Fractionalization in synthetic quantum matter

Quantum Magnetism meets Quantum Computing

CRC183
ENTANGLED STATES OF MATTER

Simon Trebst
University of Cologne

ML4Q
MATTER AND LIGHT FOR
QUANTUM COMPUTING

Pre-March $(\text{Meeting})^2$

Fine Theoretical Physics Institute, March 2024

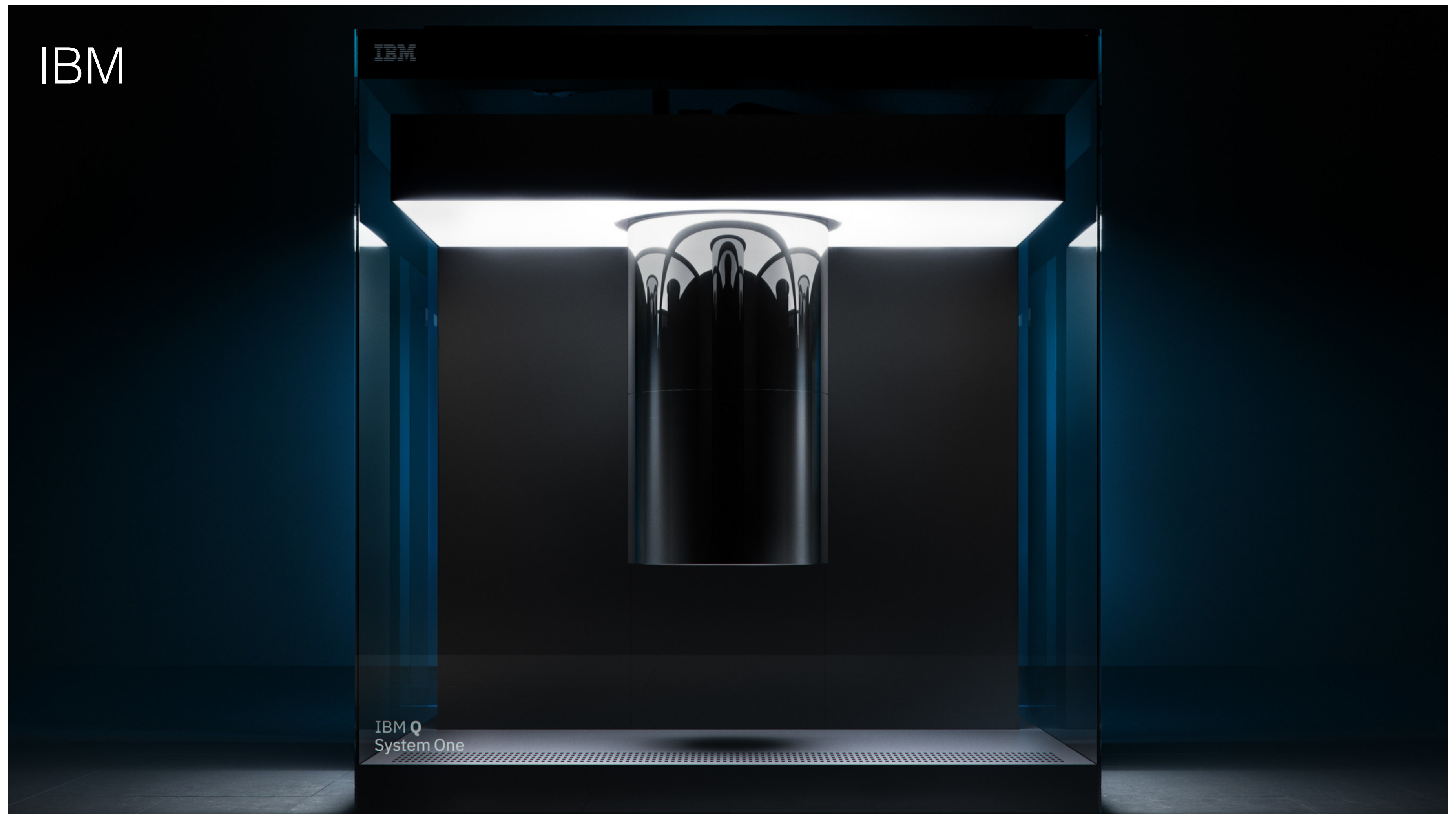
quantum hardware in the NISQ era

An experimental pivot from of a **few pristine qubits**

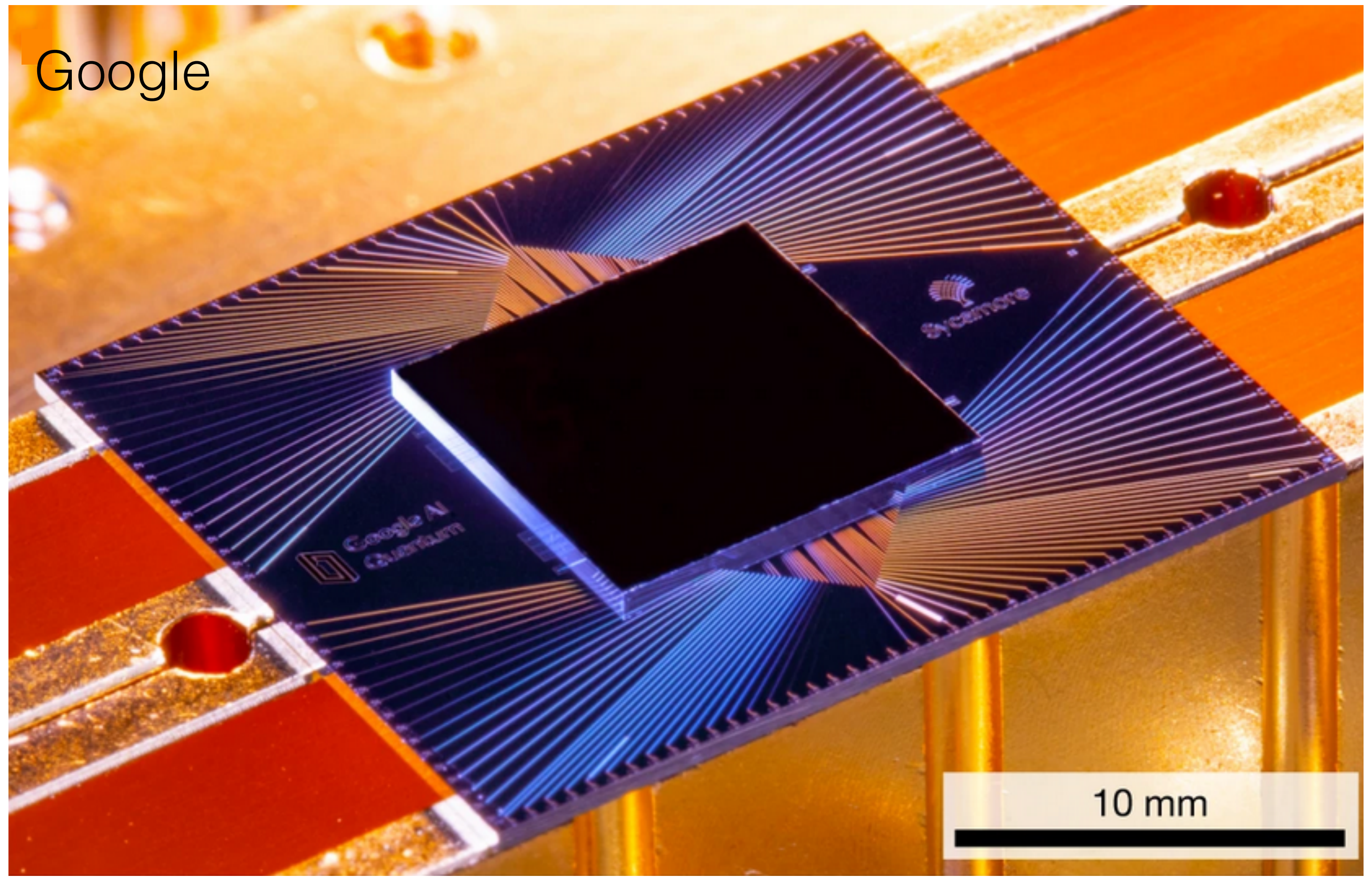
to the realization of circuit architectures of **50 … 1000 qubits**

but tolerating a significant level of **imperfections.**

noisy intermediate scale quantum (NISQ) devices



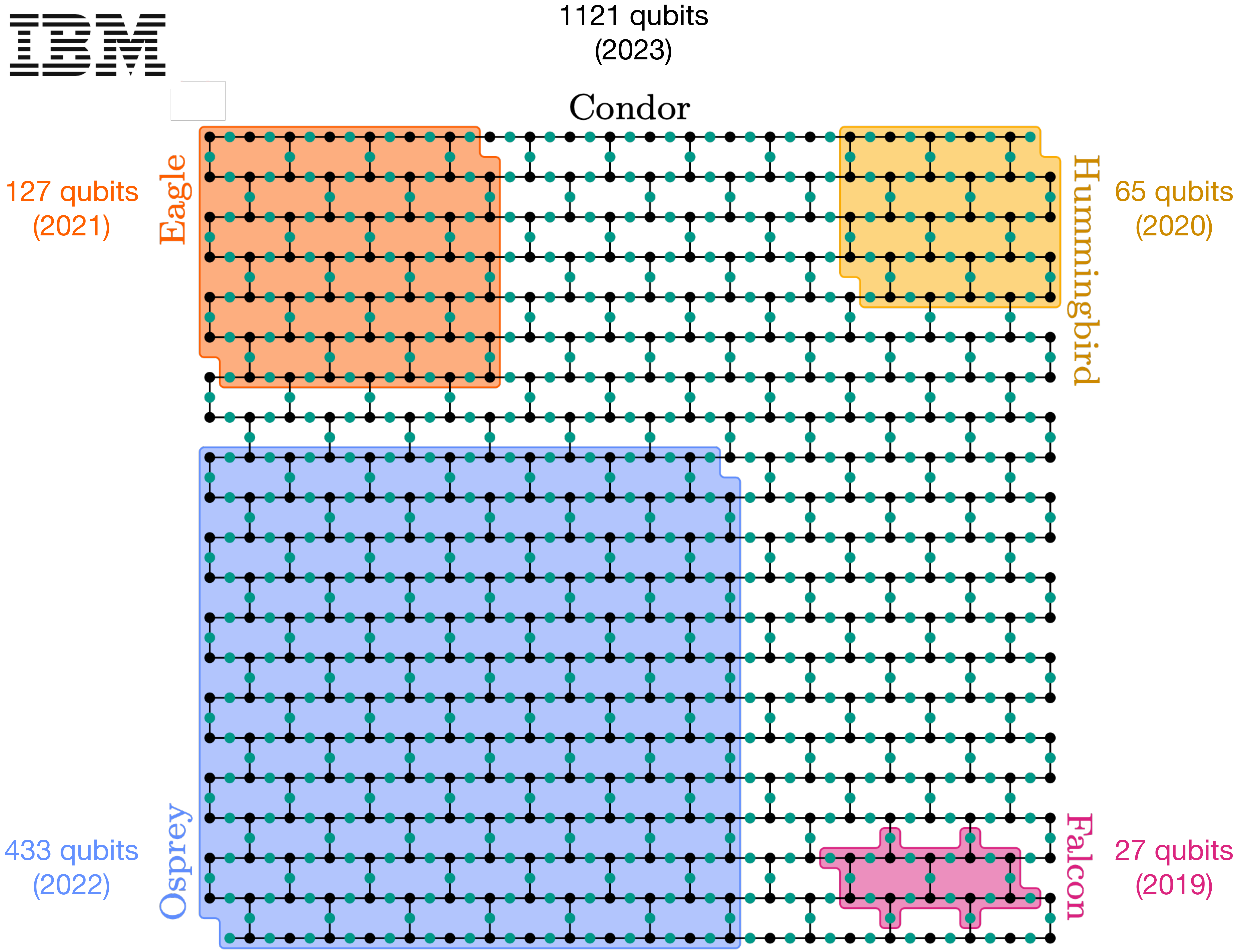
Osprey generation — 433 qubits



Sycamore chip — 53 qubits

quantum hardware in the NISQ era

IBM



(2019)

Article | Published: 23 October 2019

Quantum supremacy using a programmable superconducting processor

Frank Arute, Kunal Arya, Ryan Babbush, Dave Bacon, Joseph C. Bardin, Rami Barends, Rupak Biswas, Sergio Boixo, Fernando G. S. L. Brandao, David A. Buell, Brian Burkett, Yu Chen, Zijun Chen, Ben Chiaro, Roberto Collins, William Courtney, Andrew Dunsworth, Edward Farhi, Brooks Foxen, Austin Fowler, Craig Gidney, Marissa Giustina, Rob Graff, Keith Guerin, ... John M. Martinis + Show authors

Nature **574**, 505–510 (2019) | Cite this article

(2021)

RESEARCH ARTICLE | TOPOLOGICAL MATTER

Realizing topologically ordered states on a quantum processor

K. J. SATZINGER, Y.-J LIU, A. SMITH, C. KNAPP, [...], AND P. ROUSHAN +93 authors Authors Info & Affiliations

SCIENCE • 2 Dec 2021 • Vol 374, Issue 6572 • pp. 1237-1241 • DOI: 10.1126/science.abi8378

(2023)

Article | Open access | Published: 11 May 2023

Non-Abelian braiding of graph vertices in a superconducting processor

Google Quantum AI and Collaborators

Nature **618**, 264–269 (2023)

(2024)

Article | Published: 14 February 2024

Non-Abelian topological order and anyons on a trapped-ion processor

Mohsin Iqbal, Nathanan Tantivasadakarn, Ruben Verresen, Sara L. Campbell, Joan M. Dreiling, Caroline Figgatt, John P. Gaebler, Jacob Johansen, Michael Mills, Steven A. Moses, Juan M. Pino, Anthony Ransford, Mary Rowe, Peter Siegfried, Russell P. Stutz, Michael Foss-Feig, Ashvin Vishwanath & Henrik Dreyer

Nature **626**, 505–511 (2024) | Cite this article

computational physics in the NISQ era

(1953) — (1992) — (2019) →

classical many-body	quantum many-body	$(\text{quantum})^2$ many-body

path integrals

tensor networks

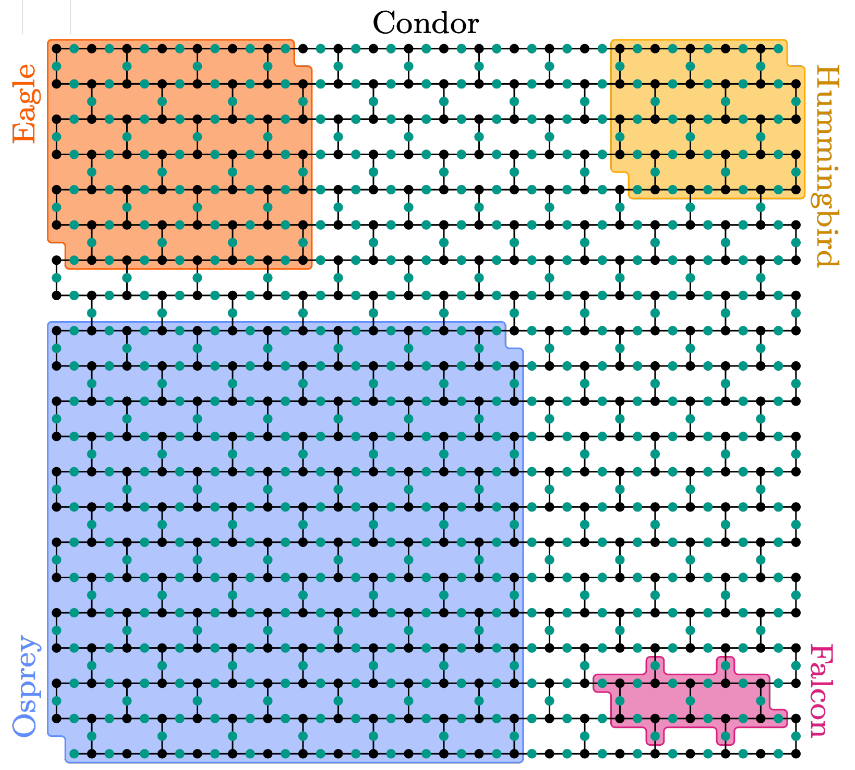
Markov chain Monte Carlo
molecular dynamics

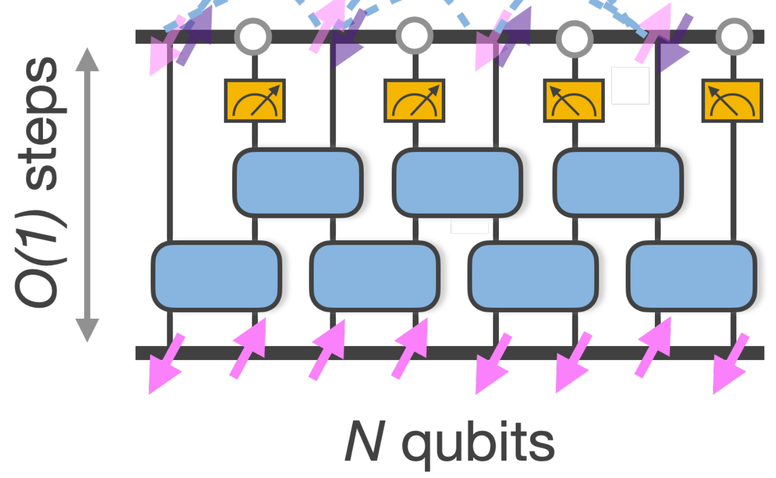
quantum Monte Carlo
DMRG, MERA, …
neural network states

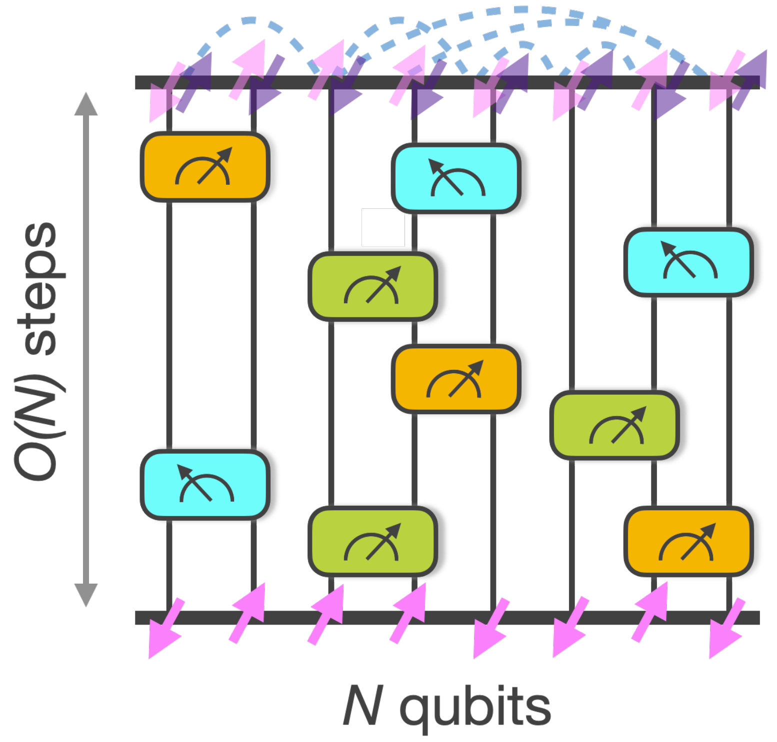
quantum circuits
adiabatic approaches
variational approaches

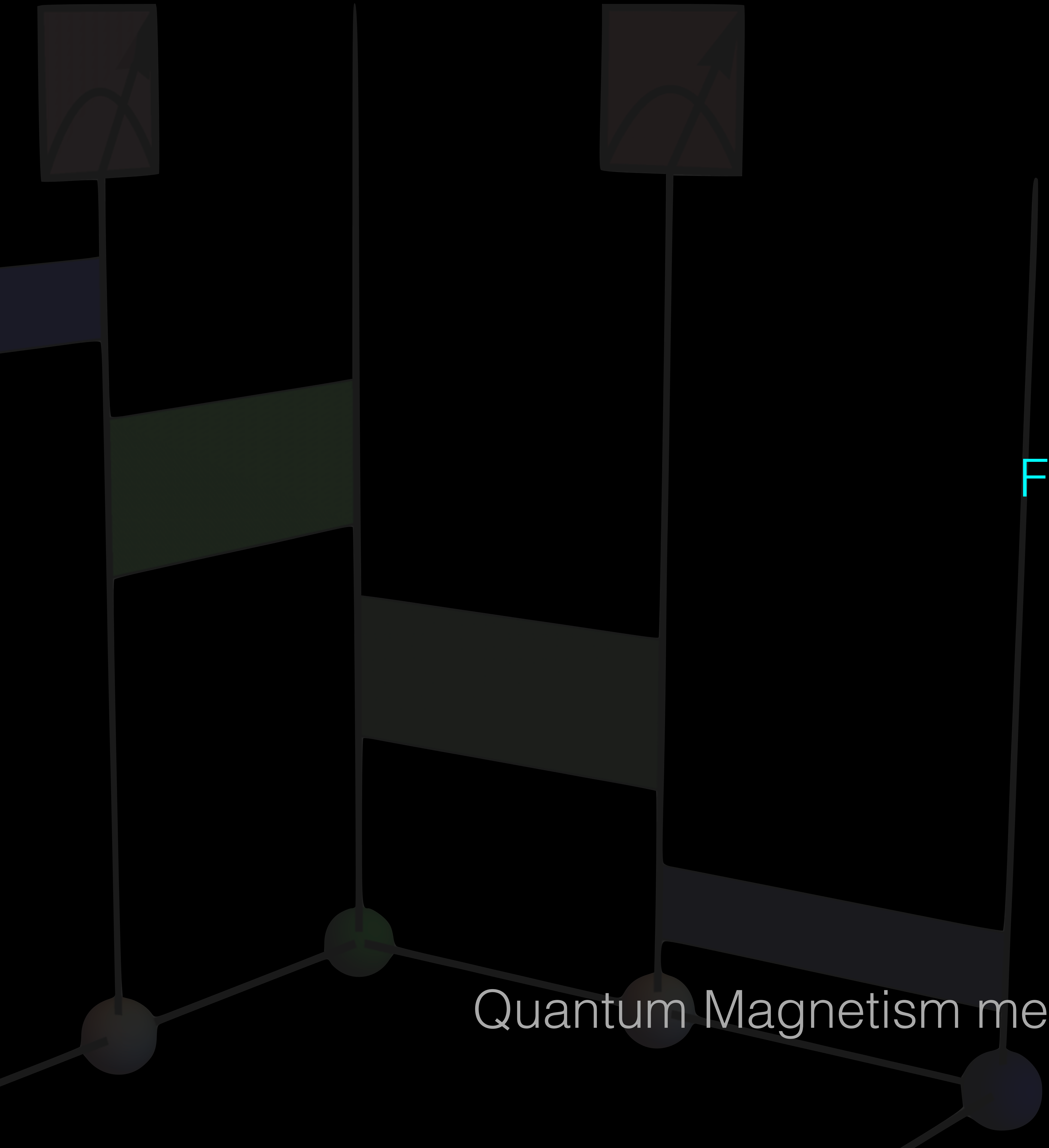
Hamiltonian
dynamics

wavefunction
dynamics









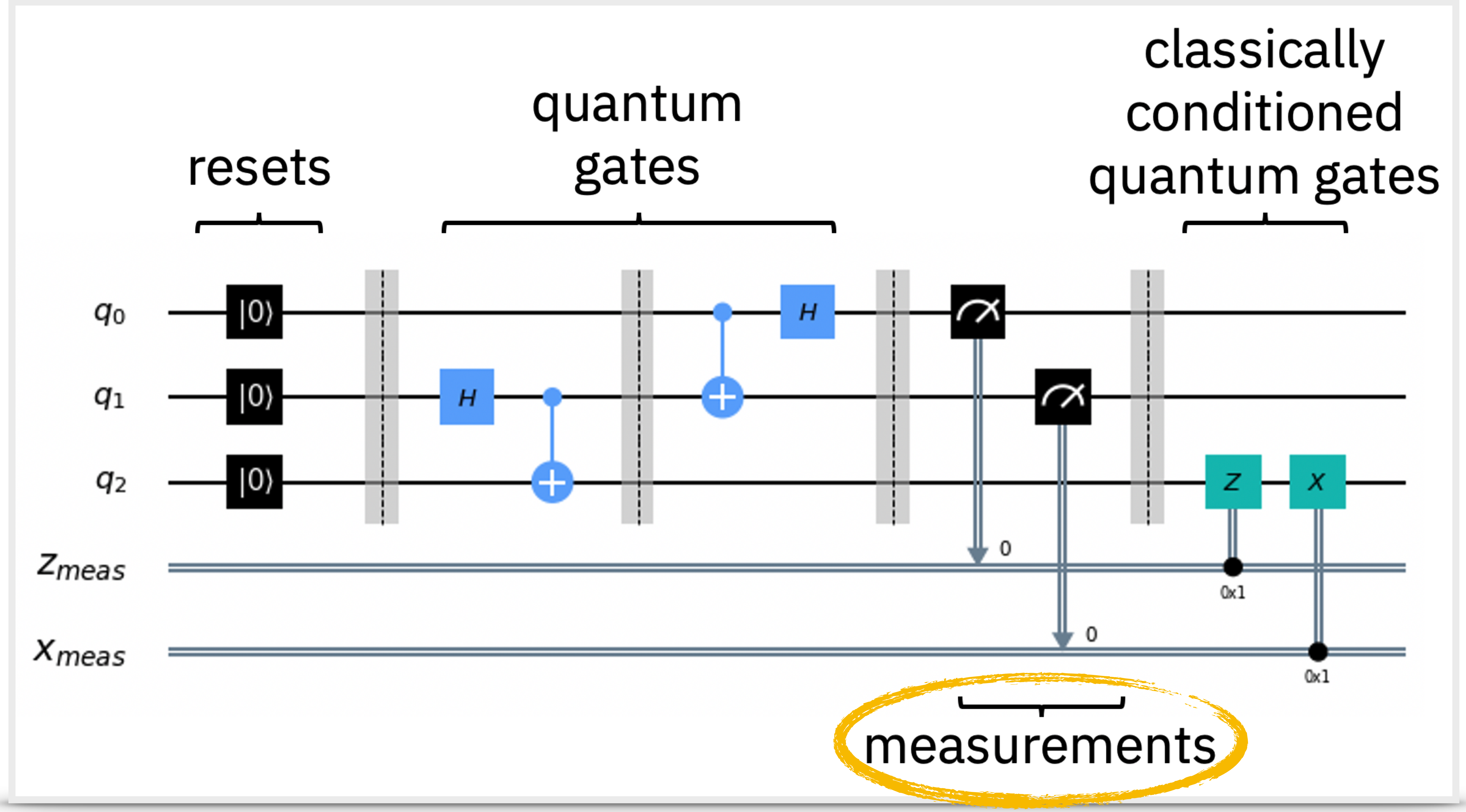
Fractionalization & Emergent Gauge Fields in **Quantum Circuits**

Quantum Magnetism meets Quantum Computing

quantum circuits in a nutshell

initial wavefunction

$|\psi_0\rangle$



Quantum computing in a nutshell, Qiskit documentation / IBM Quantum

final wavefunction(s)

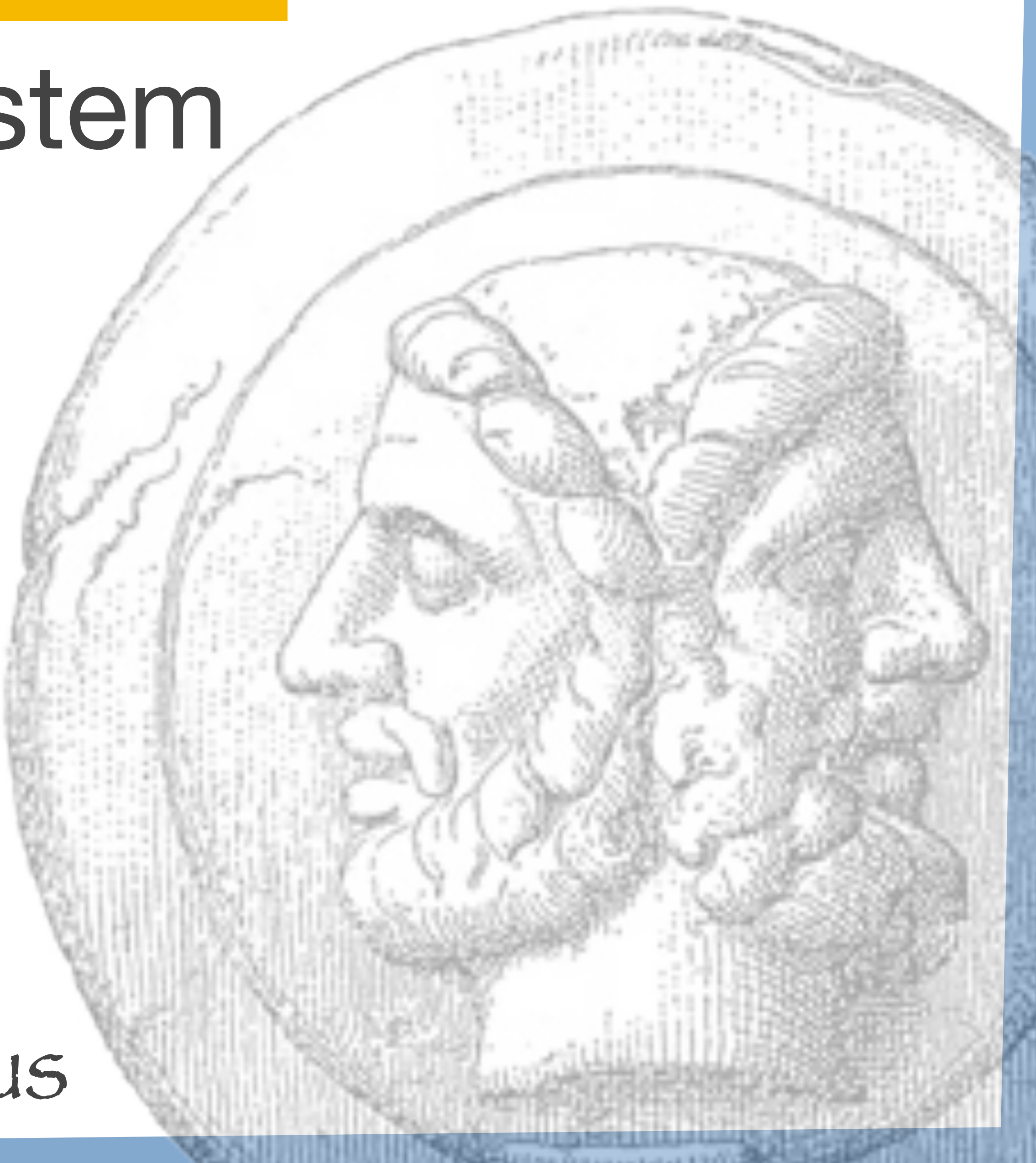
$|\psi_f\rangle$

quantum measurements

"About your cat, Mr. Schrödinger — I have good news and bad news."

Quantum measurements can

- **extract information** from a system
- **shape entanglement** of a quantum system

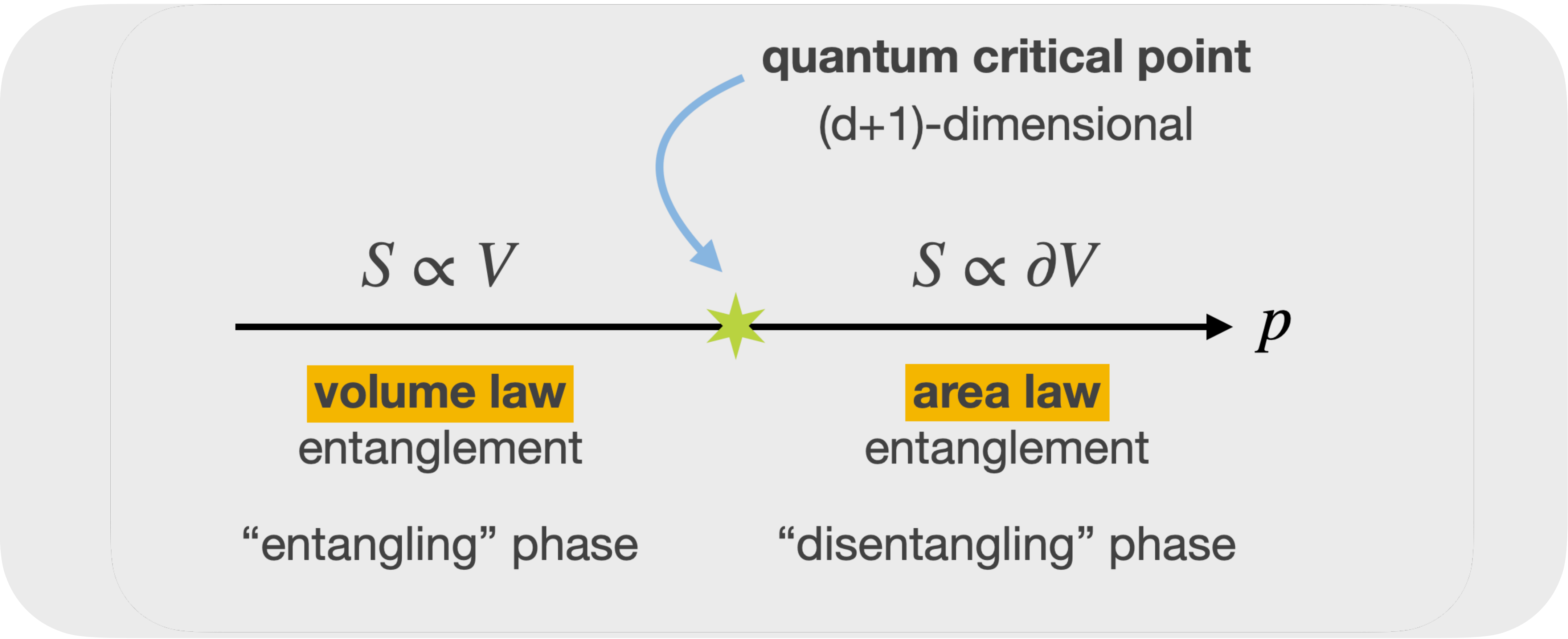


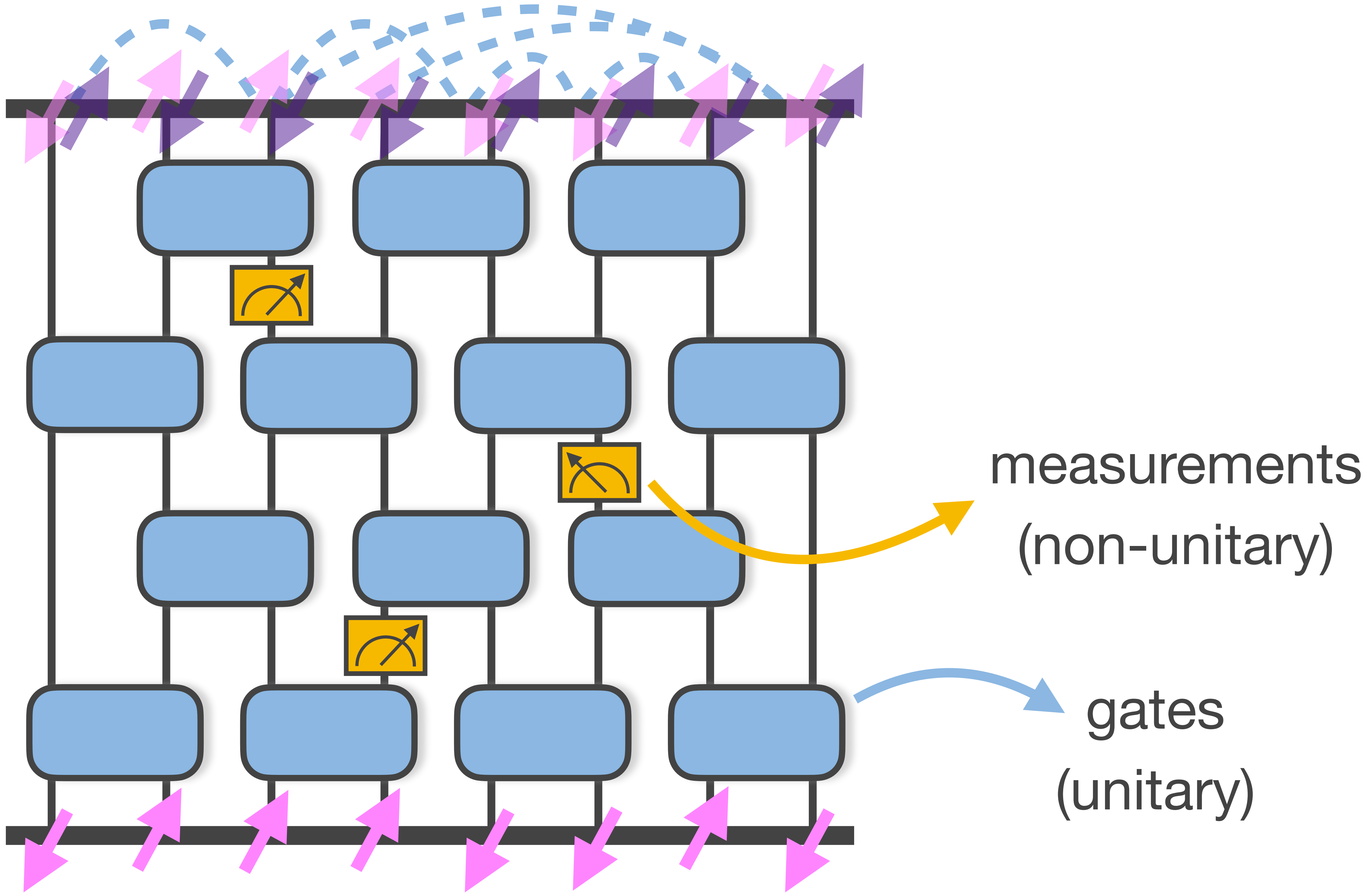
double-faced Janus

entanglement phase transitions

hybrid unitary/projective dynamics

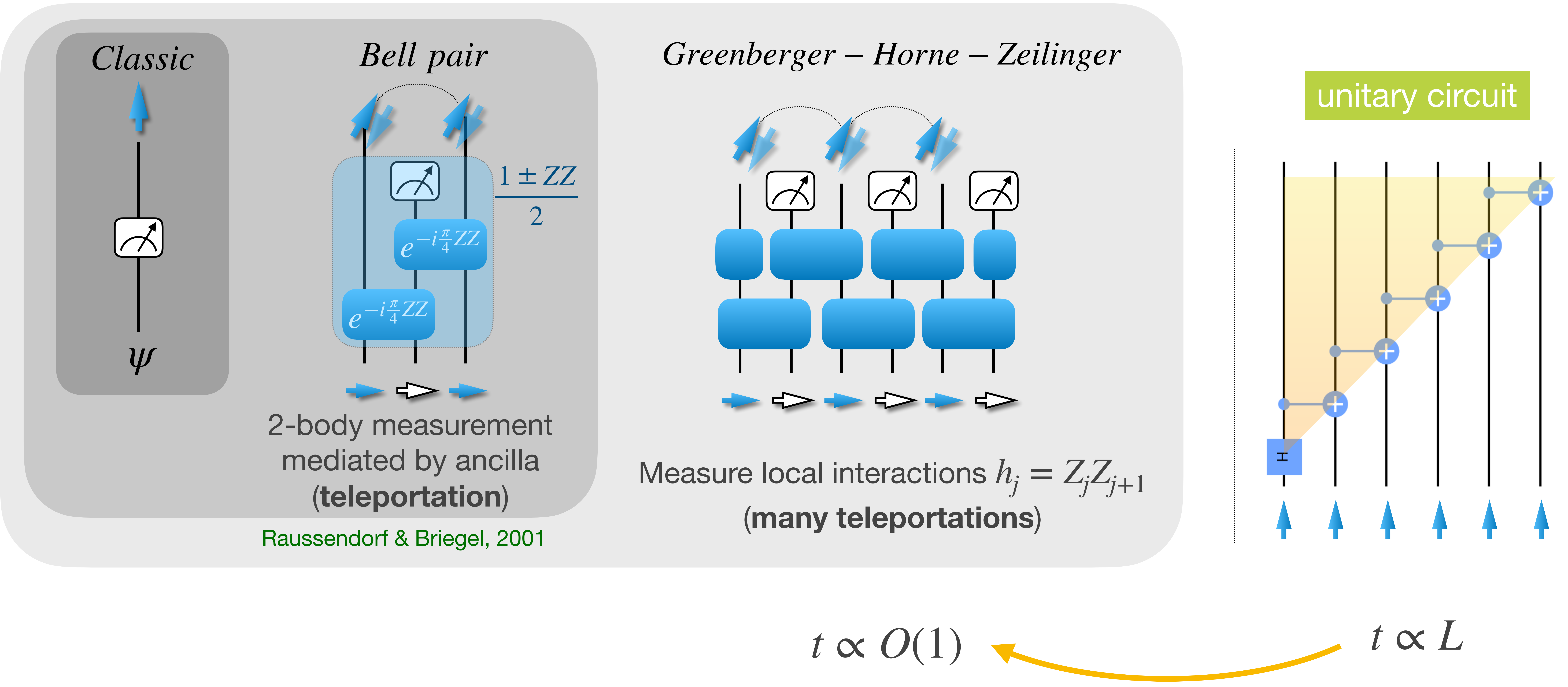
- **competition** between **scrambling** (unitary) and **disentangling** (measurement) dynamics
- entanglement dynamics along *single quantum trajectories*
- **entanglement phase transition** as function of measurement rate





A. Potter & R. Vasseur, Springer QST book series (2022)
M. Fisher, V. Khemani, A. Nahum & S. Vijay, Ann. Rev. Cond. Matt. Phys. **14**, 335 (2023)

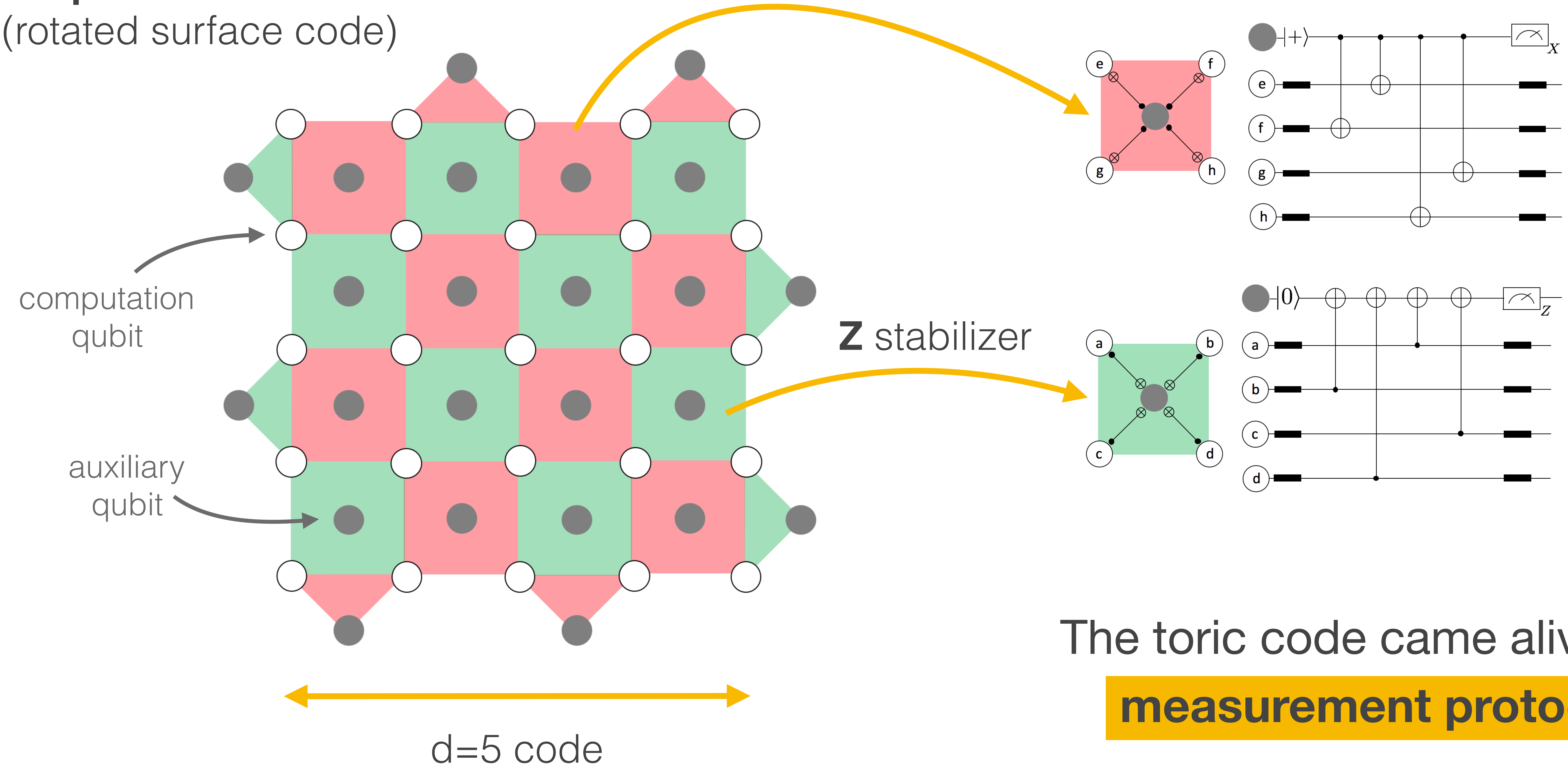
quantum states from measurements



paradigmatic example: stabilizer code

Kitaev (1997)

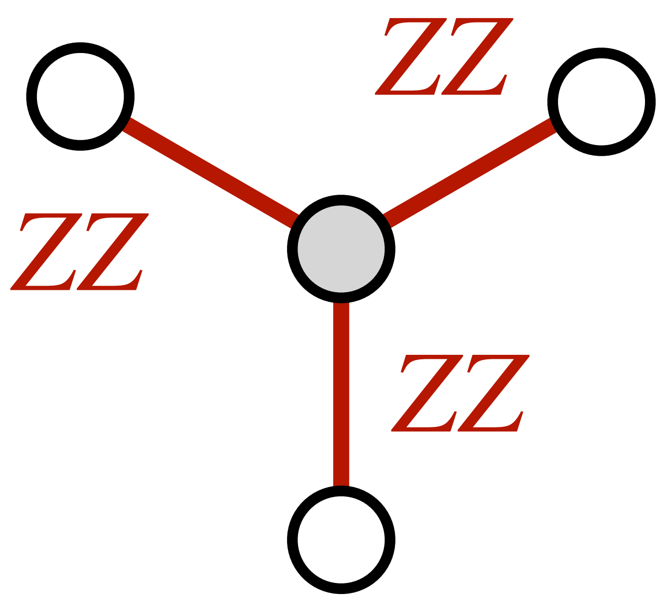
open boundaries
(rotated surface code)



The toric code came alive as a **measurement protocol.**

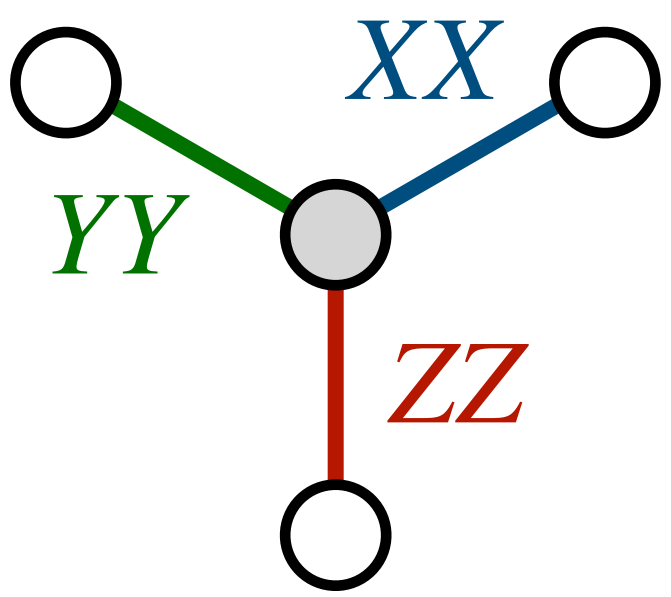
commuting vs non-commuting measurements

Guo-Yi Zhu



Nishimori's cat

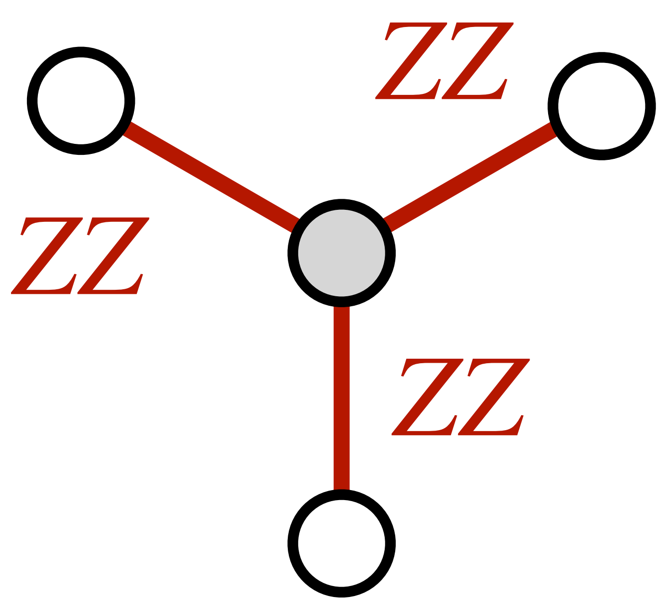
- **commuting**
- parallelized
- no dynamics



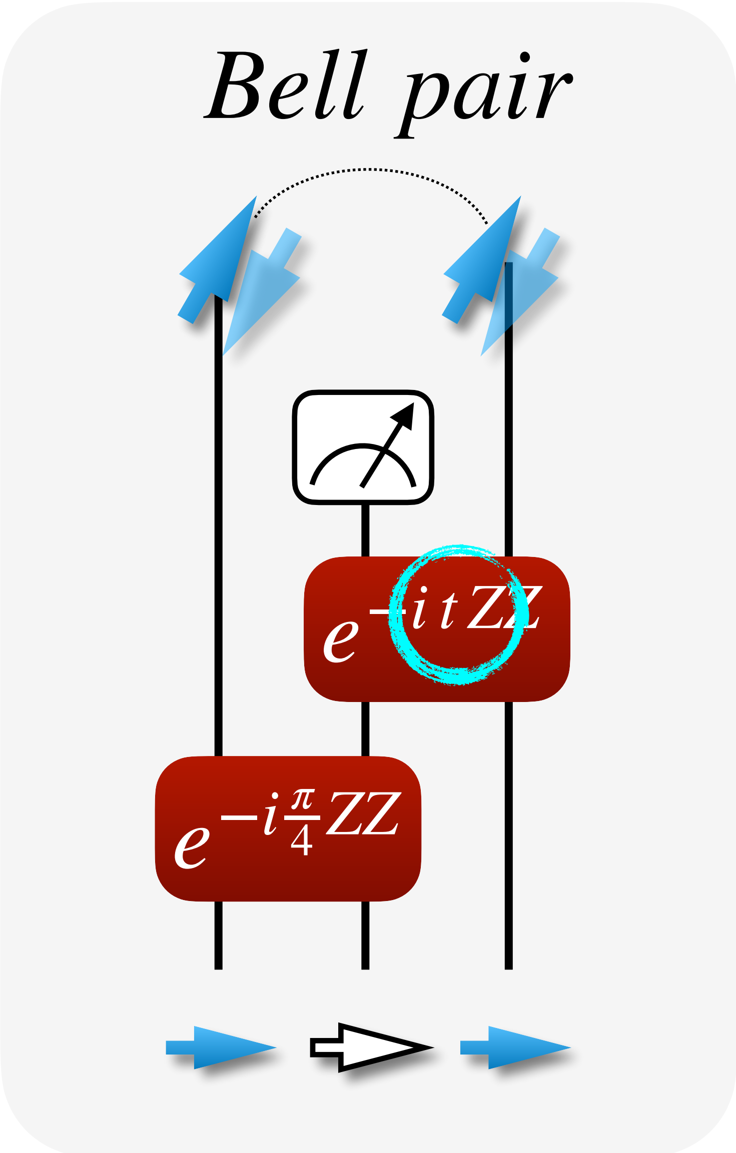
Kitaev spin liquid

- **non-commuting**
- sequential
- dynamics

Nishimori's cat



Nishimori's cat

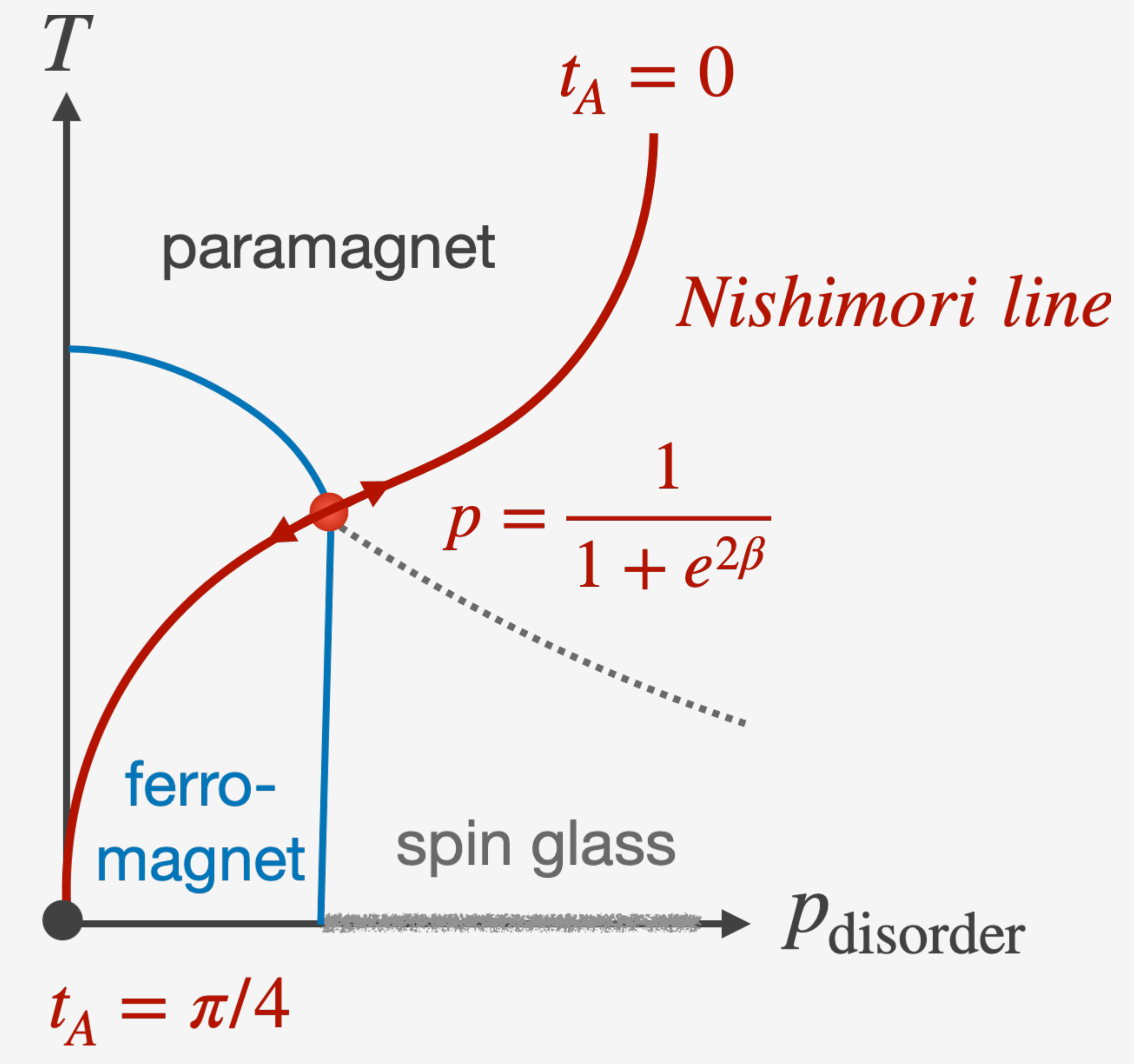


weak measurement

strong measurement

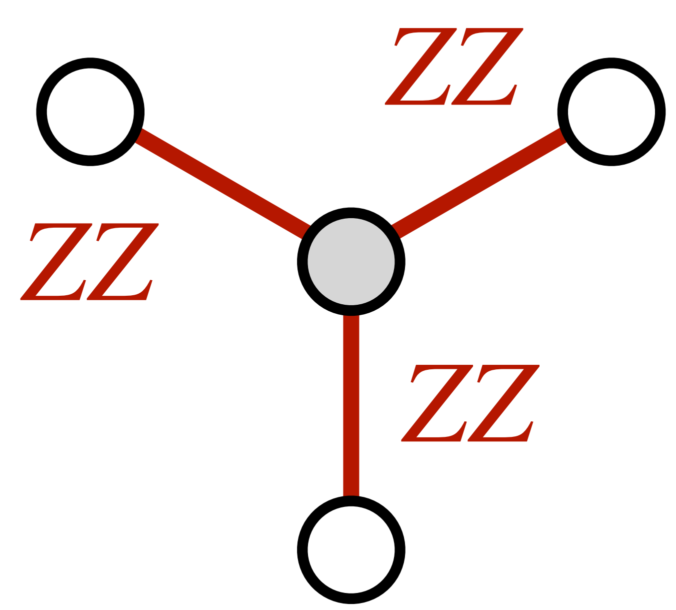
thermal fluctuations and disorder are **locked**

Nishimori (1981)



theory – Phys. Rev. Lett. 131, 200201 (2023)
experiment (IBM) – arXiv:2309.02863 (2023)

commuting vs non-commuting measurements



Nishimori's cat

- **commuting**
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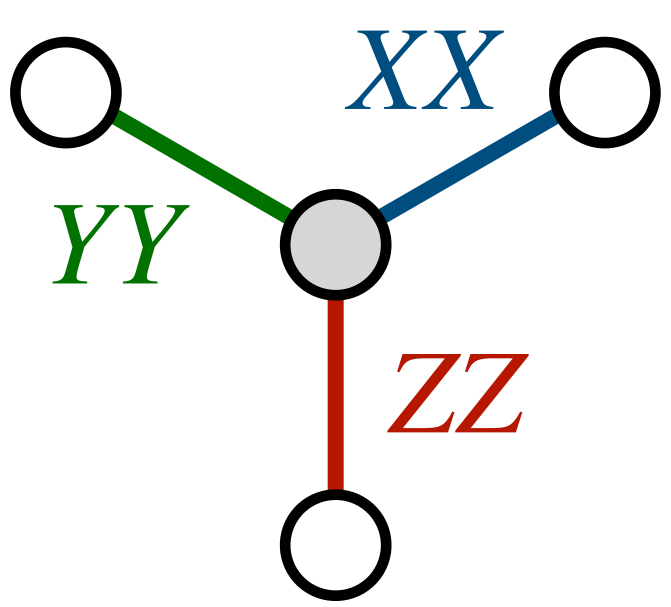
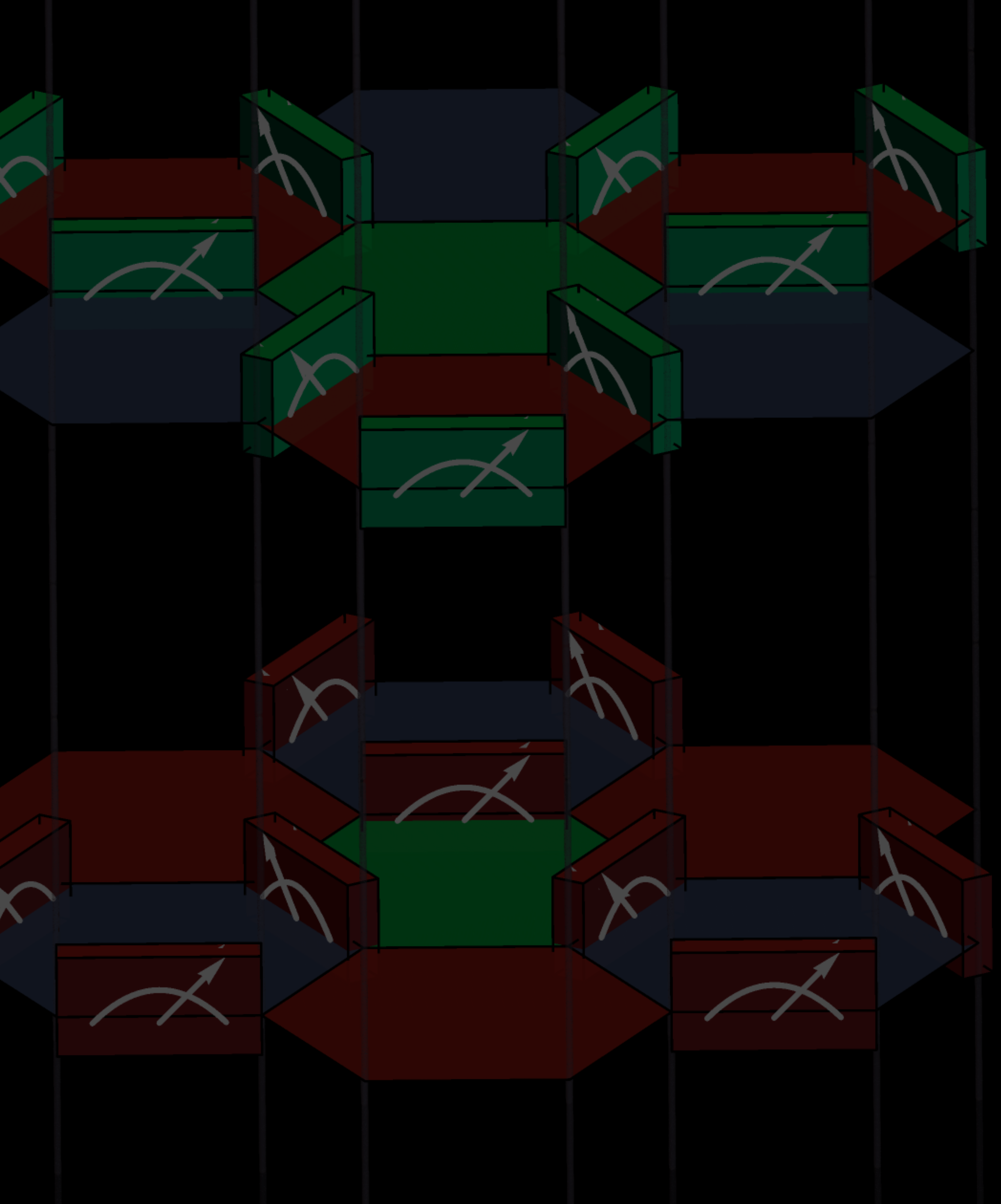


Kitaev spin liquid

- **non-commuting**
- sequential
- dynamics



Kitaev circuits

frustration and entanglement

$\mathcal{H}$

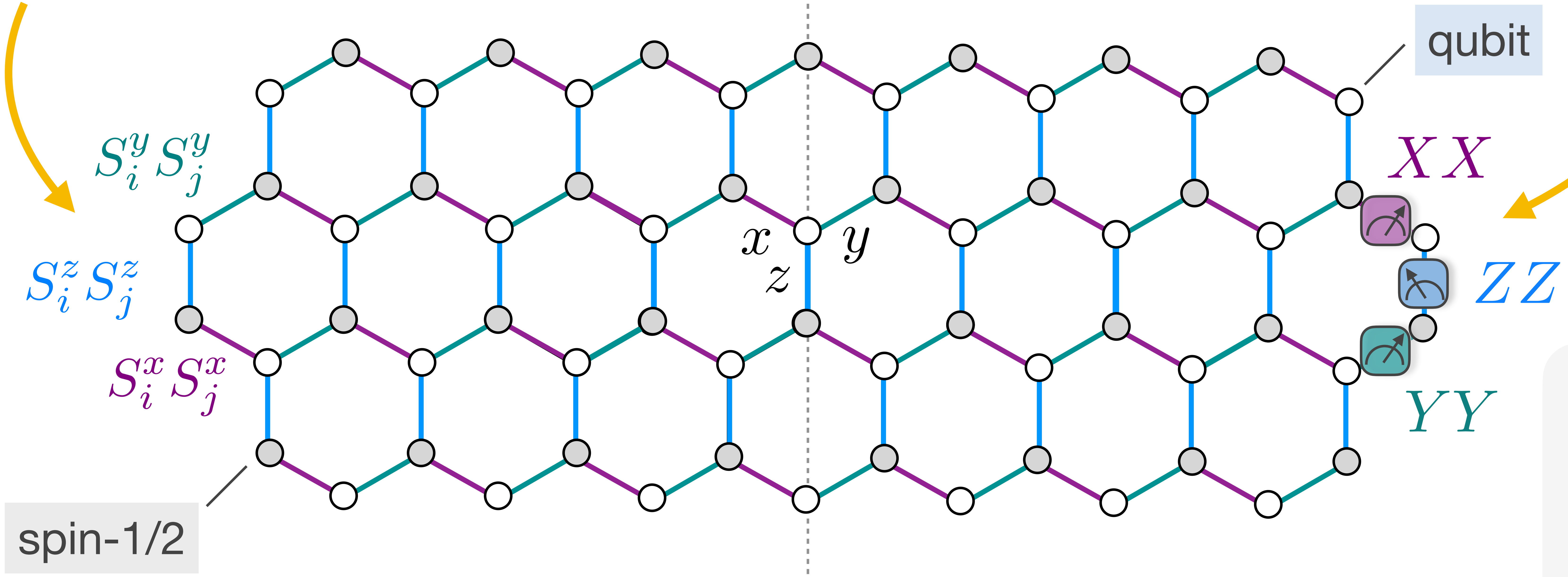
magnetic exchange

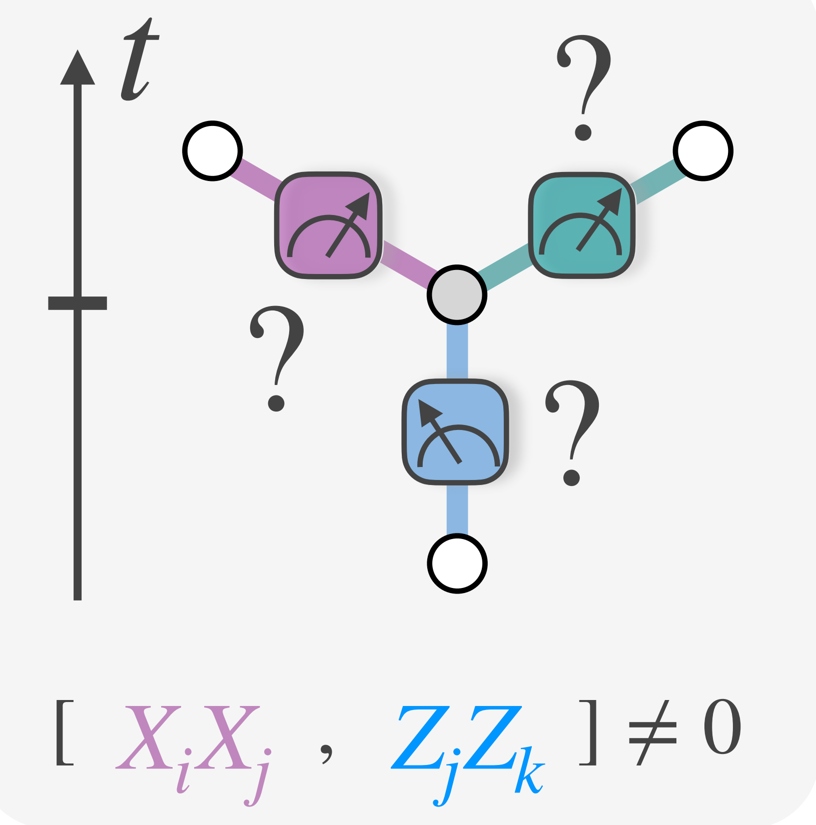
Kitaev (2006)

Hamiltonian

measurement

Clifford gates





ground state

minimizes global energy

dynamical state

eigenstate of measurements

no state can satisfy every local interaction

frustration

non-commuting operators

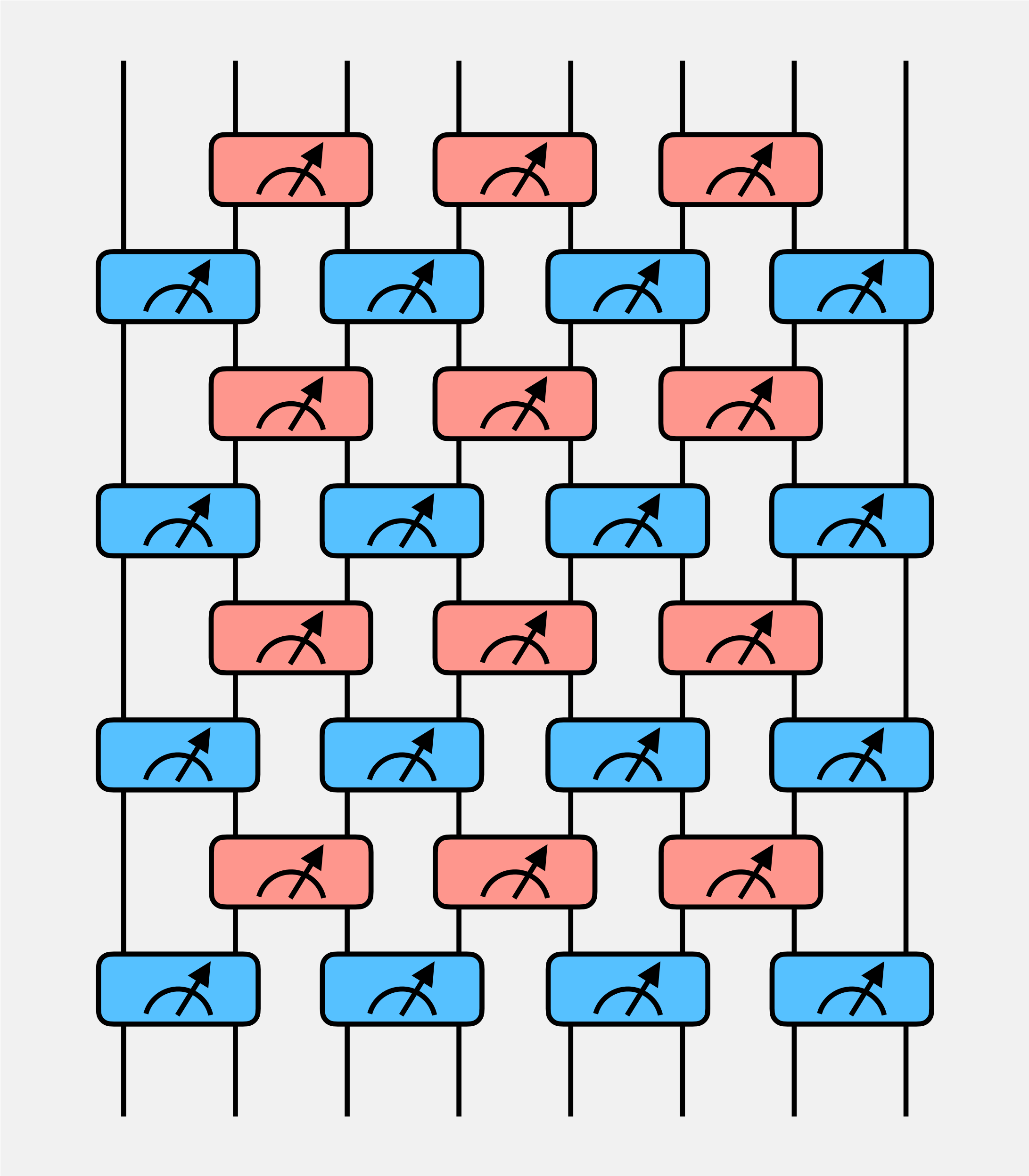
- cannot be measured simultaneously
- will be over-written

imaginary time vs. measurement-only

Hamiltonian ground state

$$e^{-\beta H}|\psi_0\rangle$$

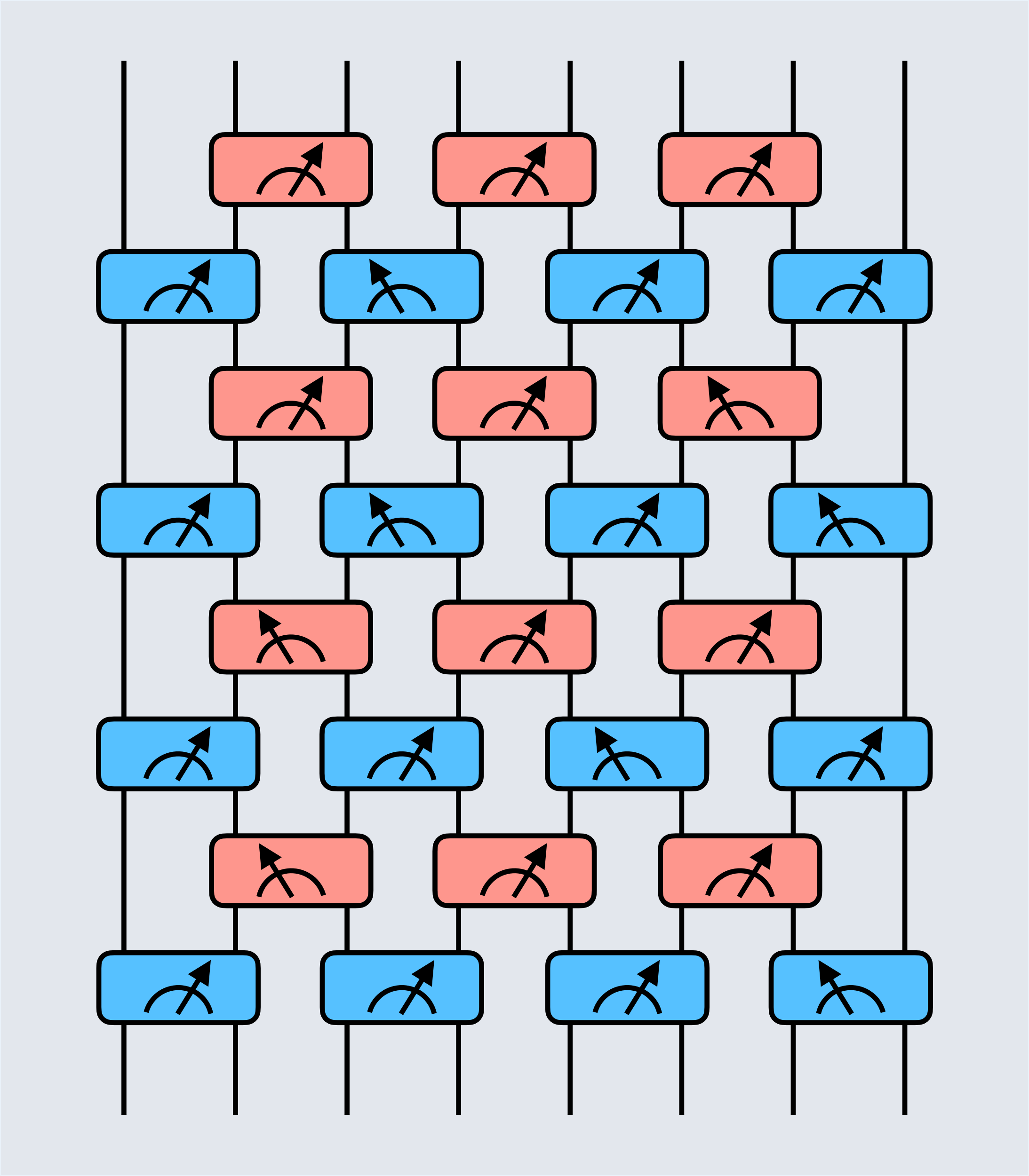
$e^{-\tau H_r}$



- **brickwall** circuit
- **no** disorder

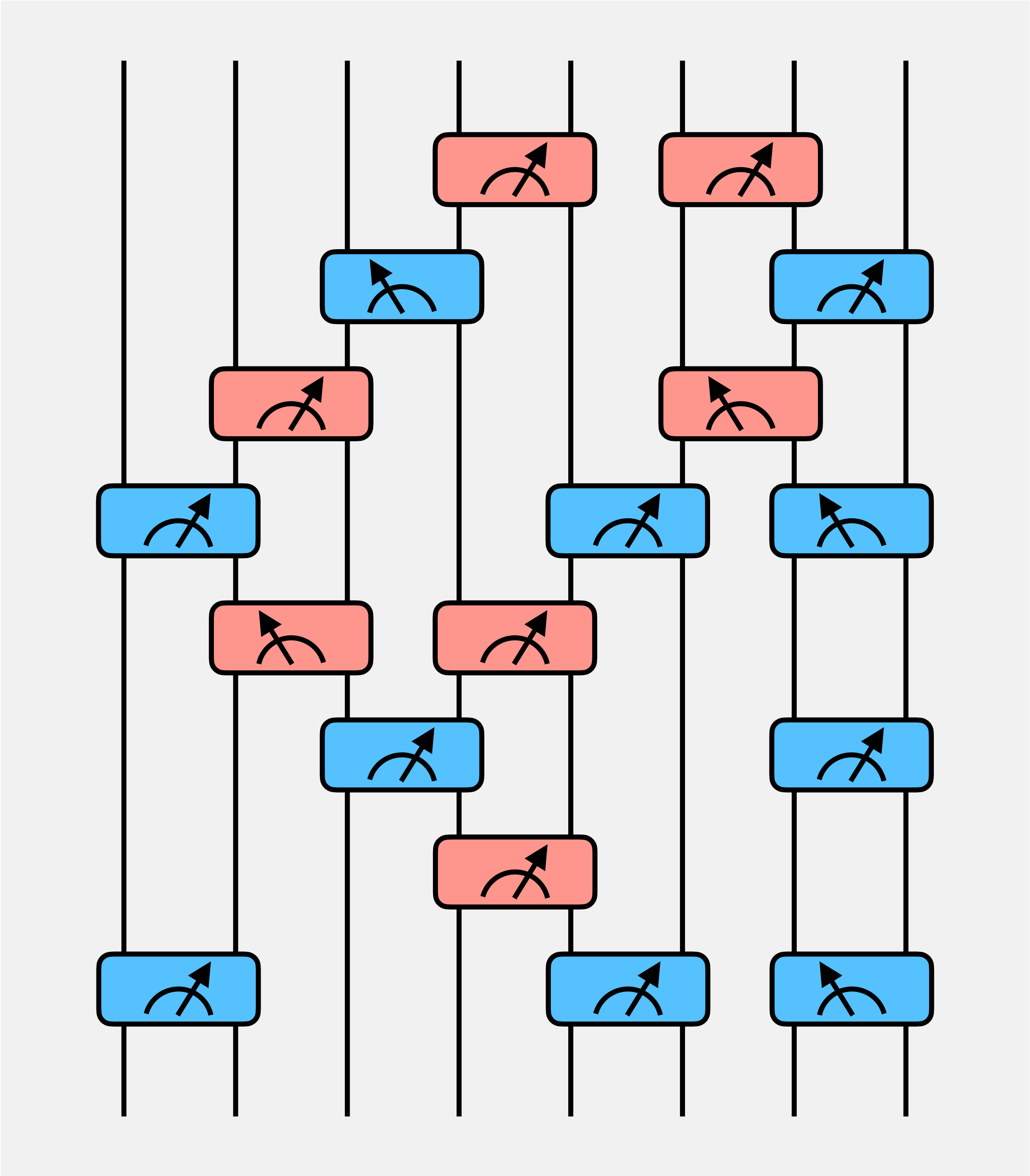
Floquet weak measurement

$$\left(e^{\mp\tau H_r}\cdots e^{\mp\tau H_0}\right)|\psi_0\rangle$$



- **brickwall** circuit
- **Born** disorder

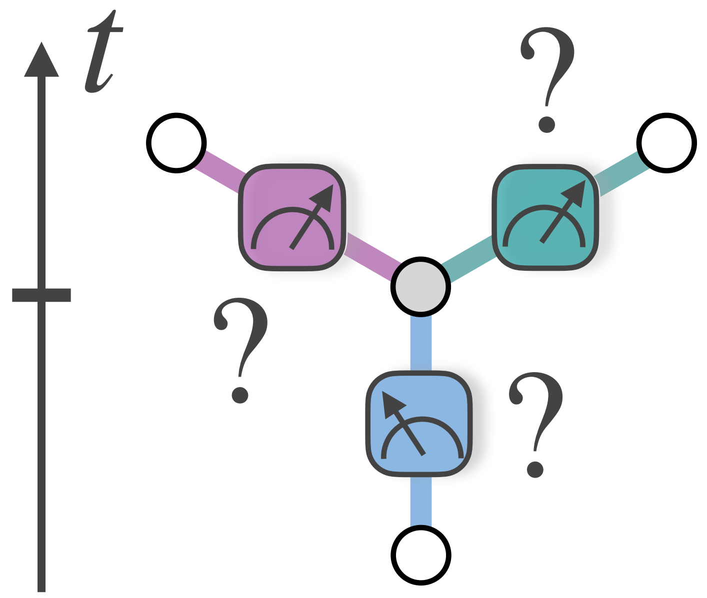
random weak/strong measurement

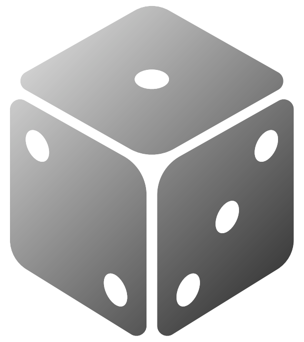


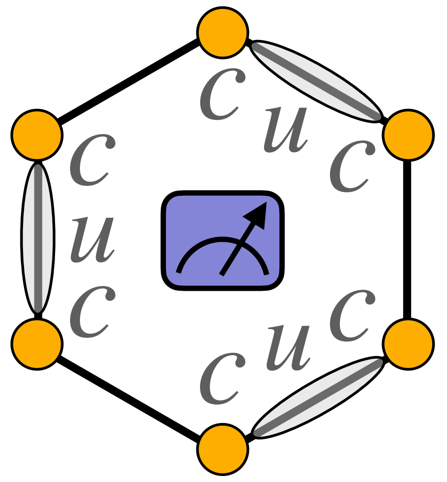
- **stochastic** circuit
- **Born** disorder

random projective Kitaev measurements

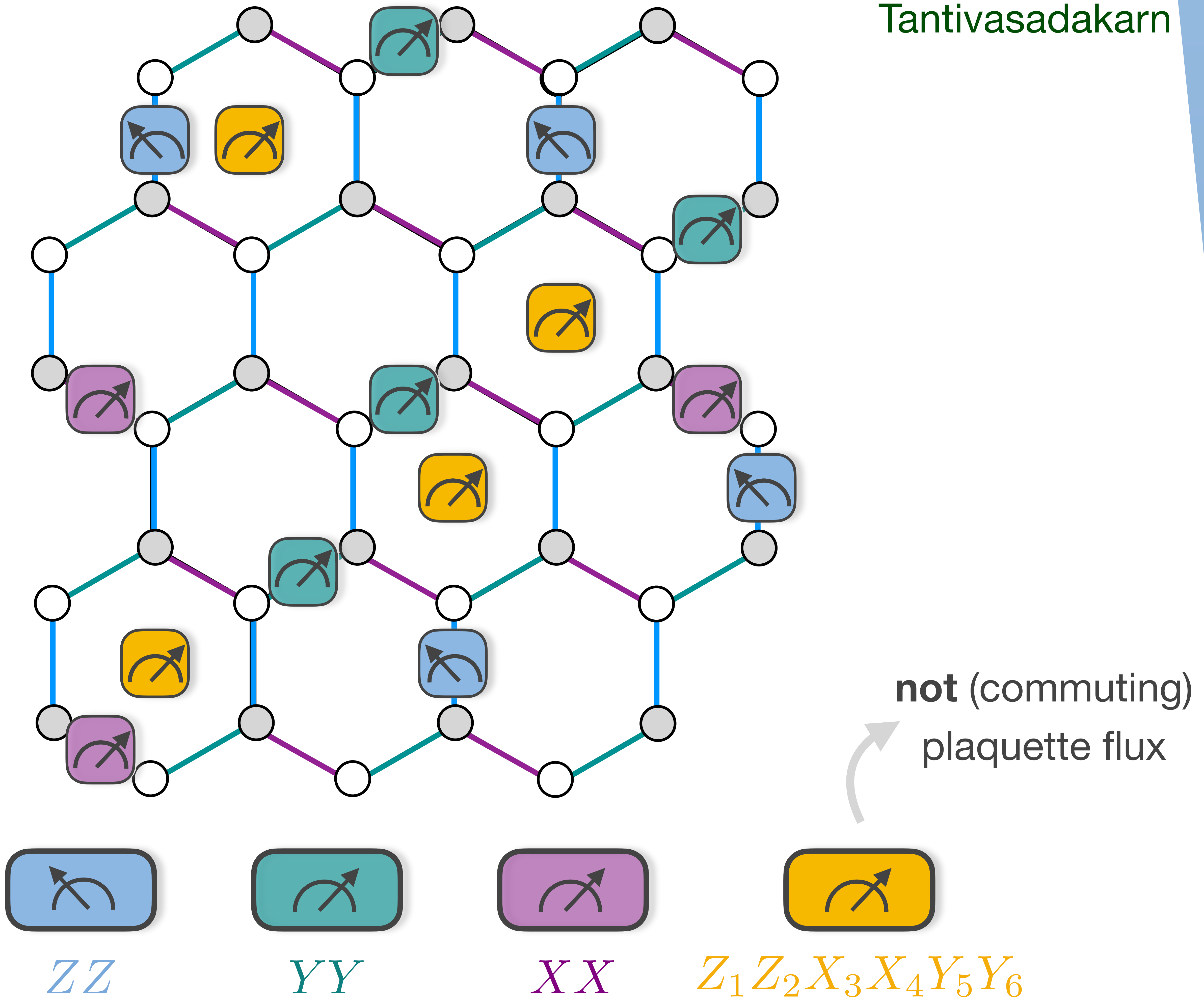
Nathanan Tantivasadakarn







Majorana interaction
→ Majorana surface code

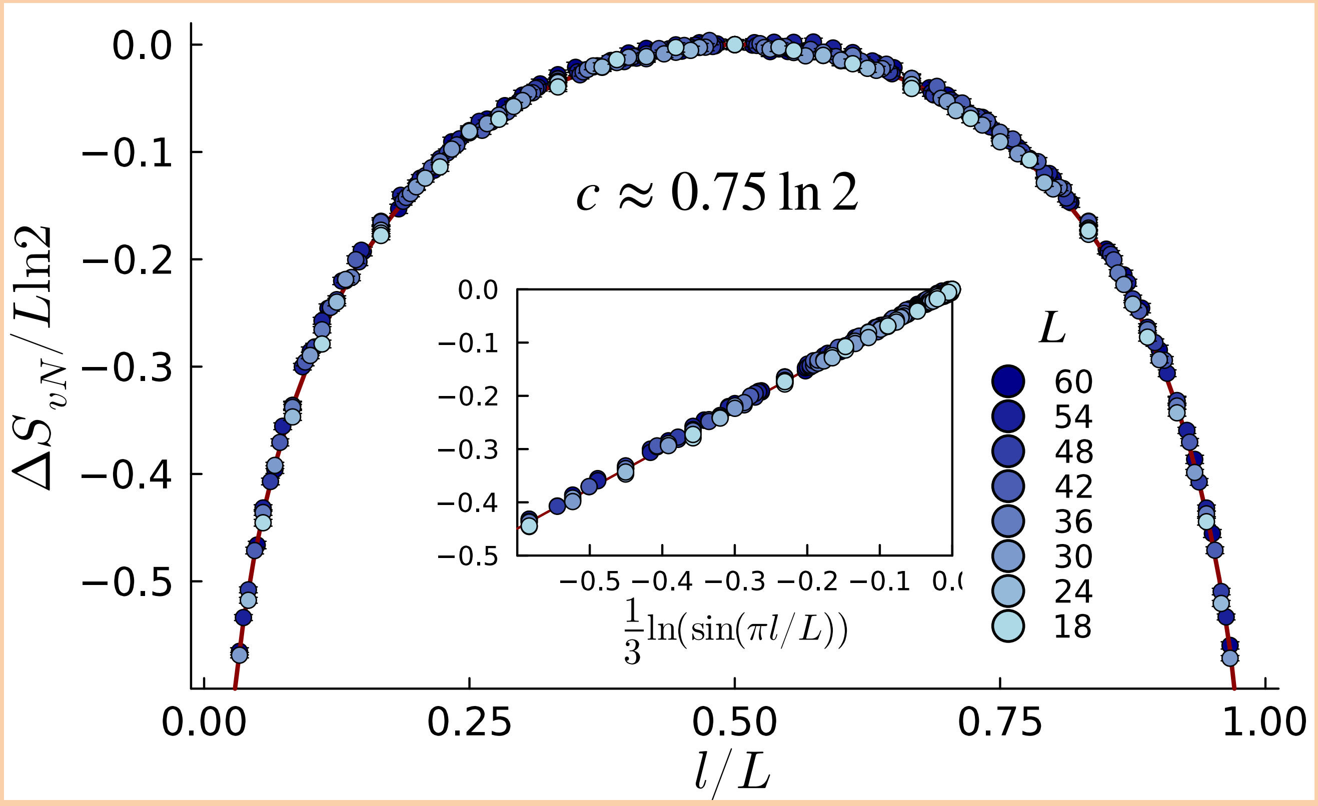


Clifford circuit

even **interacting** problem can be simulated in polynomial time (in Heisenberg picture)

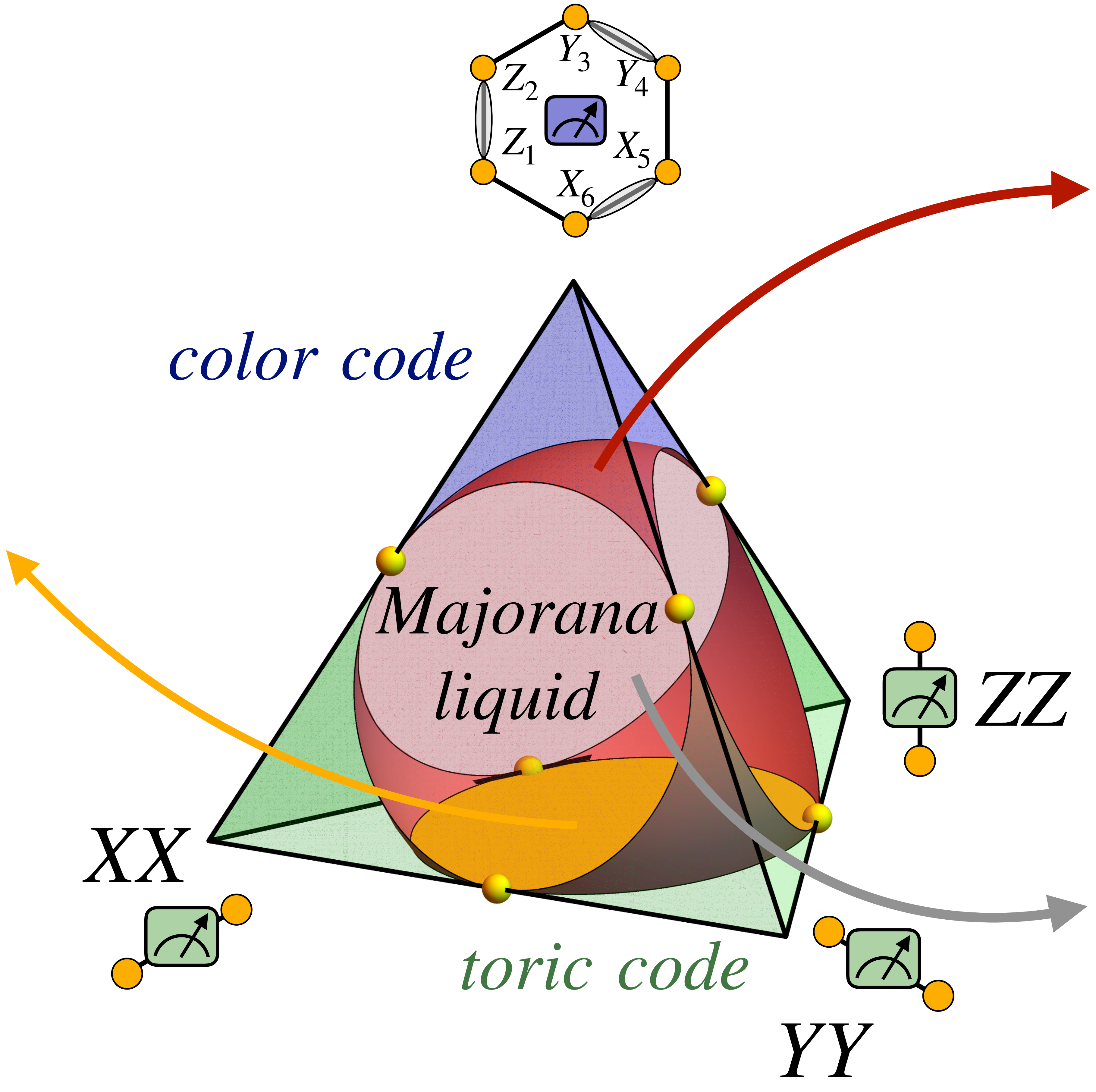
Nahum, Skinner 2020; Lavasani, Luo, Vijay 2022; Sriram, Rakovszky, Khemani, Ippoliti 2022; **Zhu, Tantivasadakarn, ST 2023: + Majorana interaction**

entanglement phase diagram

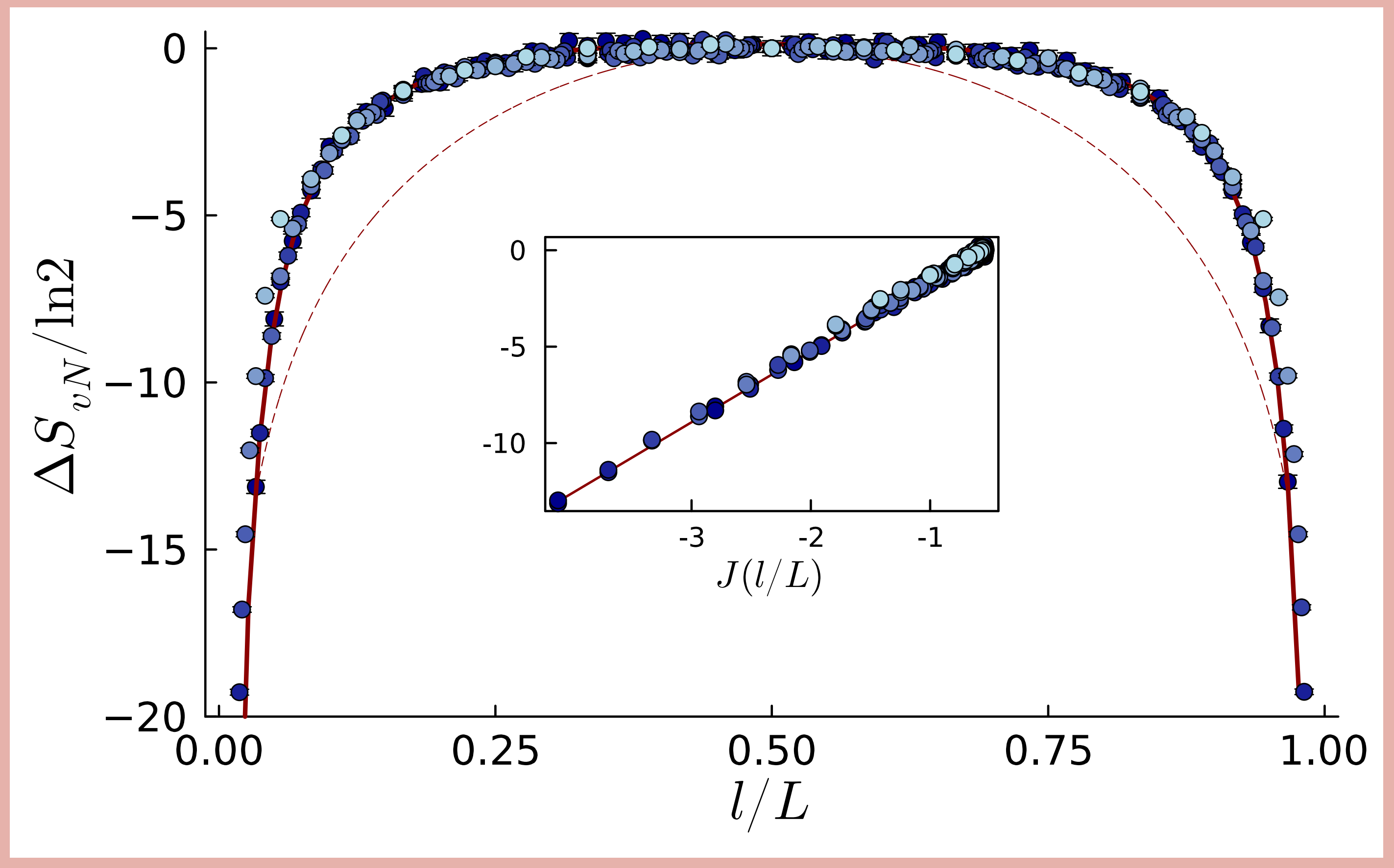


Fermi-surface scaling

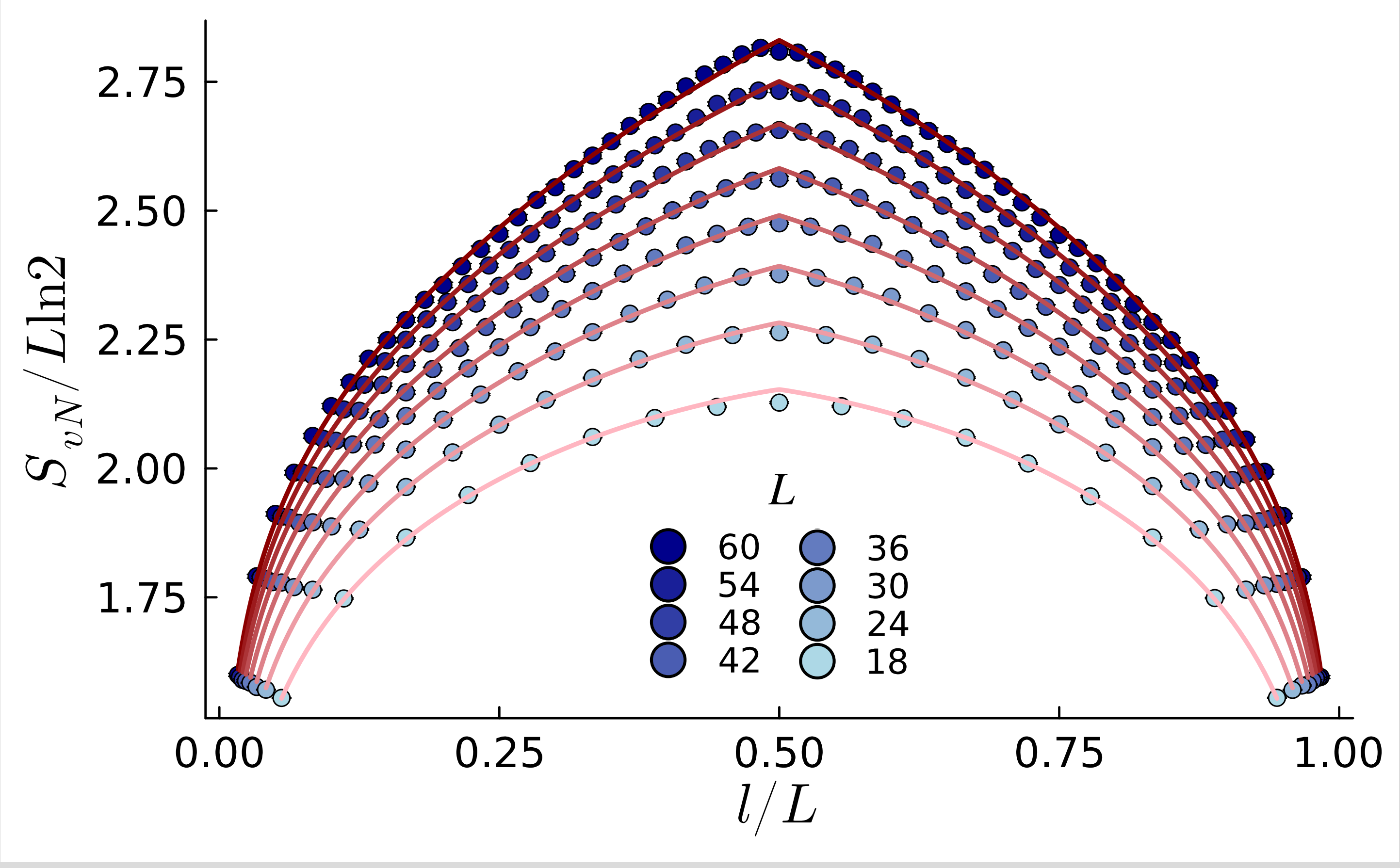
$$S_{vN}(l,L) = \frac{cL}{3}\ln\left(\frac{L}{\pi}\sin\frac{\pi l}{L}\right) + \cdots$$



quantum Lifshitz scaling



volume – law + L ln L



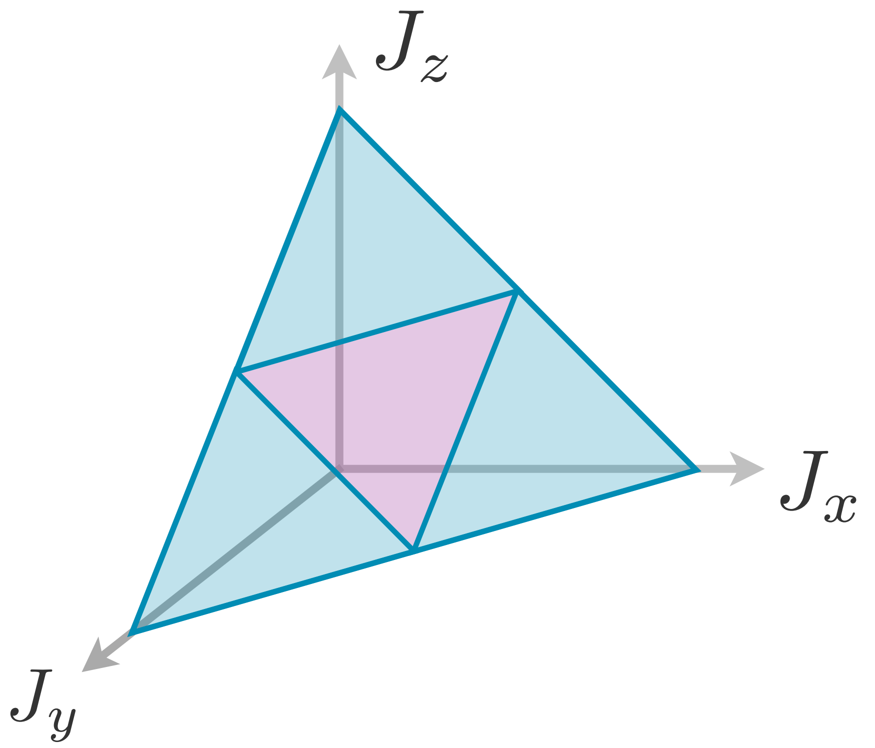
Nahum, Skinner 2020; Lavasani, Luo, Vijay 2023; Sriram, Rakovszky, Khemani, Ippoliti 2023

Zhu, Tantivasadakarn, ST 2023: + Majorana interaction

side remark: computational complexity

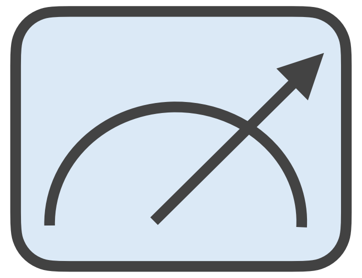
dynamics	**free** Majorana fermions	**interacting** Majorana fermions

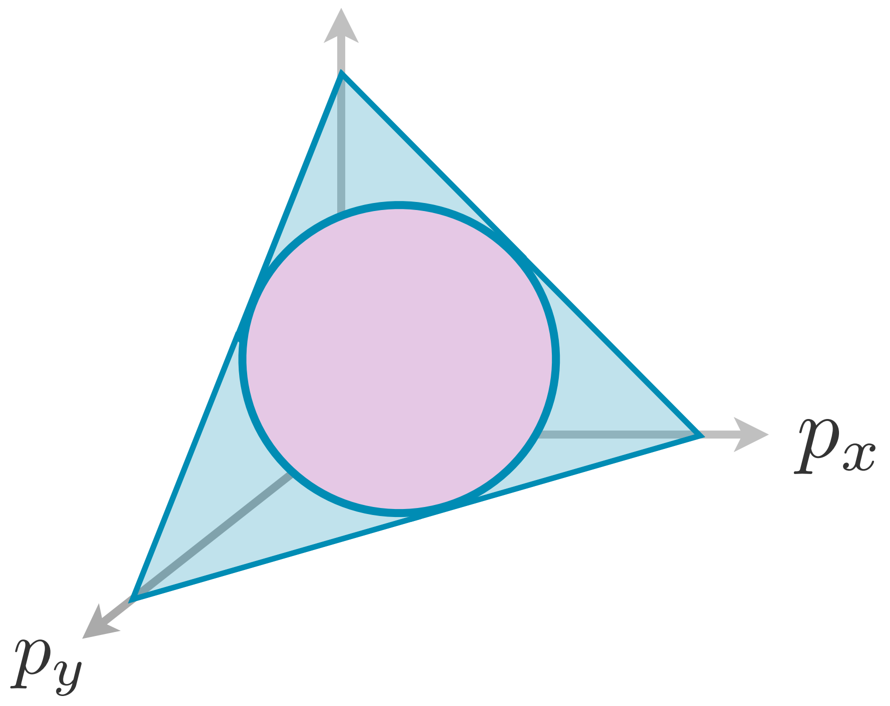
$\mathcal{H}$



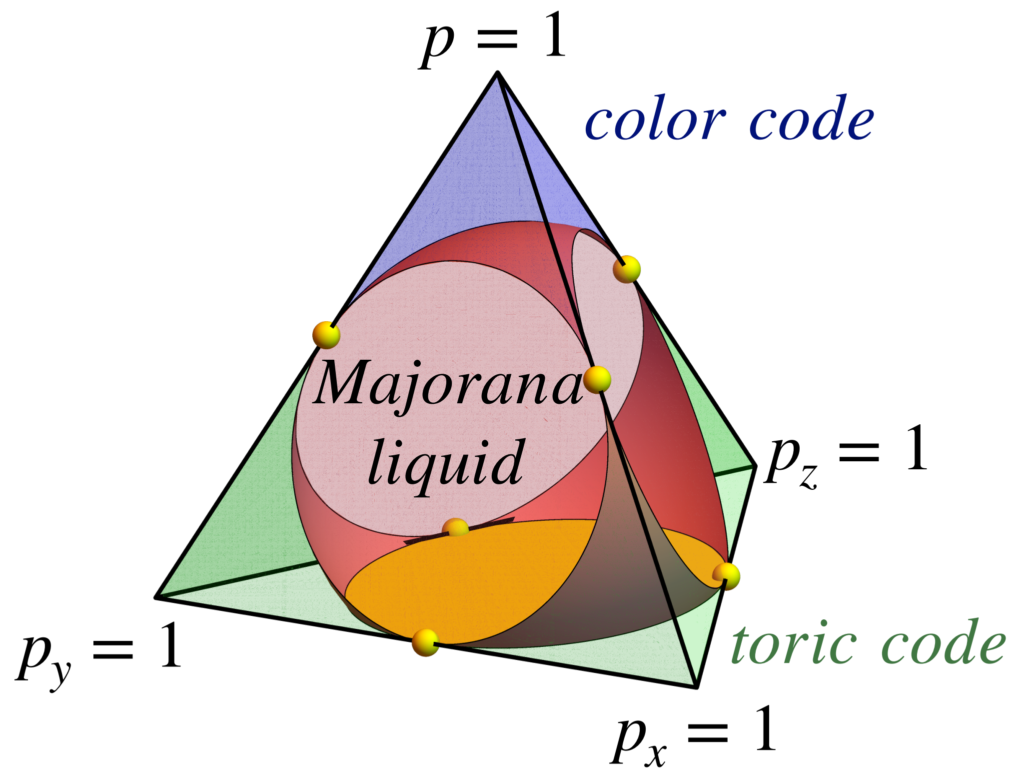
?

sign problem



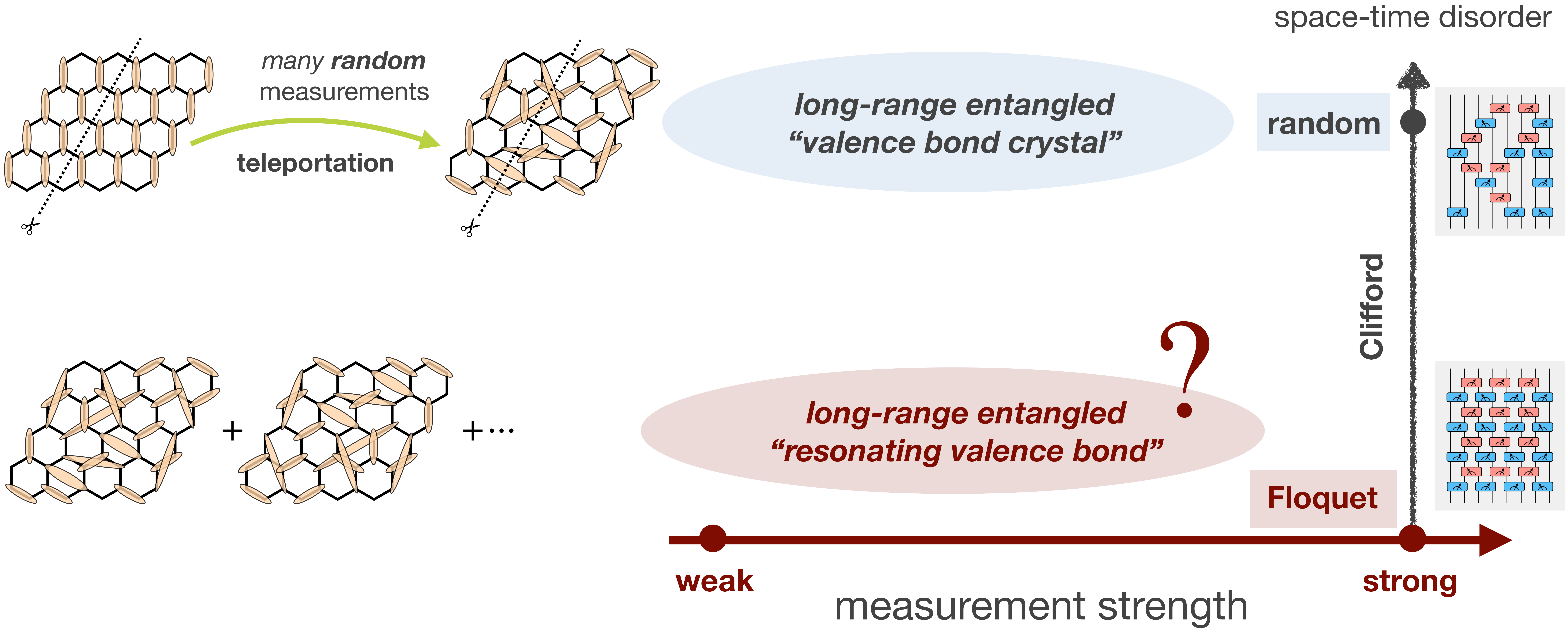


Clifford circuits

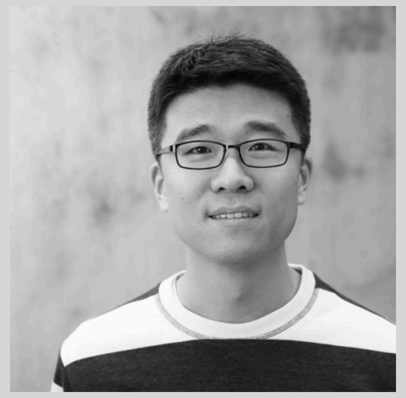


Gottesman-Knill theorem, quant-ph/9807006

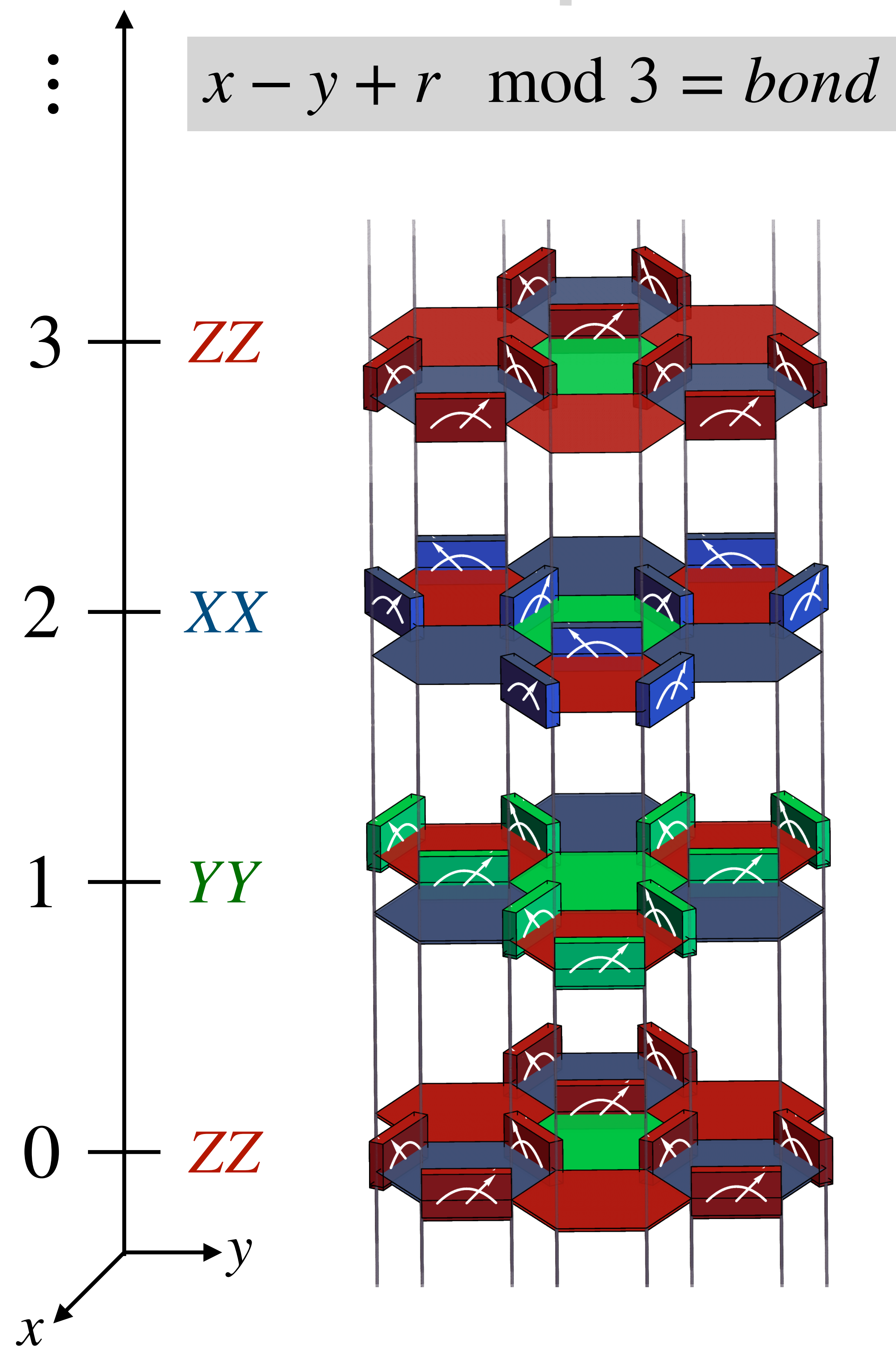
measurement, teleportation, and beyond



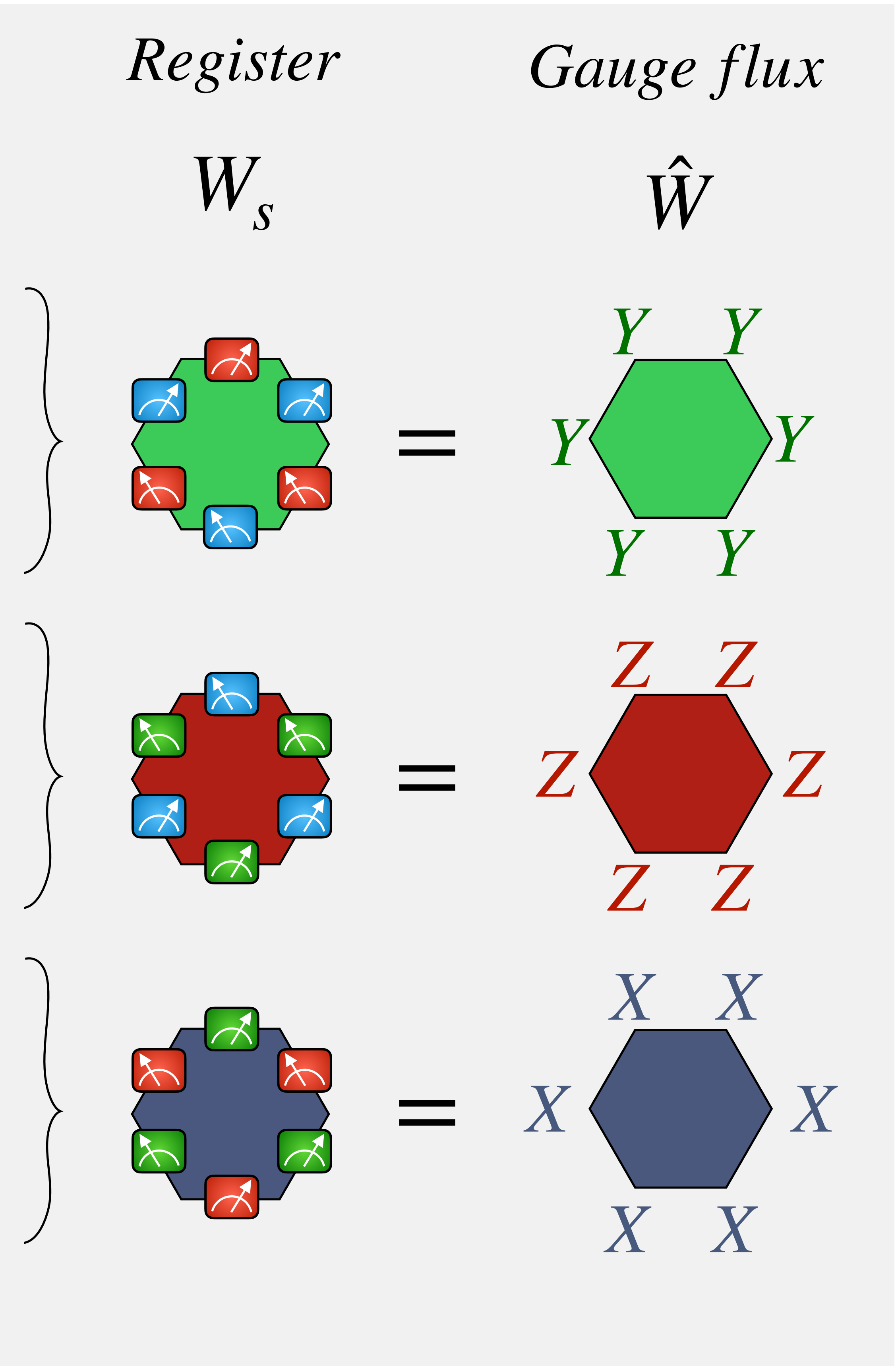
dynamical protocol

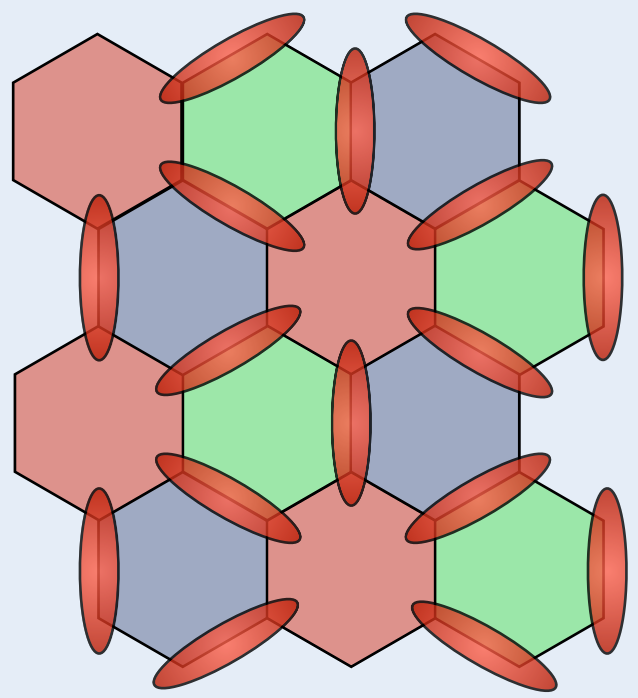


Hastings, Haah (2021)



$\rho_0 \propto \mathbb{1}$

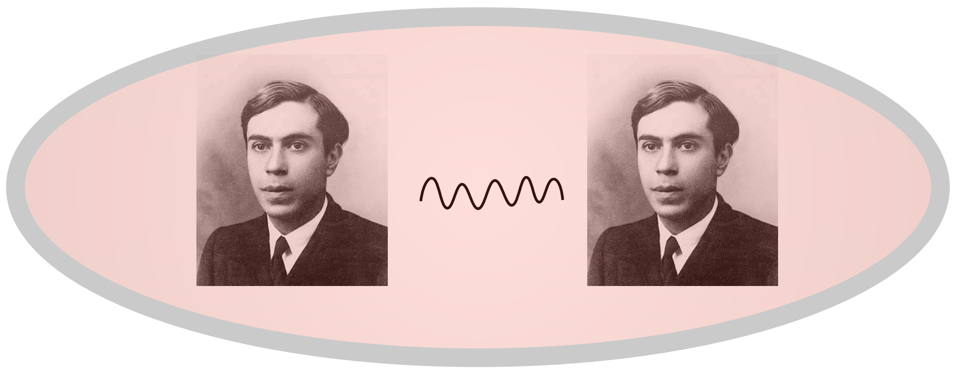




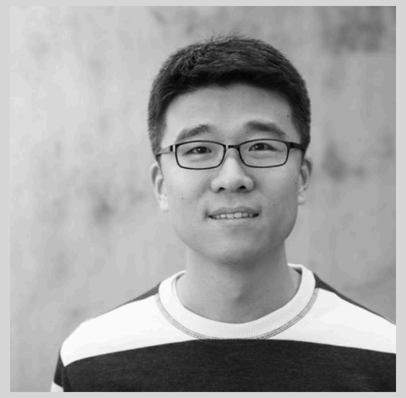
$H = \pm\infty ZZ \quad \pm YY \quad \pm XX$

Gauge flux → a glassy toric code

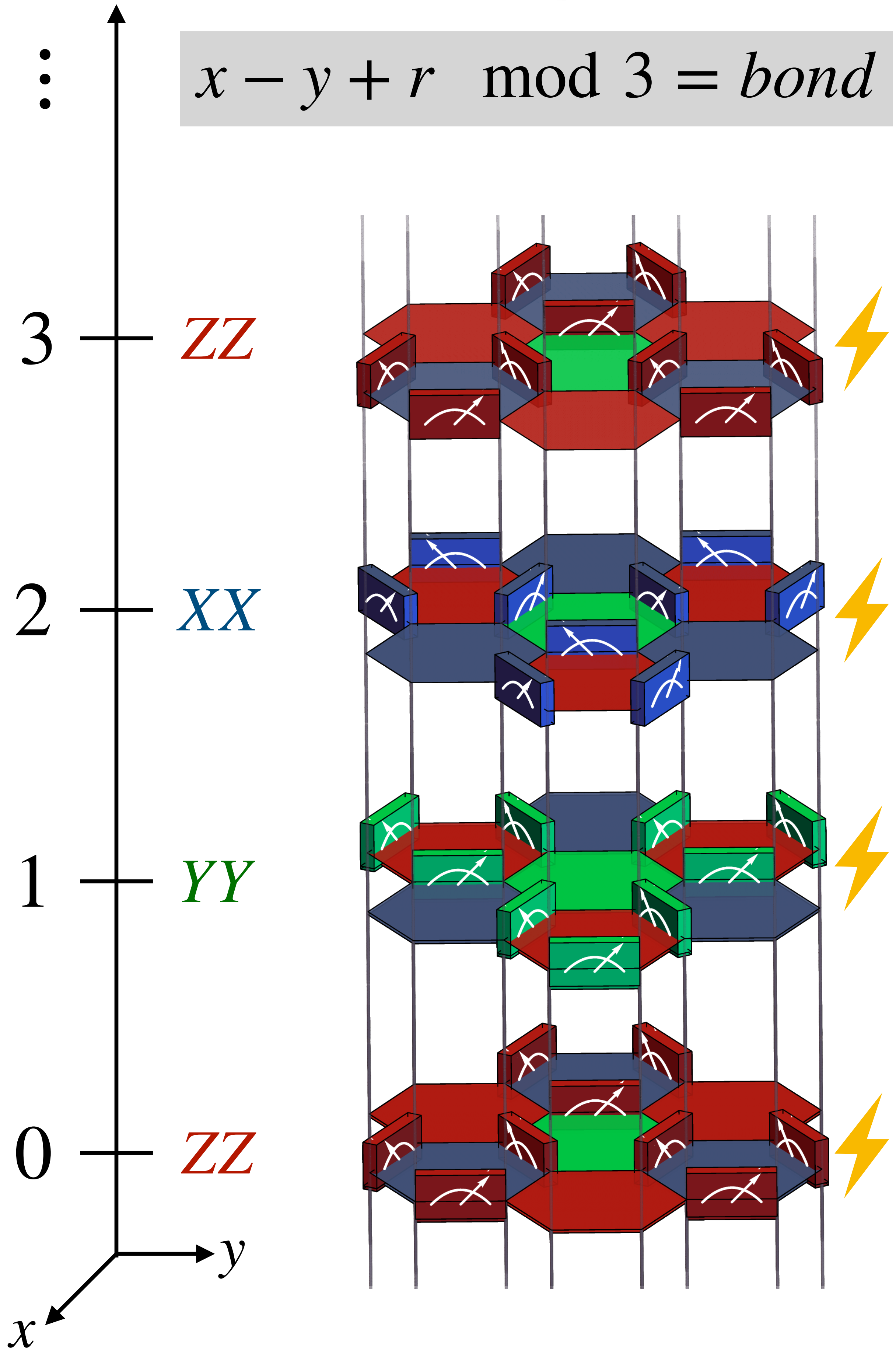
Majoranas are confined in *hard-core* dimers



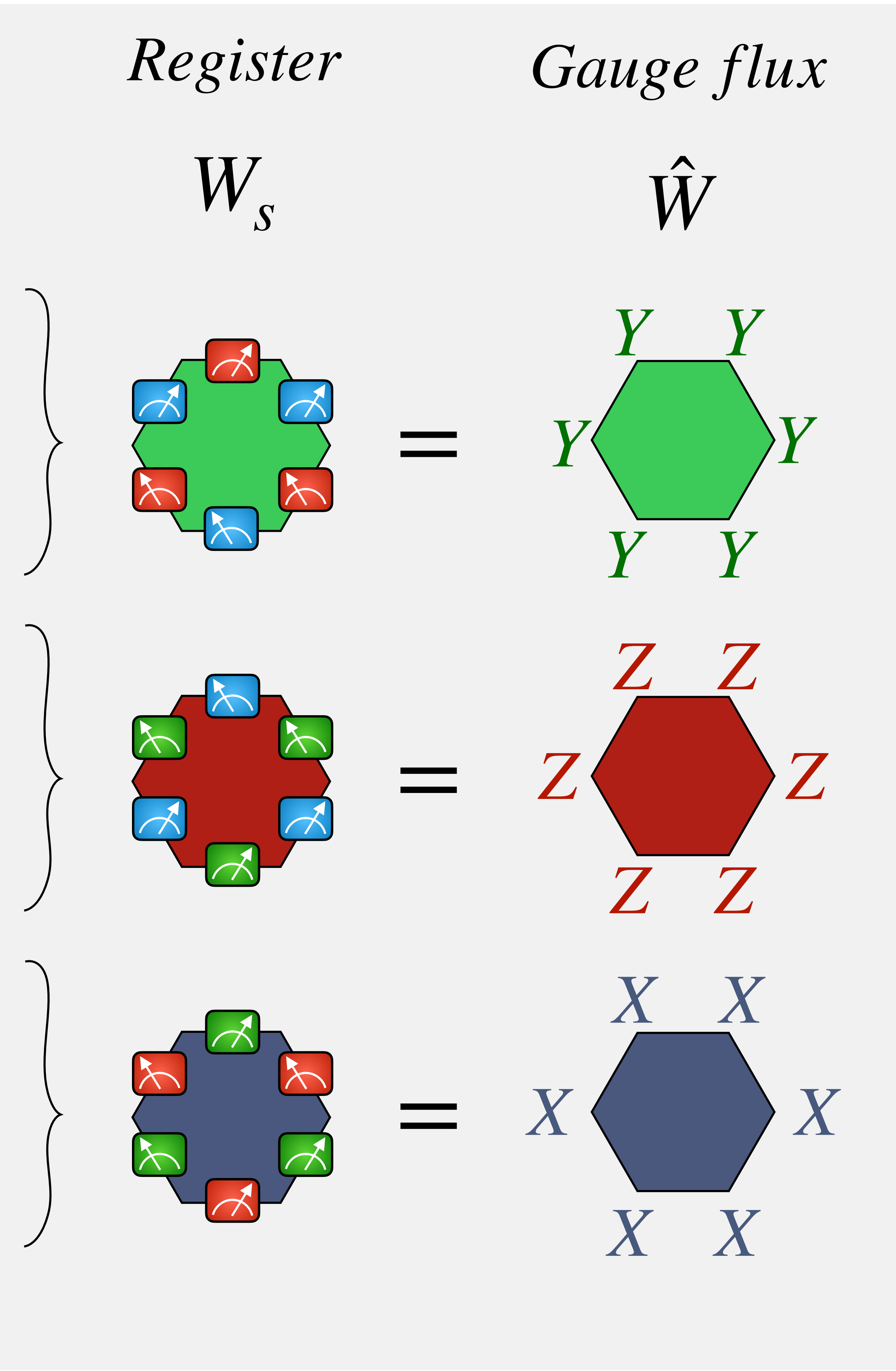
dynamical protocol

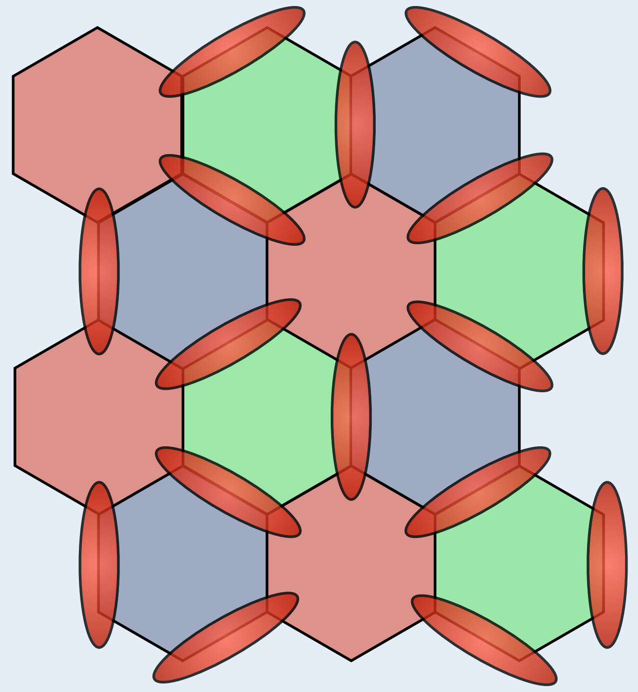



Hastings, Haah (2021)



$\rho_0 \propto \mathbb{1}$





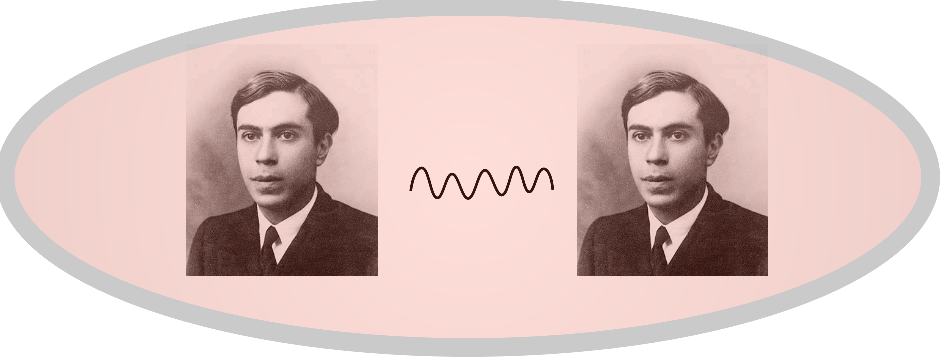
$H = \pm\infty ZZ \quad \pm YY \quad \pm XX$

Gauge flux → a glassy toric code

Majoranas are confined in *hard-core* dimers

Questions:

- How to liberate Majorana?
- Stability of the code?

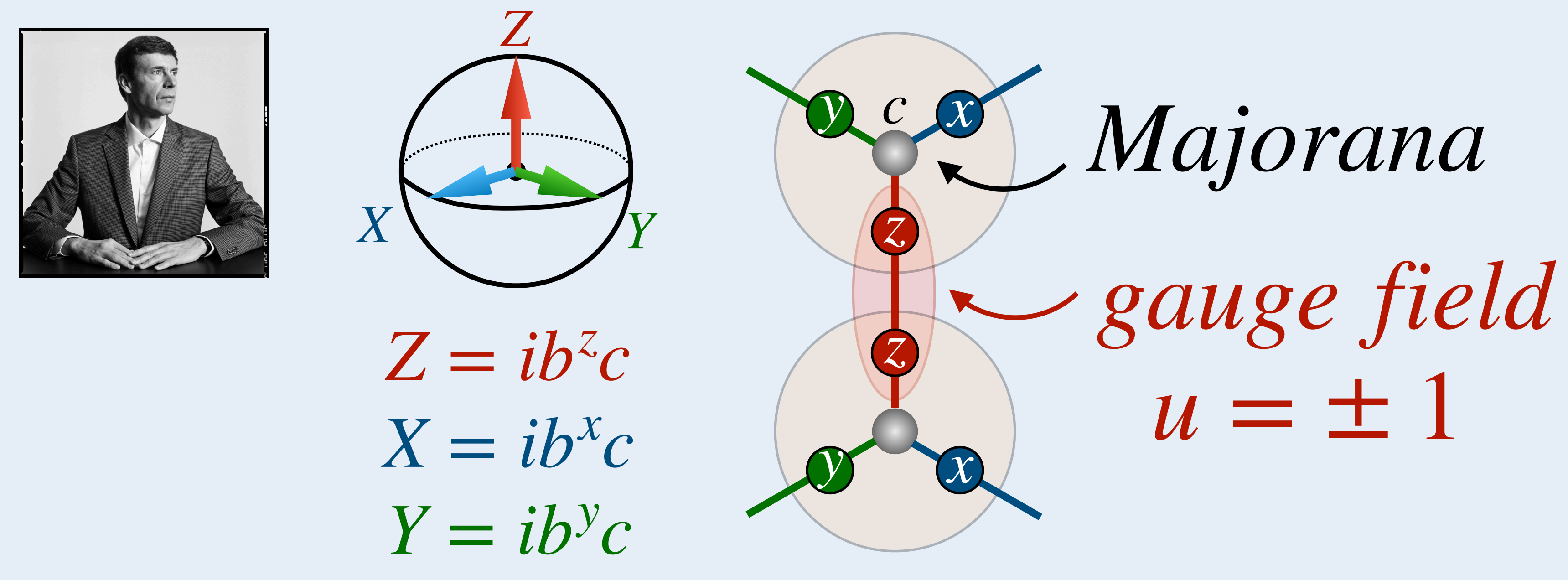


coherent error / weak measurement →

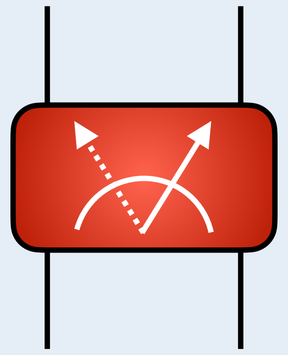
soften dimers – a channel for Majorana to escape !

Majorana, flux pillars, loops

Guo-Yi Zhu



$\exp(-\tau s ZZ) \quad (s = \pm 1)$

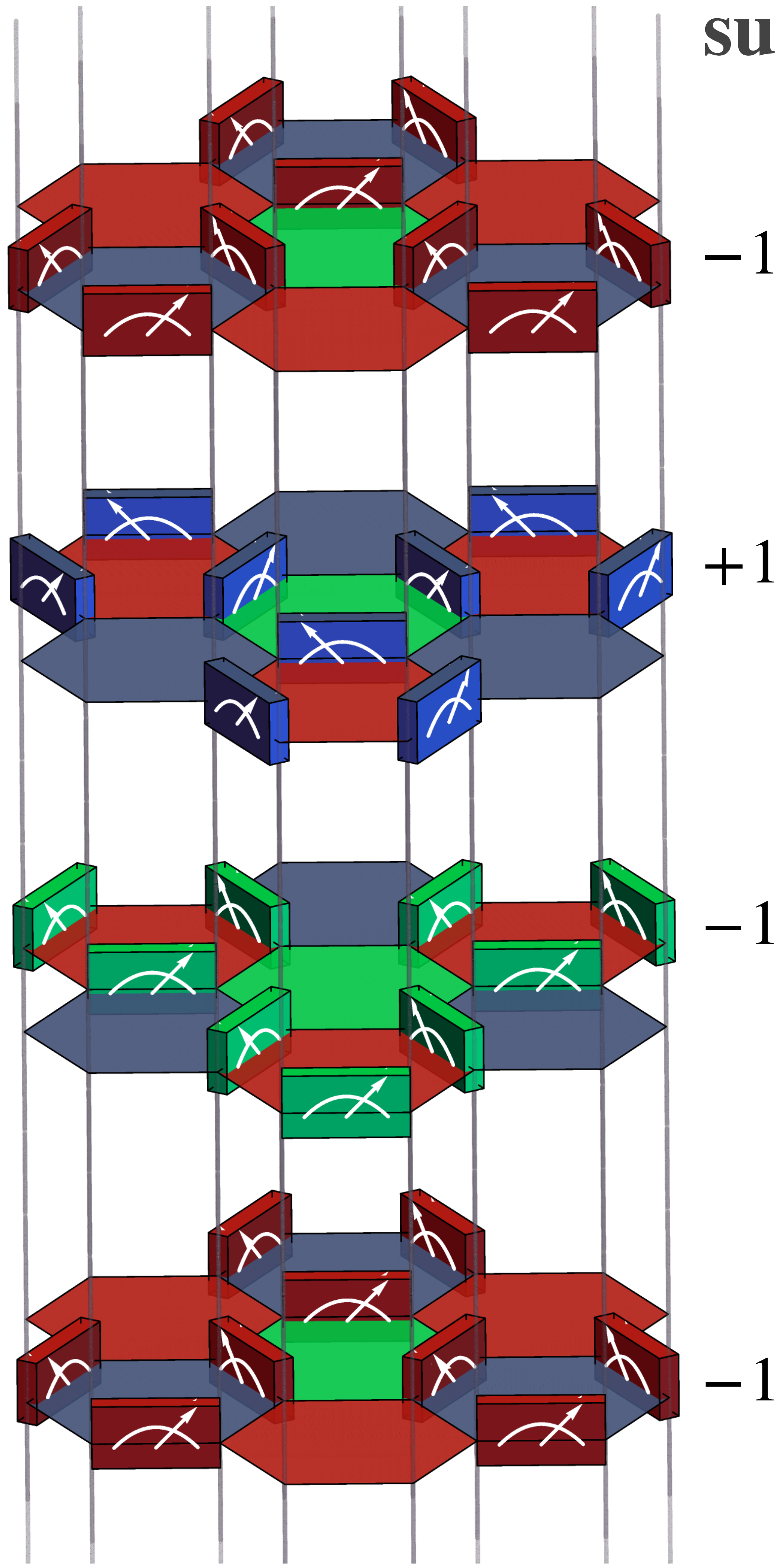


$\exp\left(-\tau \quad (su) \quad ic_A c_B\right)$

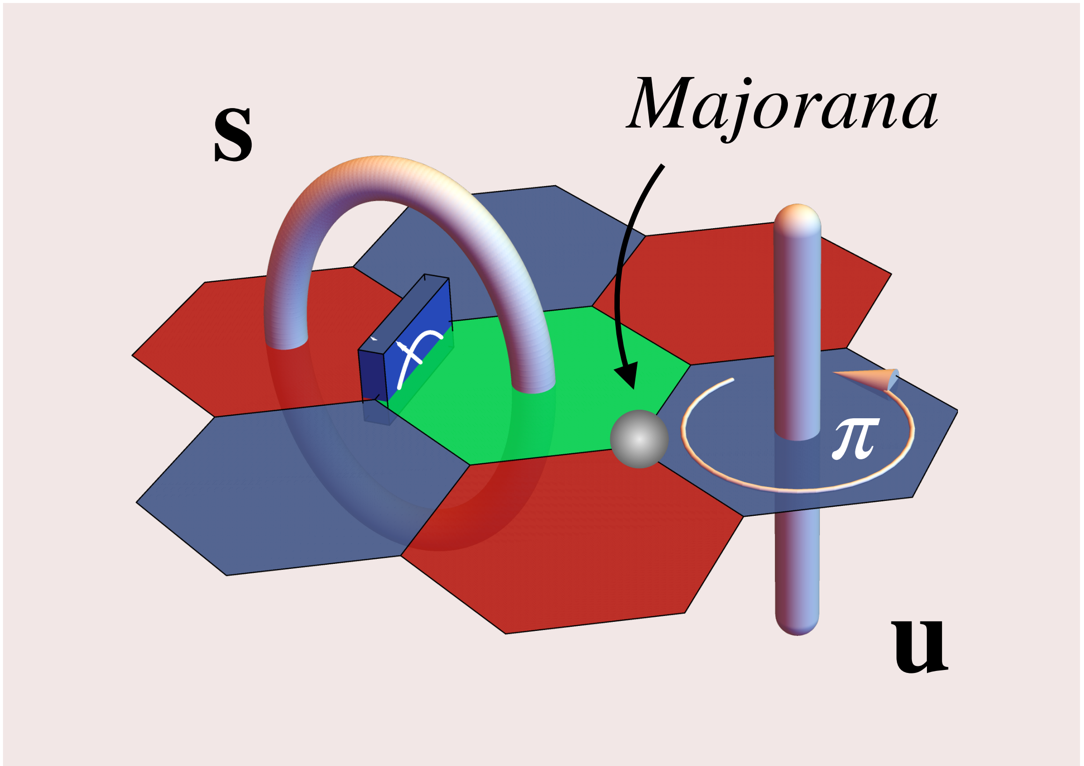
measurement strength
$\tau = \tanh^{-1}(\sin(2t))$

net gauge field

Majorana bilinear



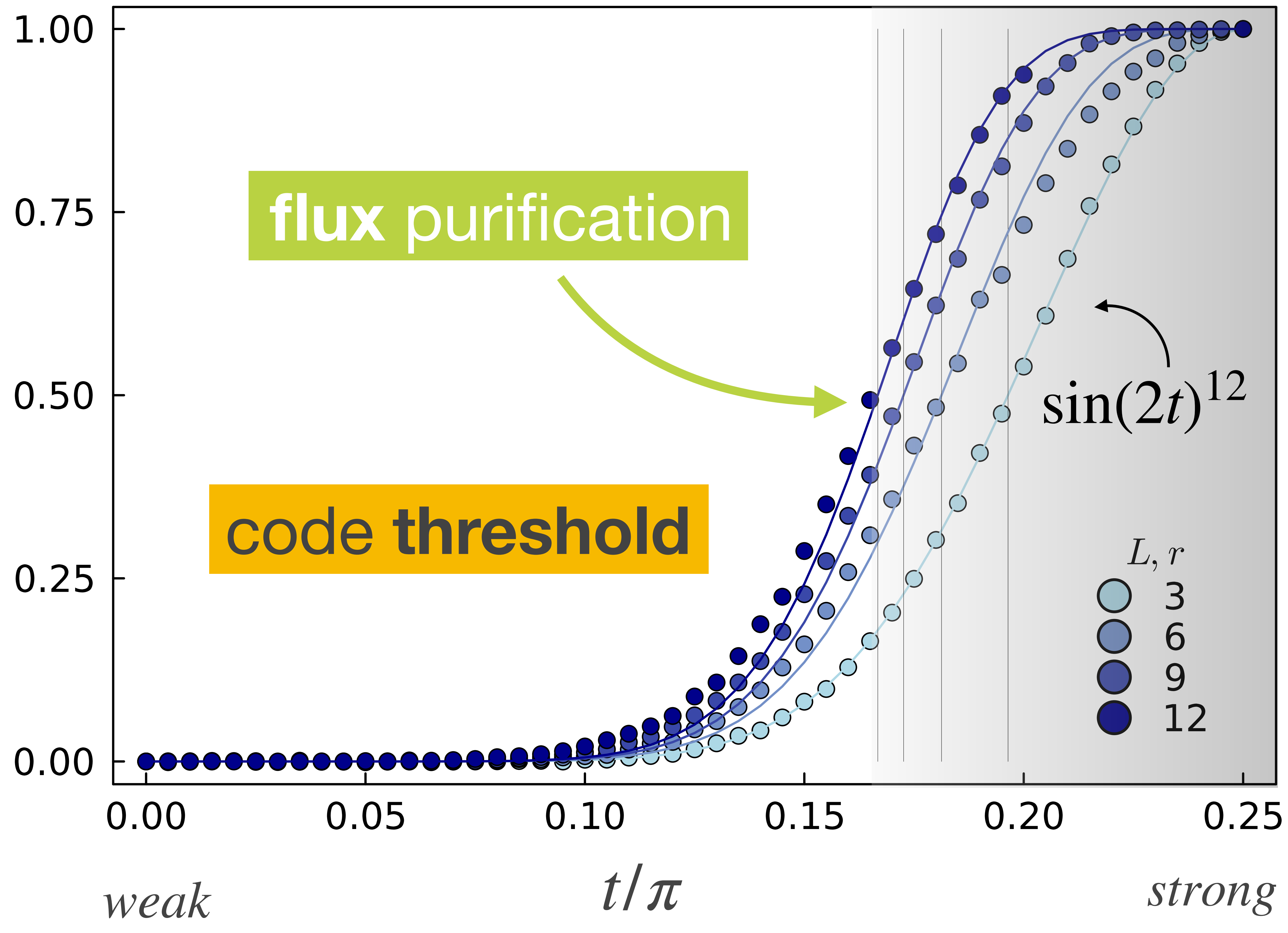
random Gaussian fermion circuit
conditioned on
gauge trajectory su



Born probability
=
Majorana partition function

purification of fluxes

flux variation $[\langle \hat{W} \rangle^2]$



- **flux** expectation value

quantum average

$$[\langle \hat{W} \rangle] = 0$$

measurement average

- **Edwards-Anderson** order parameter

$$[\langle \hat{W} \rangle^2] = \sum_{\mathbf{s}} P(\mathbf{s}) \langle \hat{W} \rangle_{\mathbf{s}}^2 = \sum_q \sum_{\mathbf{s},\mathbf{u}} \frac{p_{\mathbf{s}} \cdot p_{\mathbf{su}}}{P(\mathbf{s})} \left(\prod_{l \in q} u_l \right)$$

- **exponential** purification

$$S_u := -\log_2 \frac{1 + [\langle \hat{W} \rangle^2]}{2} \approx \left(-\log_2 \frac{1 + \sin(2t)^{12}}{2} \right)^{\frac{r+1}{4}}$$

but there is more – double-peaks

circuit depth

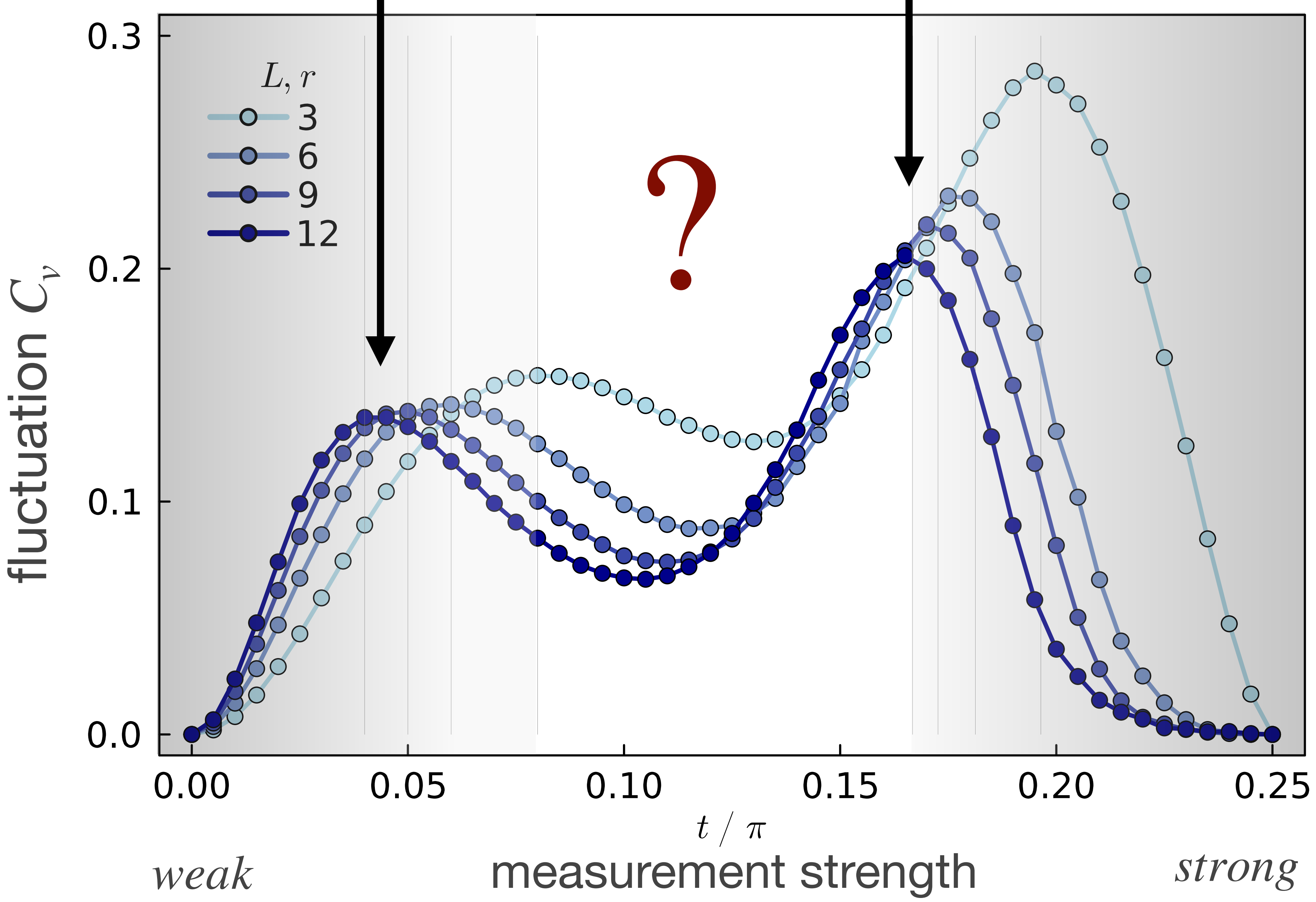
$$\rho_{\mathbf{su}} \propto \exp\left(-\frac{\beta}{4}\mathbf{c}H_{\mathbf{su}}\mathbf{c}\right)$$

effective Hamiltonian

weak measurement-only circuit

qubit fractionalization

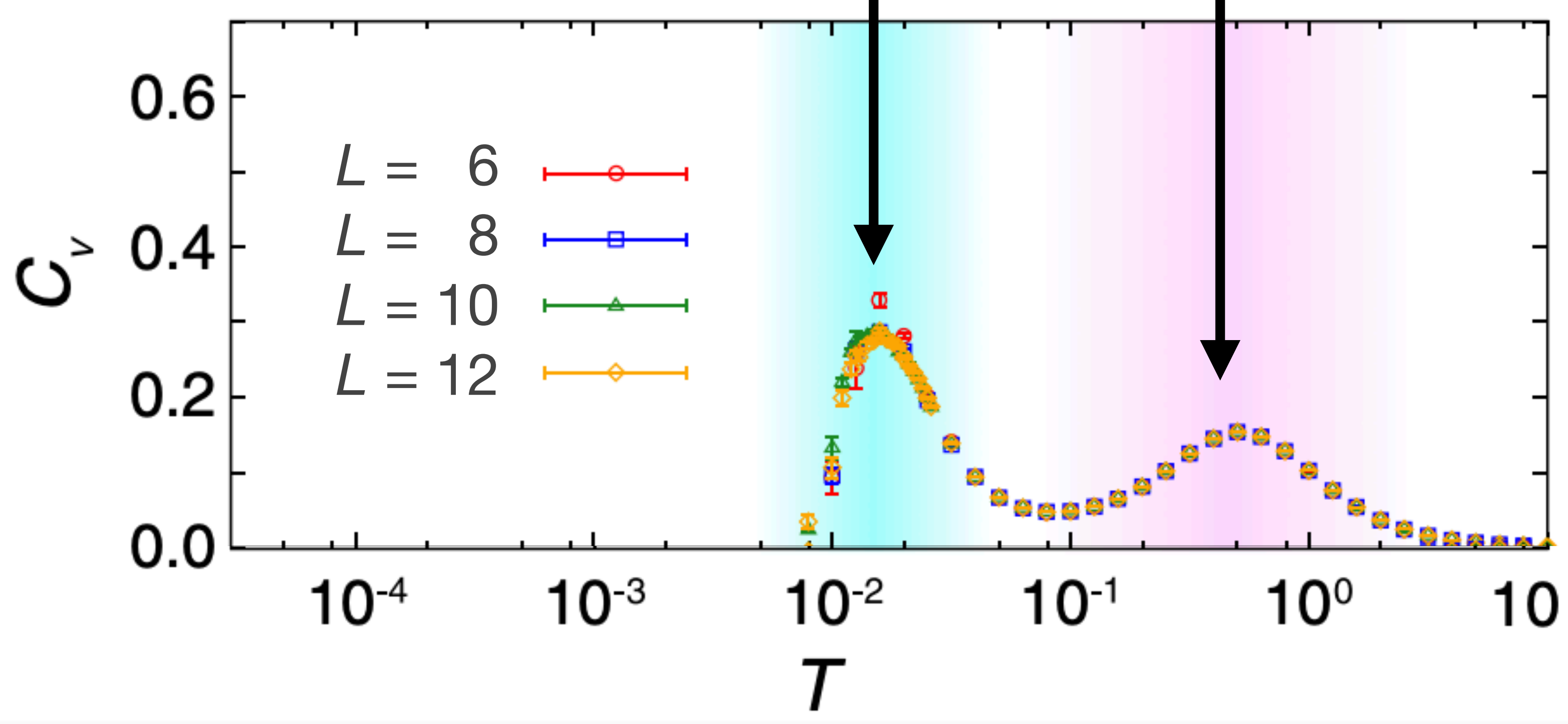
flux purification



Hamiltonian at finite temperature

flux purification

spin fractionalization



Nasu, Udagawa, Motome, 2014

PRL 113, 197205 (2014) PHYSICAL REVIEW LETTERS week ending 7 NOVEMBER 2014

Vaporization of Kitaev Spin Liquids

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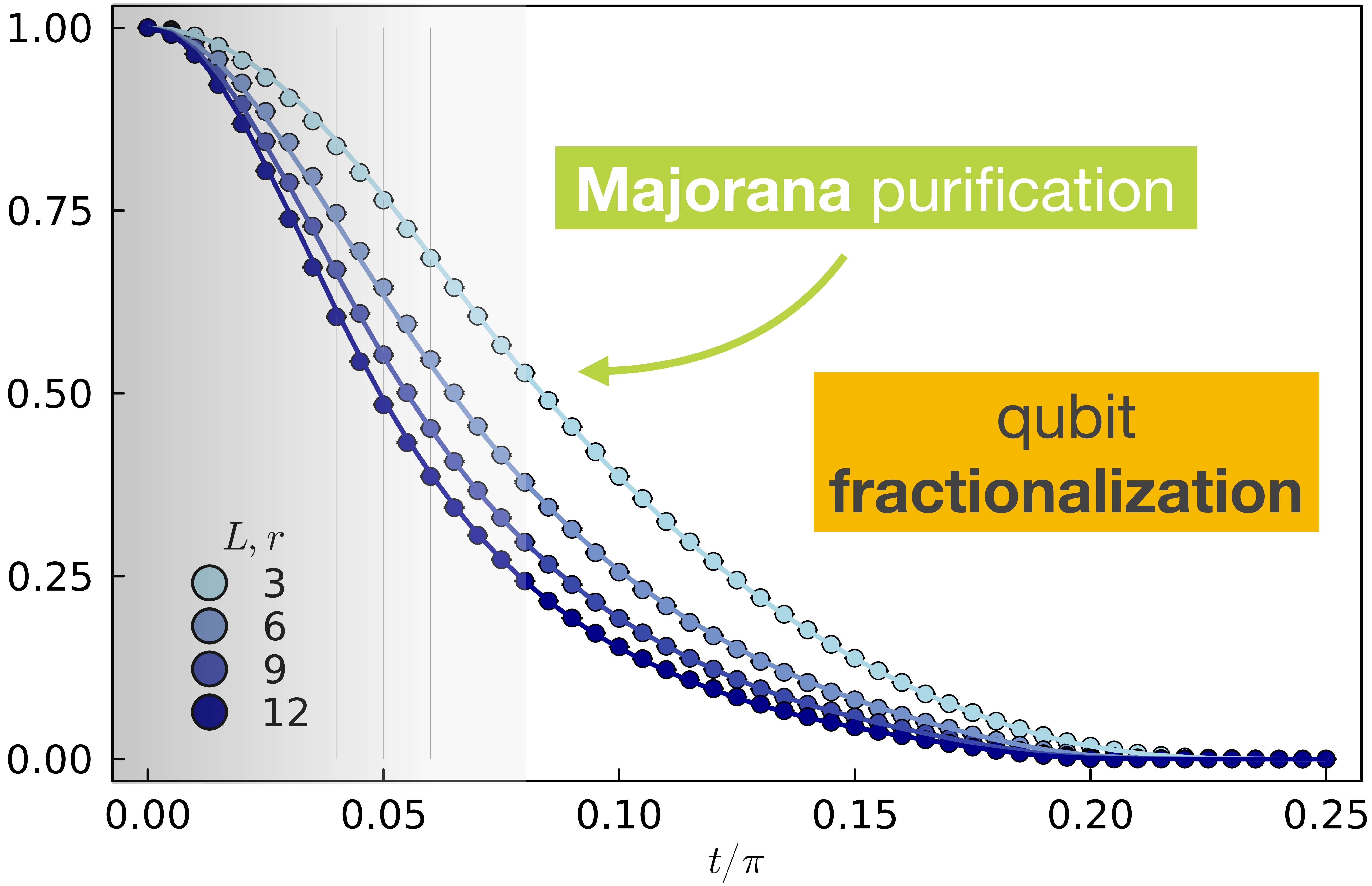
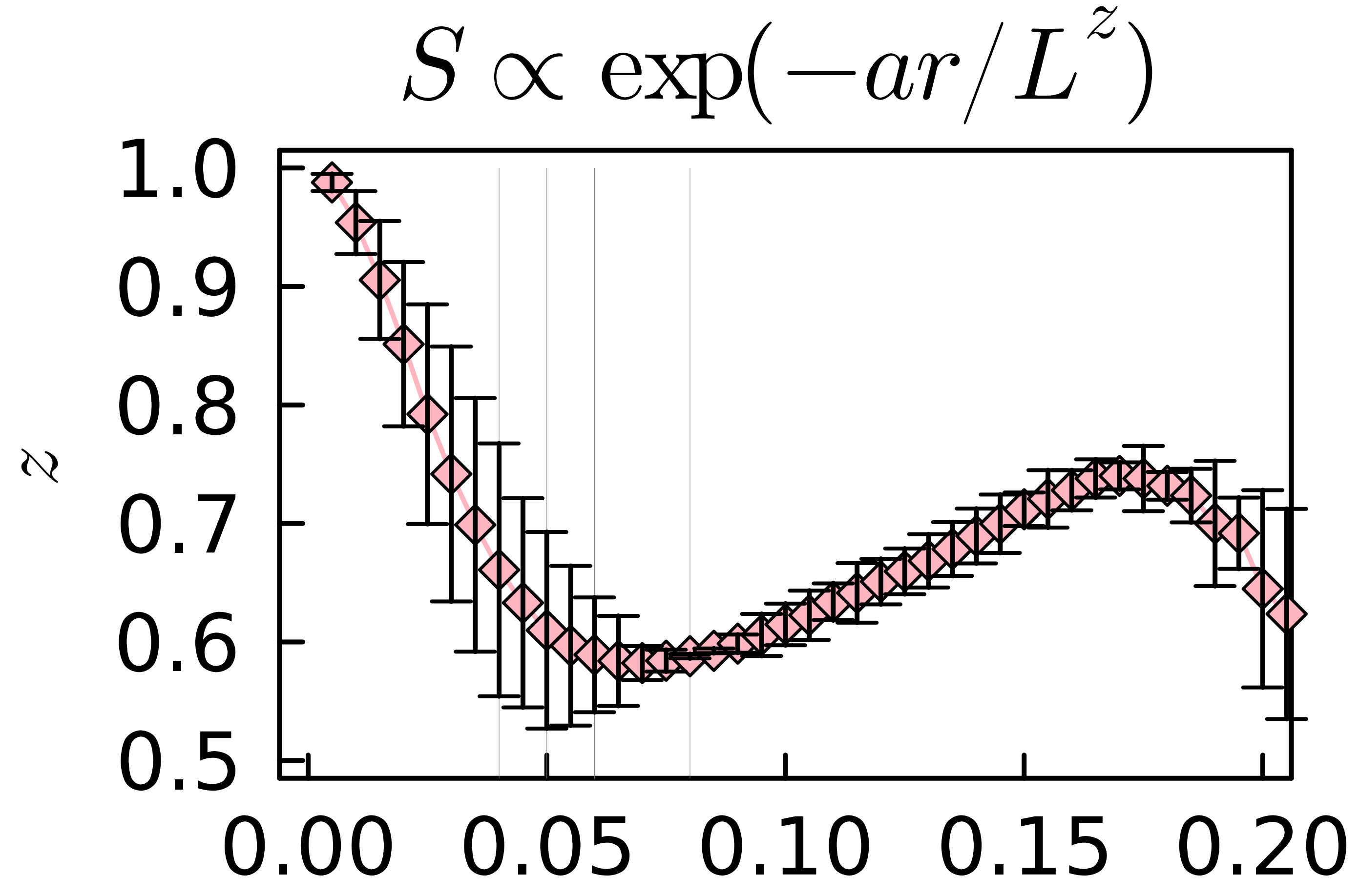
purification of Majoranas

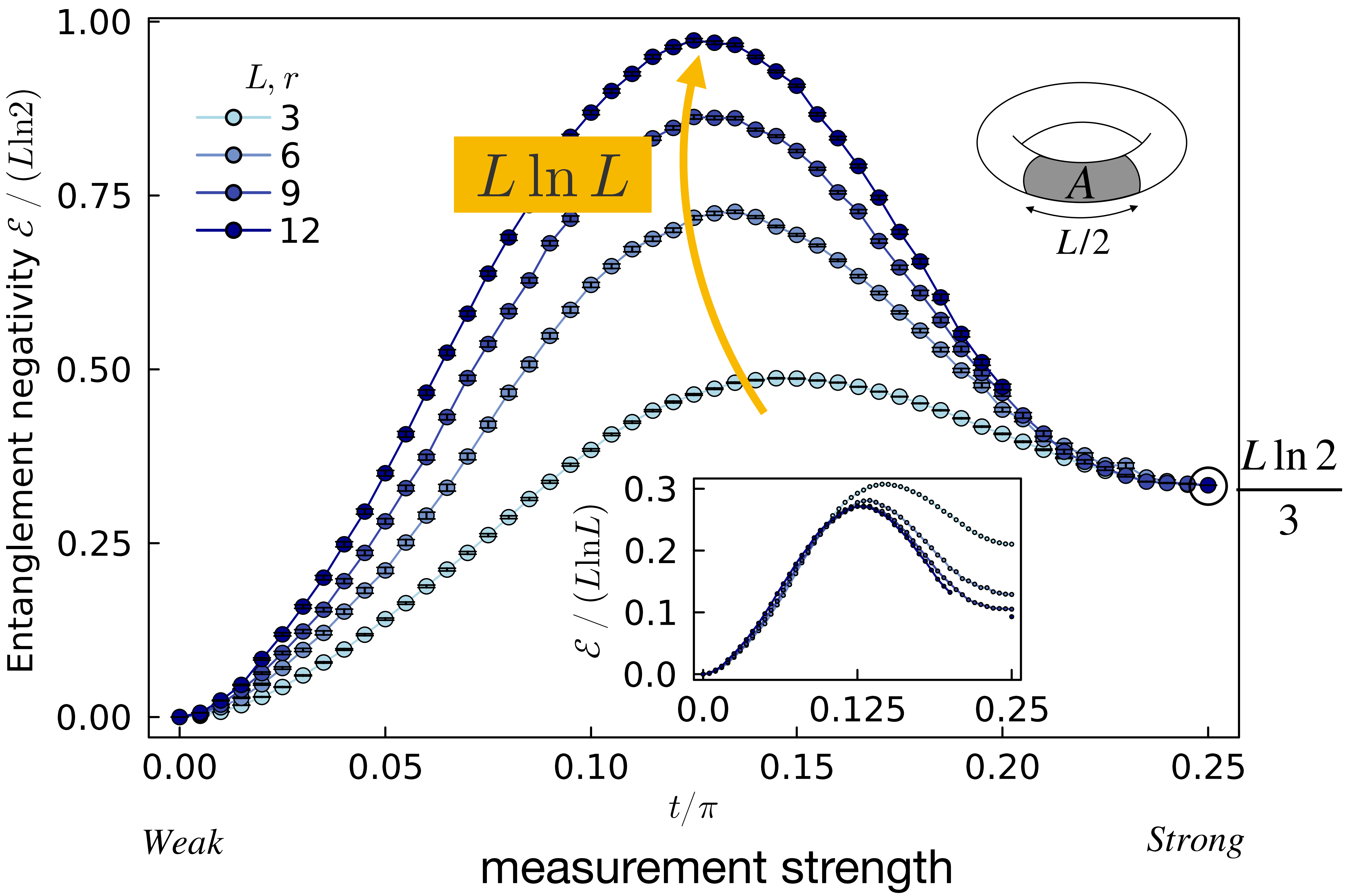
Majorana entropy density $[\ln 2]$ $S = \beta(E - F)$





dynamical critical exponent

Majorana liquid



- fermionic **entanglement negativity**
  - definition: response under ***partial*** **time reversal**
  - property: **distill out thermal entropy**
  - diagnose: **mixed state entanglement**

  Shapourian, Shiozaki, Ryu, 2017

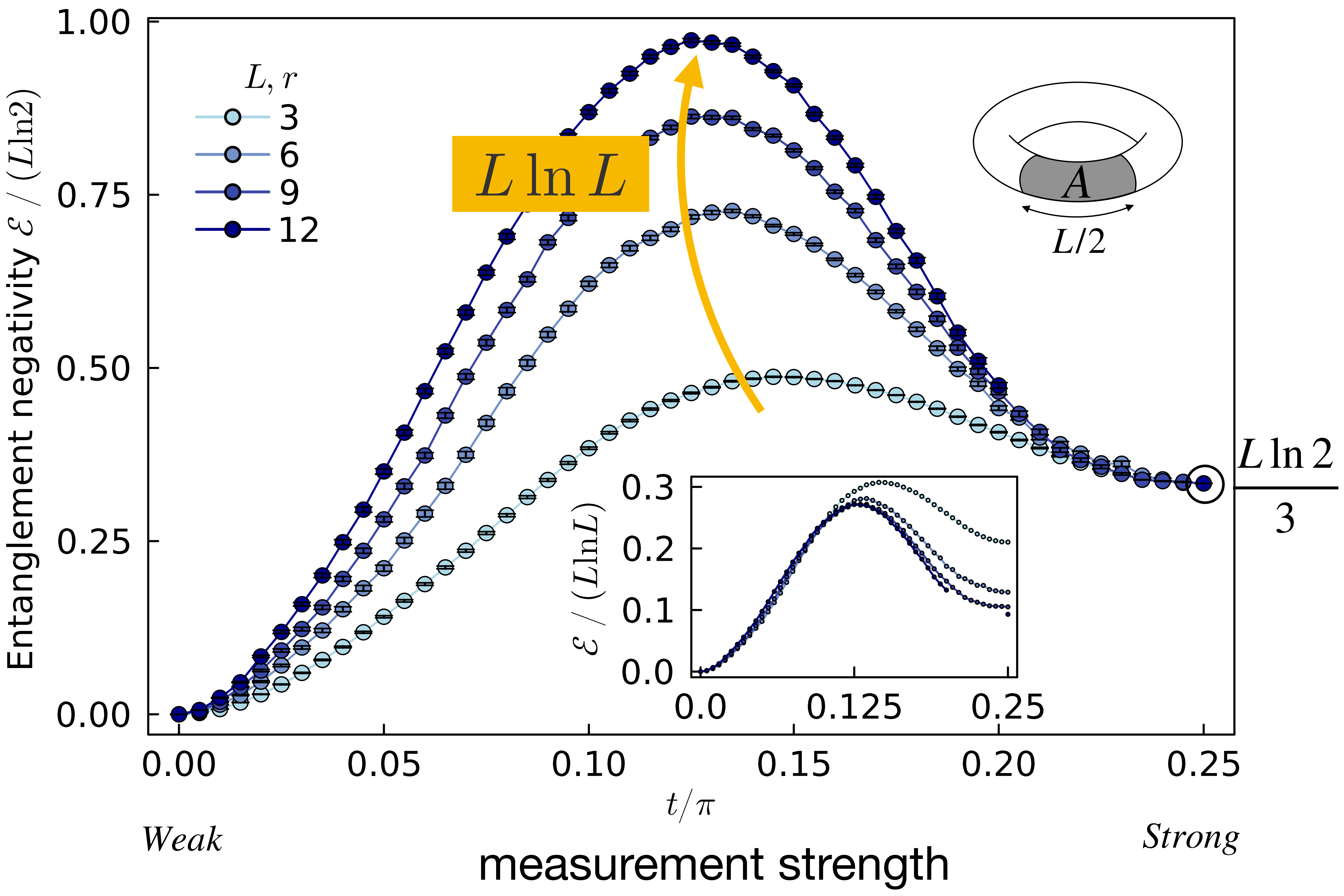
$$\mathscr{E} = \sum_{\mathbf{su}} p_{\mathbf{su}} \cdot \ln || \rho_{\mathbf{su}}^{R_A} ||_1$$

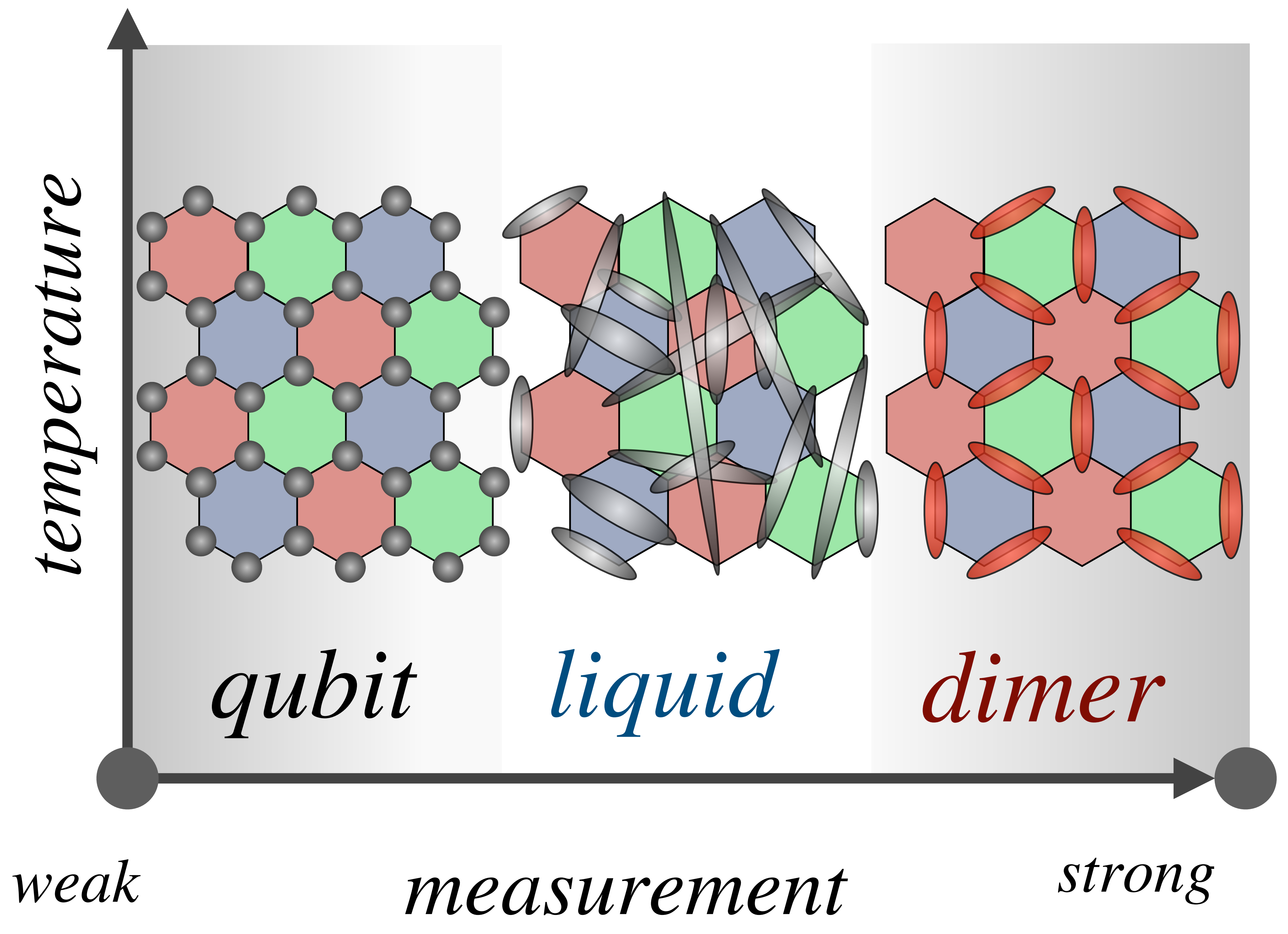
- **entanglement phase transition**

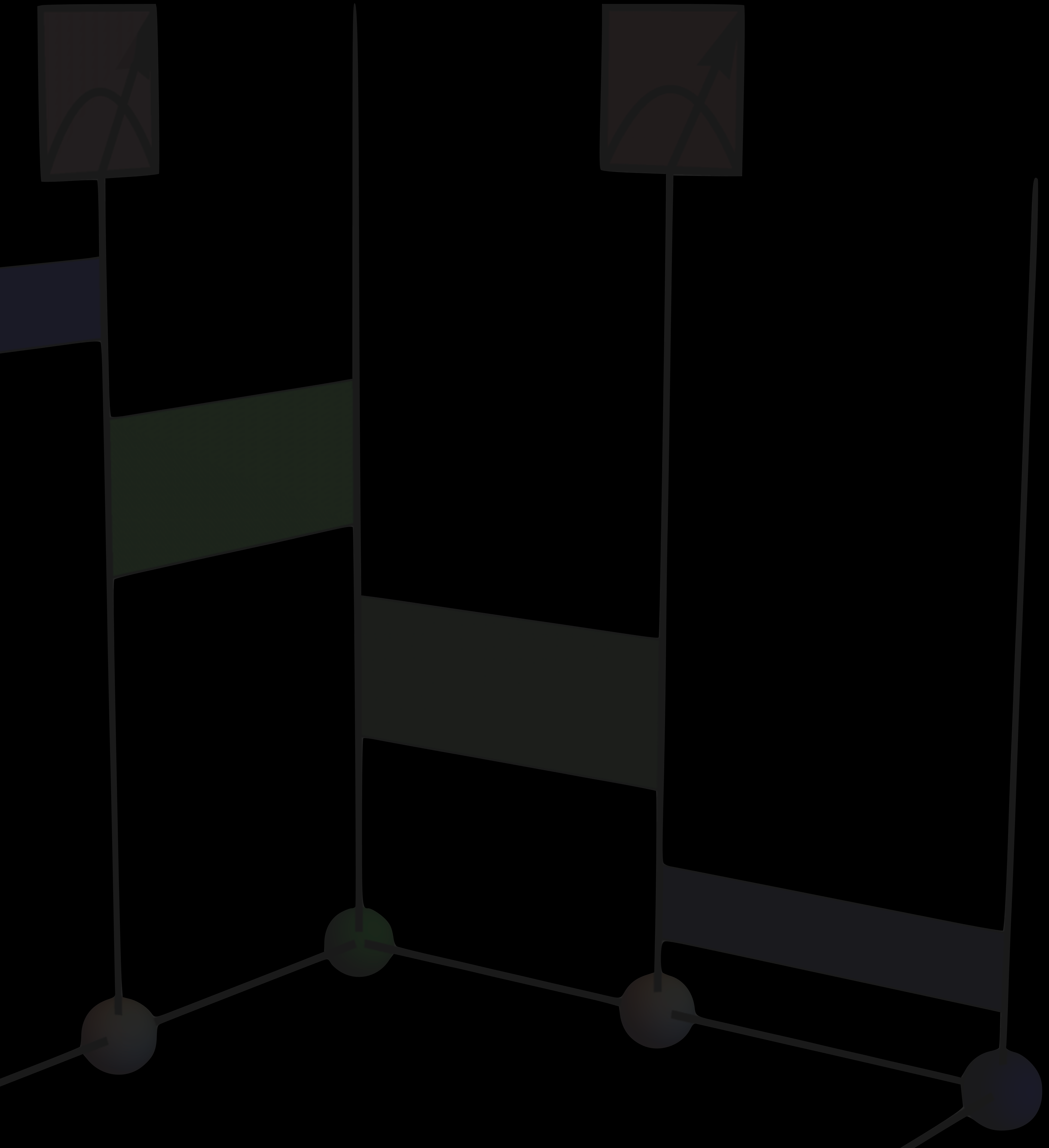
  stable fermion phase with $\mathbf{L} \ln \mathbf{L}$ **entanglement**

  Fava, Piroli, Swann, Bernard, Nahum, NLσM, 2023

Majorana liquid





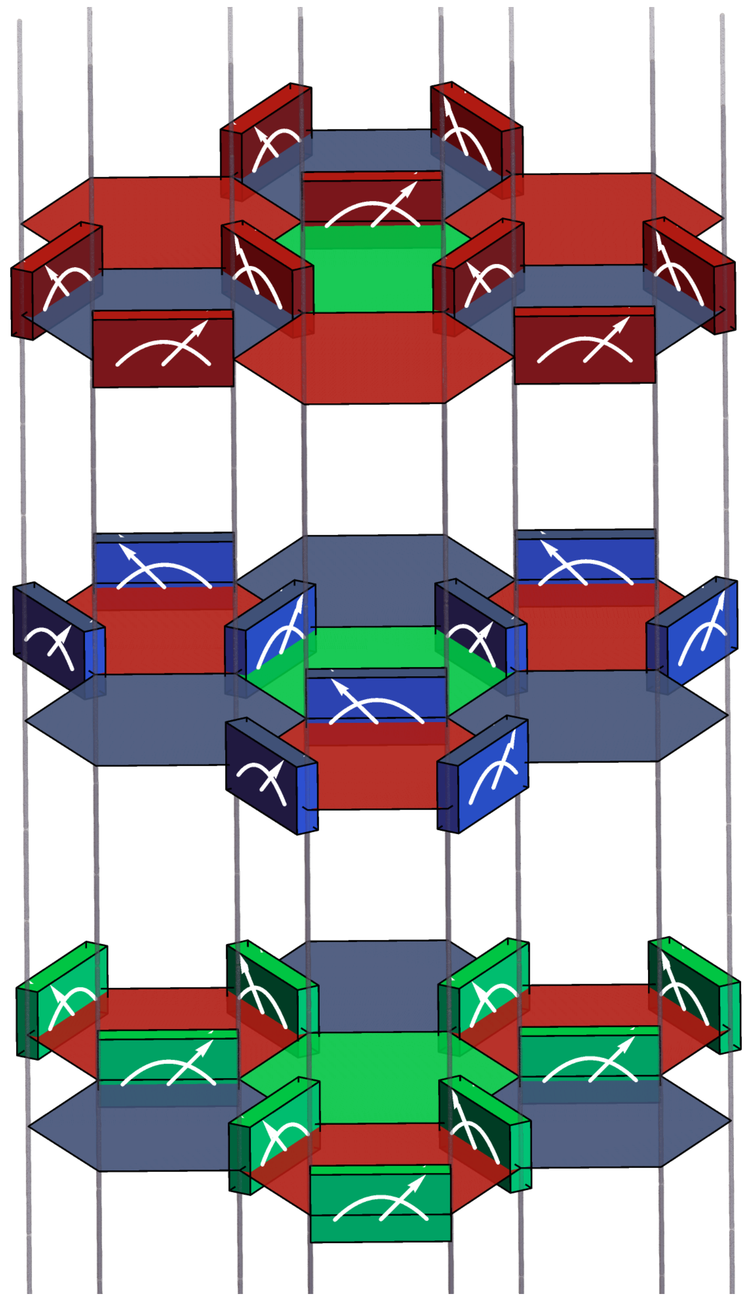


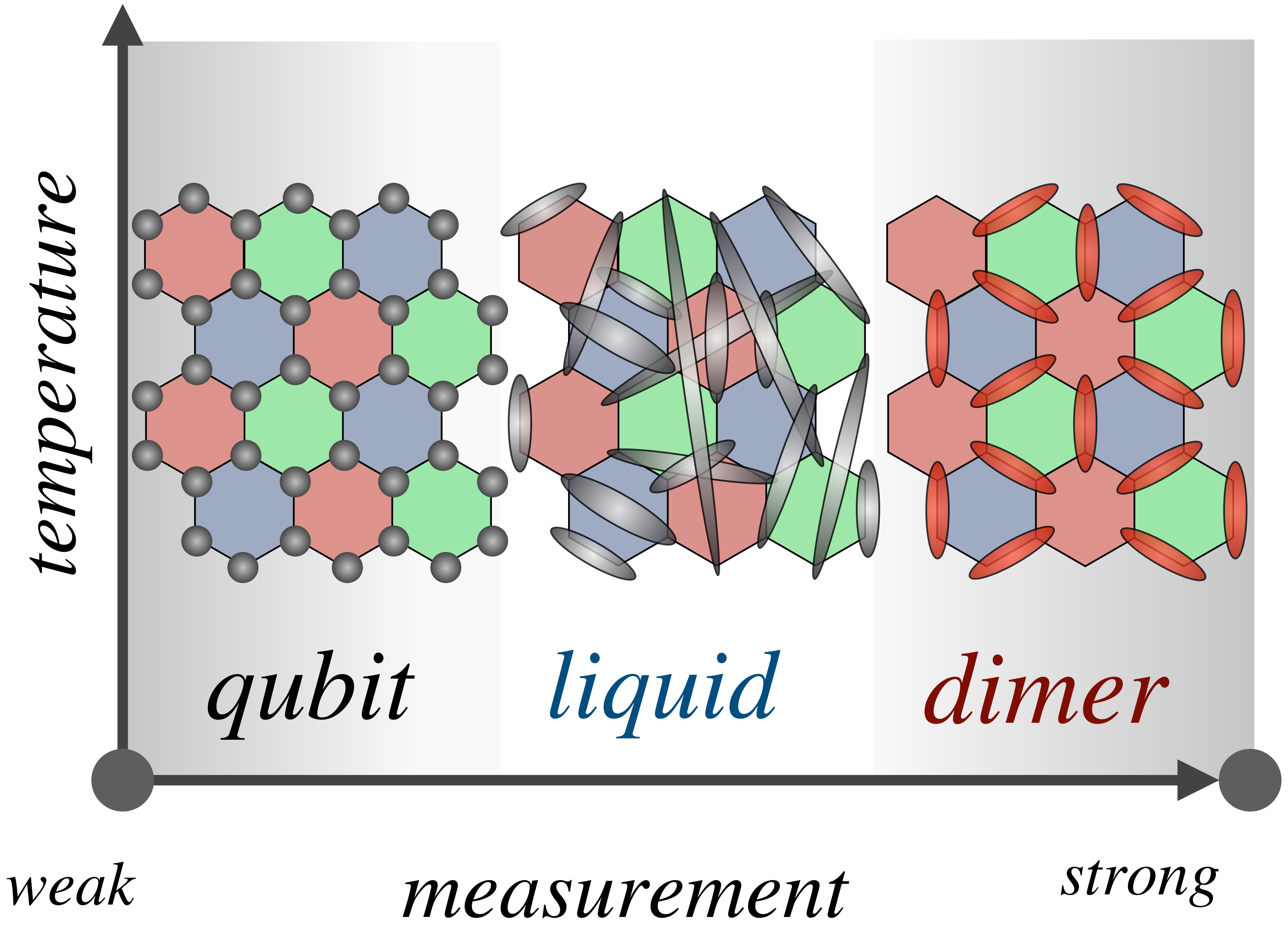
summary

Floquet code — conclusions

Guo-Yi Zhu

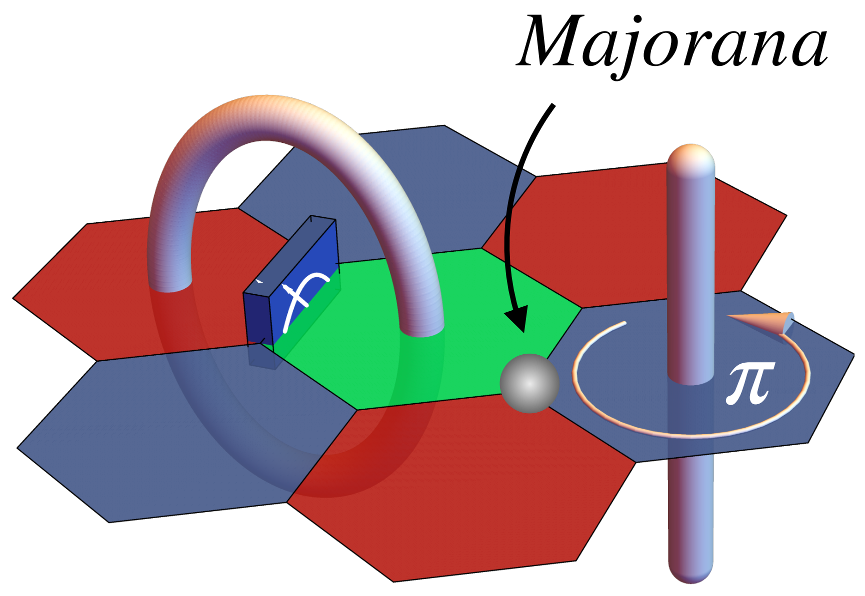
- **frustration** & **qubit fractionalization** by tunable weak measurement
- Floquet code **breakdown** to non-trivial state under coherent error
- **Majoranas** escape confinement and form long-range entangled **liquid**





Outlook

- **Feed-forward** deterministic preparation?
- topological phase transition from a parent **color code** (+ Majorana interaction)?



Guo-Yi Zhu & ST, arXiv: 2311.08450

Hamiltonian vs. monitored dynamics

Hamiltonian dynamics

- **equilibrium** dynamics of **isolated** systems
- **unitary** evolution
- energy **conserved**
- quantum **ground states**
- **area-law entanglement** structures
- macroscopic entanglement (spin liquids)

measurement dynamics

- **out-of-equilibrium** dynamics of **open** systems
- **non-unitary** evolution
- energy **not** conserved
- long-time **steady states**
- **plethora of entanglement** structures
- macroscopic entanglement (spin liquids)