Collective states of interacting non-Abelian anyons

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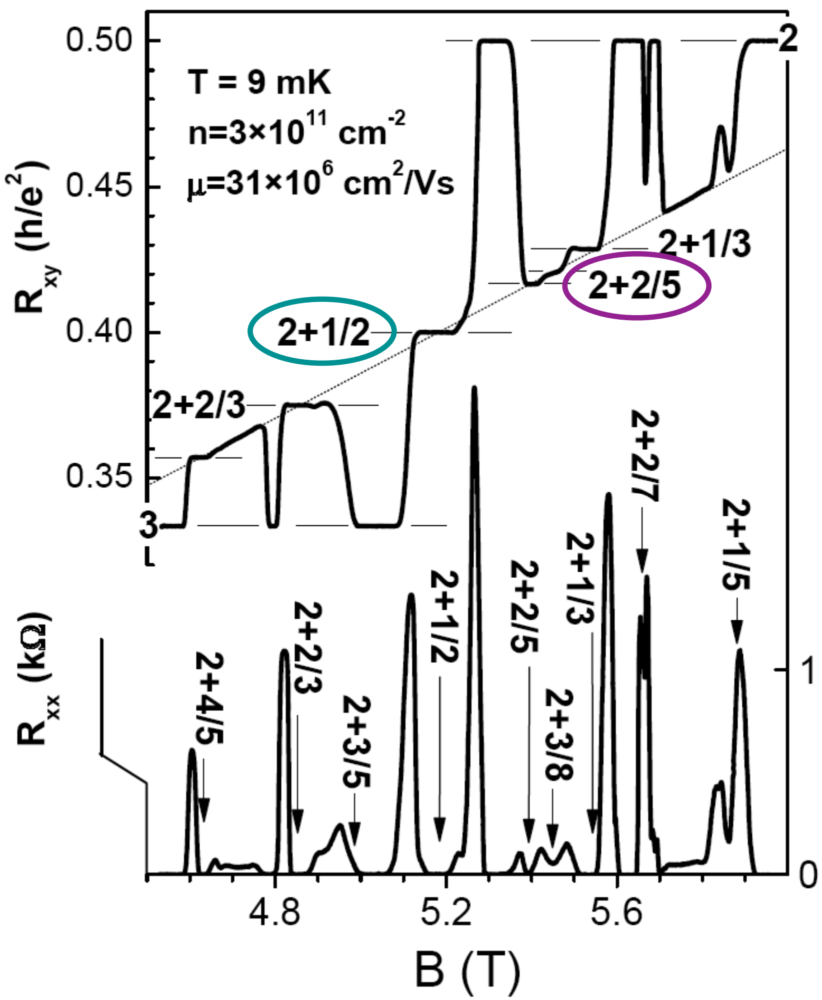
Fractional quantum Hall liquids



J.S. Xia *et al.*, PRL (2004)

"Pfaffian" state

Moore & Read (1994)

Charge e/4 quasiparticles
Ising anyons

Nayak & Wilzcek (1996)

$SU(2)_2$

"Parafermion" state

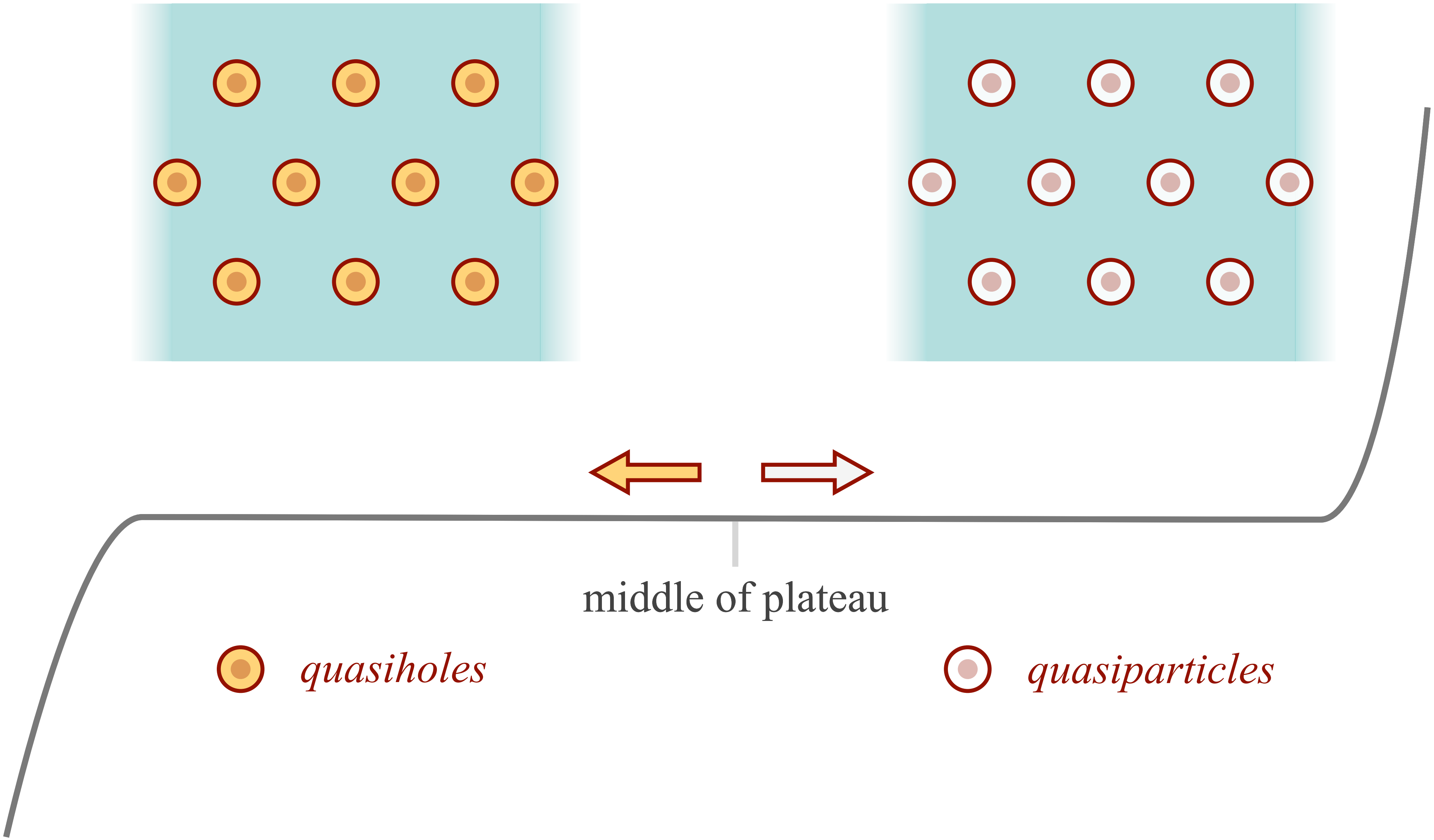
Read & Rezayi (1999)

Charge e/5 quasiparticles
Fibonacci anyons

Slingerland & Bais (2001)

$SU(2)_3$

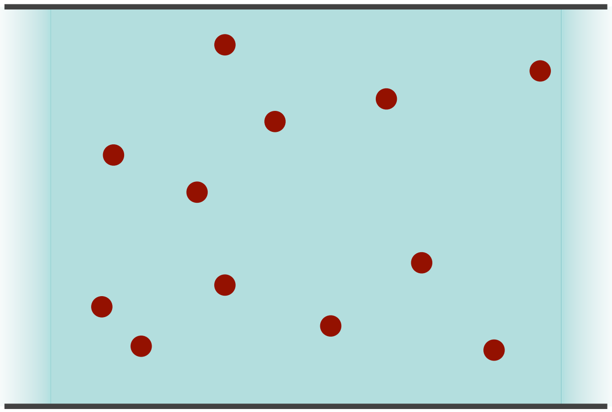
Quantum Hall plateaus



Abelian vs. non-Abelian anyons

Consider a set of ‘**pinned**’ anyons at fixed positions.

Abelian

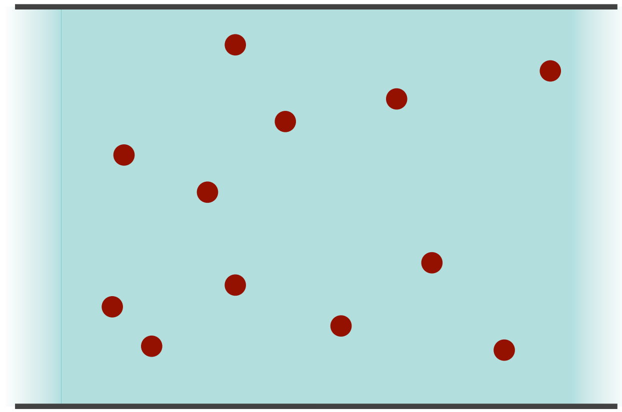


single state

$$\psi(x_2, x_1) = e^{i\pi\theta} \cdot \psi(x_1, x_2)$$

fractional phase

non-Abelian



(degenerate) **manifold** of states

matrix

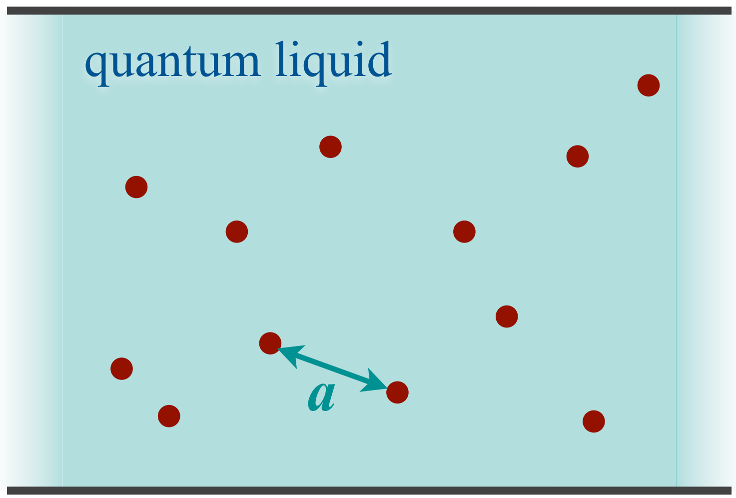
$$\psi(x_1 \leftrightarrow x_3) = M \cdot \psi(x_1, \ldots, x_n)$$

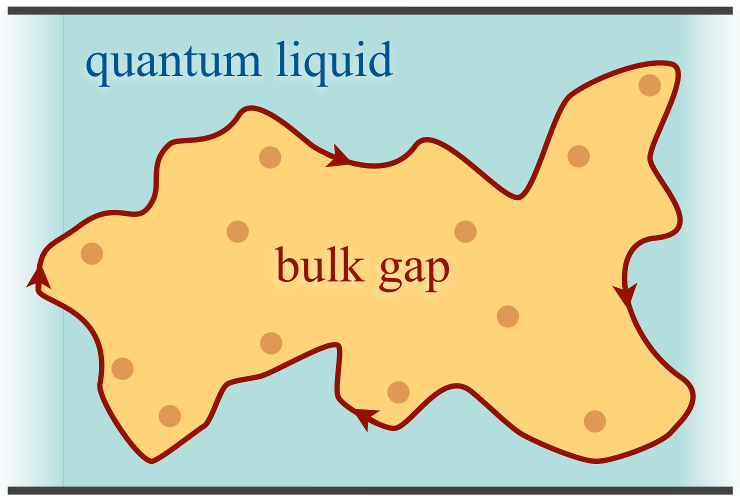
$$\psi(x_2 \leftrightarrow x_3) = N \cdot \psi(x_1, \ldots, x_n)$$

In general *M* and *N* do not commute!

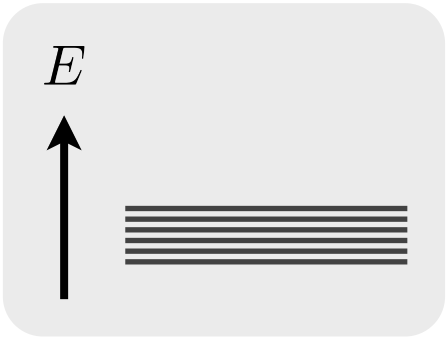
A soup of non-Abelian anyons

Phys. Rev. Lett. **98**, 160409 (2007); Phys. Rev. Lett.**103**, 070401 (2009); arXiv:1003.3453



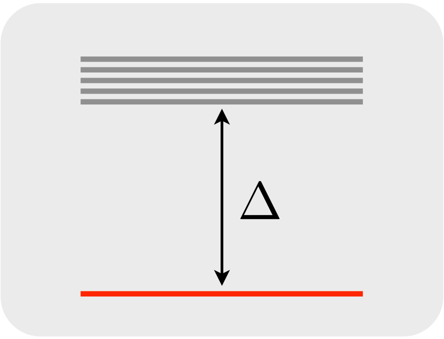


$a \gg \xi_m$



anyon-anyon interactions →

$a \approx \xi_m$



macroscopic degeneracy

unique ground state

Anyon-anyon interactions

Anyon quantum numbers

$SU(2)_k$ = ‘deformation’ of SU(2)

with finite set of representations

$$0, \frac{1}{2}, 1, \frac{3}{2}, 2, \ldots, \frac{k}{2}$$

fusion rules

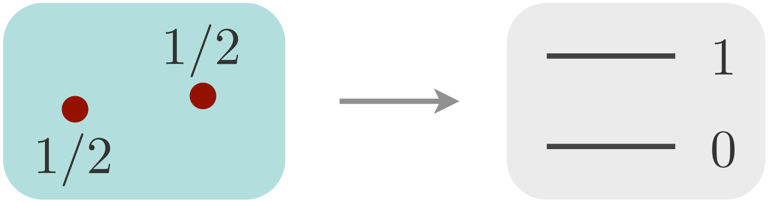
$$j_1 \times j_2 = |j_1 - j_2| + (|j_1 - j_2| + 1) + \ldots + \min(j_1 + j_2, k - j_1 - j_2)$$

example k = 2

$$1/2 \times 1/2 = 0 + 1$$

Anyon pair

$$1/2 \times 1/2 = 0 + 1$$

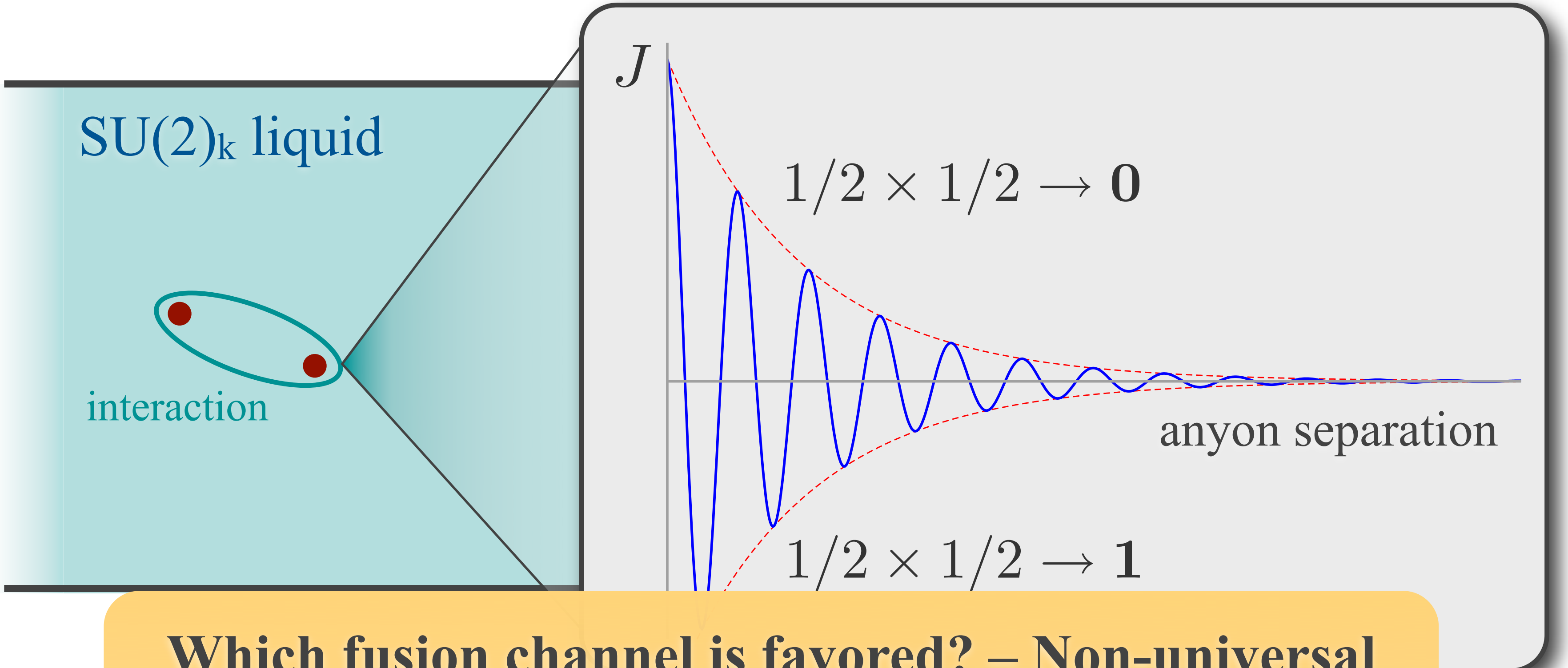


Energetics for many anyons

$$H = J \sum_{\langle ij \rangle} \prod_{ij}^{0}$$

“Heisenberg Hamiltonian”
for vortices

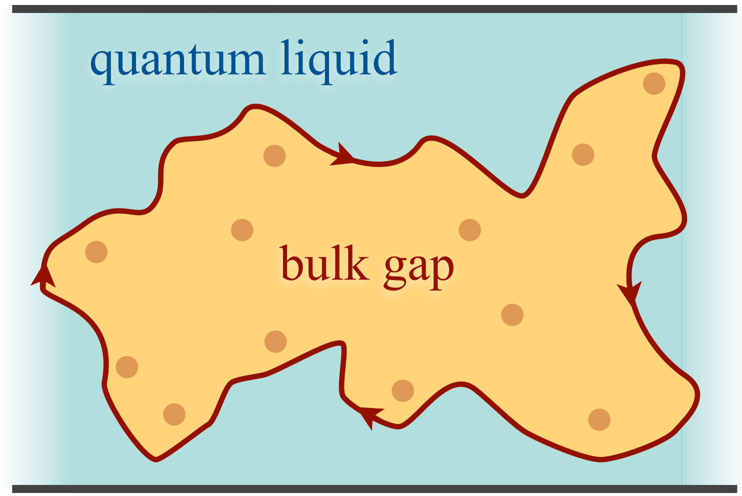
Microscopic (pair) splitting

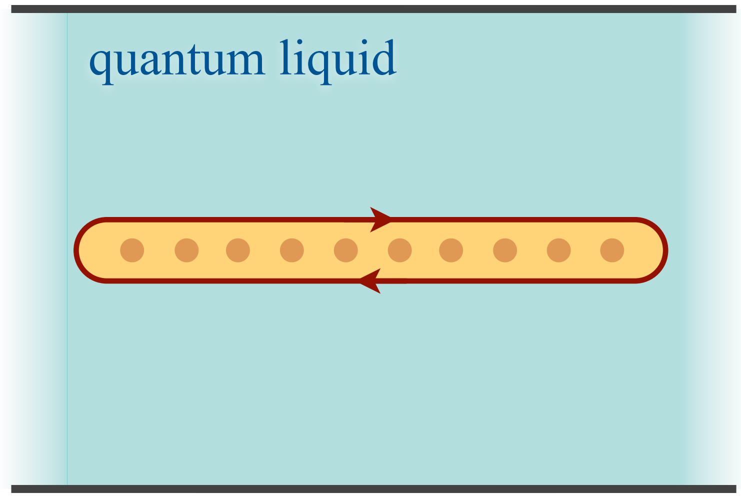


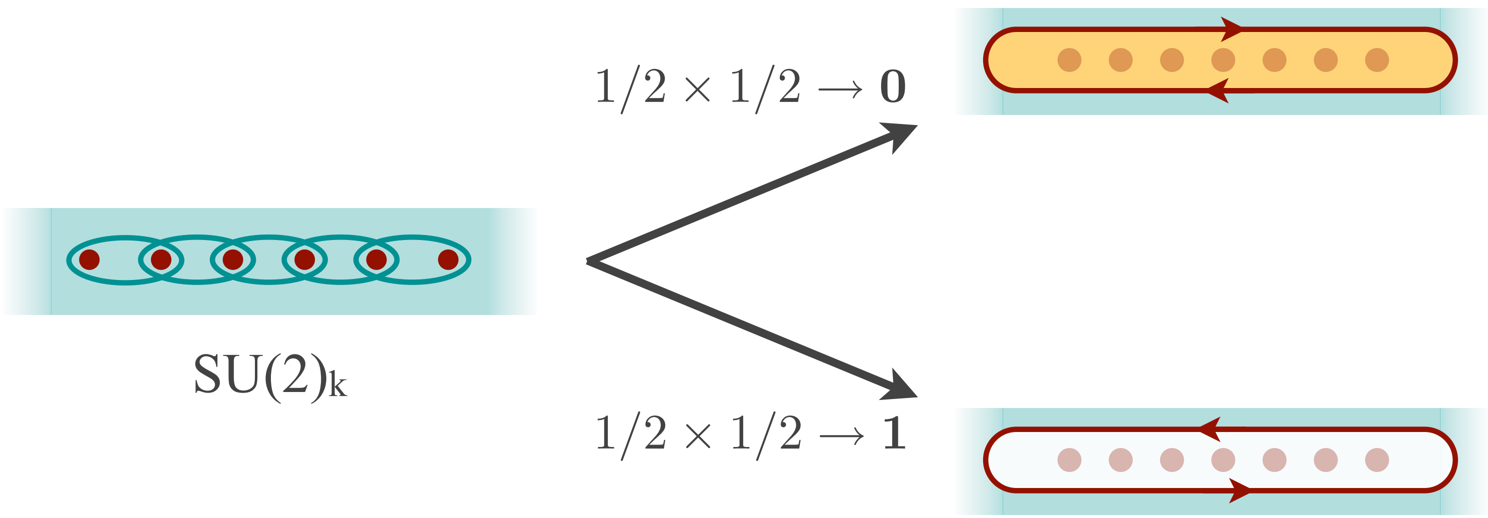
Which fusion channel is favored? – Non-universal

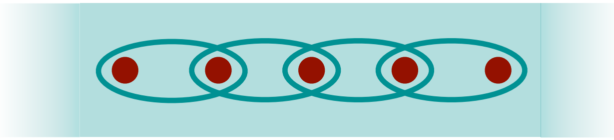
p-wave superconductor M. Cheng *et al.*, PRL (2009)	$1/2 \times 1/2 \rightarrow \mathbf{0}$ short distances, then oscillates
Moore-Read state M. Baraban *et al.*, PRL (2009)	$1/2 \times 1/2 \rightarrow \mathbf{1}$ short distances, then oscillates
Kitaev's honeycomb model V. Lahtinen *et al.*, Ann. Phys. (2008)	$1/2 \times 1/2 \rightarrow \mathbf{0}$

Approaching the many-anyon state







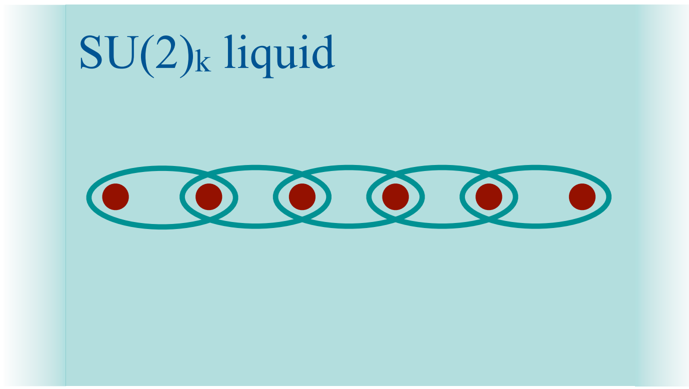


Gapless edge modes

level k	$1/2 \times 1/2 \rightarrow \mathbf{0}$ 'antiferromagnetic'	$1/2 \times 1/2 \rightarrow \mathbf{1}$ 'ferromagnetic'
2	Ising c = 1/2	Ising c = 1/2
3	tricritical Ising c = 7/10	3-state Potts c = 4/5
4	$\frac{SU(2)_{k-1} \times SU(2)_1}{SU(2)_k}$	$\frac{SU(2)_k}{U(1)}$
5		
k	**k-critical Ising** c = 1-6/(k+1)(k+2)	**Z_k-parafermions** c = 2(k-1)/(k+2)
∞	Heisenberg AFM c = 1	Heisenberg FM c = 2

Gapless modes & edge states

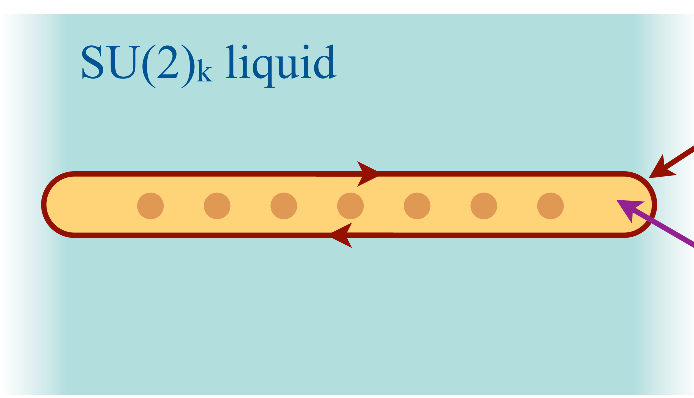
Phys. Rev. Lett. **103**, 070401 (2009).



critical theory
(AFM couplings)

$$\frac{SU(2)_{k-1} \times SU(2)_1}{SU(2)_k}$$

finite density
interactions



gapless modes = edge states

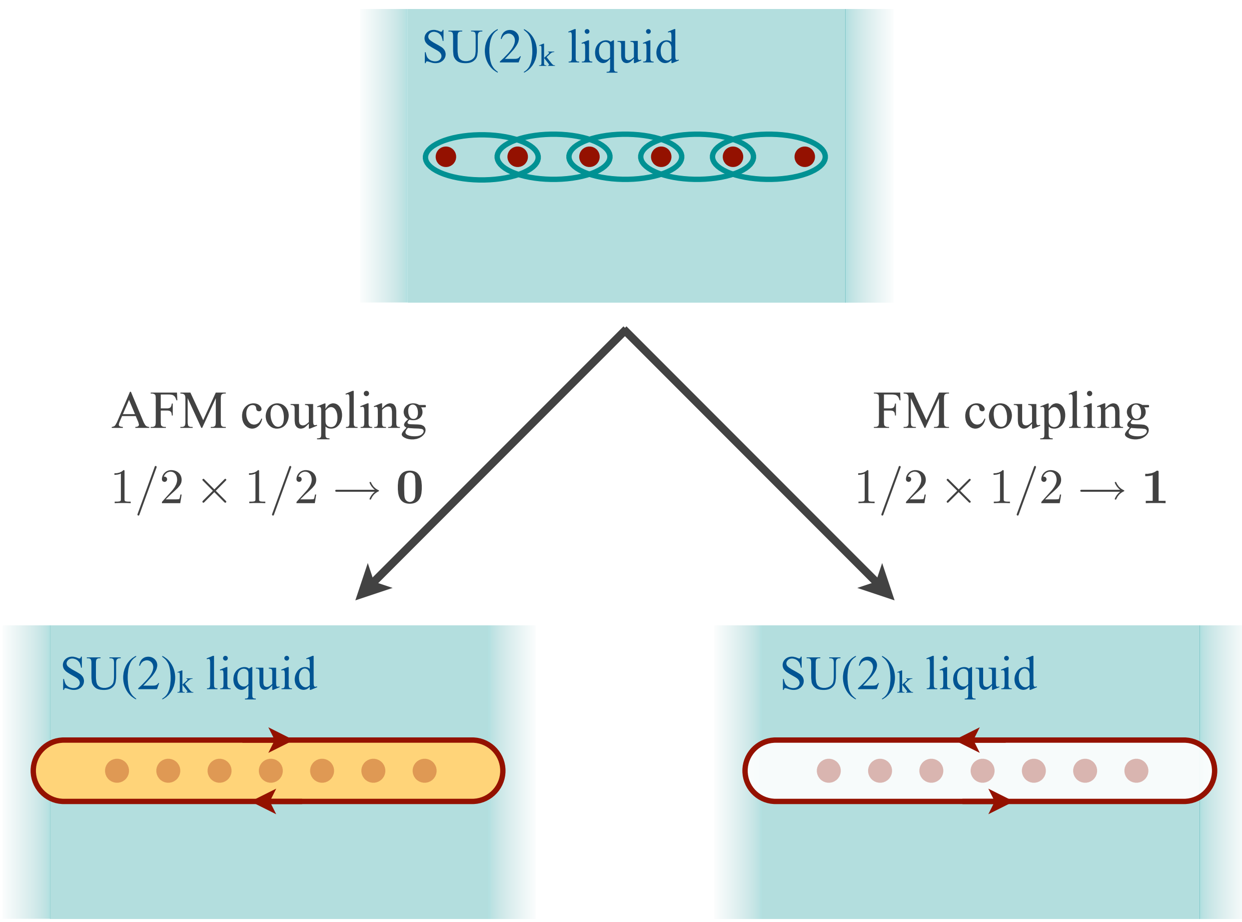
$$\frac{SU(2)_{k-1} \times SU(2)_1}{SU(2)_k}$$

nucleated liquid

$$SU(2)_{k-1} \times SU(2)_1$$

A powerful correspondence

Phys. Rev. Lett. **103**, 070401 (2009).



nucleated liquid
$SU(2)_{k-1} \times SU(2)_1$

nucleated liquid
$U(1)$

collective states
of anyonic spin chains

↓

edge states
of topological liquids

↓

nucleation of novel
topological liquids

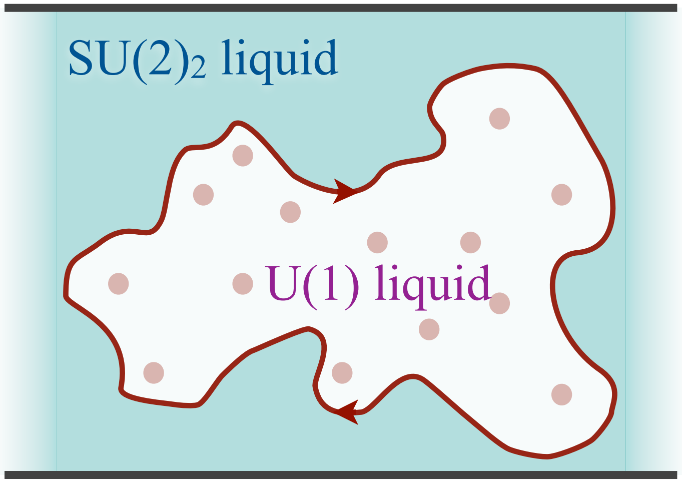
Which liquid is nucleated?

	$1/2 \times 1/2 \rightarrow \mathbf{0}$ 'antiferromagnetic'	$1/2 \times 1/2 \rightarrow \mathbf{1}$ 'ferromagnetic'
bosonic quantum Hall	$SU(2)_2 \downarrow SU(2)_1 \times SU(2)_1$ 220 Halperin state	$SU(2)_2 \downarrow U(1)$
fermionic quantum Hall	$Z_2 \times U(1) \downarrow U(1) \times U(1)$ 331 Halperin state	$Z_2 \times U(1) \downarrow U(1)$
p-wave superconductor	$Z_2 \downarrow U(1)$	$Z_2 \downarrow \emptyset$

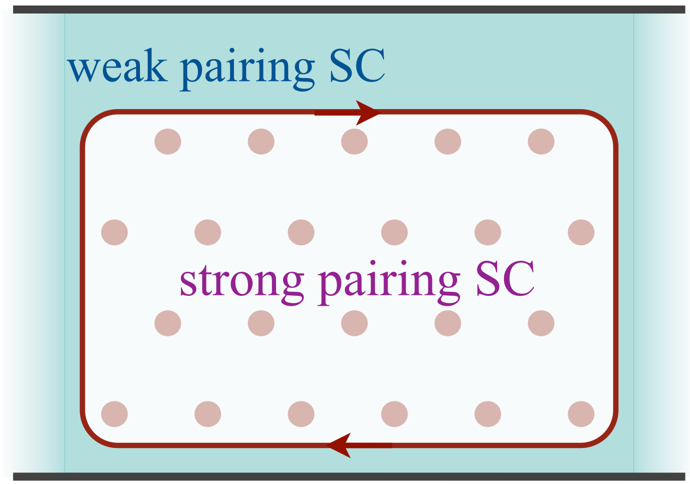
$$SU(2)_k = Z_k \times U(1)$$

Earlier work for Majorana fermions

Read & Ludwig PRB (2000)



Grosfeld & Stern PRB (2006)



Grosfeld & Schoutens PRL (2009)

Kitaev unpublished (2006)
Levin & Halperin PRB (2009)
Bais, Slingerland & Haaker PRL (2009)

2D anyon systems

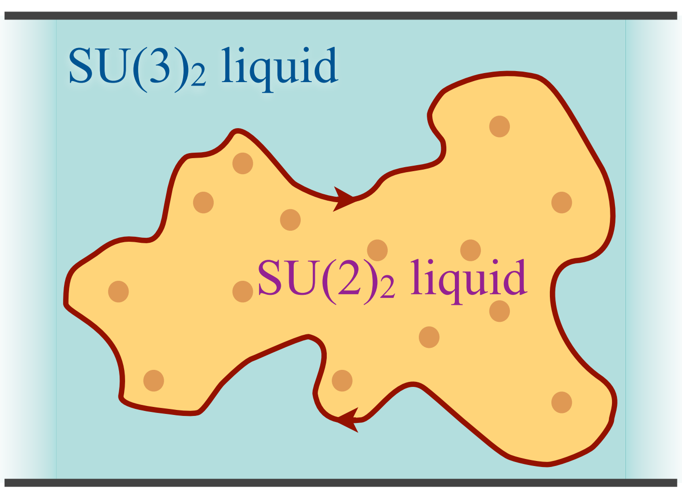
All of these previous results fit into our more general framework.

Recent work for Fibonacci anyons

Read & Ludwig PRB (2000)

Grosfeld & Stern PRB (2006)

Grosfeld & Schoutens PRL (2009)

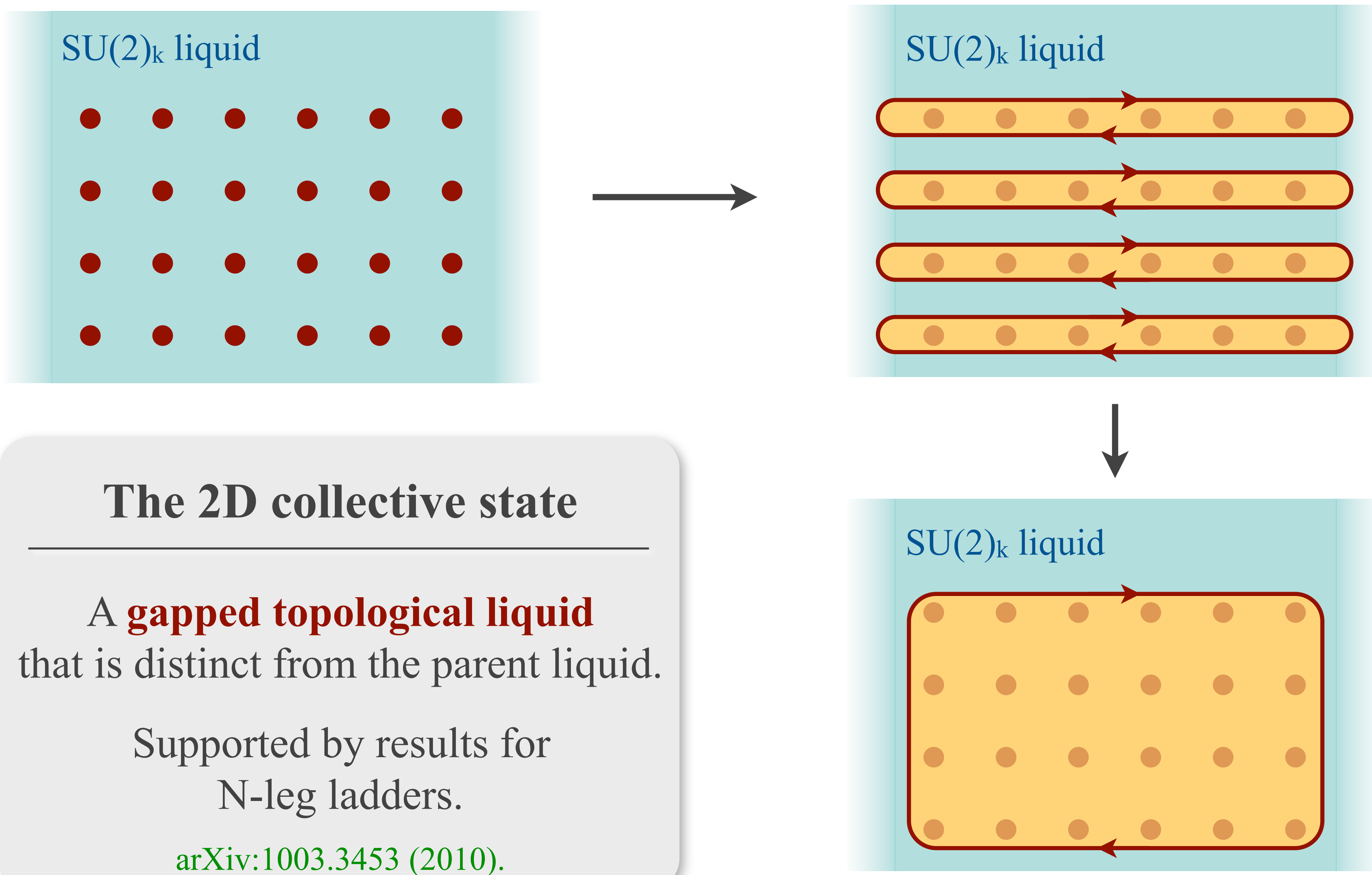


Kitaev unpublished (2006)
Levin & Halperin PRB (2009)
Bais, Slingerland & Haaker PRL (2009)

2D anyon systems

All of these previous results fit into our more general framework.

Approaching two dimensions



The 2D collective state

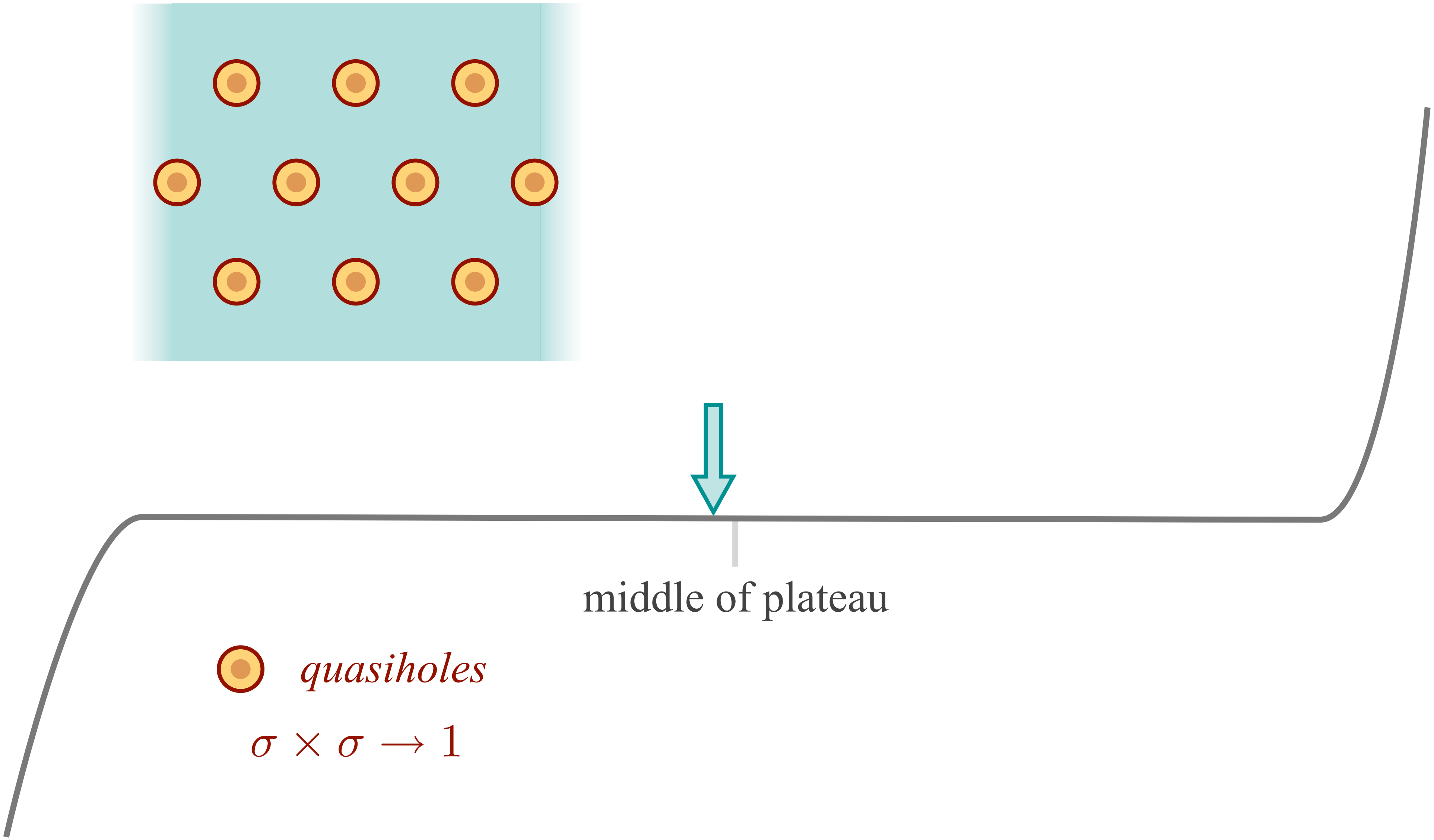
A **gapped topological liquid** that is distinct from the parent liquid.

Supported by results for N-leg ladders.

arXiv:1003.3453 (2010).

Quantum Hall plateaus

$a \gg \xi_m$

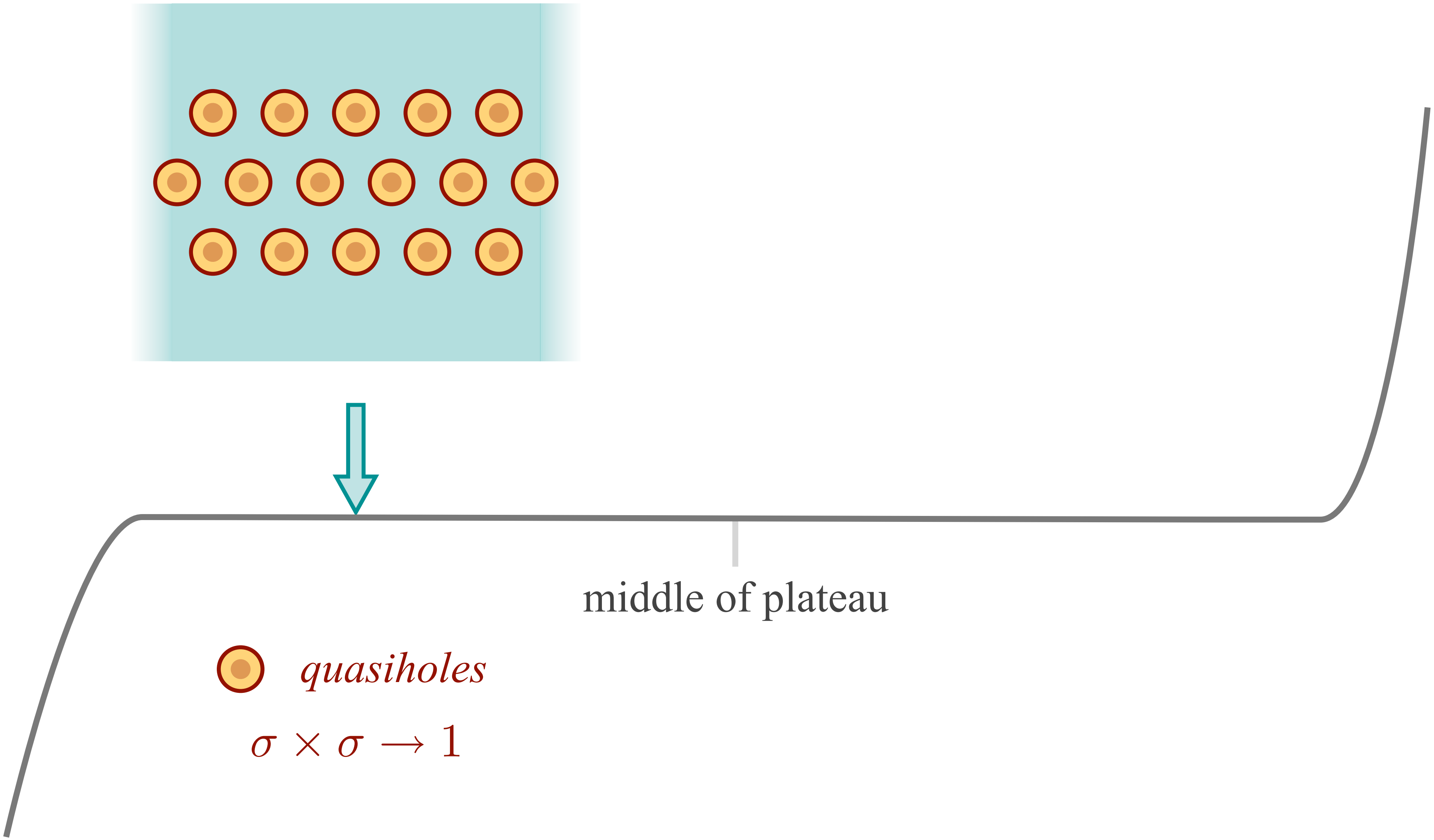


quasiholes

$\sigma \times \sigma \rightarrow 1$

Quantum Hall plateaus

$a \approx \xi_m$

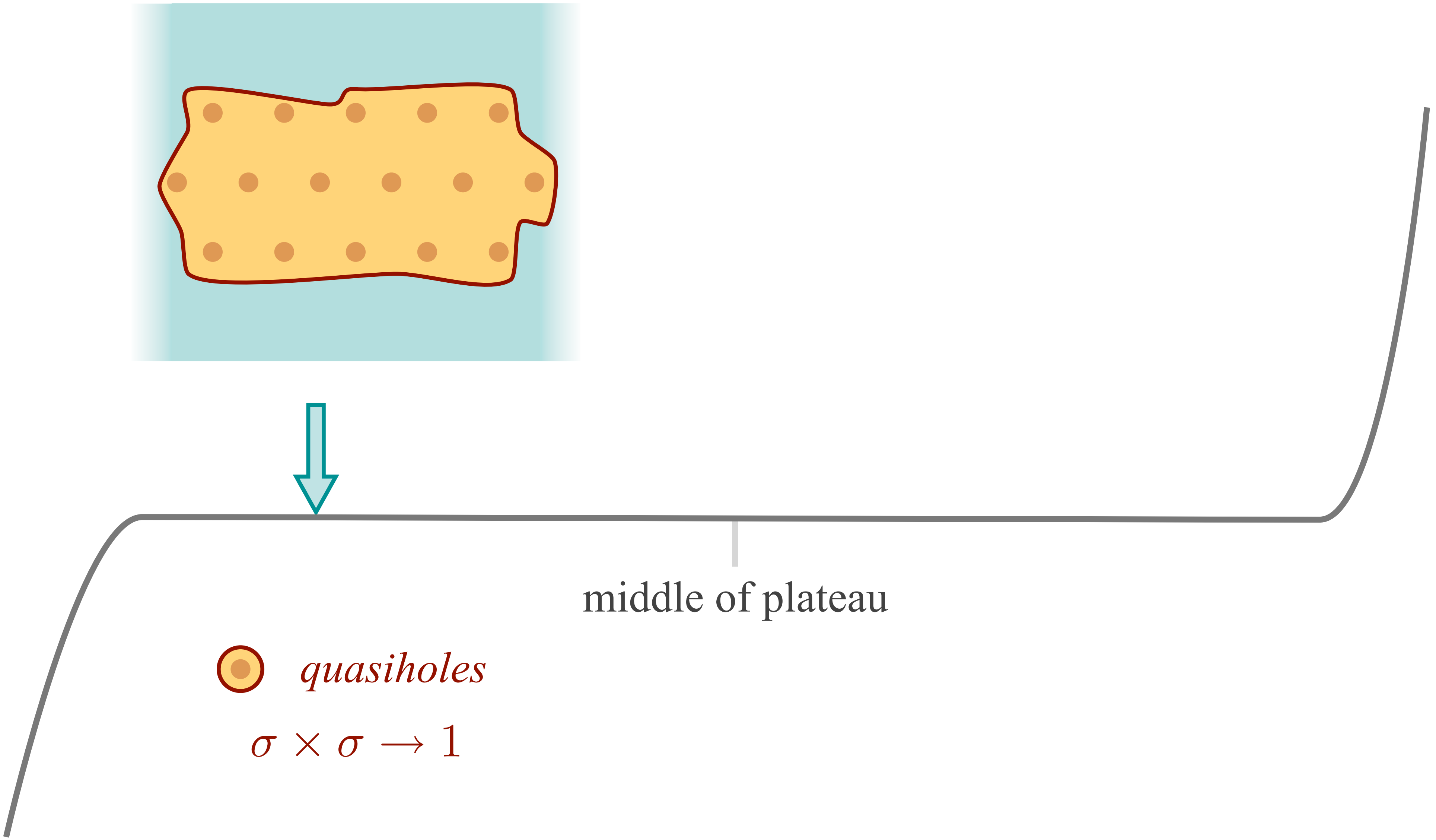


quasiholes

$\sigma \times \sigma \to 1$

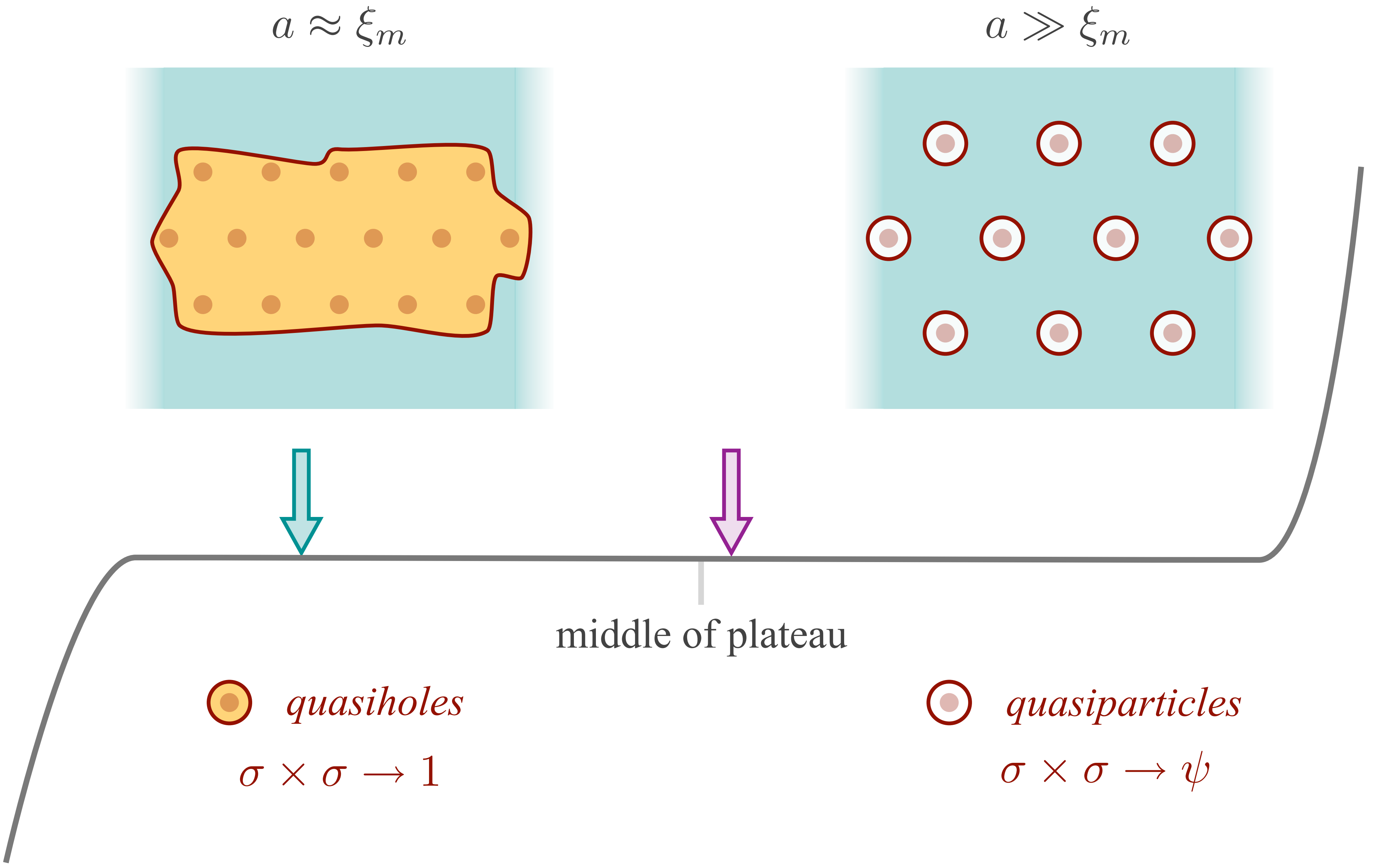
Quantum Hall plateaus

$a \approx \xi_m$

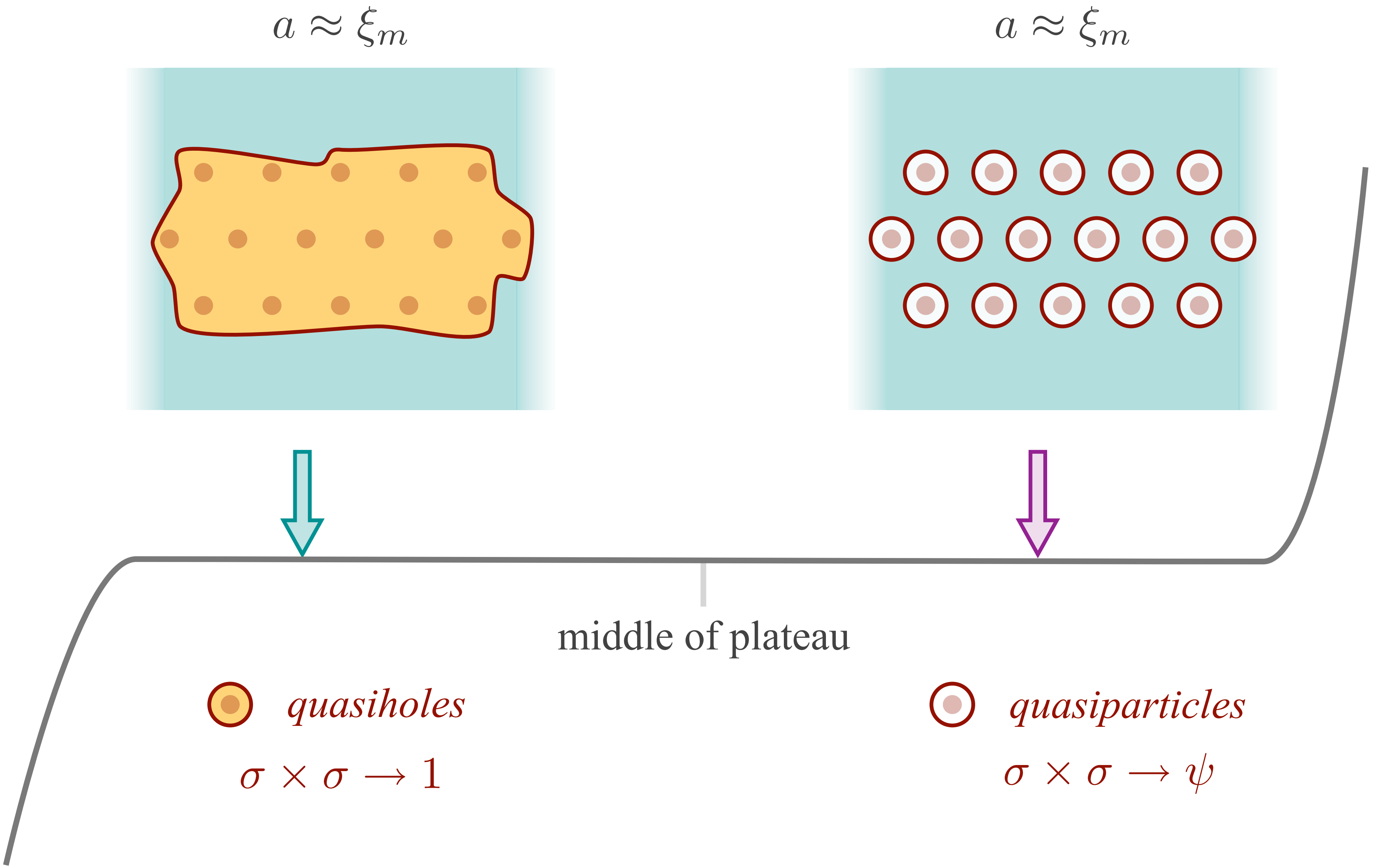


$\sigma \times \sigma \to 1$

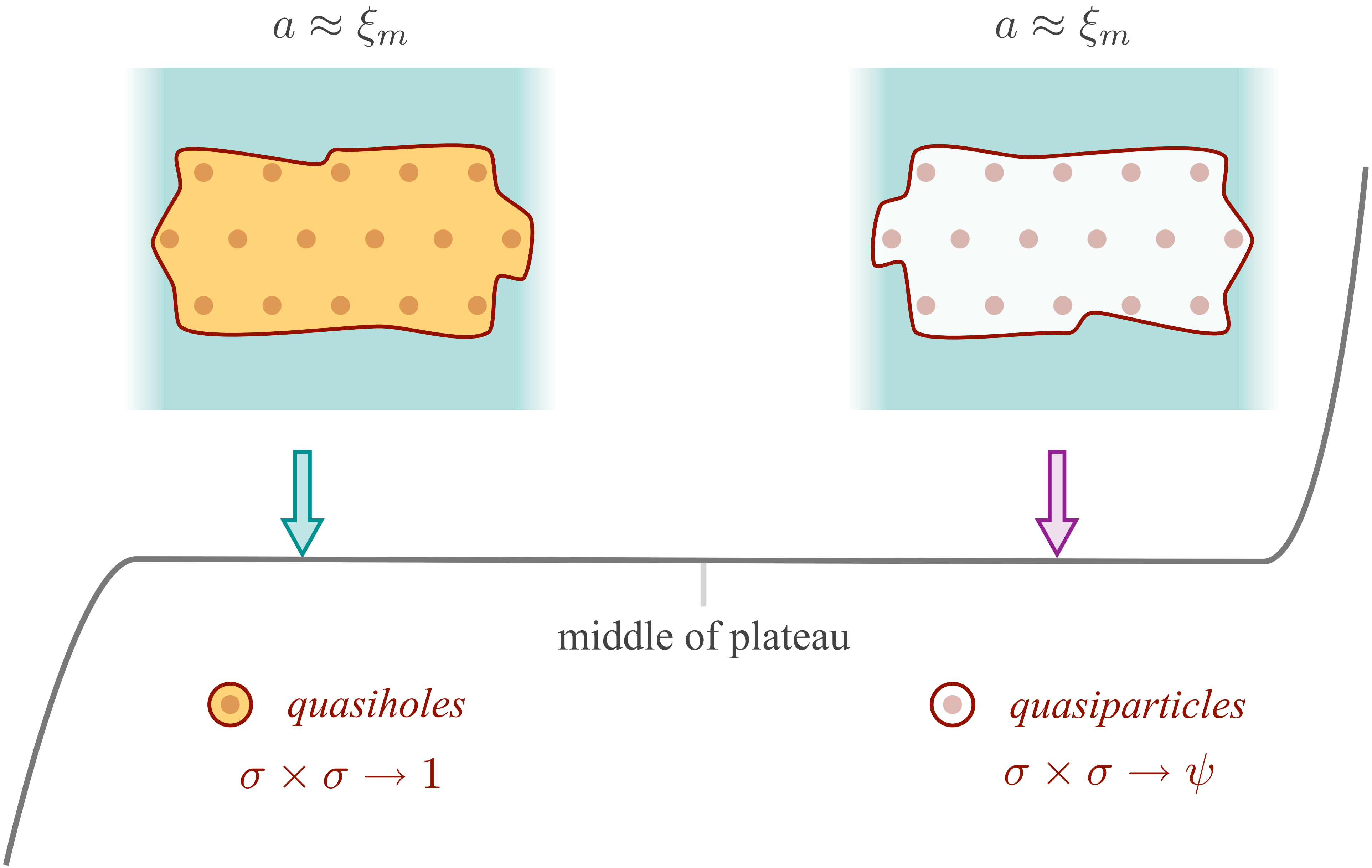
Quantum Hall plateaus



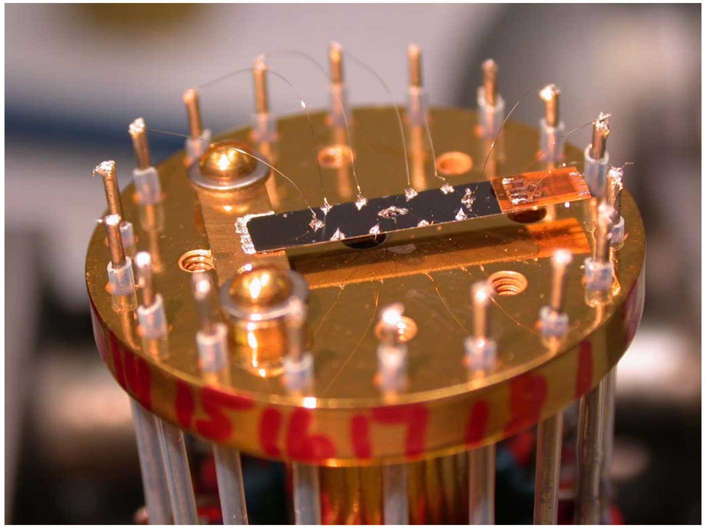
Quantum Hall plateaus



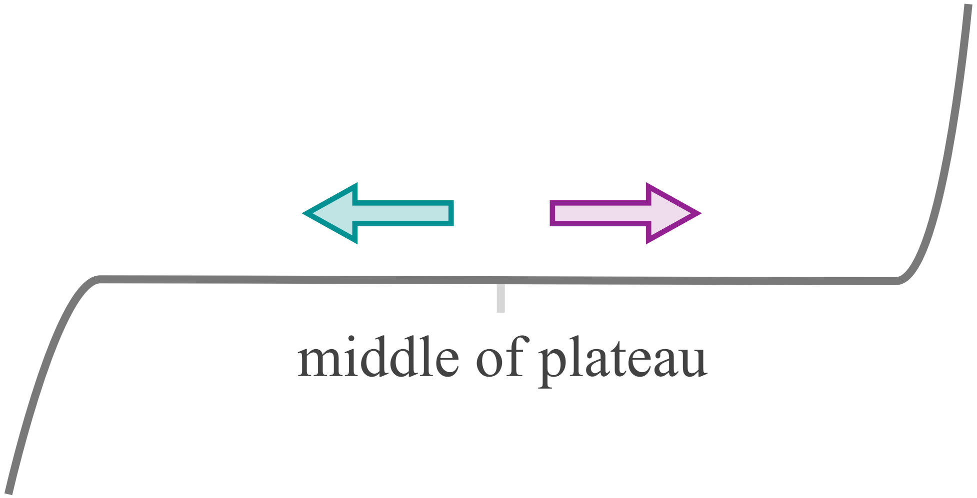
Quantum Hall plateaus



Experimental consequences



Caltech thermopower experiment

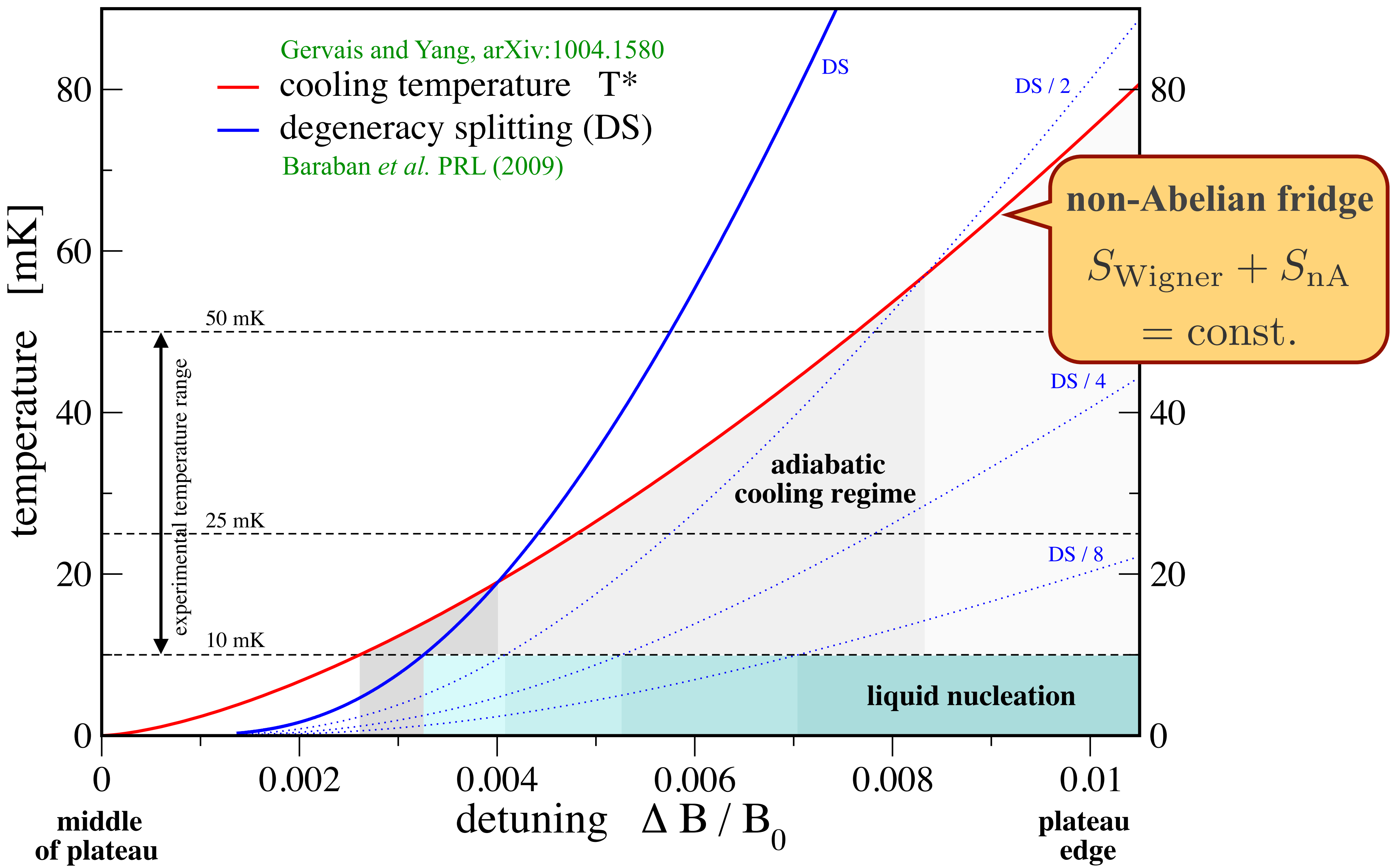


What changes (experimentally) as we move on the plateau?

electrical transport — **unchanged** – remain on the plateau

heat transport (neutral modes) — **changes** – evidence of the new liquid

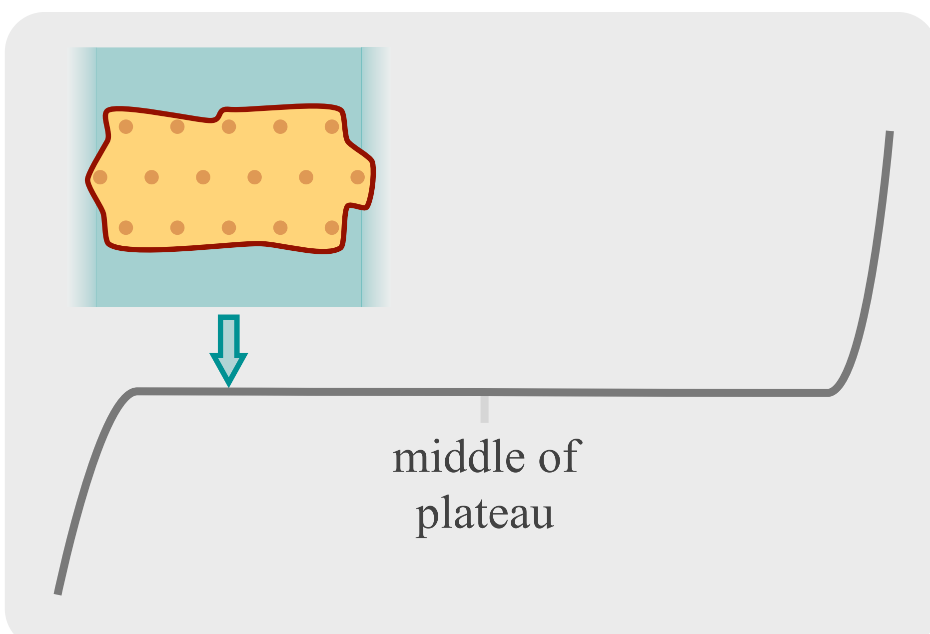
Microscopic estimates



Conclusions

$a \gg \xi_m$ $a \approx \xi_m$

macroscopic degeneracy **unique** ground state



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