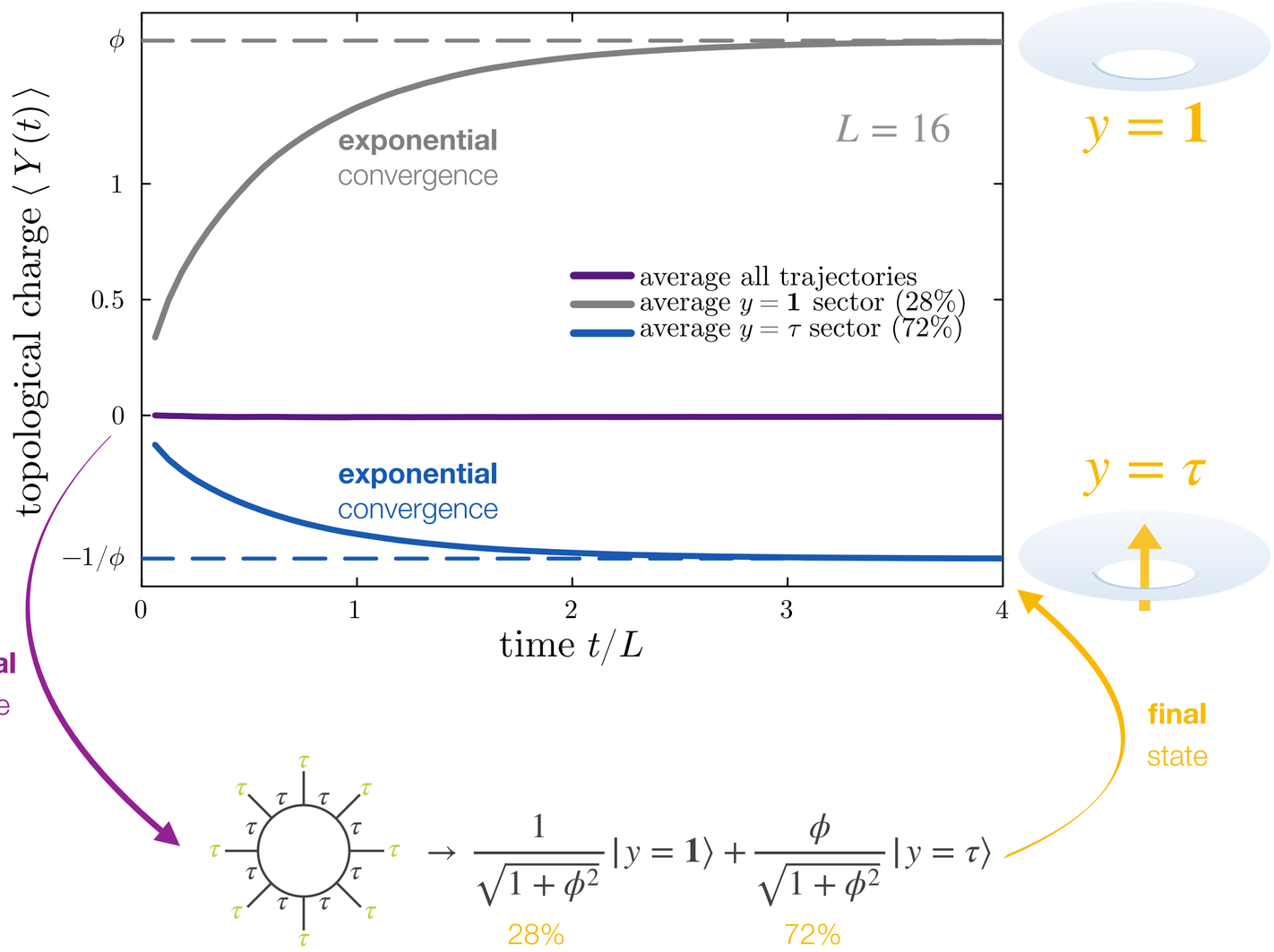
learning the topological charge





A fusion basis state has fluctuating topological symmetry charge $\langle \hat{Y} \rangle = 0$

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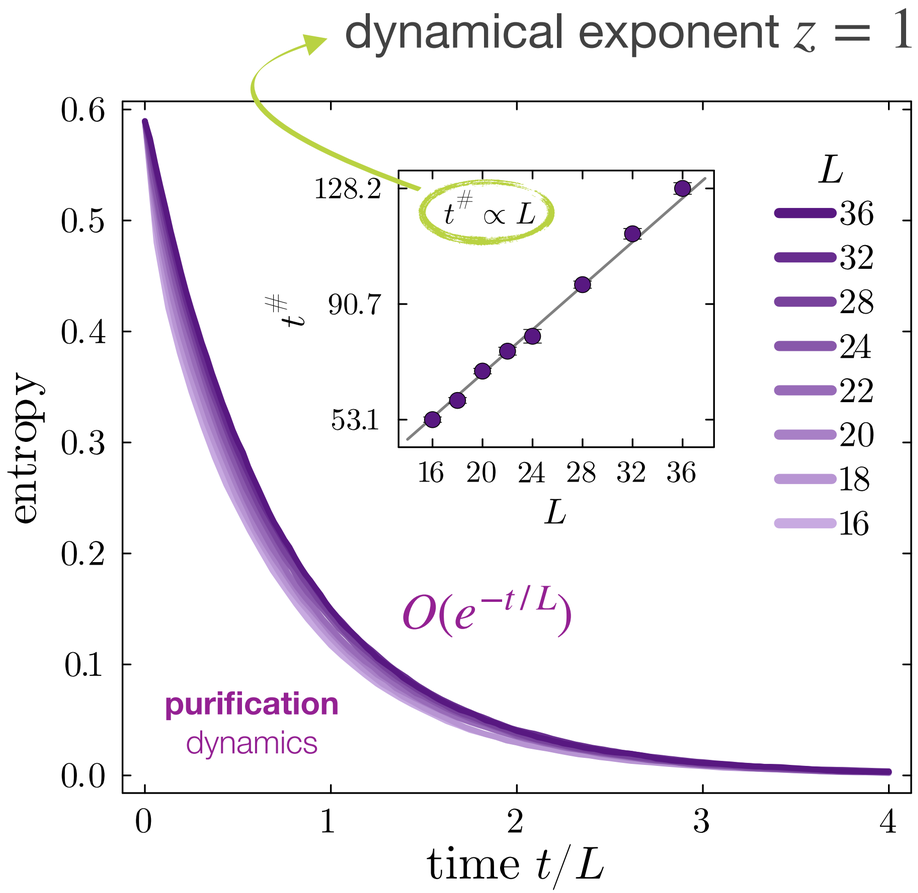
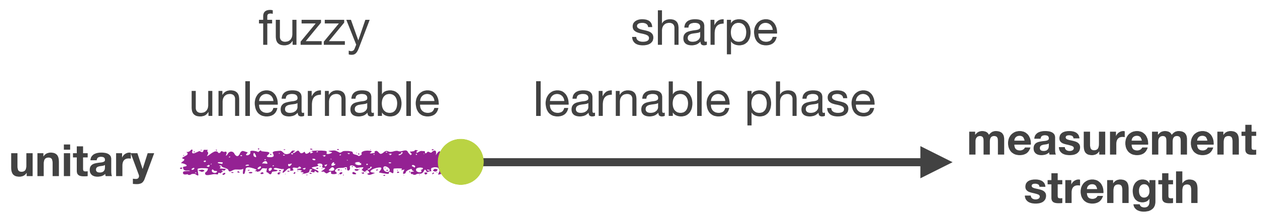
learning the topological charge

arXiv:2610.18965

charge **sharpening**





- $U(1)$ **charge** $\quad t_{\#} \propto O(1)$ non-critical
- Non-Abelian τ **charge** $\quad t_{\#} \propto L$ $z = 1$ conformal critical
- $SU(2)$ **spin** $\quad t_{\#} \propto L^2$ $z = 2$ Lifshitz-like critical

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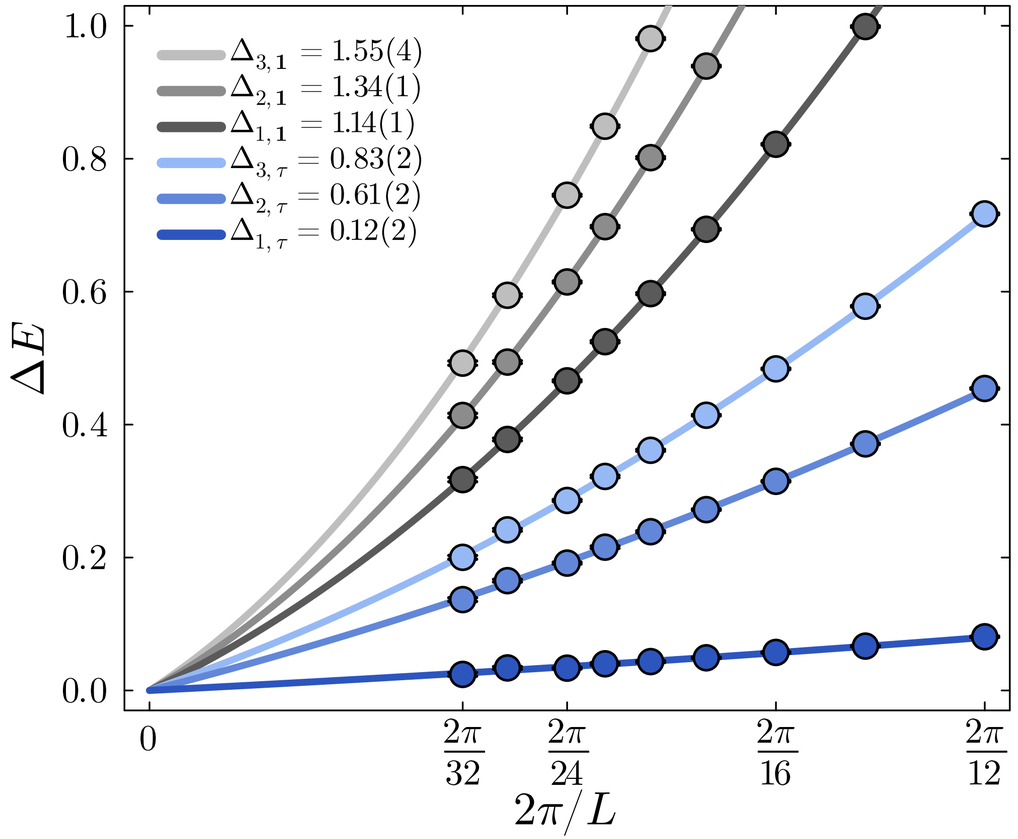
monitored criticality

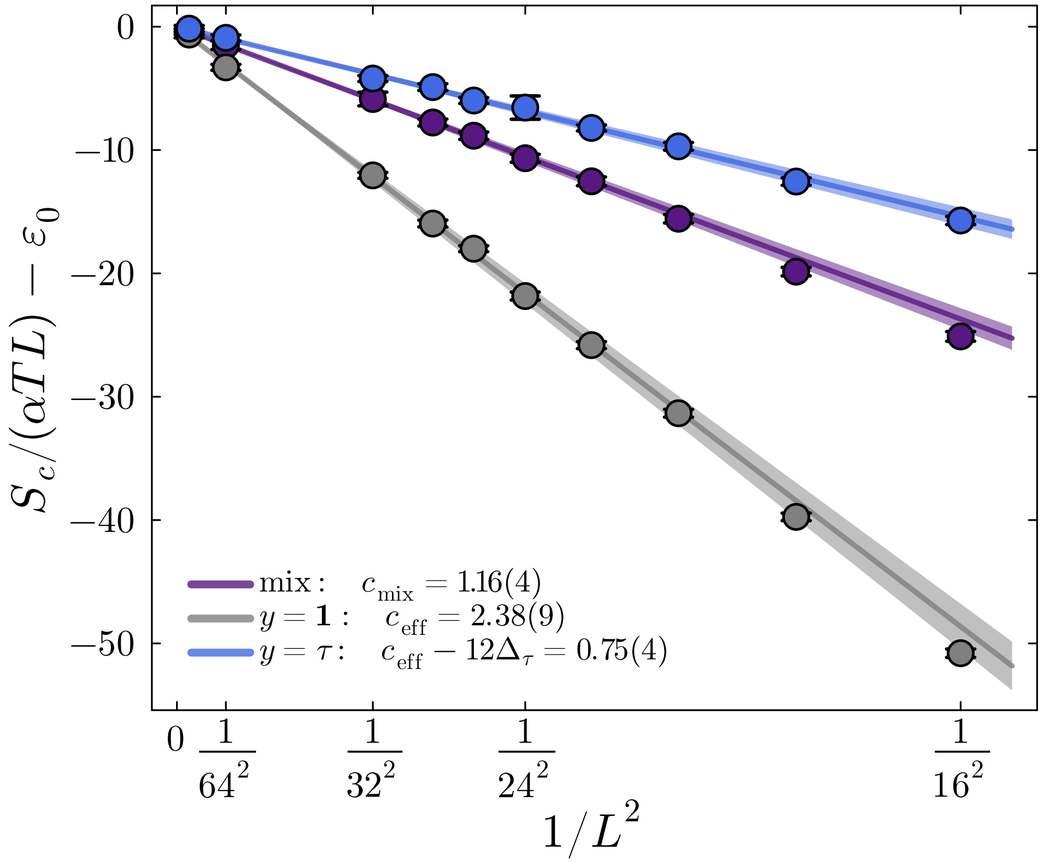
arXiv:2610.18965

Lyapunov spectrum



$$E_n = \frac{2\pi}{L}\left(\Delta_n - \frac{c_{\text{eff}}}{12}\right) + \dots$$

Shannon entropy scaling



$$\frac{S_c}{\alpha T} \equiv E_0 = \epsilon_0 L - \frac{\pi c_{\text{eff}}}{6L} + \dots$$

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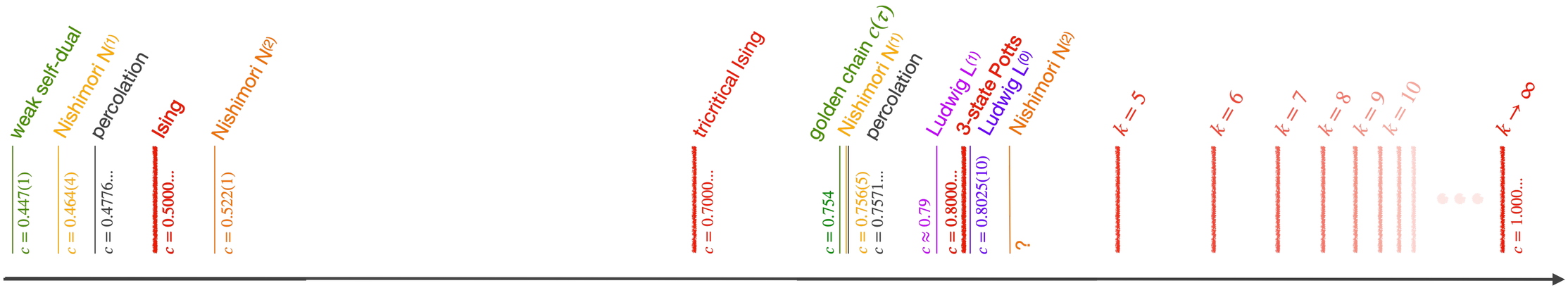
monitored criticality

The **golden chain** and its su(2)$_k$ generalizations are described by the sequence of **minimal models**.

$$c = 1 - \frac{6}{(k+1)(k+2)} = \frac{1}{2}, \frac{7}{10}, \frac{4}{5}, \frac{6}{7}, \ldots, 1$$



Ising fine structure

$\mathbb{Z}_2$ toric code

tricritical **Ising**

3-state **Potts** fine structure

$\mathbb{Z}_3$ toric code

4-state **Potts**

arXiv:2608.2026

The **monitored golden chain** and its su(2)$_k$ generalizations are described by a family of **non-unitary CFTs**.

Why this ordering matters? The Casimir central charge obeys a **"c-effective" theorem under RG flows** (akin to the c-theorem for unitary CFTs).

R. Patil & A.W.W. Ludwig arXiv:2507.07959

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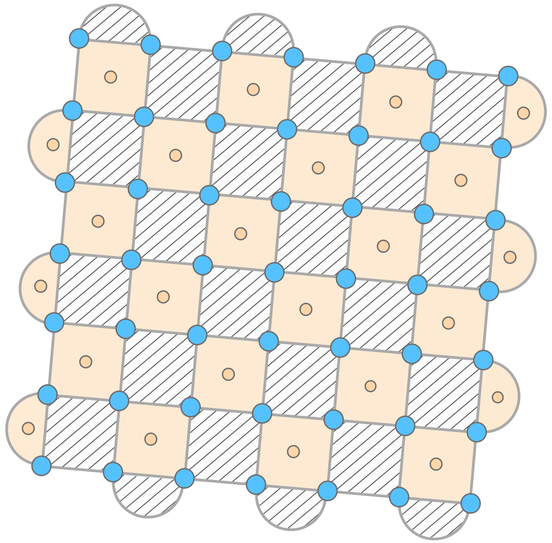
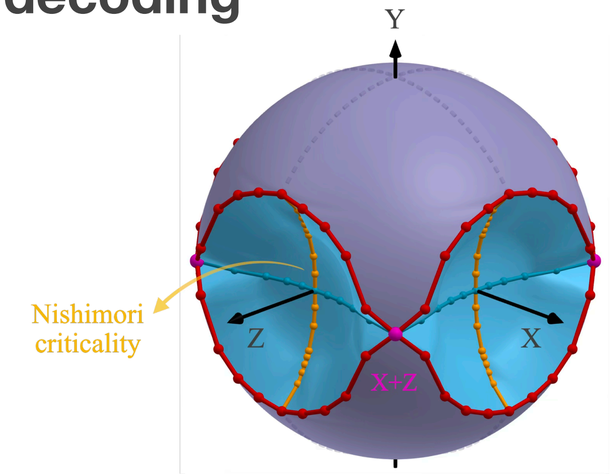
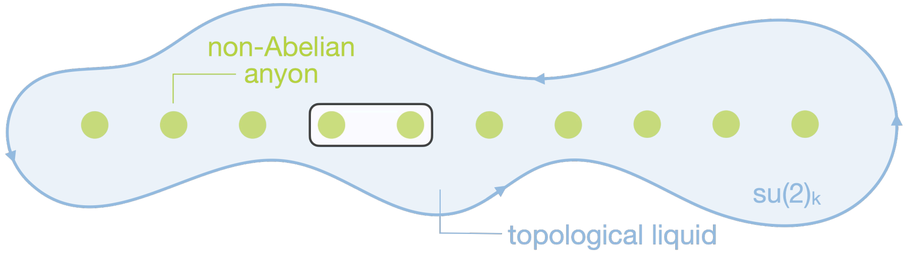
closing remarks

summary

- **measurements** are a powerful tool to create entangled states
  - **shallow** circuit constructions for cat & toric code states
  - trade-off in form of **auxiliary qubits** and **decoding**
- **monitoring** topology
  - **stability** of protocols & thresholds
  - **criticality** & **Nishimori** physics
- **golden** chain
  - **non-Abelian learning** problem
  - **generalized symmetry** & **non-unitary** CFT







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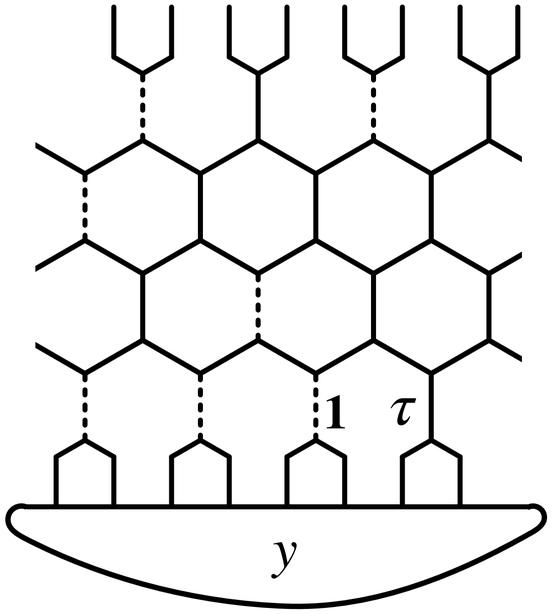
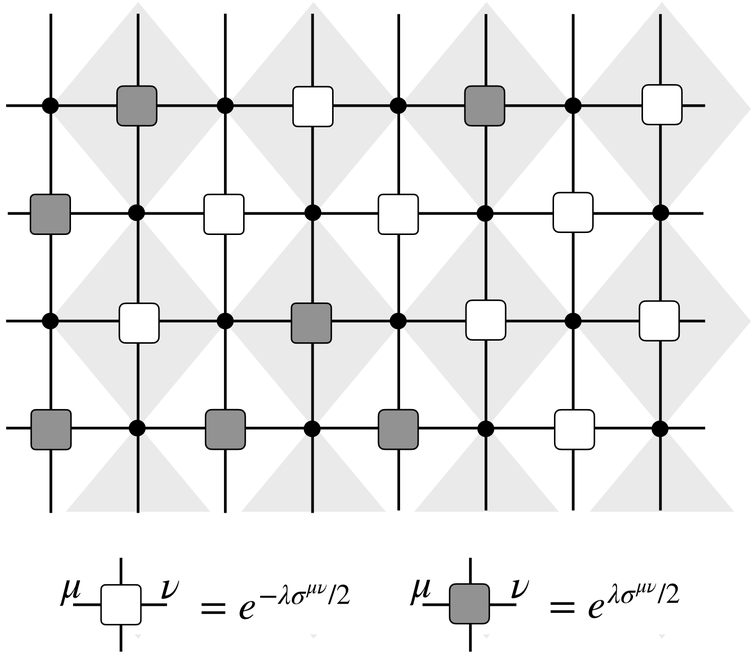
Thanks!

golden chain – stat mech model

fusion tree diagram



random tensor network



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