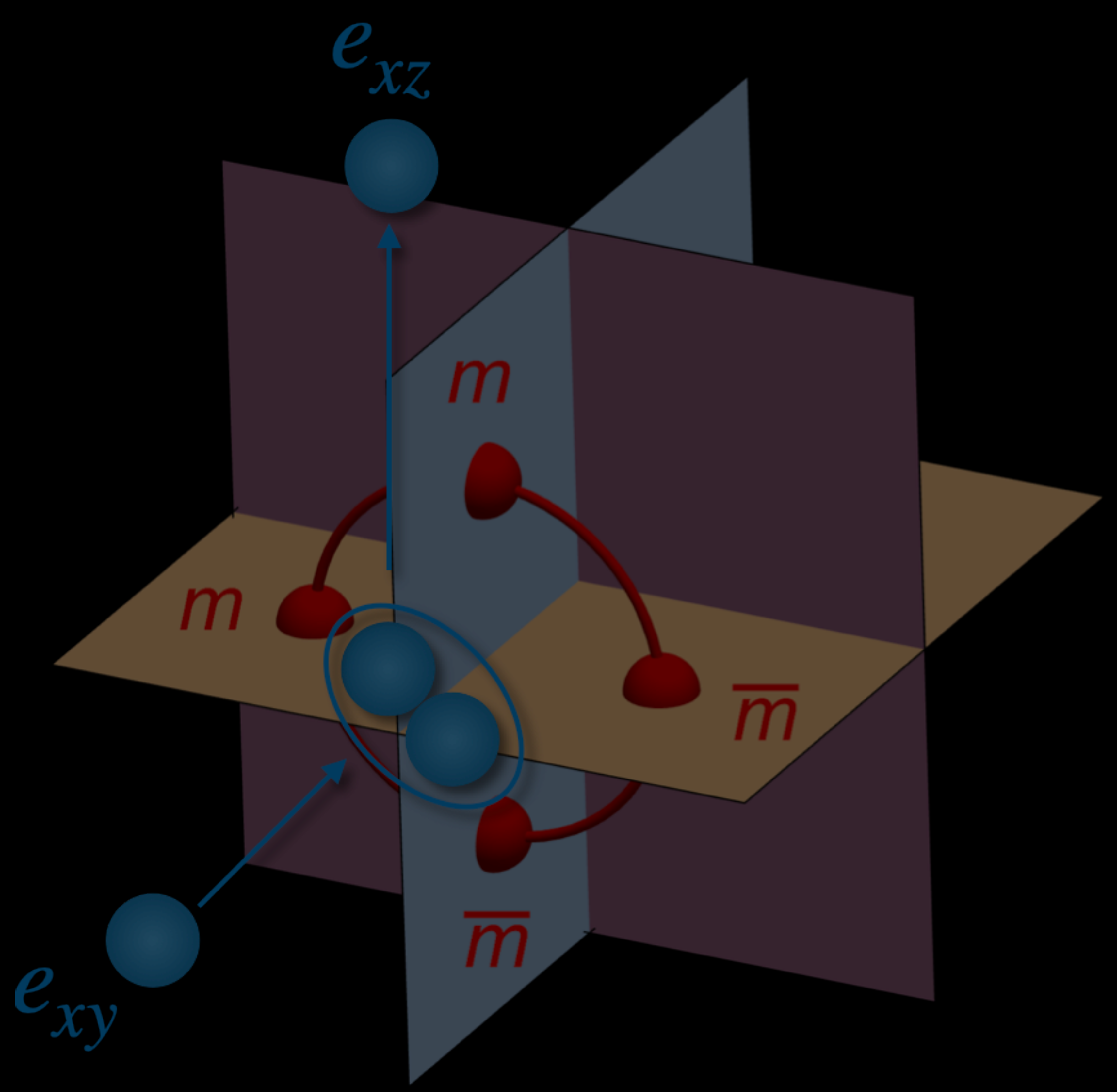
Topological fracton quantum phase transitions

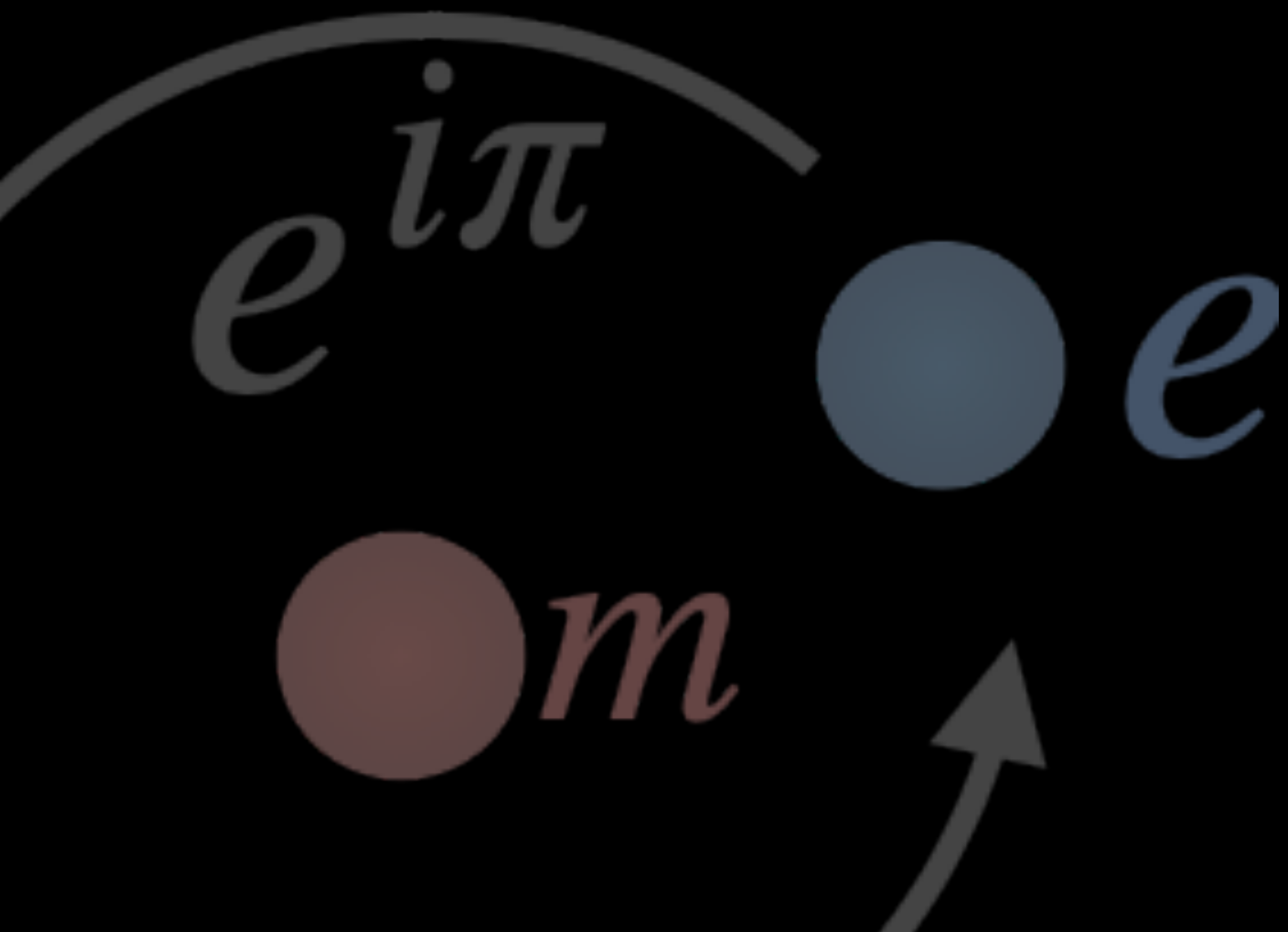
from exact tensor network deformations



Simon Trebst
University of Cologne

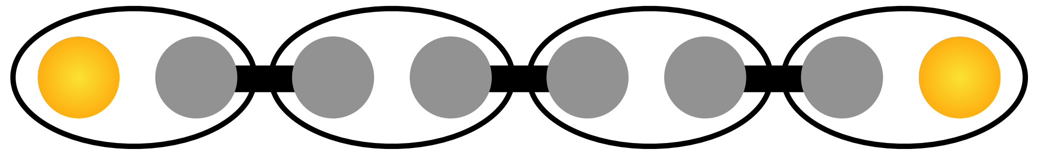
CRC183 ENTANGLED STATES OF MATTER

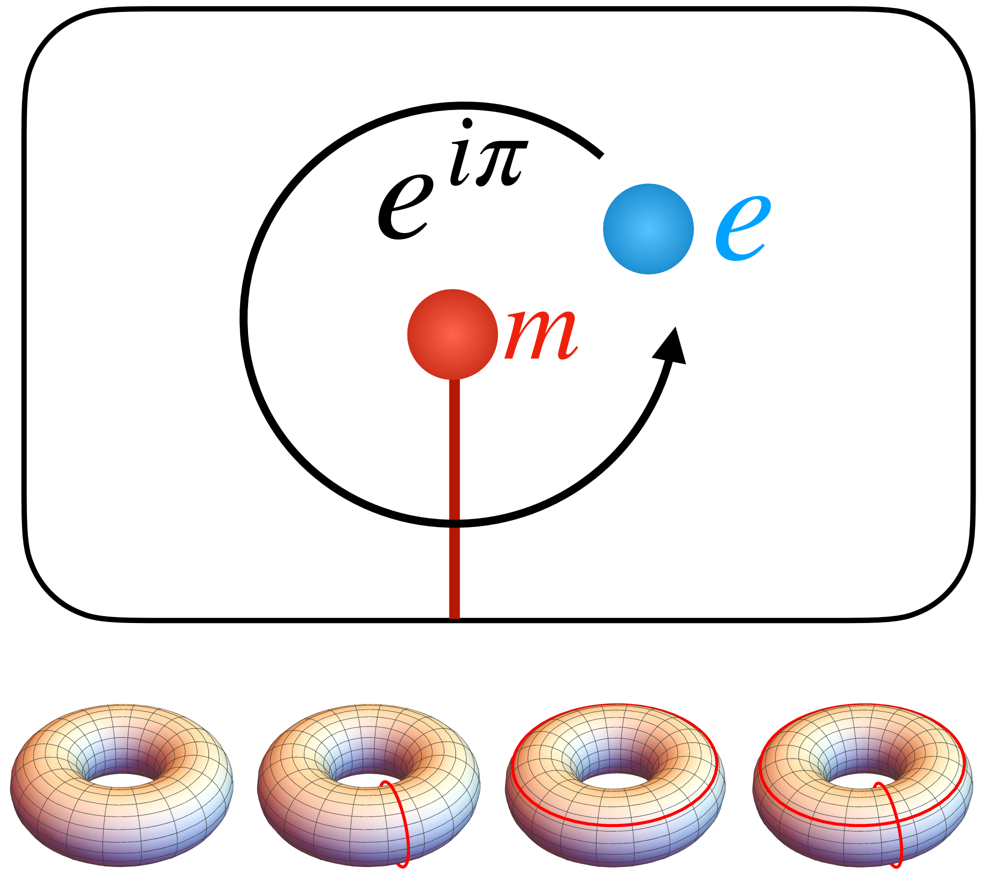
Topological Quantum Phases of Matter Beyond Two Dimensions
Sorbonne Université Paris, October 2022

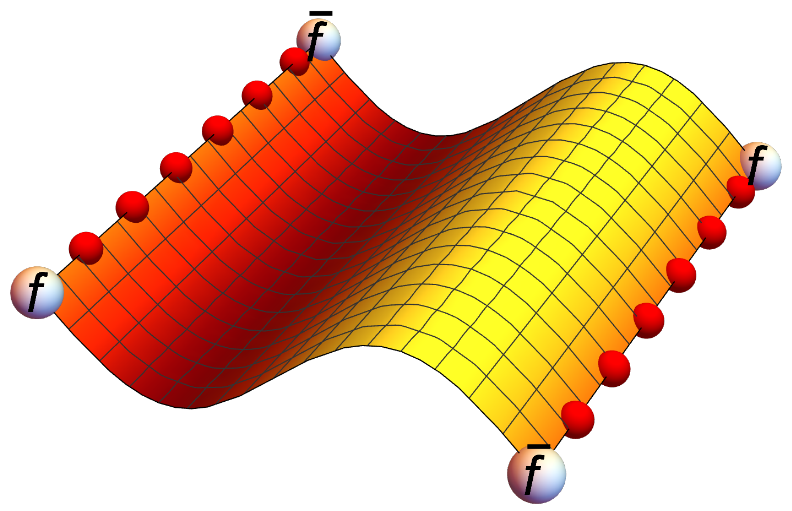


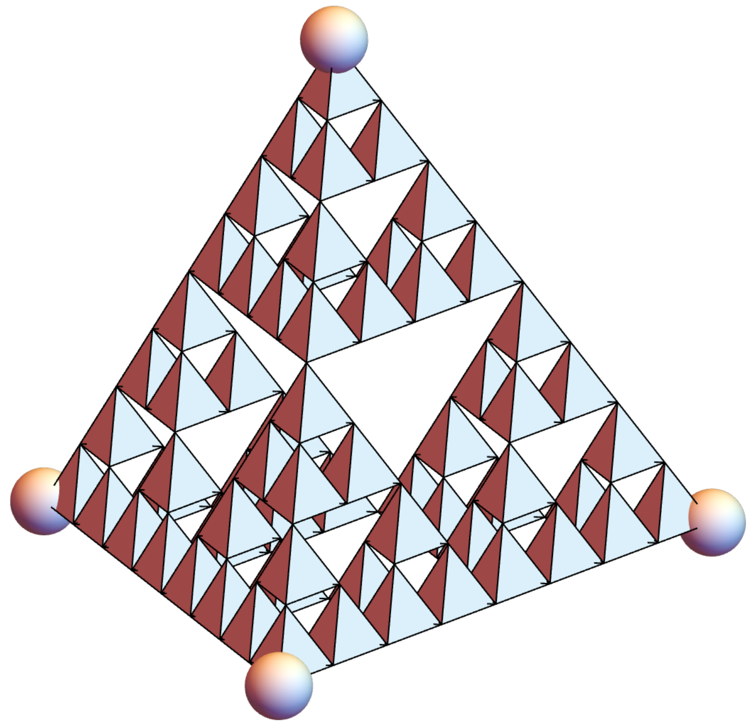
intrinsic topological order

topological quantum liquids







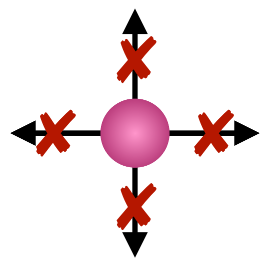


dimensionality

1D	2D	3D
symmetry protected	*intrinsic* topological order & **anyons**	*geometric* topological order & **fractons**
topological state	vector gauge field	tensor gauge field

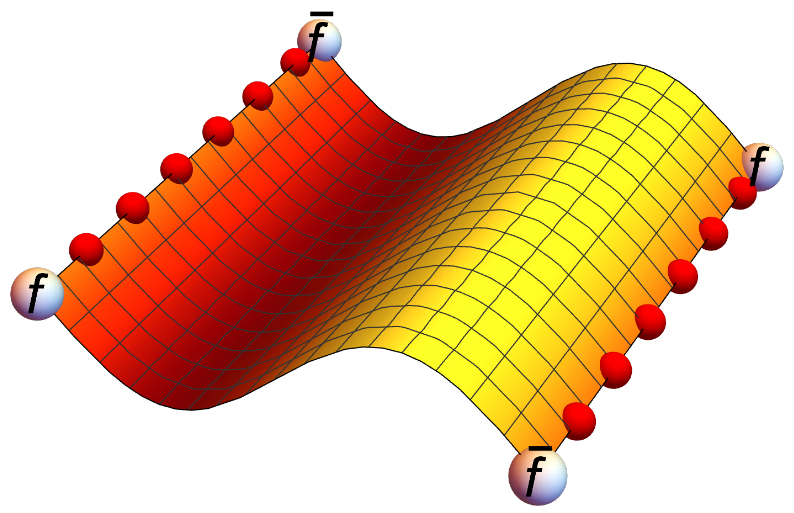
Reviews: Wen 2019; Nandkishore, Hermele 2018; Pretko, Chen, You 2020

fracton order

- fracton excitation: restricted mobility 
- robust quantum memory against temperature 
- tensor gauge theory (space dim >= 3) $\partial_i \partial_j E_{ij} = \rho$

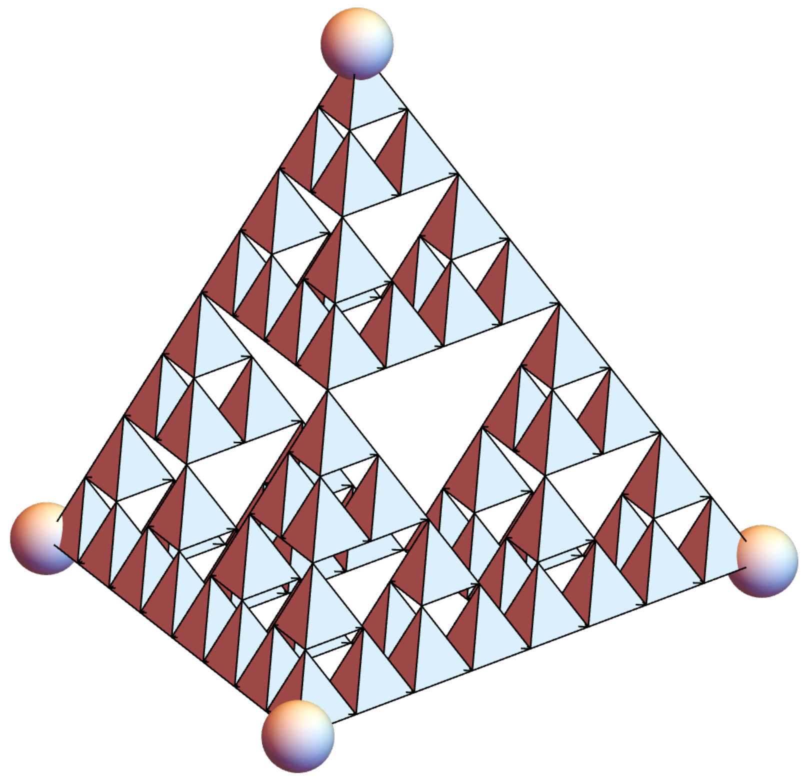
string-net condensate

3D generalizations



“membrane" condensate

type-I



“fractal” condensate

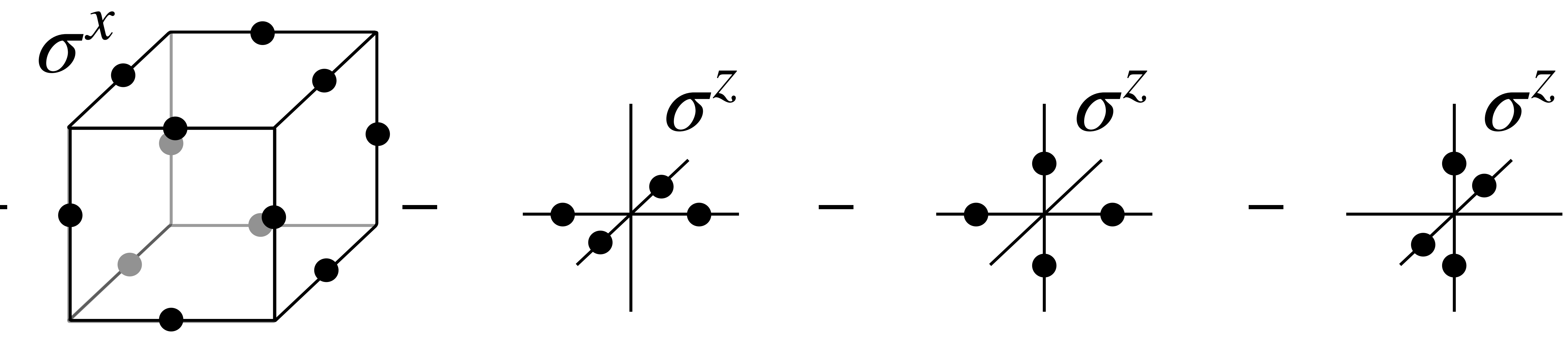
type-II

Chamon 05; Haah 11; Yoshida 13; Vijay, Haah, Fu, 16; Pretko 17; …

X-Cube model

Vijay, Haah, Fu 2016

Hamiltonian: $H_{XC} = -$



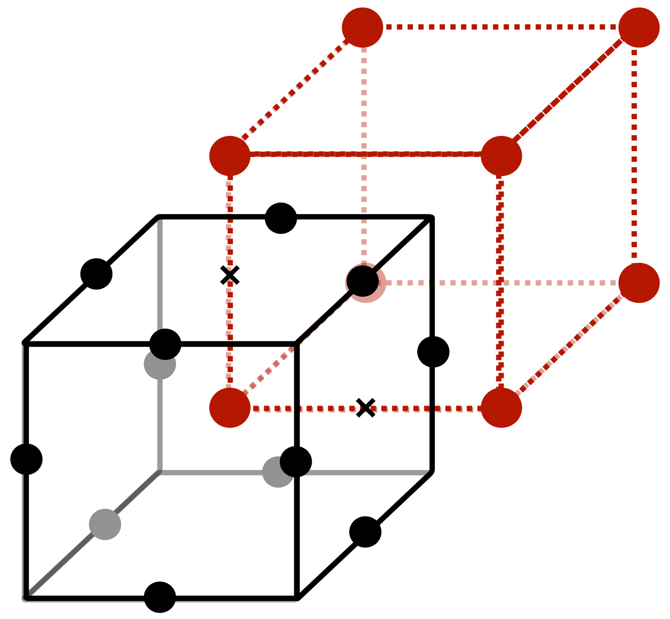
gauge theory

duality

gauging / duality

cube ⟷ vertex

link ⟷ face



exactly solvable

matter theory

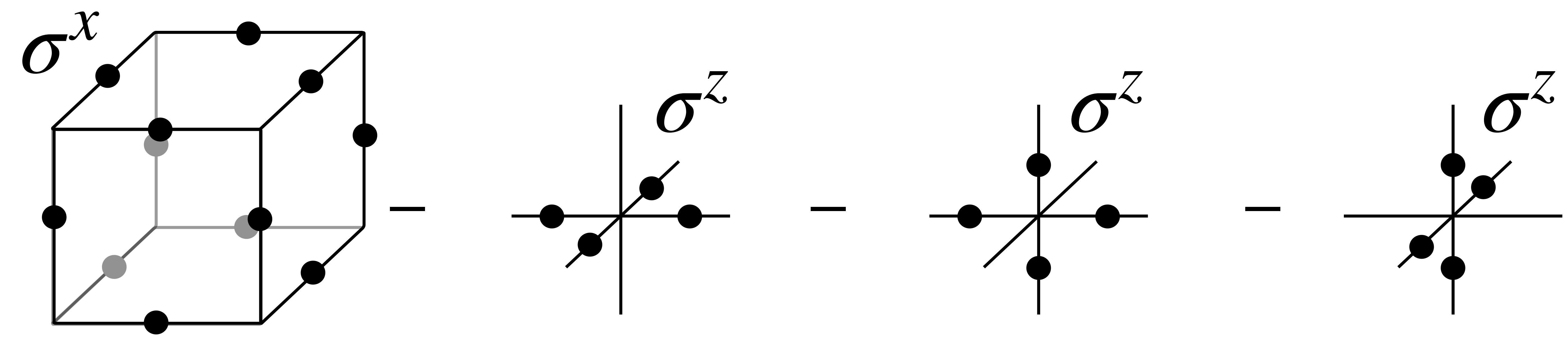
$$H_{\mathrm{PIM}} = -\sum_{(ijkl)\in\square} s_i^z s_j^z s_k^z s_l^z + h\sum_j s_j^x$$

plaquette Ising model

X-Cube model

Vijay, Haah, Fu 2016

Hamiltonian: $H_{XC} = -$

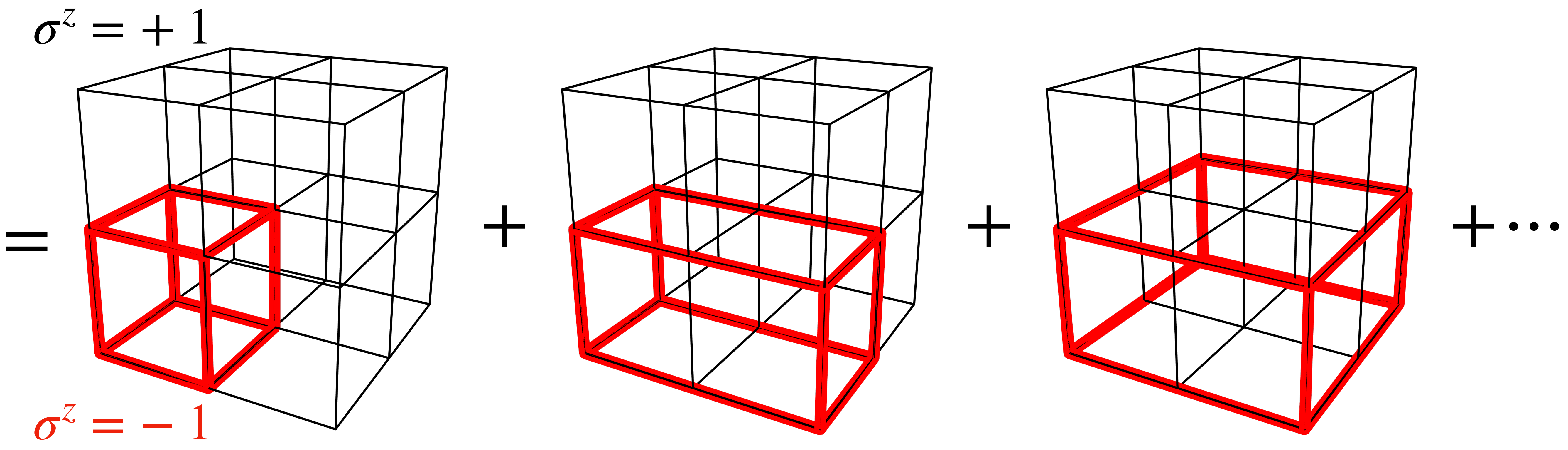


dual to plaquette Ising model

exactly solvable

ground states

$|\psi_{XC}\rangle =$



subextensive manifold

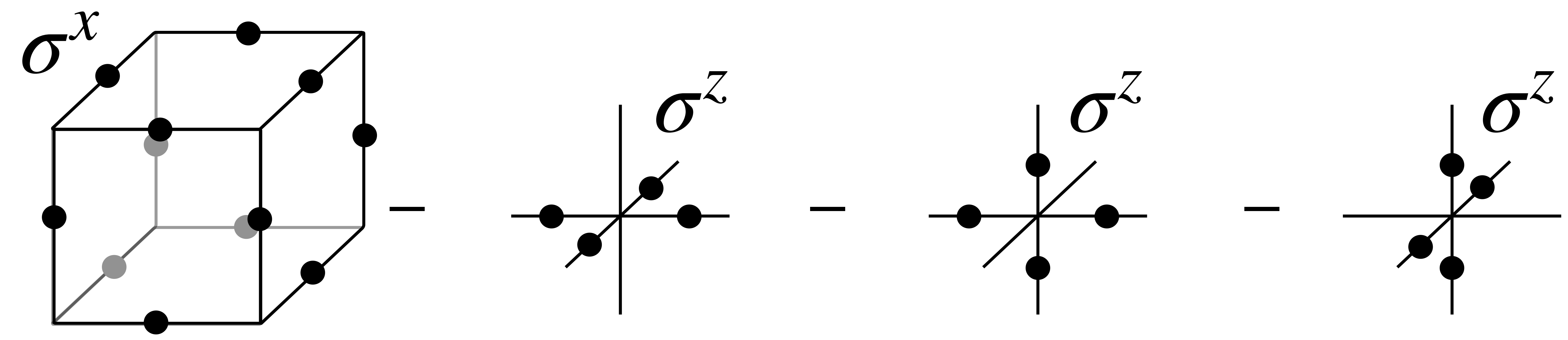
tensor network state representation

He, Zheng, Bernevig, Regnault 2018

X-Cube model

Vijay, Haah, Fu 2016

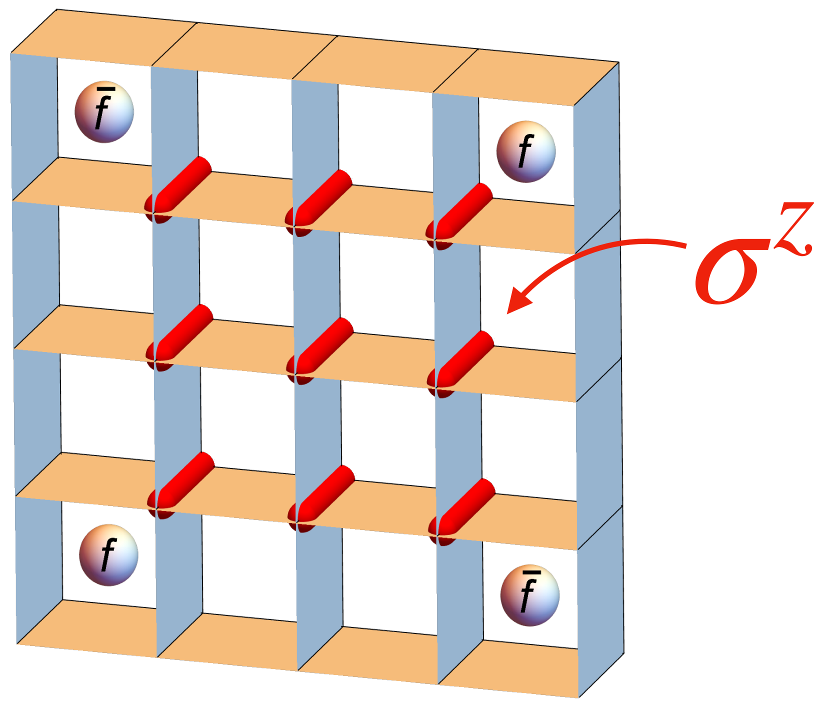
Hamiltonian: $H_{XC} = -$



dual to plaquette Ising model

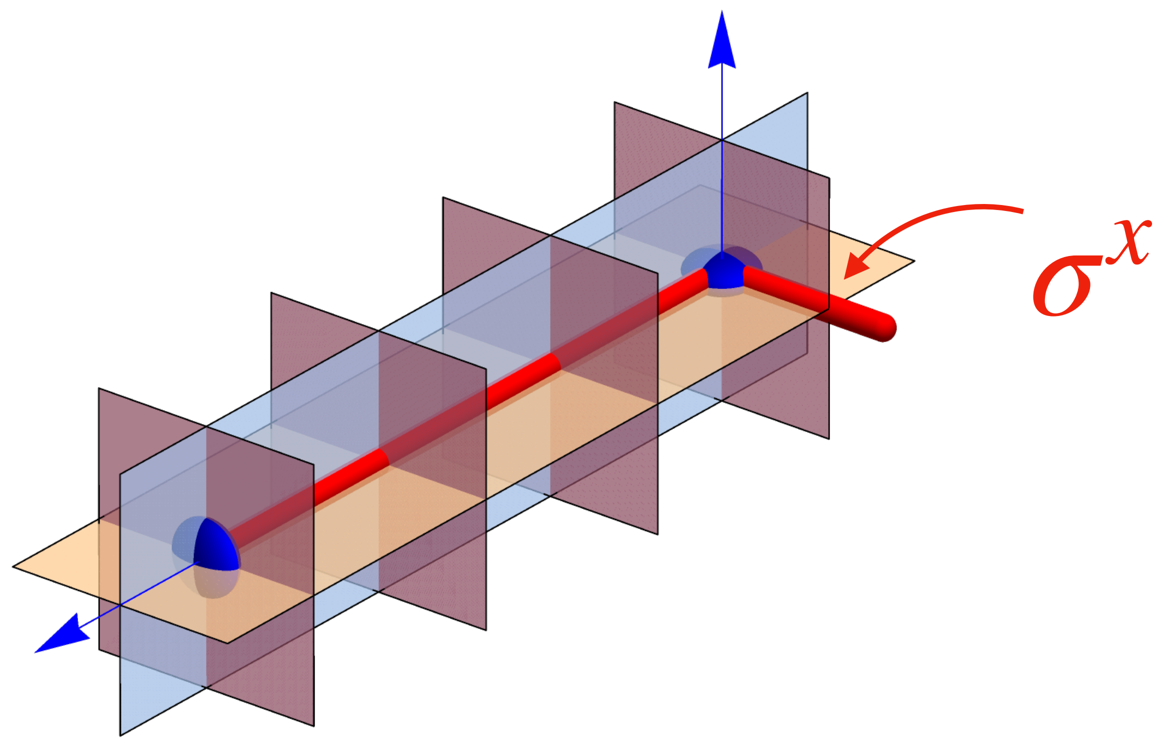
exactly solvable

excitations



electric scalar charge

fracton

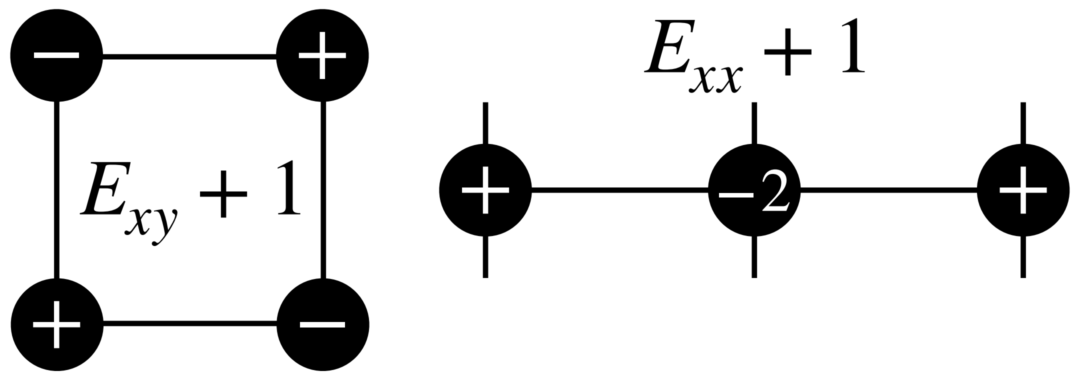


magnetic vector monopole

lineon

U(1) tensor gauge theory

- Organising principle: charge **dipole conservation**
- Partial confinement: **subsystem** charge conservation
- **Higgs** U(1) → Z_2 = X cube



$$U(1):\ \partial_i\partial_j E_{ij} = \rho$$

subsystem symmetry

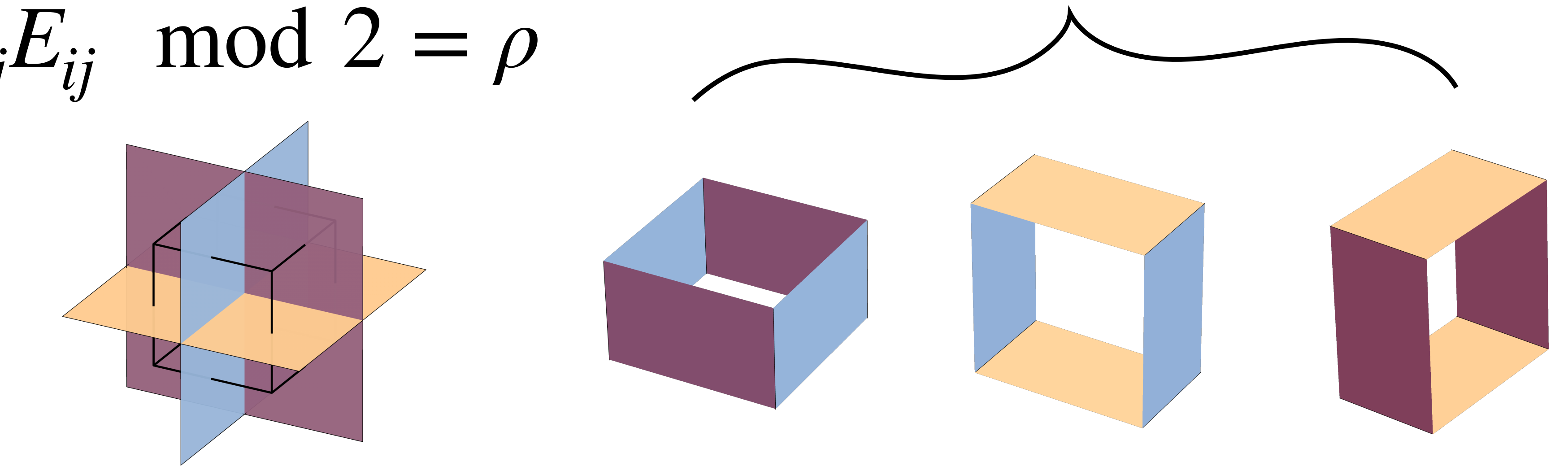
$$E_{xx} = E_{yy} = E_{zz} = 0$$

$$\rho \mod 2$$

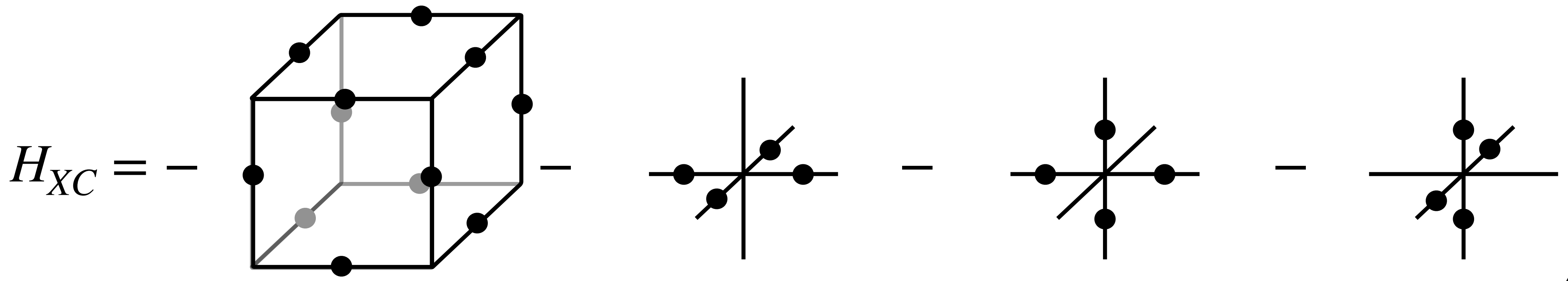
Gauss law: charge

$$\partial_i\partial_j E_{ij} \mod 2 = \rho$$

vector monopoles



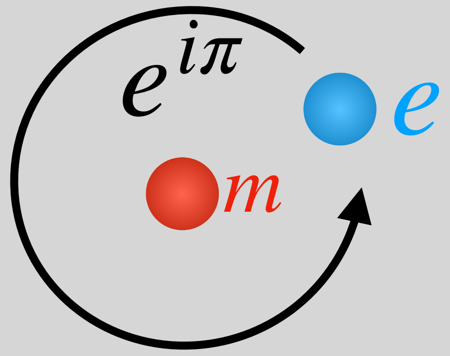
$$\sigma^x \sim e^{i\pi E},\quad \sigma^z \sim e^{iA}$$

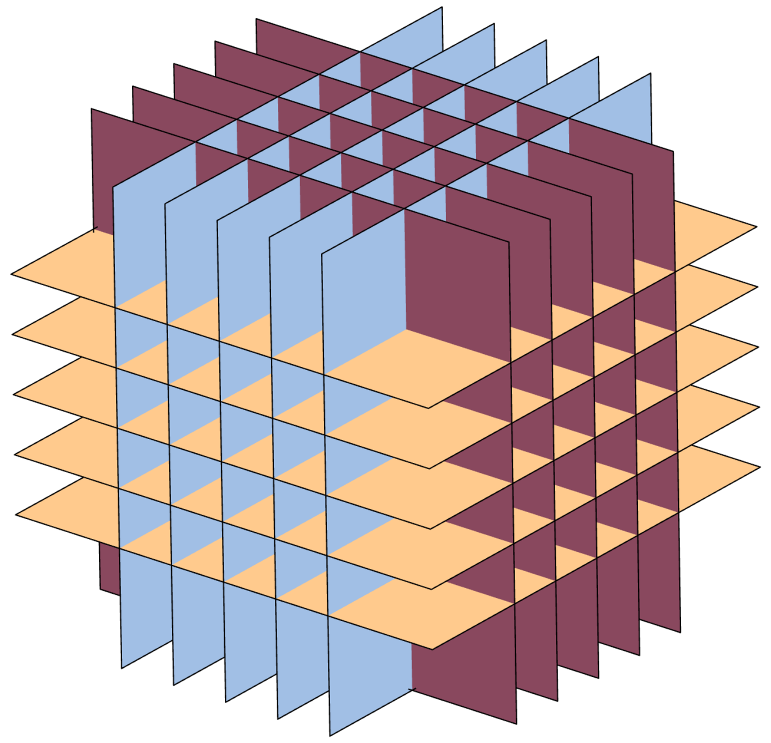


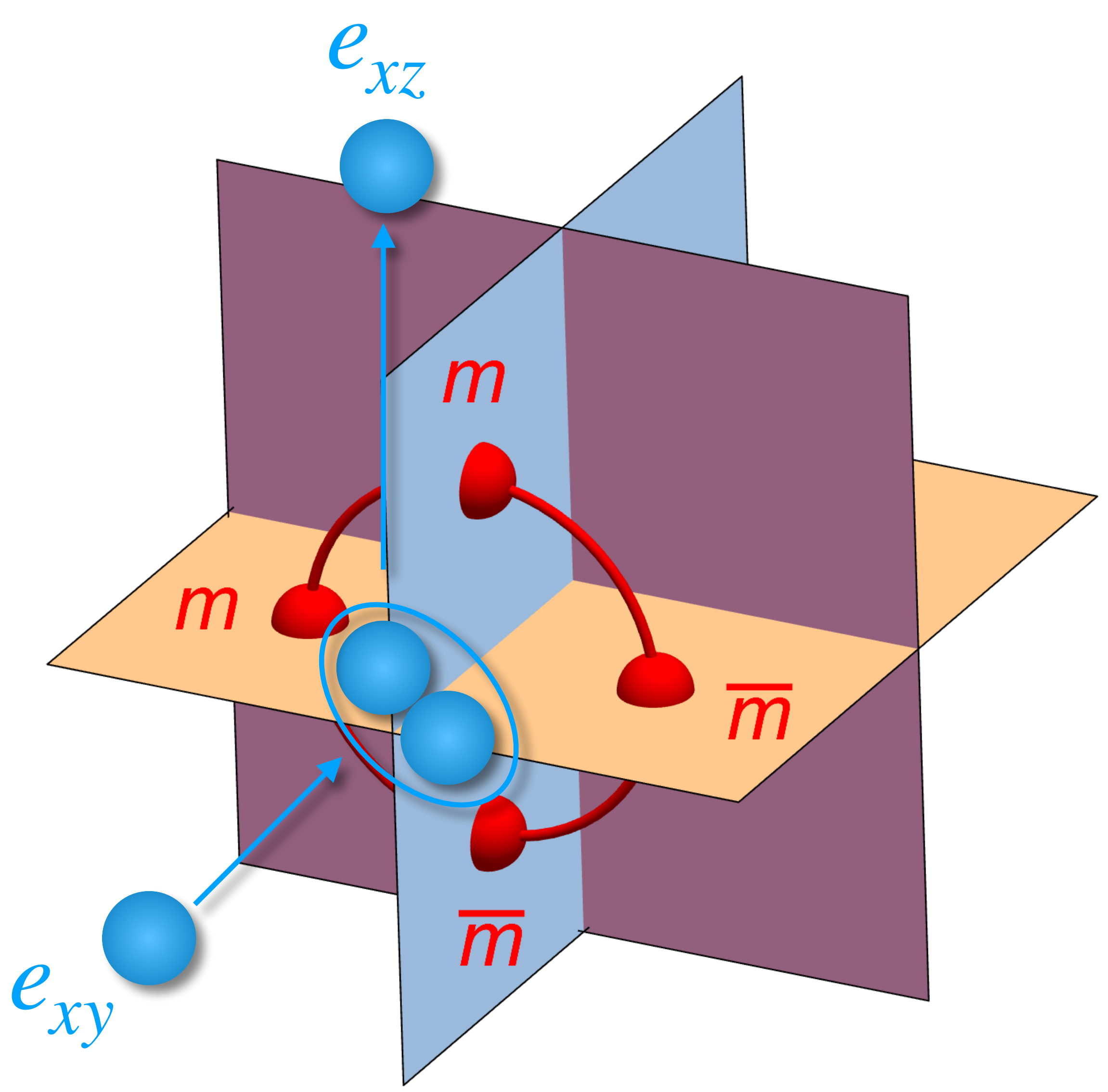
U(1) (symmetric) tensor gauge theory

Xu, Wu, 08; Pretko 17; Ma, Hermele, Chen 18; Bulmash Barkeshli 18; Seiberg, Shao, 21; …

anyon condensation







2D toric code layers

(building blocks)

e & m anyons

(bound in layers)

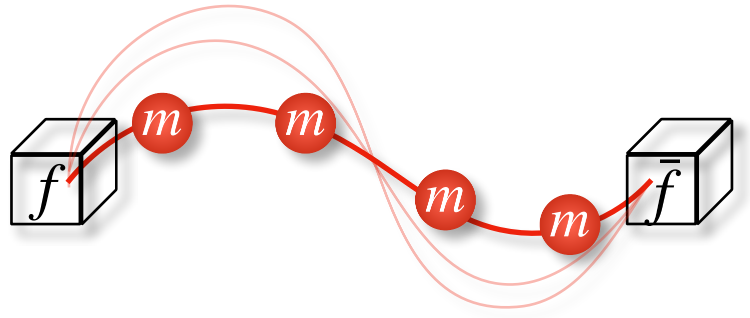
***m*-loop condensate**

e confined

ee-pair condensate

fracton confined

3D X-Cube



3D Toric Code

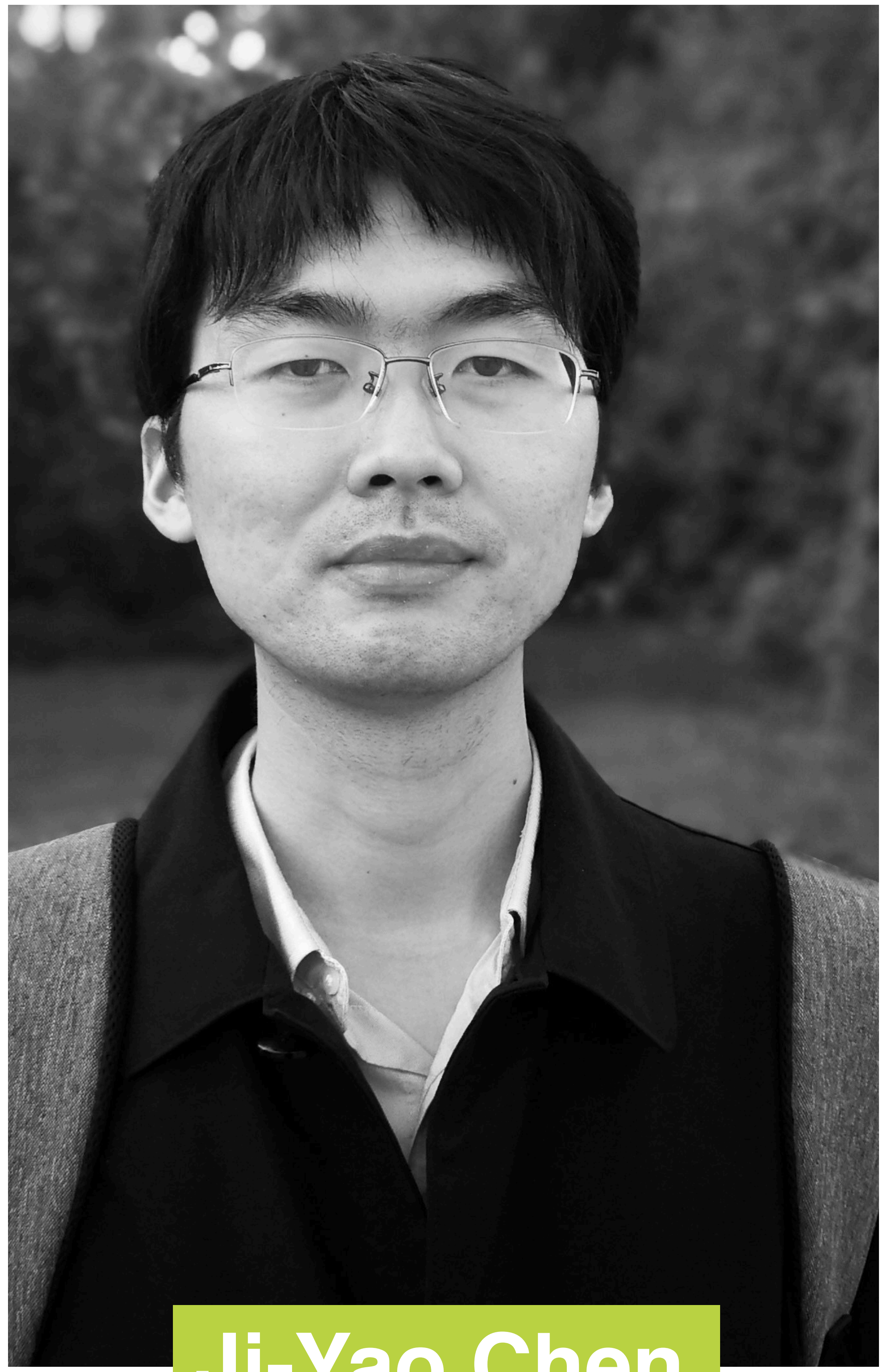
$e_{xy} \simeq e_{xz} \simeq e_{yz} \simeq e$

Interesting phase transitions?

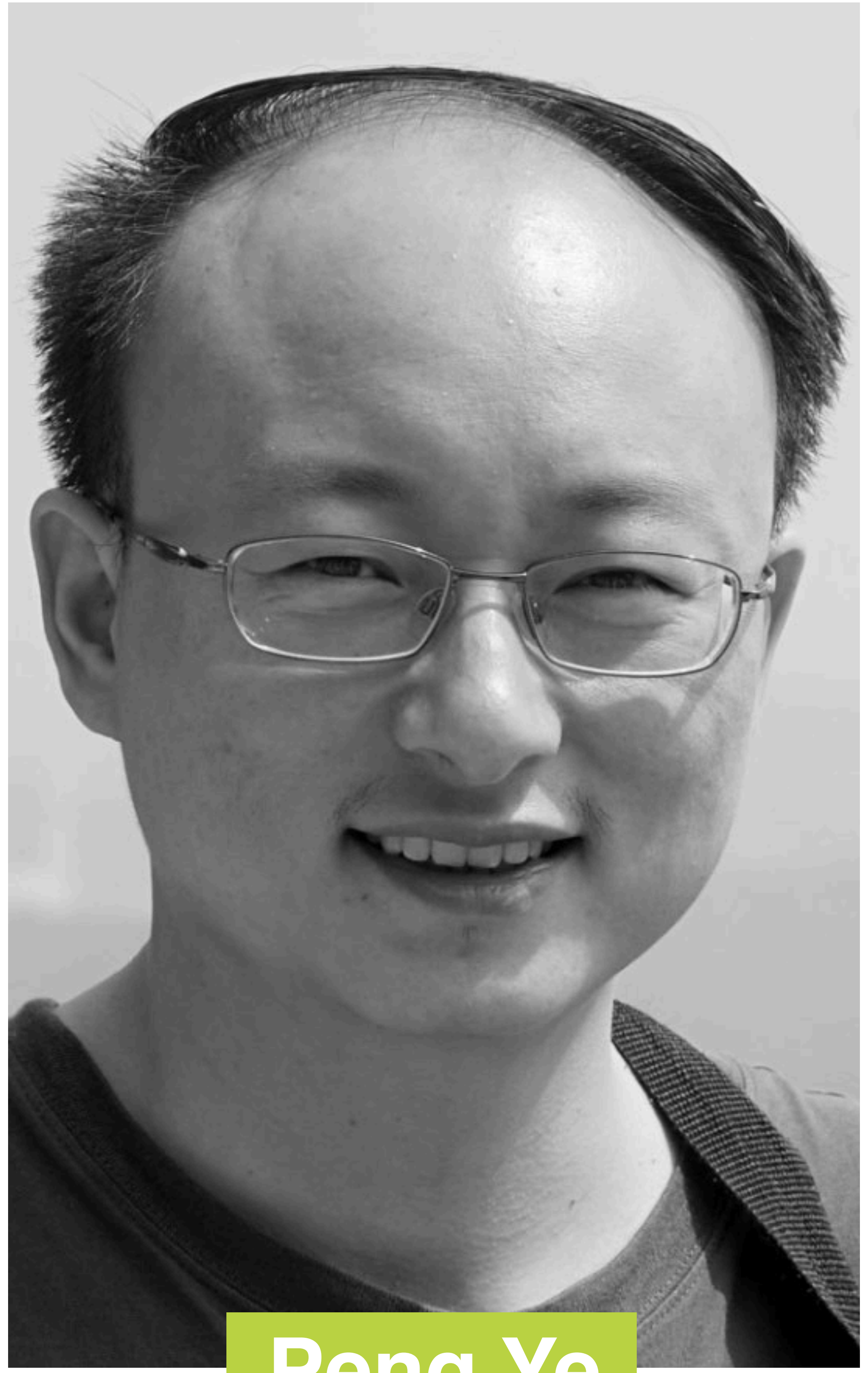
Ma, Lake, Chen, Hermele 2017; Vijay 2017

meet the team

Guo-Yi Zhu



Ji-Yao Chen



Peng Ye

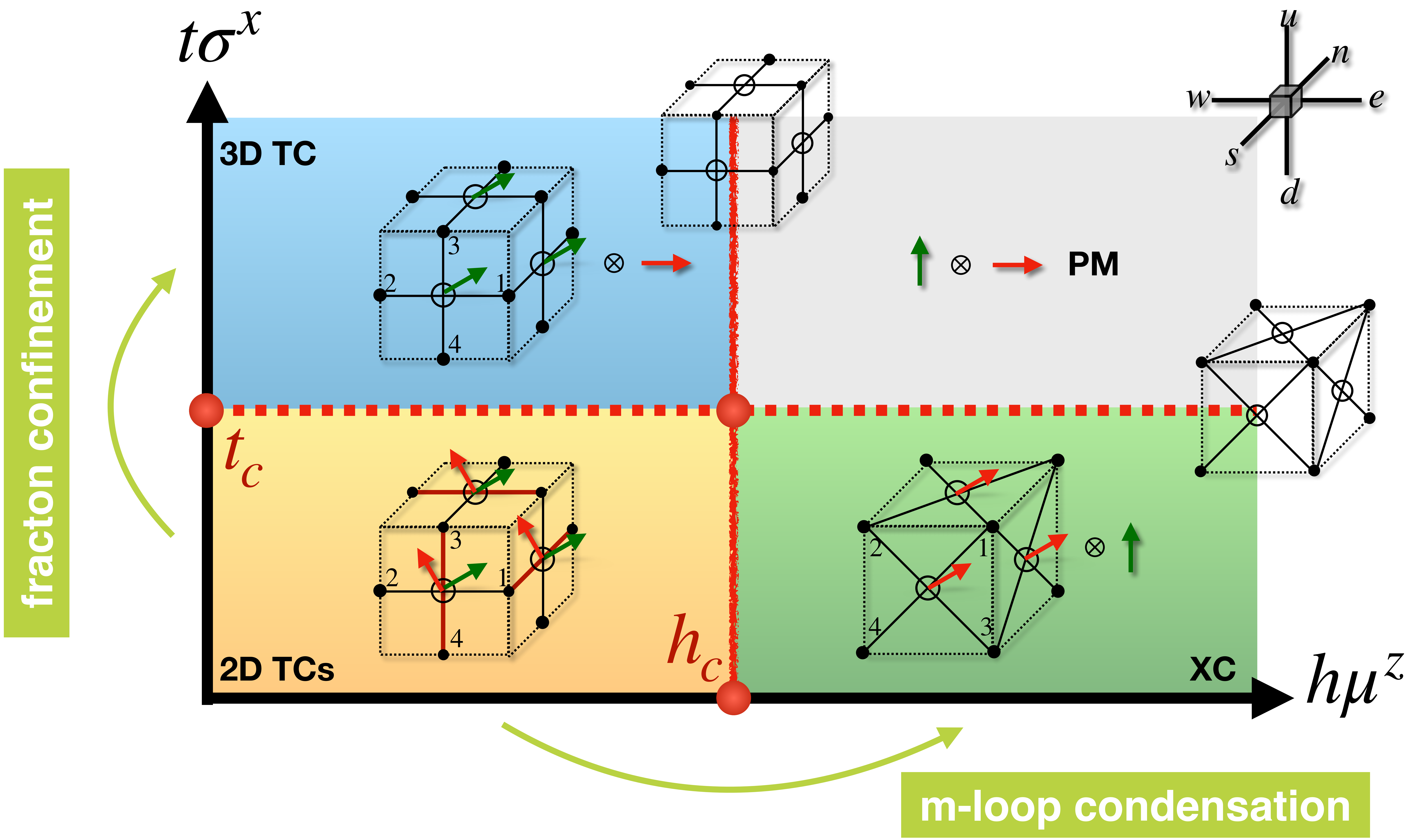
University of Cologne

arXiv:2203.00015

Sun Yat-sen University, Guangzhou

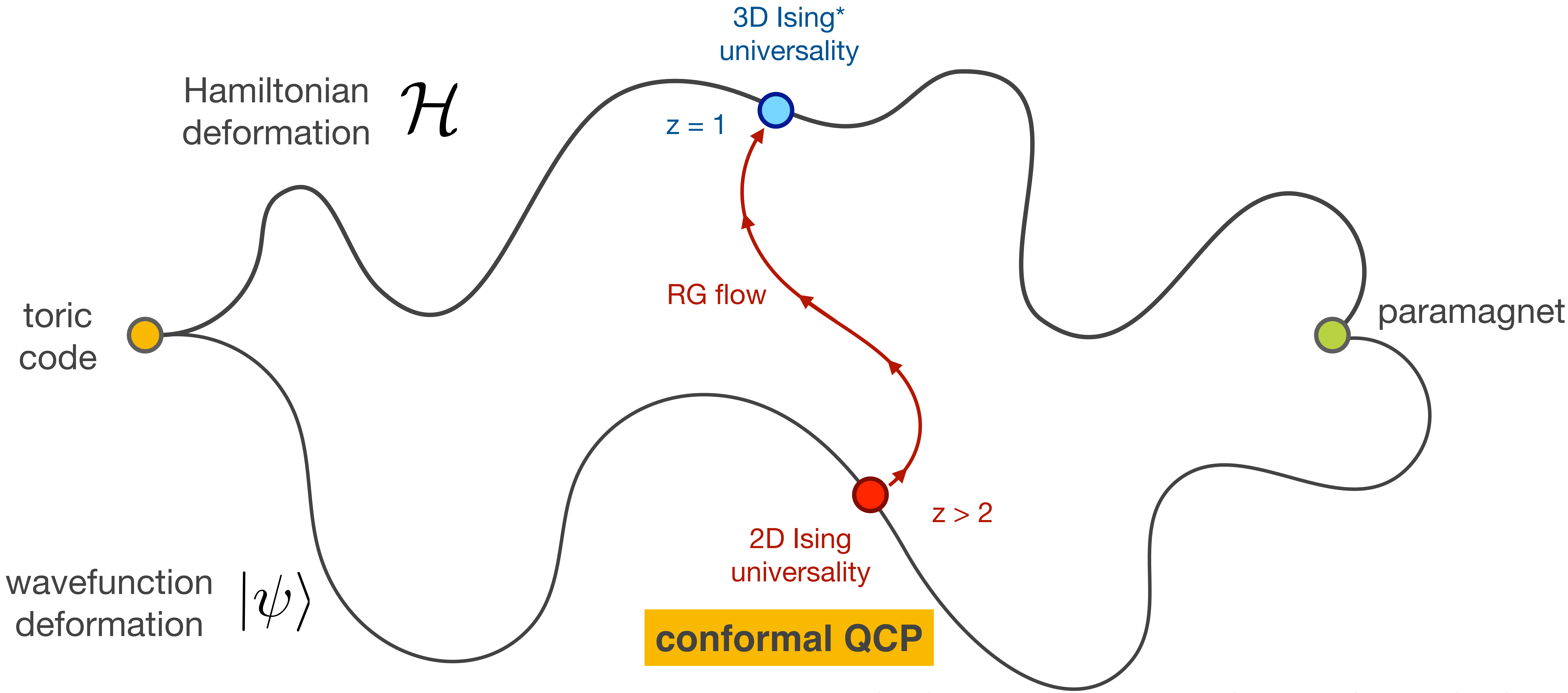
wavefunction deformations

$$|\psi(t,h)\rangle = \exp\left(\frac{1}{2}\sum_l h\mu_l^z + t\sigma_l^x\right)|\psi_0\rangle$$



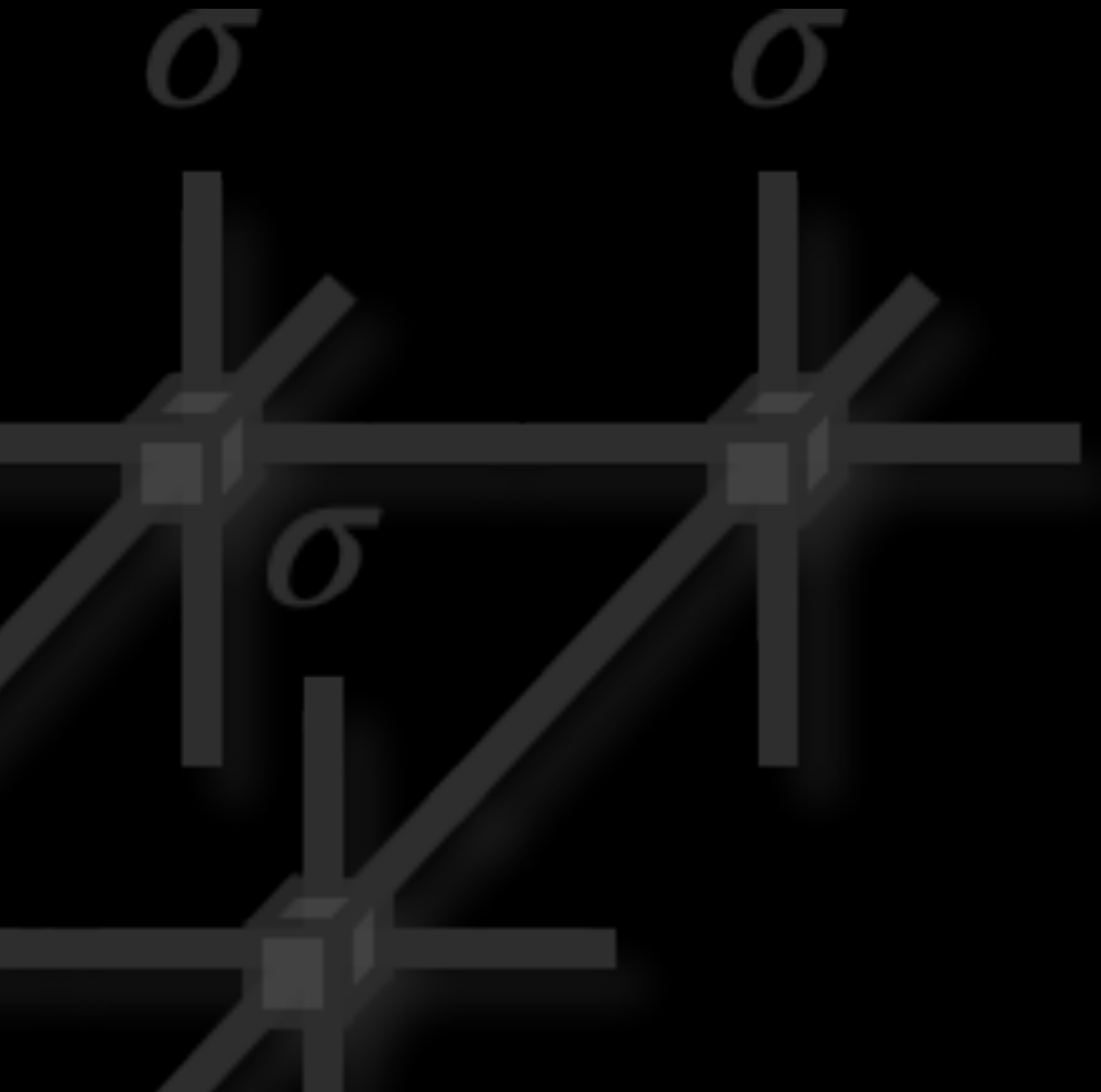
arXiv:2203.00015

phase transitions & deformations



Ardonne, Fendley, Fradkin (2004),
Castelnovo & Chamon (2008), Fendley (2008)

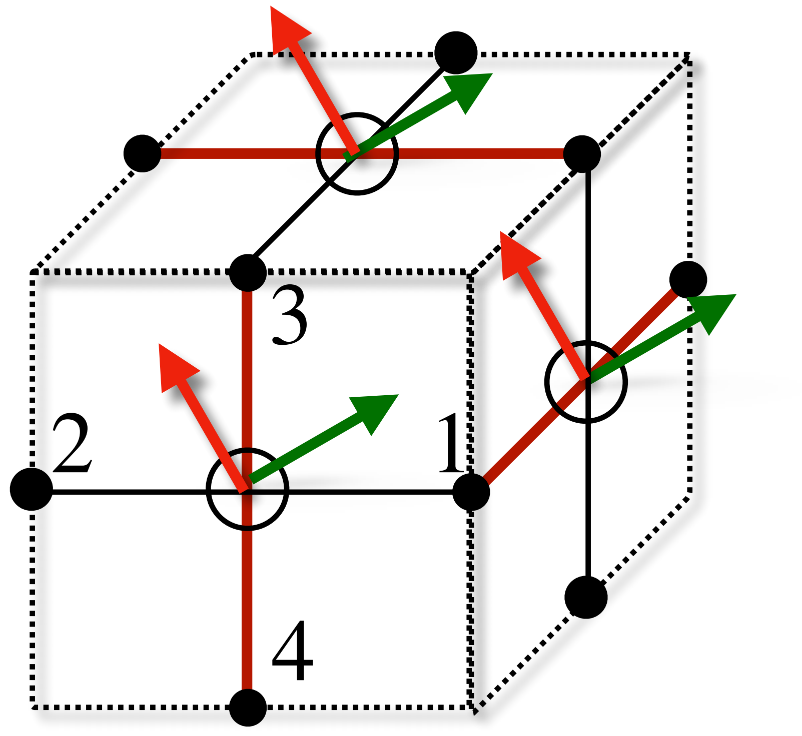
Castelnovo, ST, Troyer (2010),
Isakov, Fendley, Ludwig, ST, Troyer (2011)



wavefunction deformations

tensor network wavefunction

$$|\psi(t,h)\rangle = \exp\left(\frac{1}{2}\sum_{l} h\mu_l^z + t\sigma_l^x\right)|\psi_0\rangle$$



(dual cubic lattice)

$|\psi_0\rangle \sim |\text{2D Toric Code}\rangle^{\otimes L_x+L_y+L_z}$

physical indices

$\mu^z = (-1)^{n_1-n_2-n_3+n_4}$

$\sigma^z = (-1)^{n_4-n_3}$

virtual indices $n = 0,1$

$$\prod_{l \in \square} \sigma_l^x = 1$$

$$\prod_{l \in *} \mu_l^z = 1$$

3D X-cube fracton-free

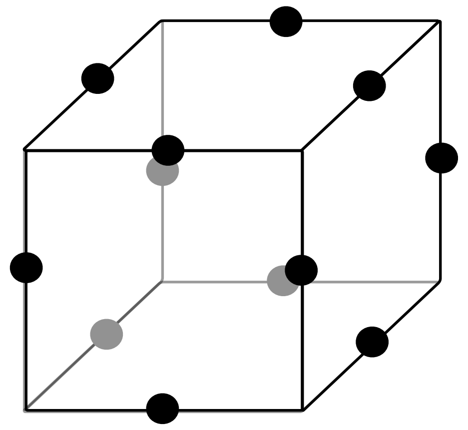
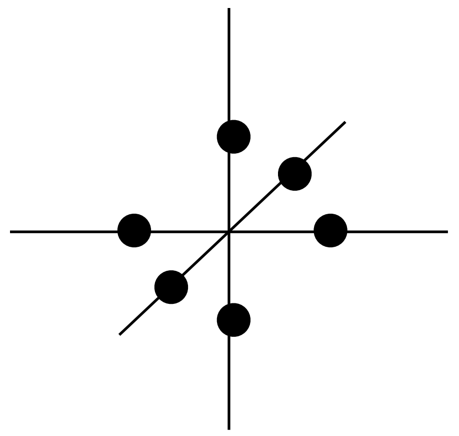
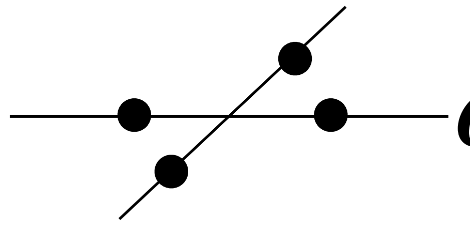
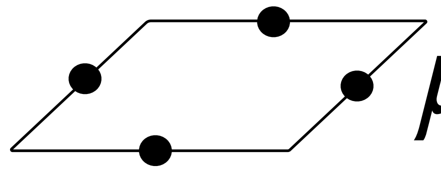
(tensor gauge **Gauss law**)

3D toric code charge-free

(vector gauge **Gauss law**)

an entangled X-Cube & toric code state

$$|\psi\rangle \sim \sum_{\text{magnetic excitations}} |\text{3D X-Cube}\rangle \otimes |\text{3D Toric-Code}\rangle$$

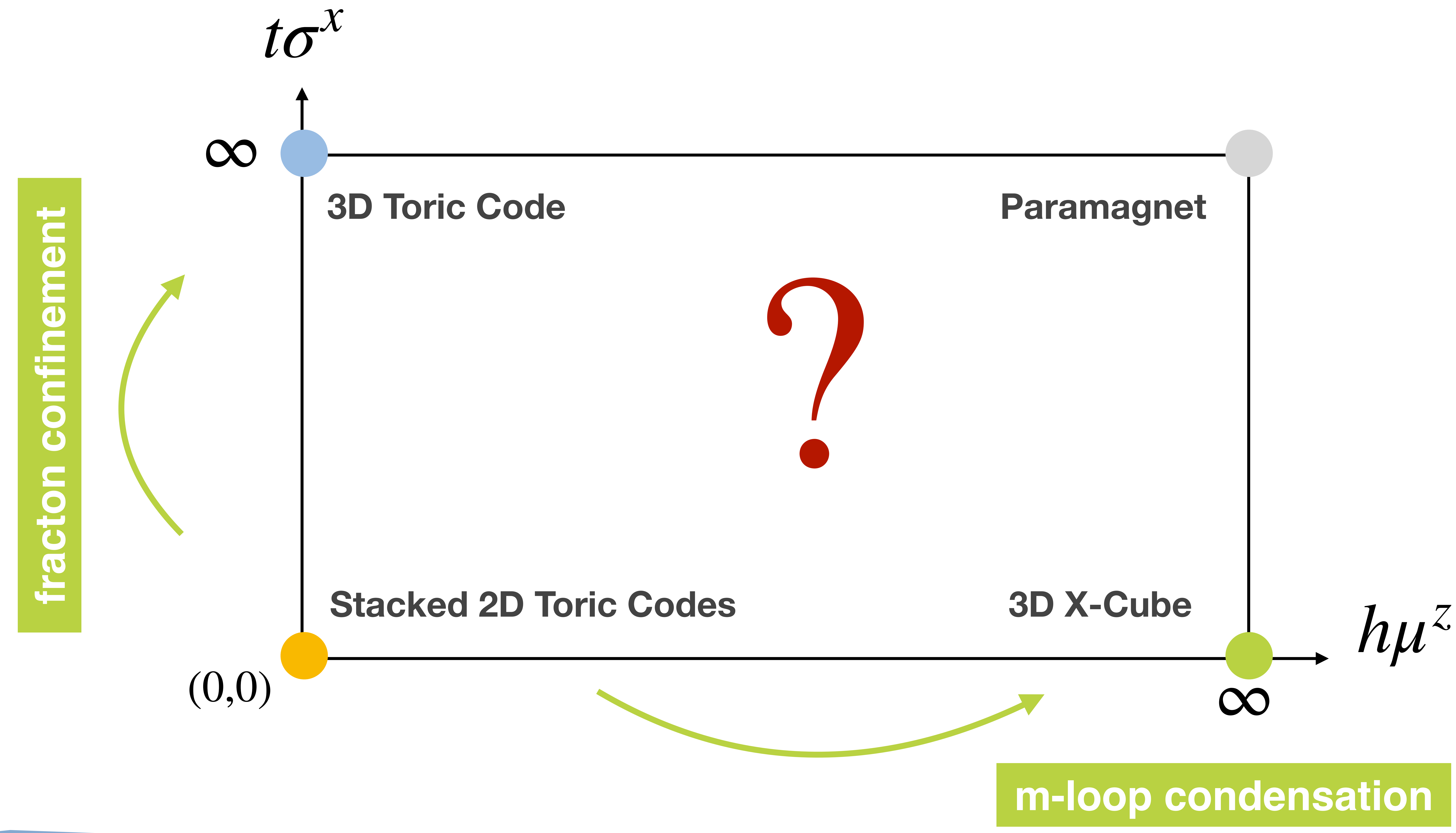
 $\sigma^x = 1$ **fracton free**	 $\mu^z = 1$ **charge free**
 $\sigma^z = 0$ vector **monopole** proliferate	 $\mu^x = 0$ magnetic **flux loop** proliferates

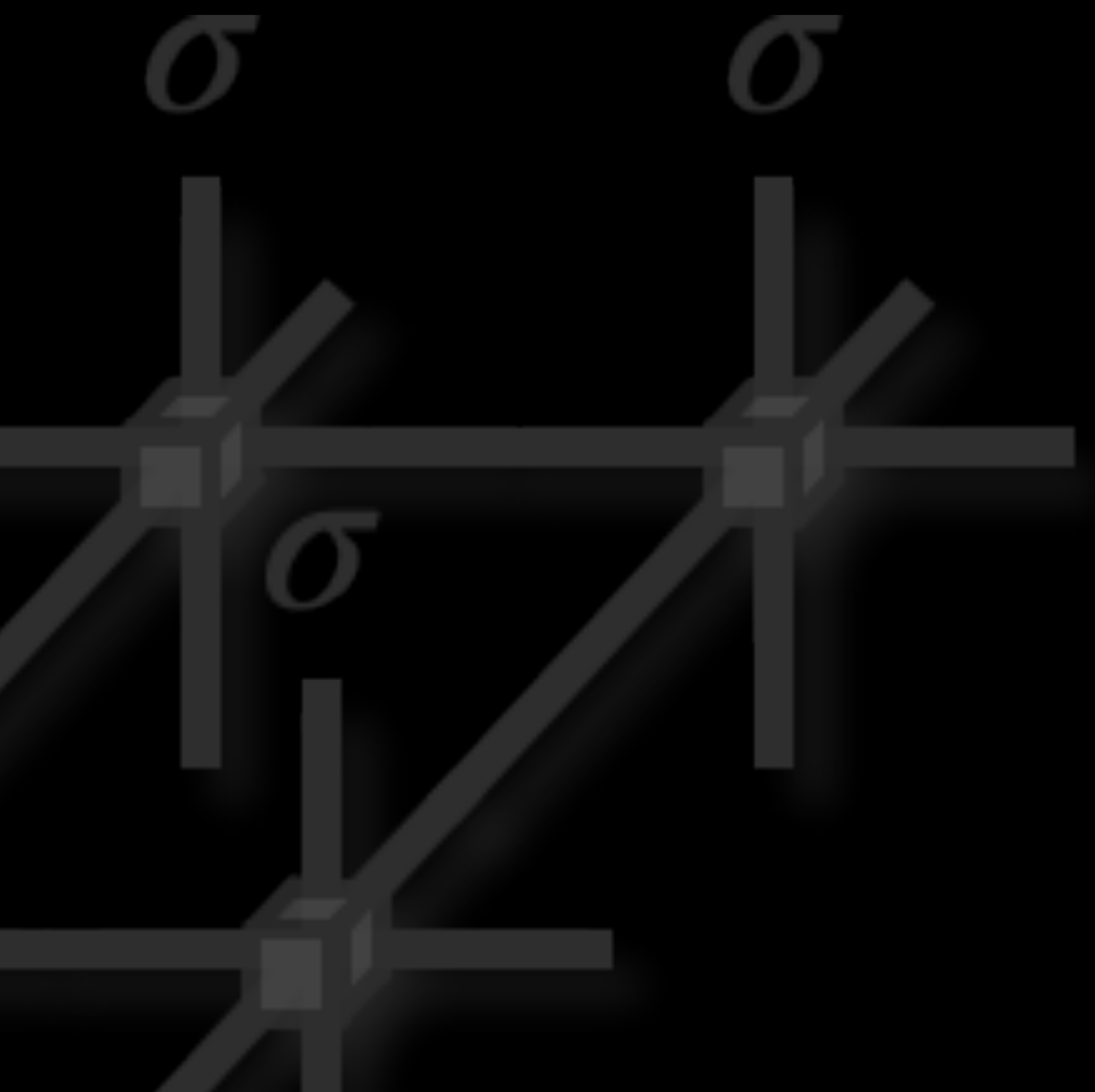
$e^{-t\sigma^x}$ fluctuate

fluctuate $e^{-h\mu^z}$

solvable limits

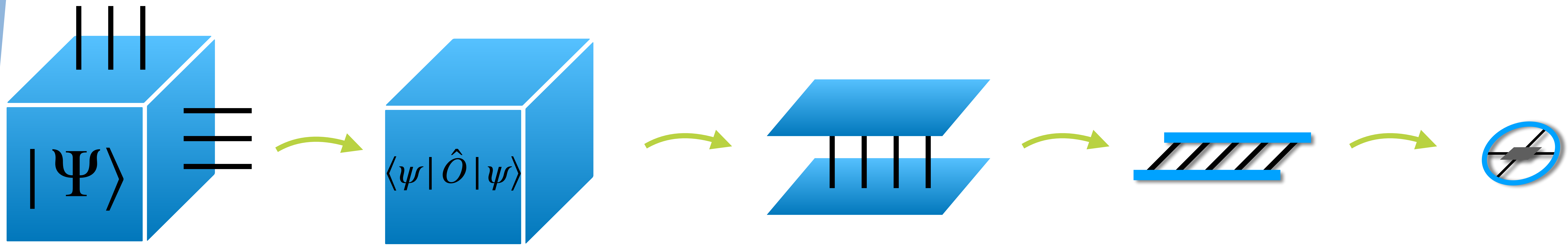
$$|\psi(t,h)\rangle = \exp\left(\frac{1}{2}\sum_l h\mu_l^z + t\sigma_l^x\right)|\psi_0\rangle$$





tensor network calculations

tensor network compression

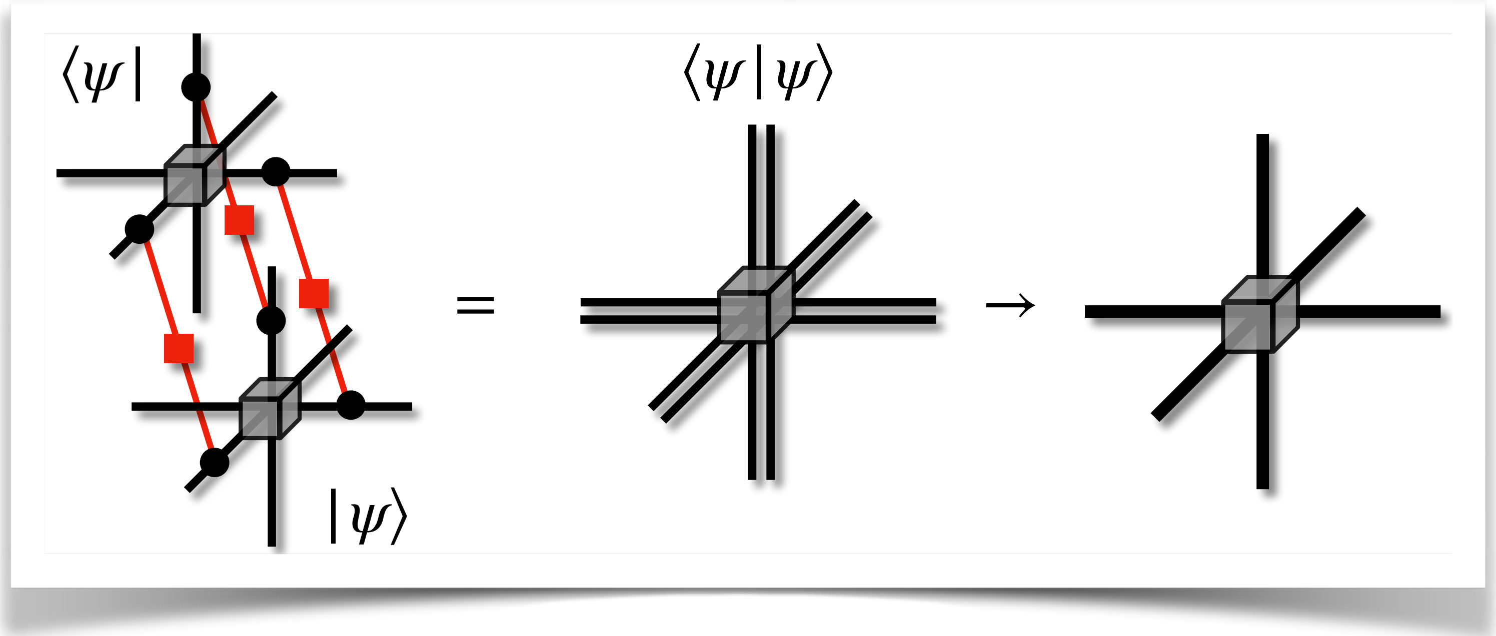


3D PEPS	3D tensor network	2D iPEPS	1D MPS	0D number
wavefunction	“classical” model	boundary fixed point	boundary fixed point	order parameter

tensor network state representation
He, Zheng, Bernevig, Regnault 2018

2D iPEPS optimization
Vanderstraeten, Haegeman, Corboz & Verstraete 2016

classical models & factorization



$$\langle\psi|\psi\rangle = \sum_{\{s\}} e^{-\epsilon_g} \times \sum_{\{\tau\}} e^{-\epsilon_p}$$

$$\epsilon_g = -h \sum_{\square} s\, s\, s\, s$$

gauge symmetry

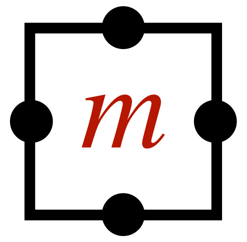
$$\epsilon_p = -t' \sum_{\square} \tau\, \tau\, \tau\, \tau$$

subsystem symmetry

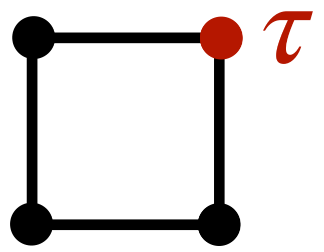
$s = \pm 1, \; \tau = \pm 1$

$t' \equiv \frac{1}{2} \ln \coth t$

3D Ising gauge model
describes m-loop fluctuation



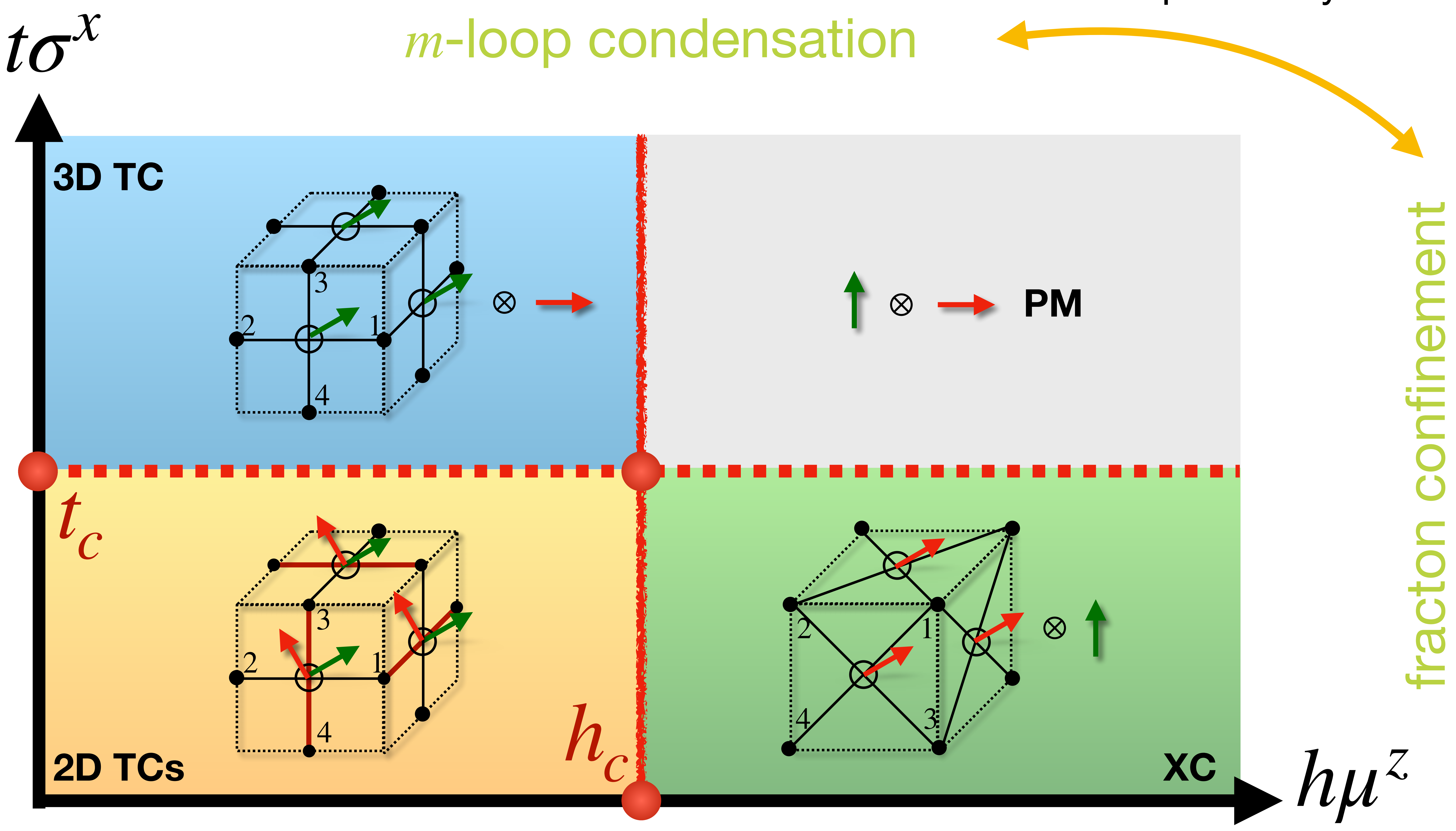
dual 3D plaquette Ising model
describes fracton confinement



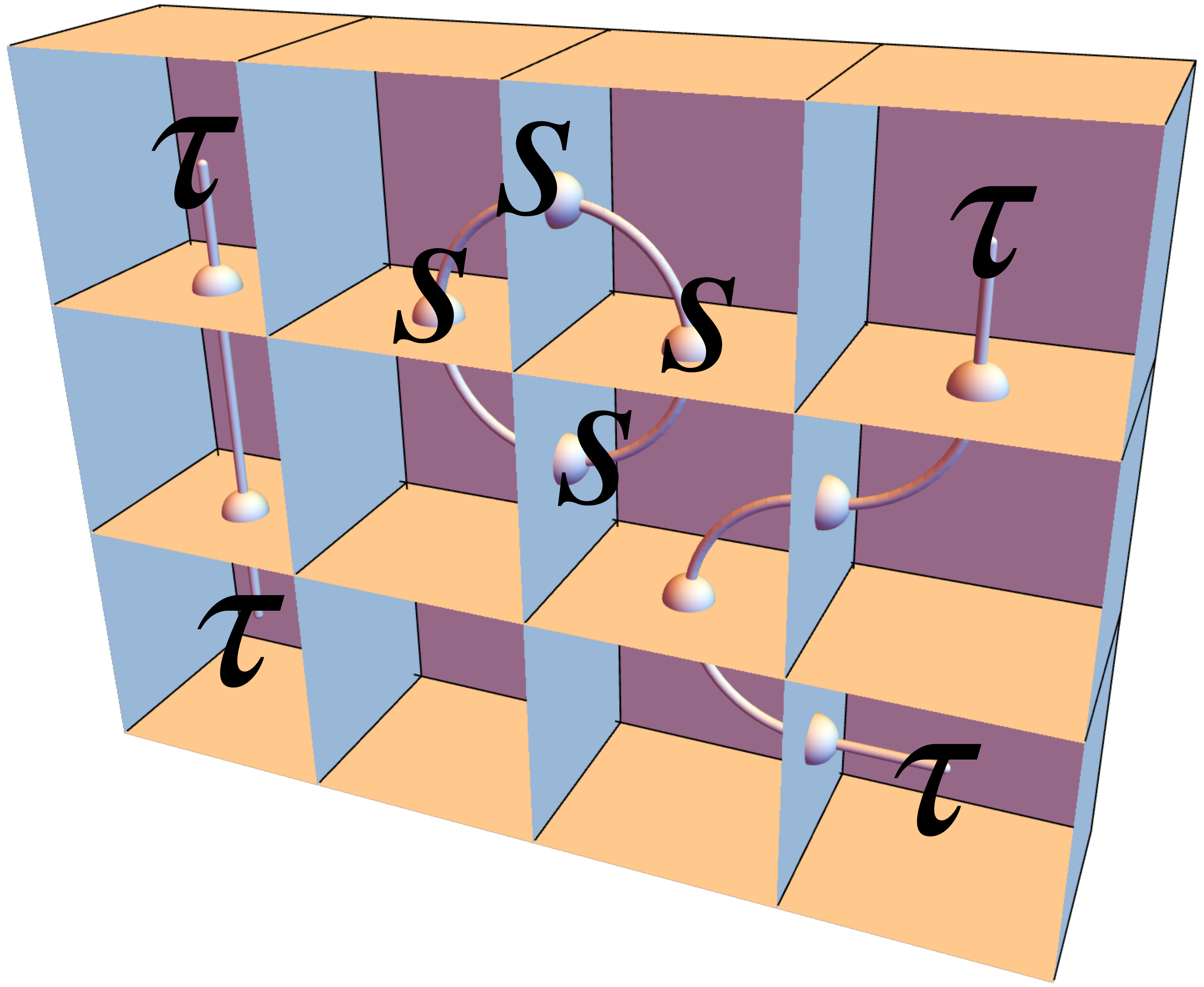
fracton test charge

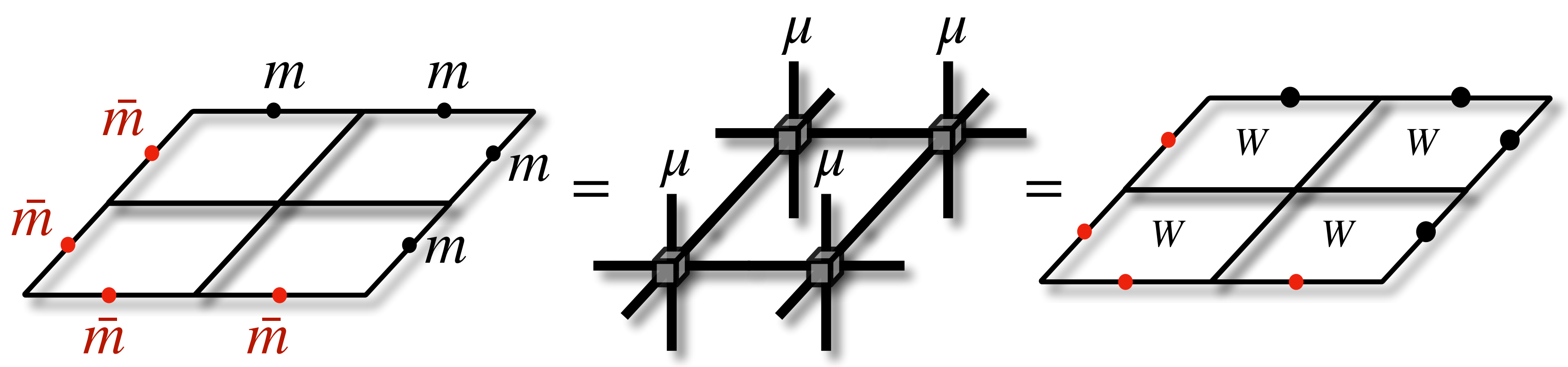
schematic phase diagram

fluctuations commute
independently controlled



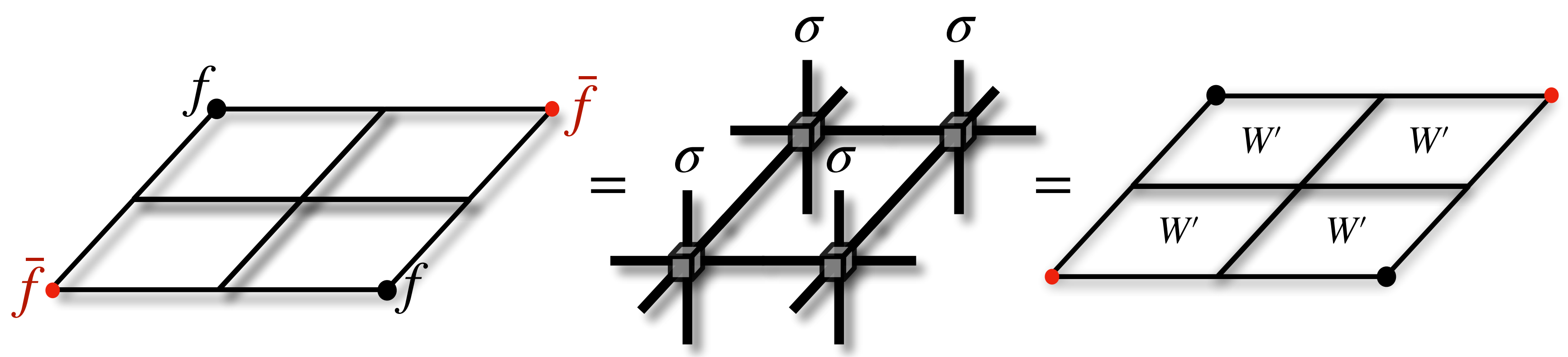
quantum-classical mapping & diagnostics

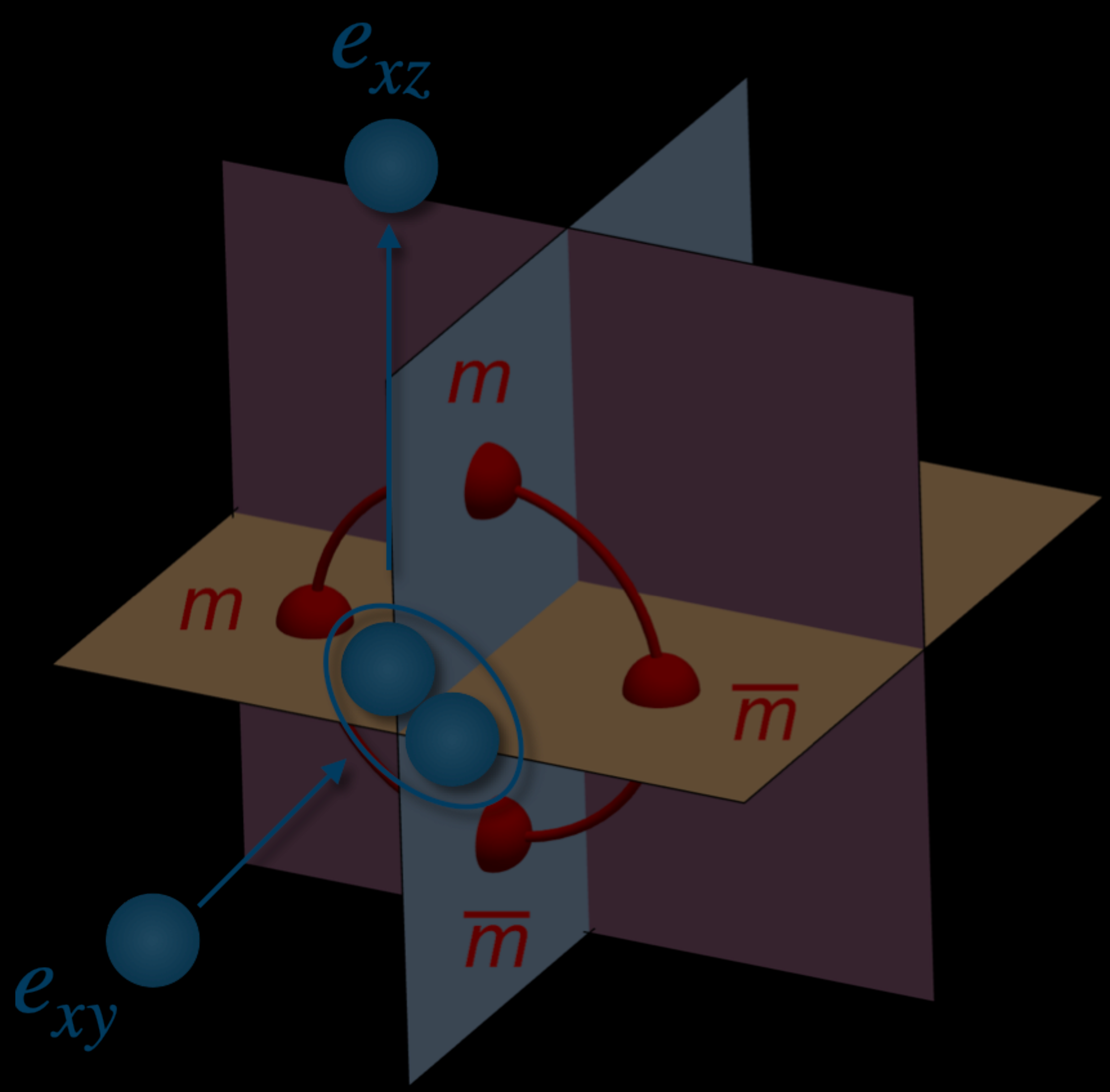




Quantum toric-code	Tensor-network	Classical gauge
μ^z $\langle e \vert e \rangle$ $\langle \psi \vert \prod_{p \in \partial M} m_p \rangle$	$Z_\square$ $X_\square$ $\prod_{\square \in M} Z_\square$	$W_\square$ 't Hooft string Wilson loop

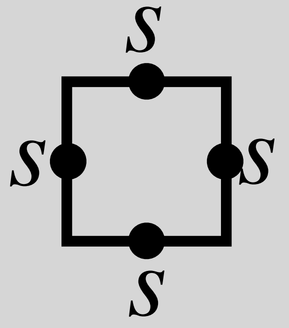
Quantum fracton	Tensor-network	Classical plaquette
quadrupole $\langle \psi \vert$monopole$\rangle$ $\Vert \vert \prod_{j \in \partial\partial M} f_j \rangle \Vert^2$ $-\ln \langle \psi \vert \psi \rangle$	$Z_\square$ $X_\square$ $\prod_{\square \in M} Z_\square$ -ln tTr $\prod_j \hat{T}(j)$	$W'_\square$ twist defect $\prod_{j \in \partial\partial M} \tau_j$ free energy

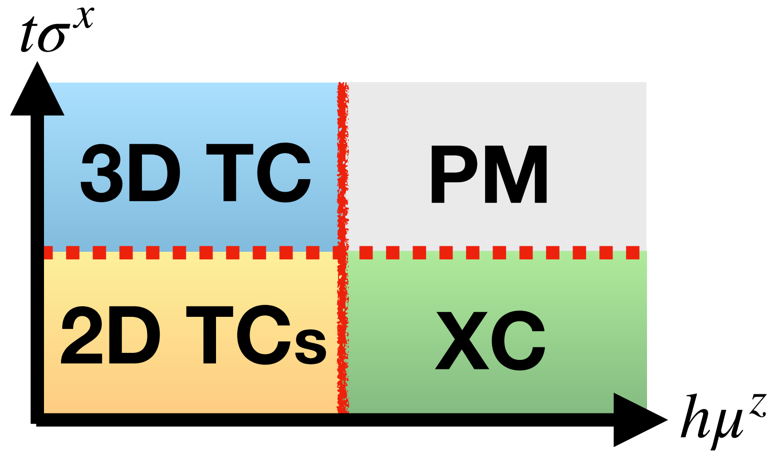




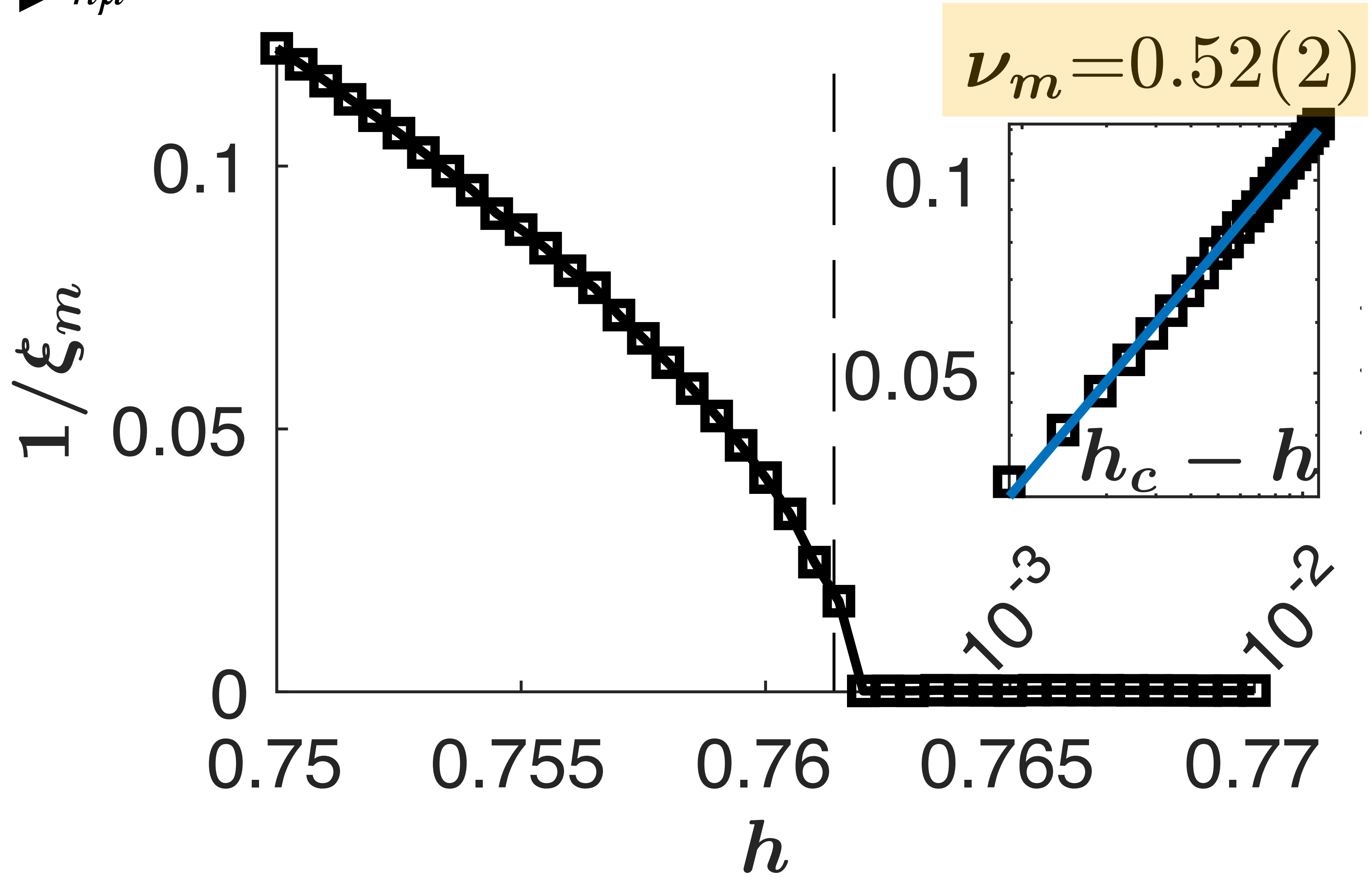
Z_2 model

m-loop condensation

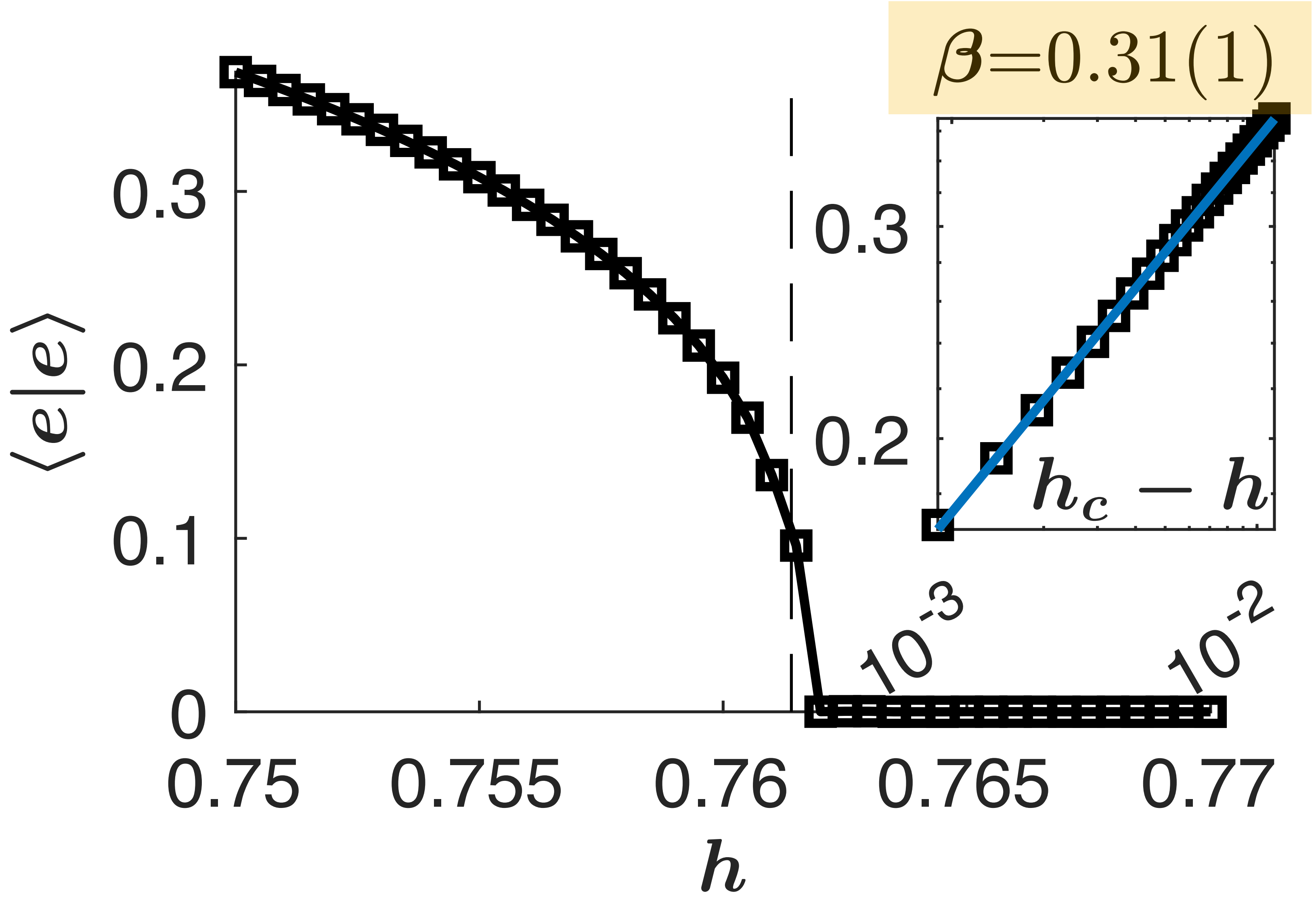
$\epsilon_g = -h \sum_{\square}$ 

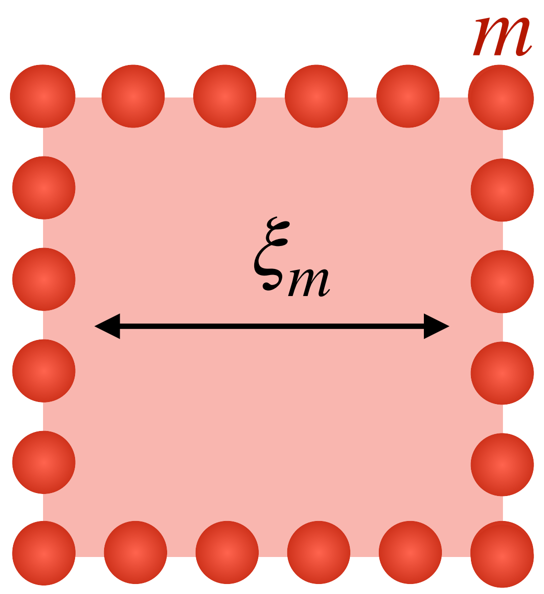


condensation length scale



deconfined charge fraction (dual Ising order)



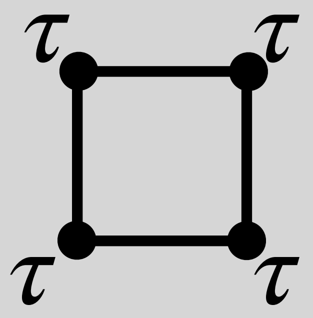


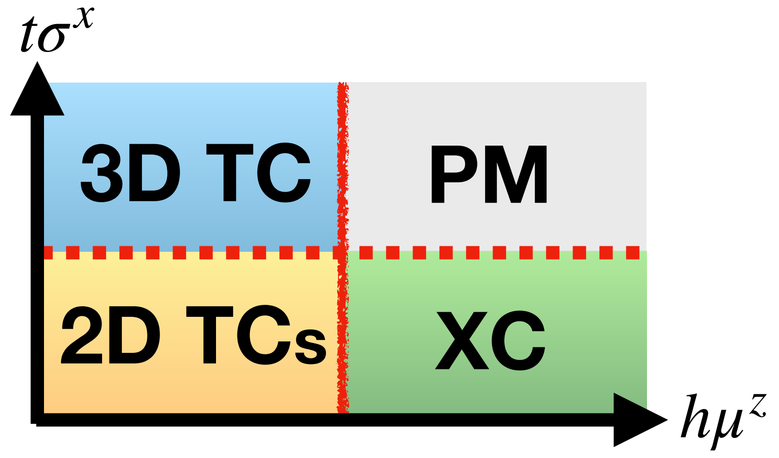
$$\langle\psi| \prod_{p\in\partial M} m_p\rangle \equiv e^{-|M|/\xi_m^2}$$

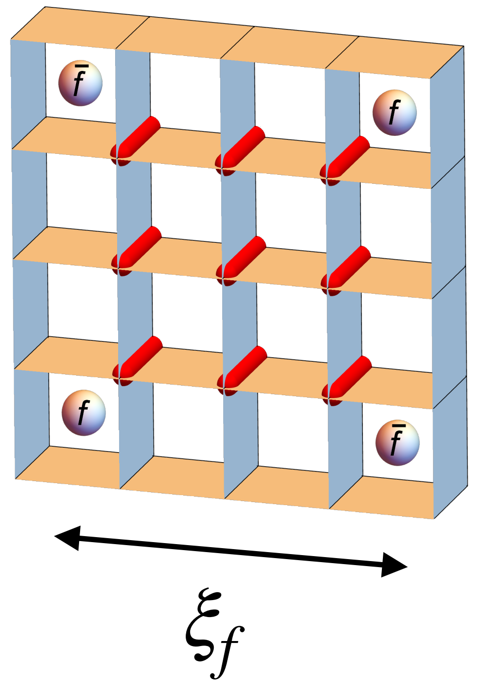
3D Ising* universality class

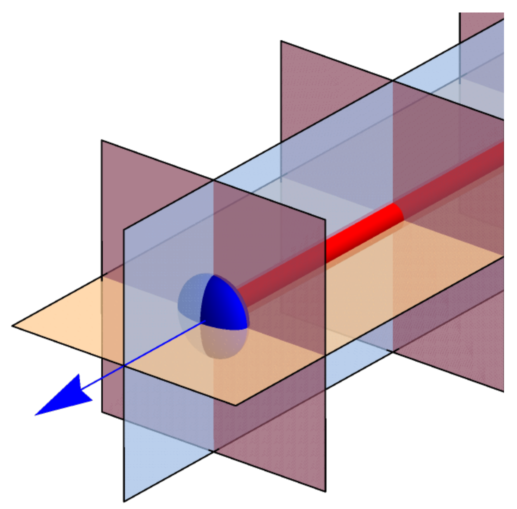
iPEPS D=3, vuMPS chi=72

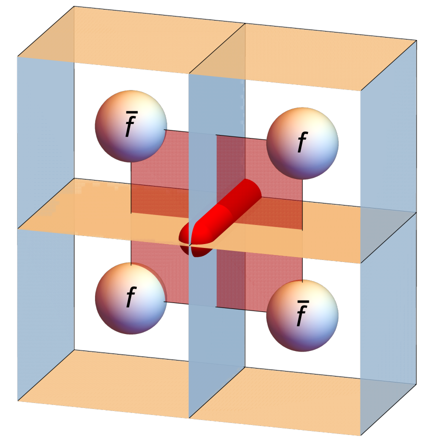
fracton confinement

$\epsilon_p = -t' \sum_{\square}$ 







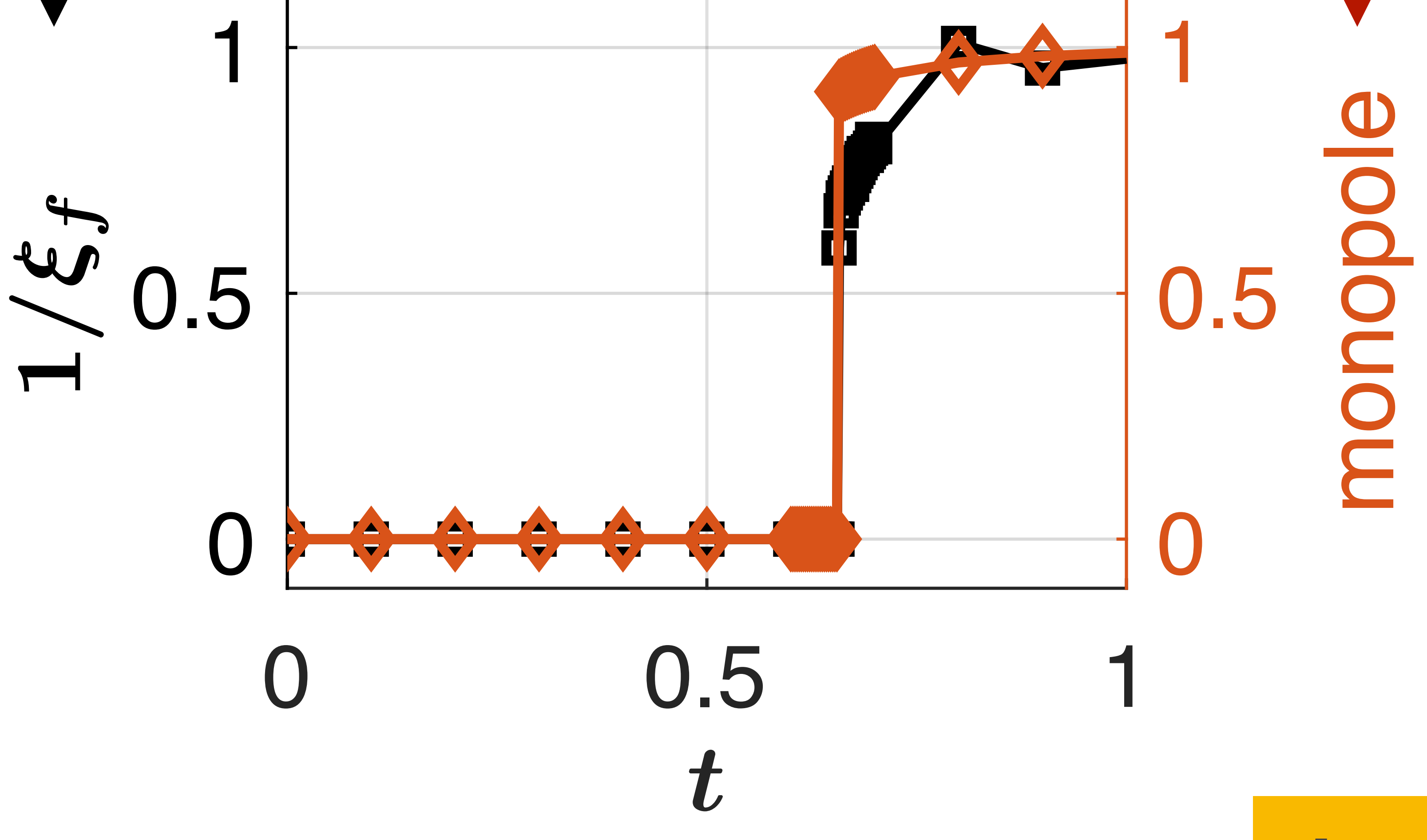


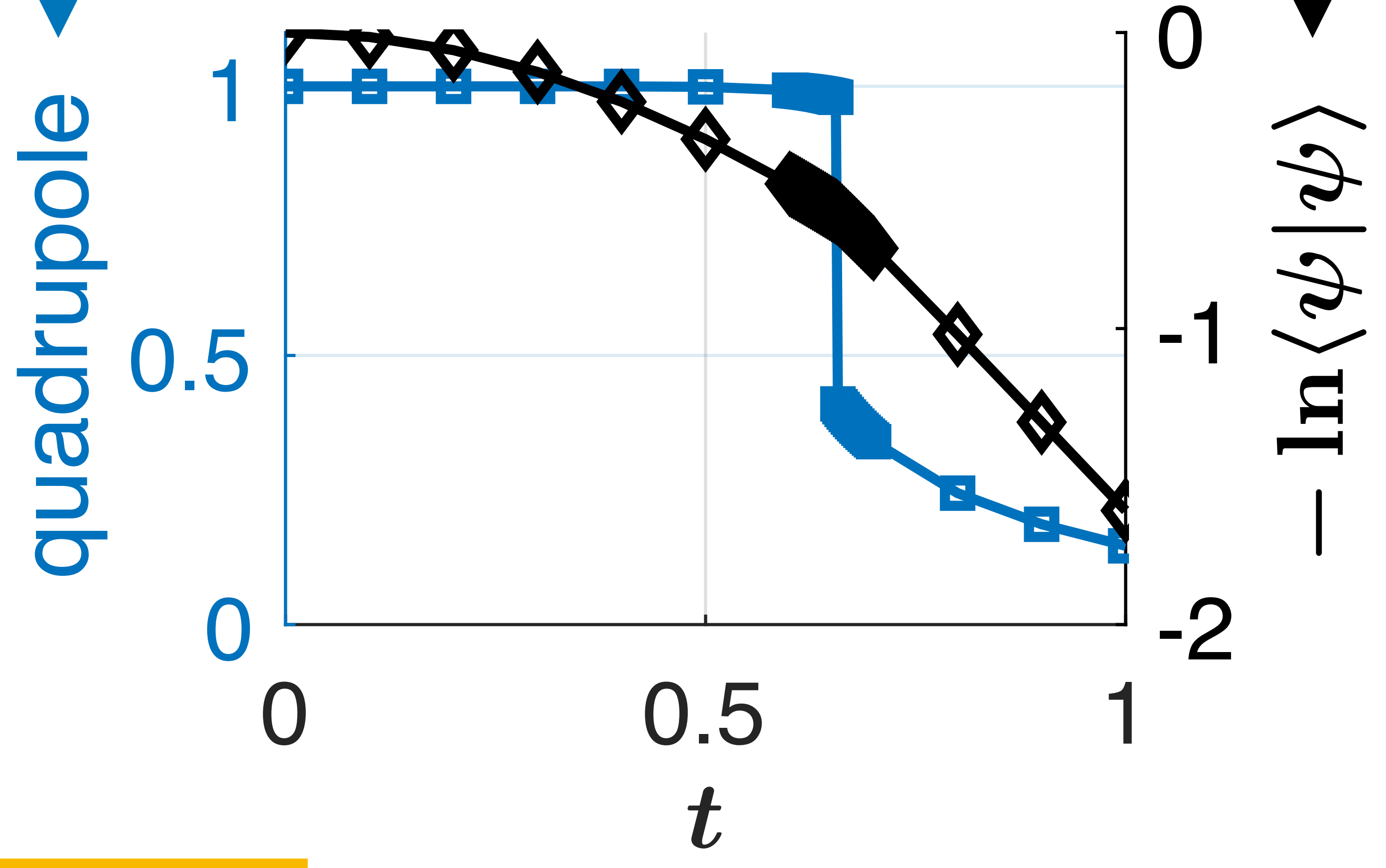
confinement length scale

monopole condensate

fracton quadrupole

“free energy”

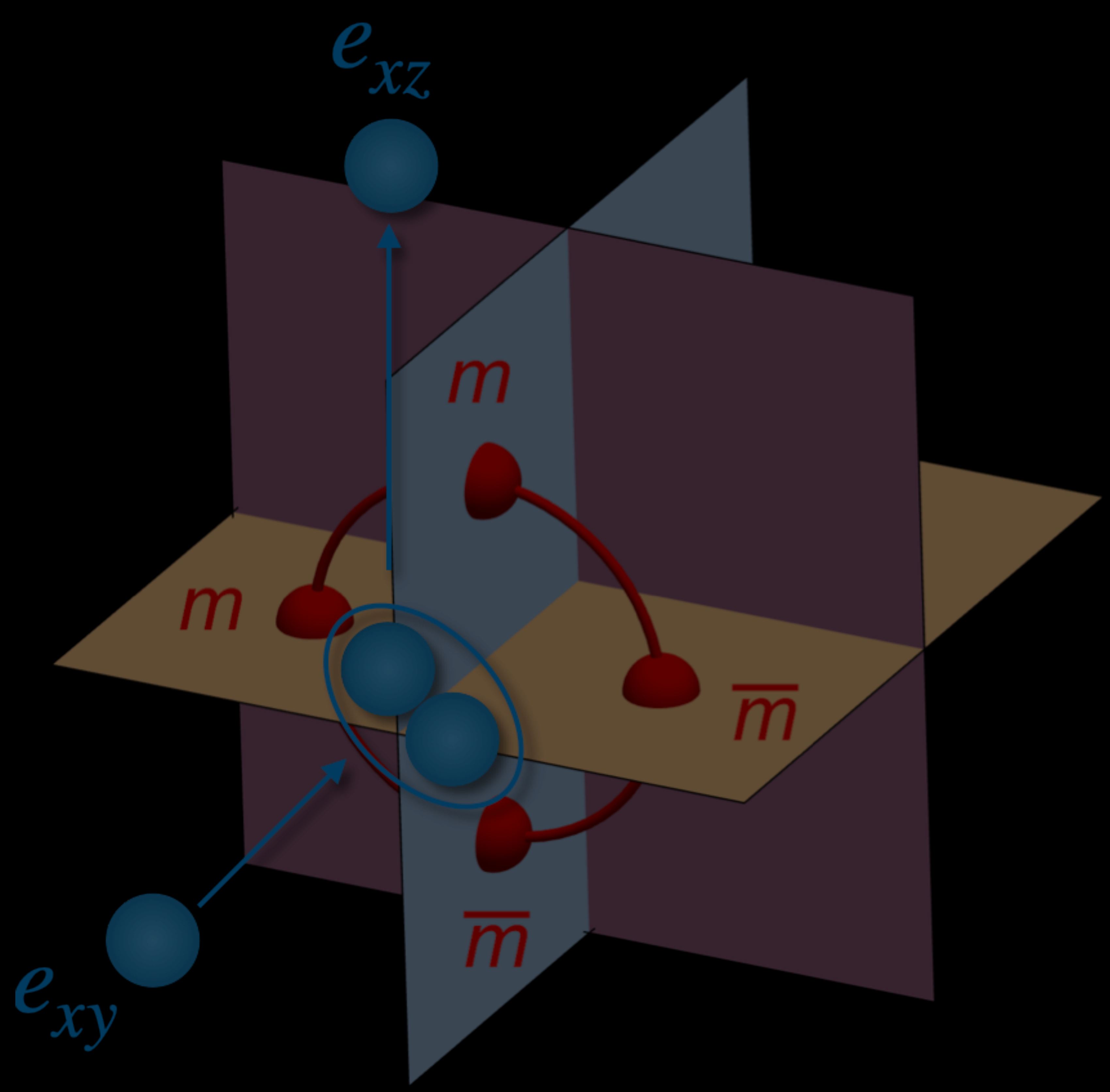




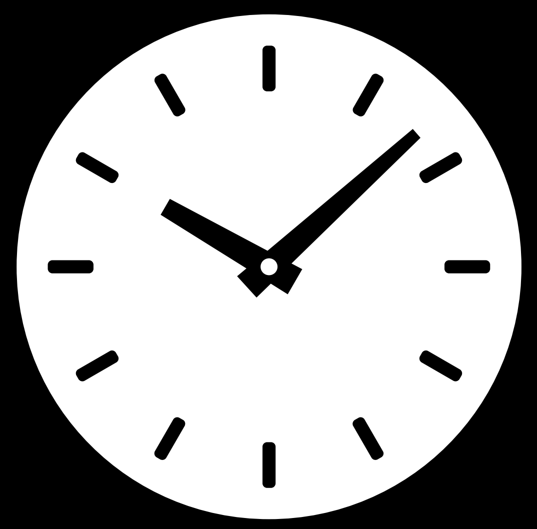
1st order transition

Mueller, Janke, Johnston, 14

iPEPS D=2



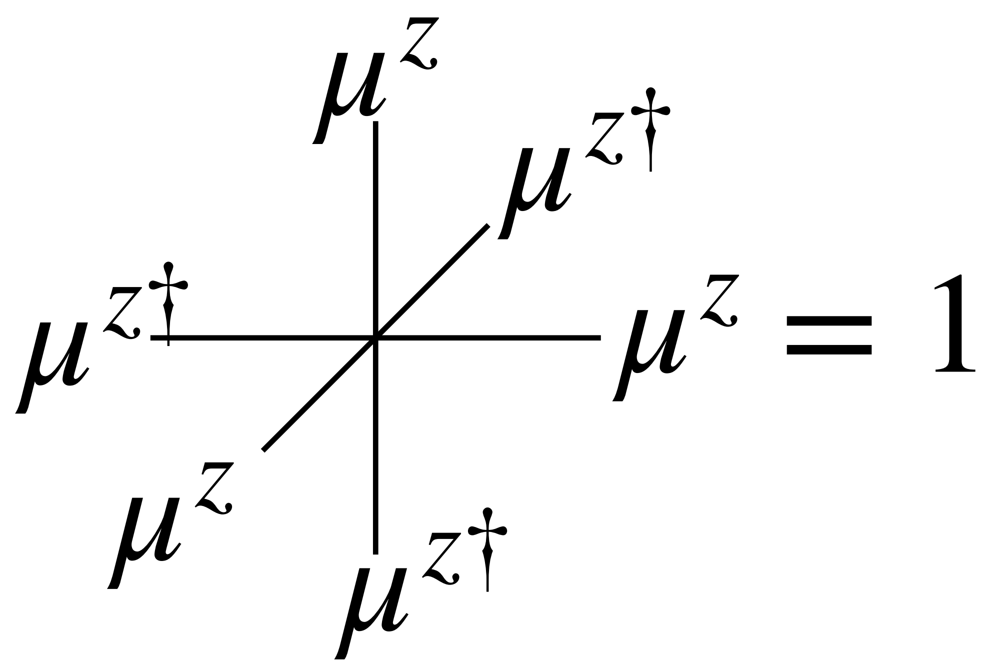
Z_N model



Z_N X-Cube & toric code

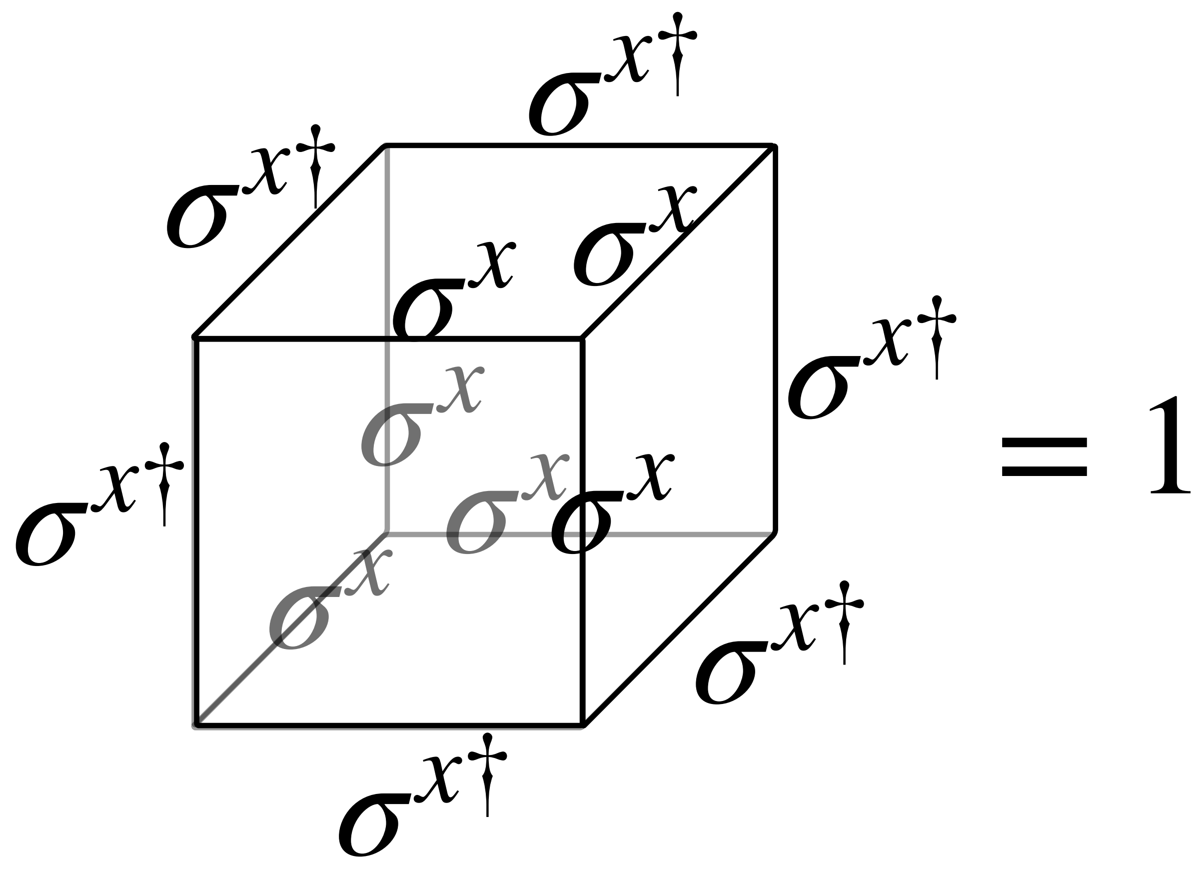
$$ZX = \omega XZ, \quad \omega = e^{i\frac{2\pi}{N}}$$

e.g. $$Z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, \quad X = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$



Z_N toric code star stabilizer

(vector gauge Gauss law)



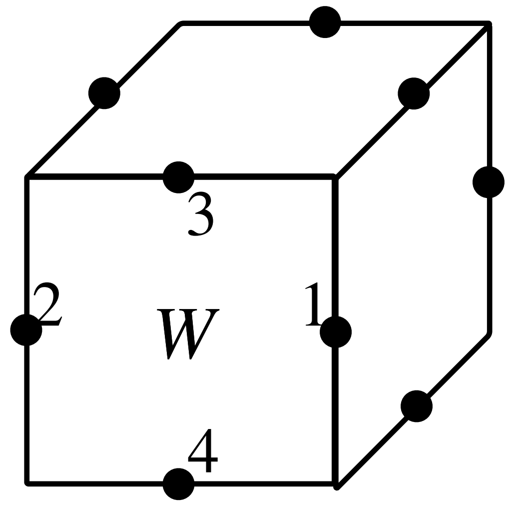
Z_N X-cube stabilizer

(tensor gauge Gauss law)

Z_N wavefunction

$$|\psi(t,h)\rangle = \exp\left(\frac{1}{2}\sum_l h\mu_l^z + t\sigma_l^x\right)|\psi_0\rangle \longrightarrow$$

$$\epsilon_g = -\frac{h}{2}\sum_{\square} W_{\square} + h.c.$$

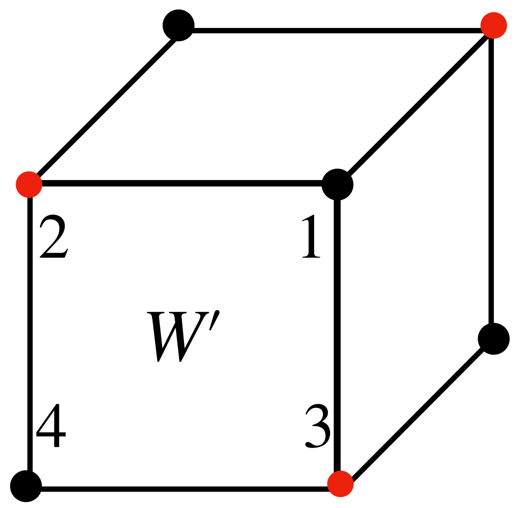


Z_N vector gauge model

$$W = \omega^{n_1 - n_2 - n_3 + n_4}$$

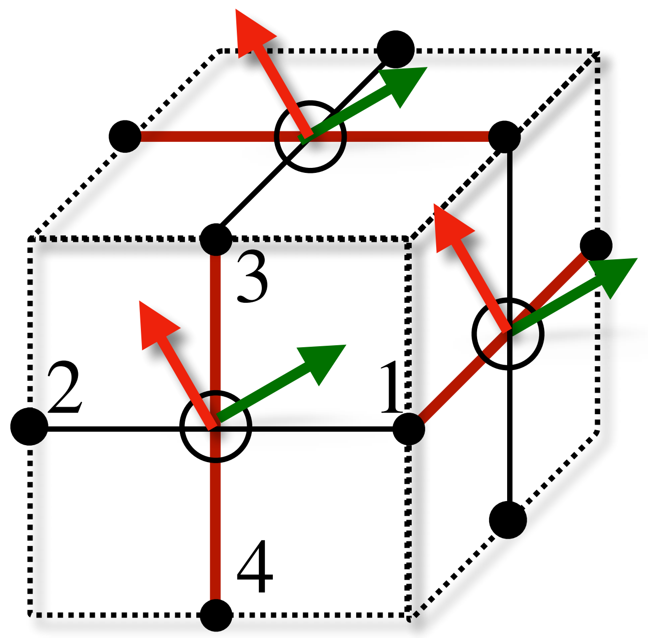
$$\otimes$$

$$\epsilon_p = -\sum_{\square} \ln\left(\frac{1}{N}\sum_{k=0}^{N-1} e^{t\cos\frac{2\pi k}{N}} W'^{k}_{\square}\right)$$



Z_N plaquette clock model

$$W' = \omega^{m_1 - m_2 - m_3 + m_4}$$



physical indices

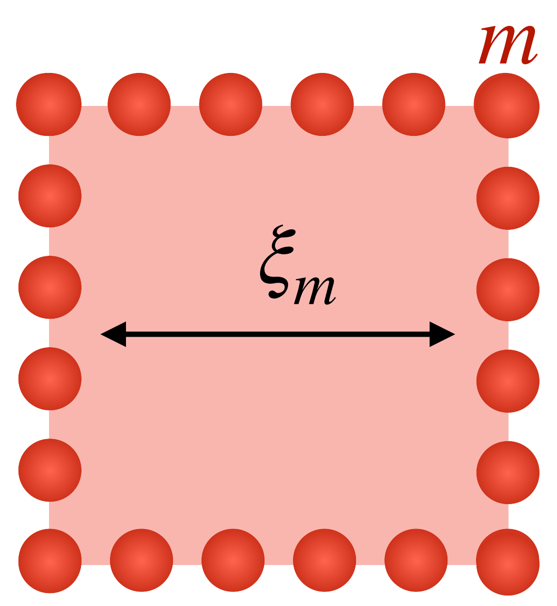
$$\mu^z = (-1)^{n_1 - n_2 - n_3 + n_4}$$

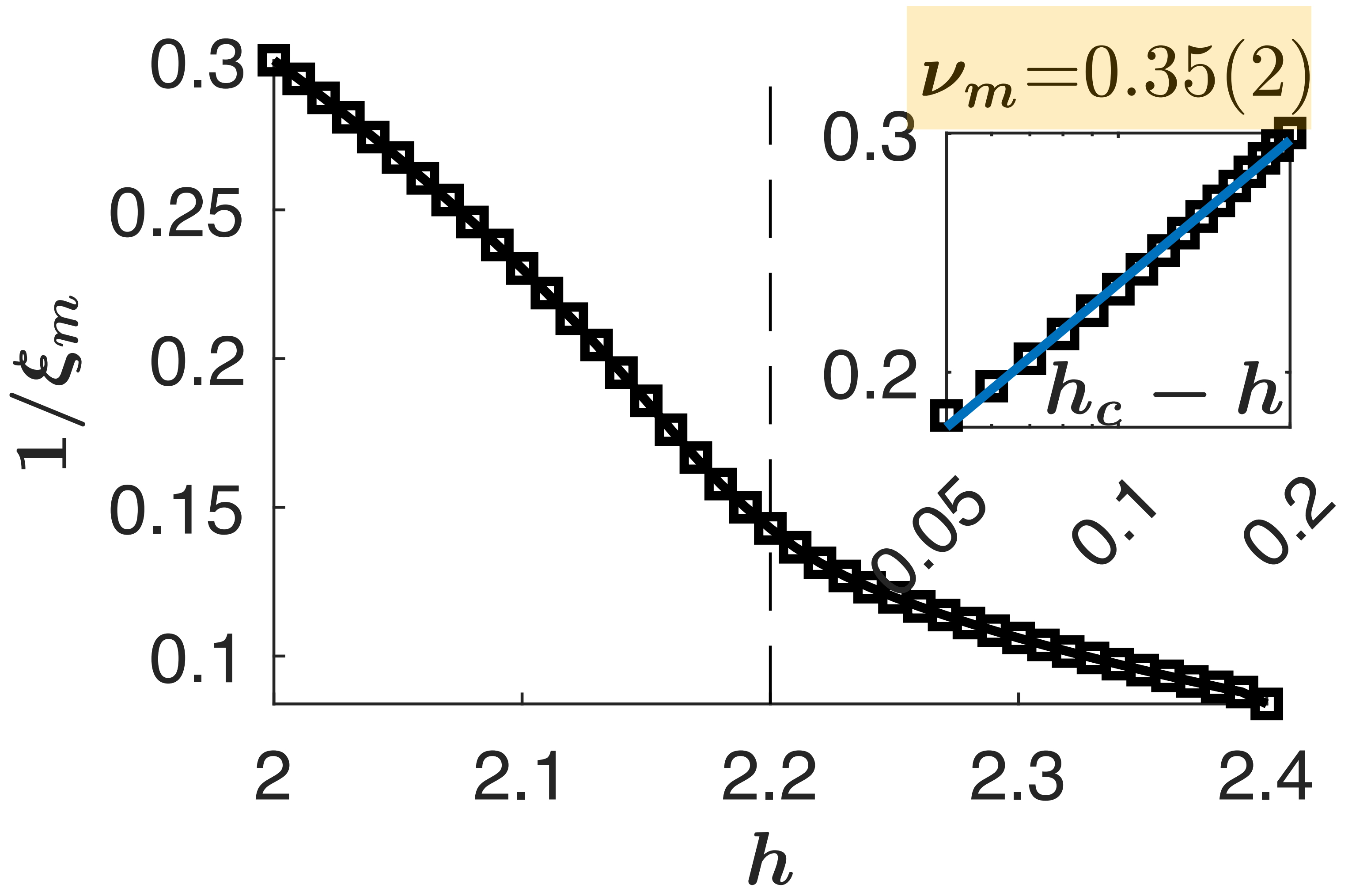
$$\sigma^z = (-1)^{n_4 - n_3}$$

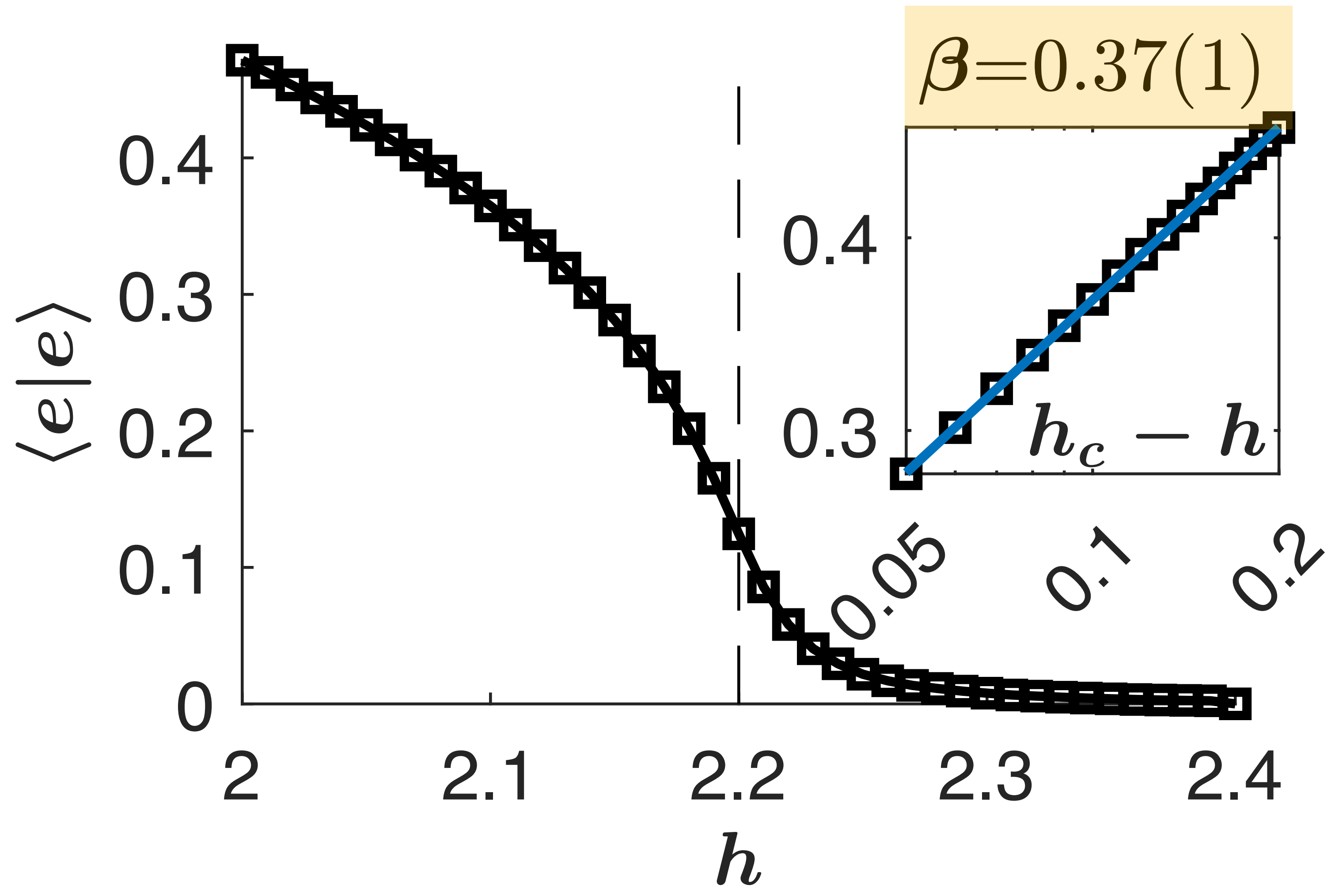
virtual indices $n = 0, 1, \cdots, N-1$

m-loop condensation

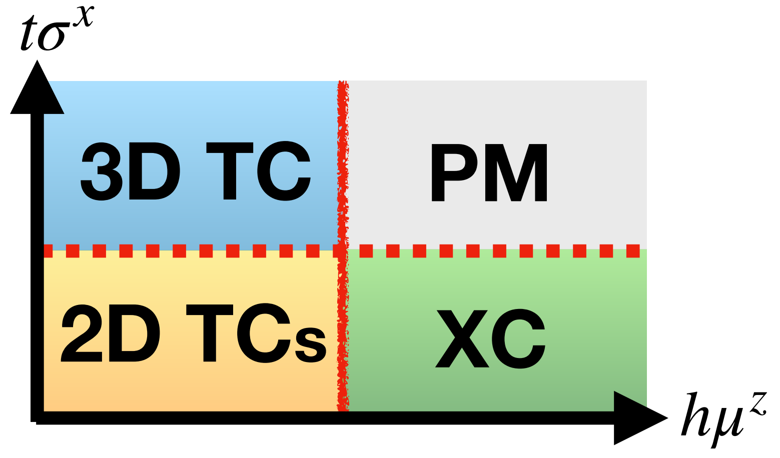
dual to 3D Z_5 clock model







iPEPS D=2, chi=80

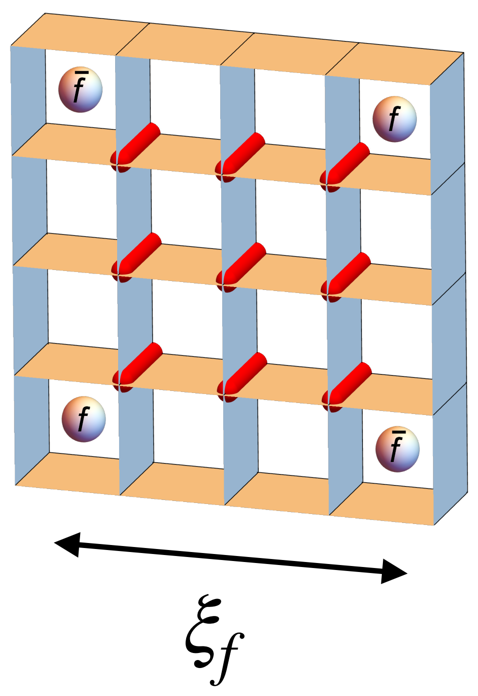


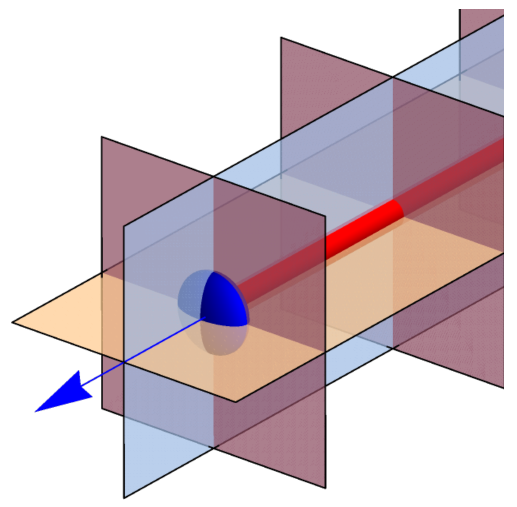
3D Z_N vector gauge model

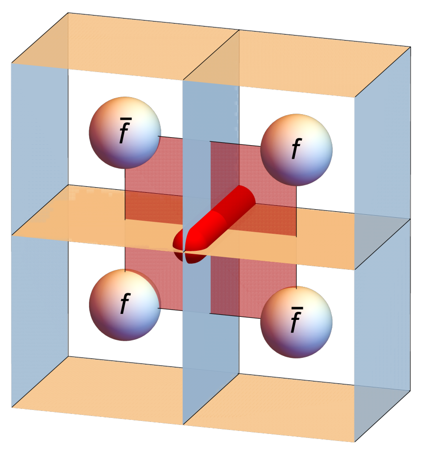
- N=2, Ising*
- N=3, weak 1st order
- N=4, Ising*^2
- N>4, XY*

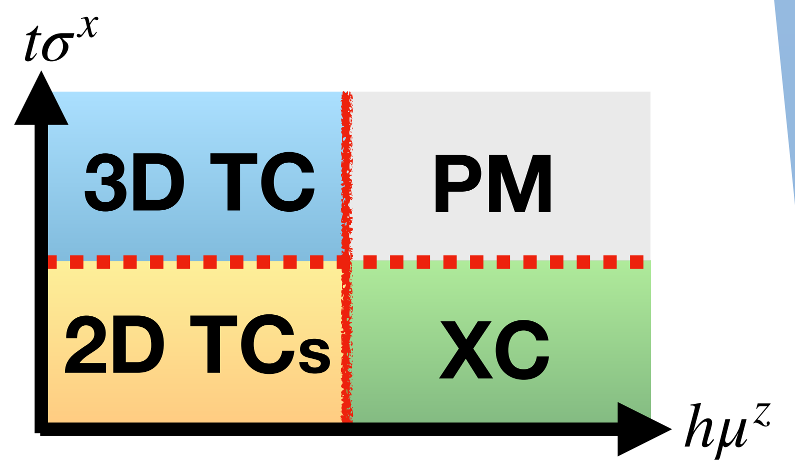
Bhanot & Creutz, 1980
Borisenko, Chelnokov, Cortese, Gravina, Papa & Surzhikov, 2014

fracton confinement







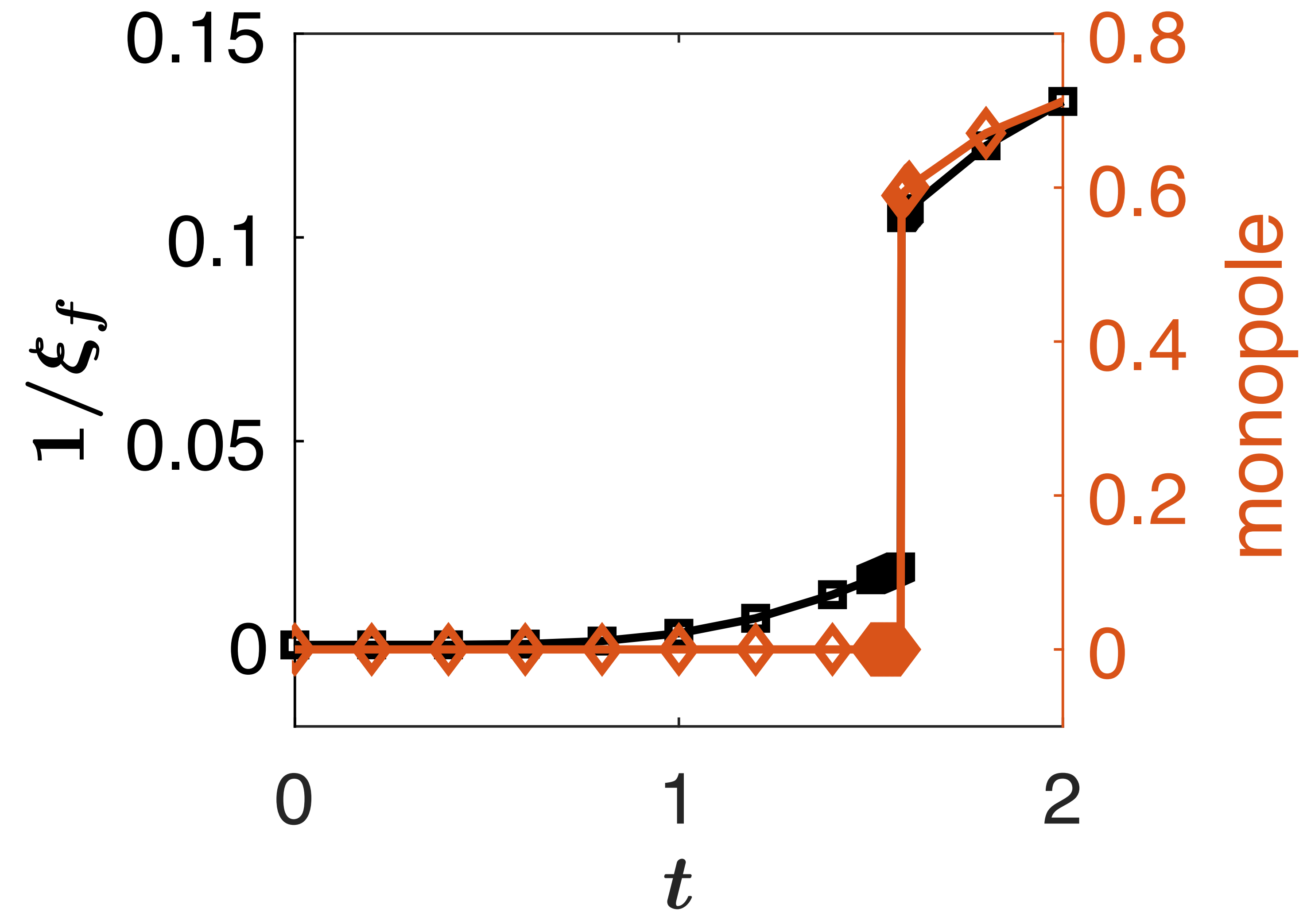


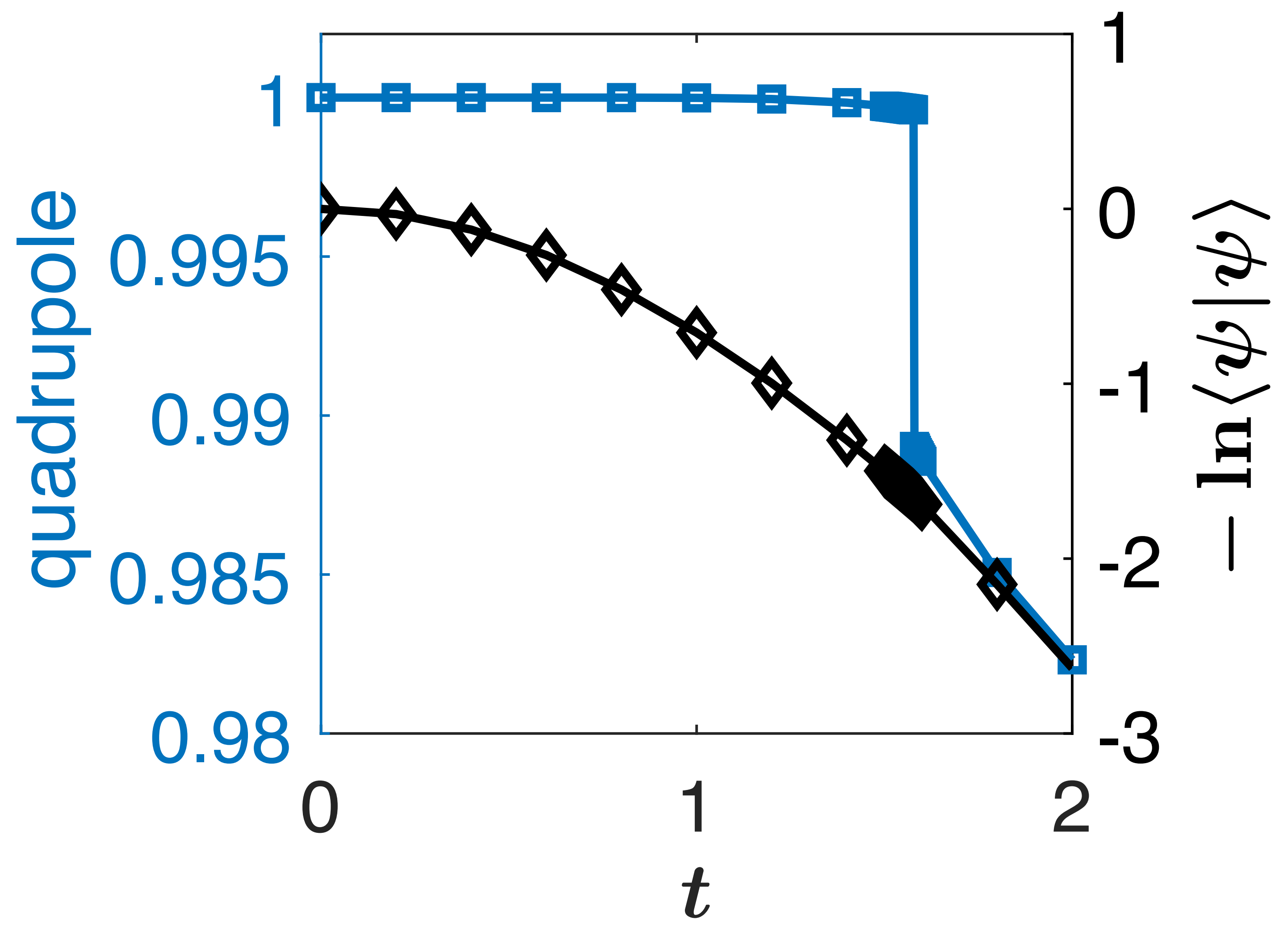
confinement lengthscale

monopole condensate

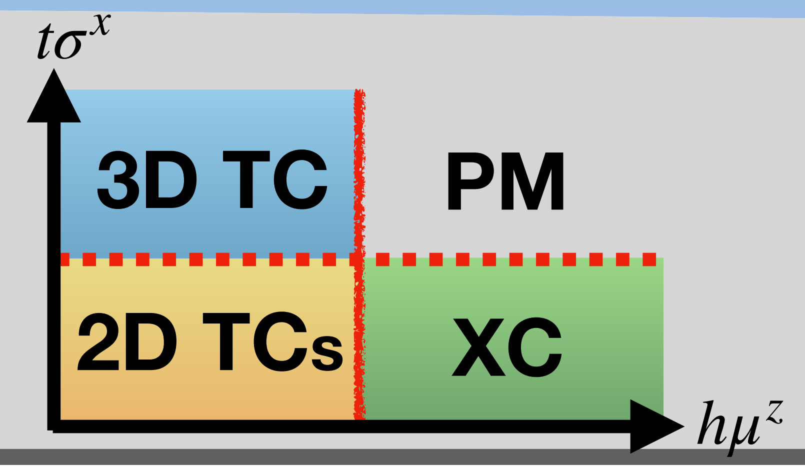
fracton quadrupole

"free energy"



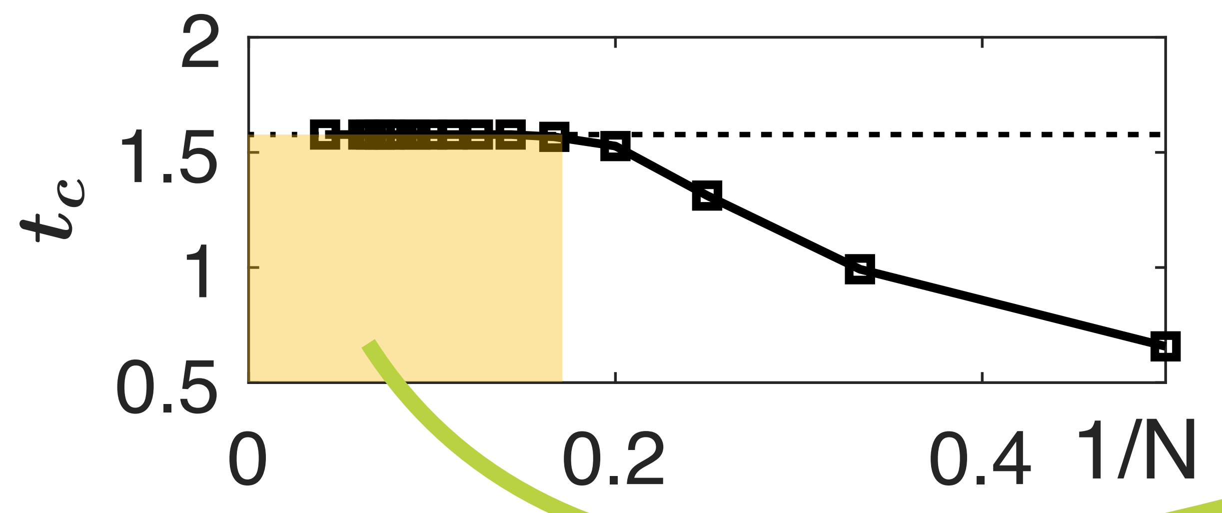


iPEPS D=2, chi=24

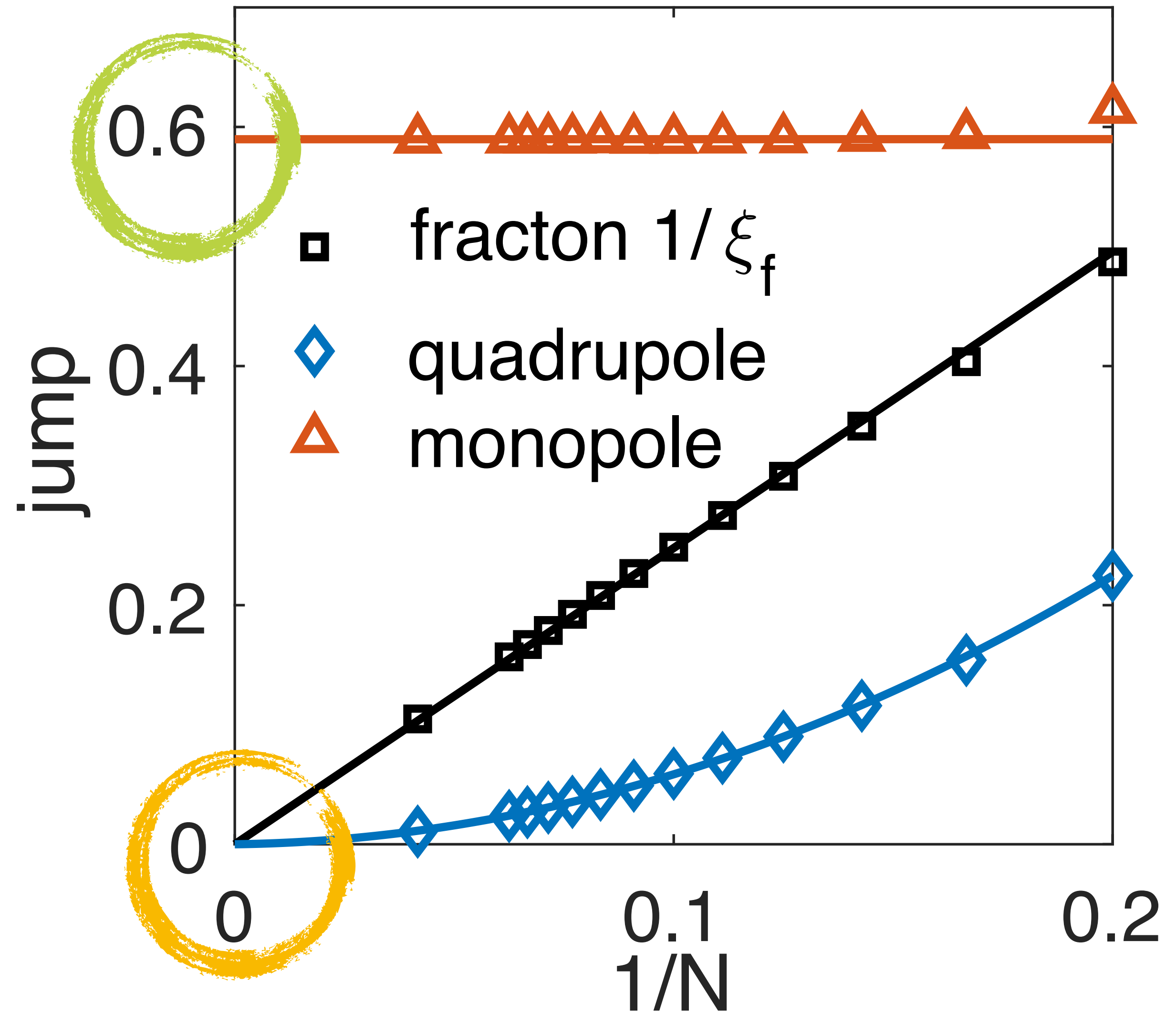


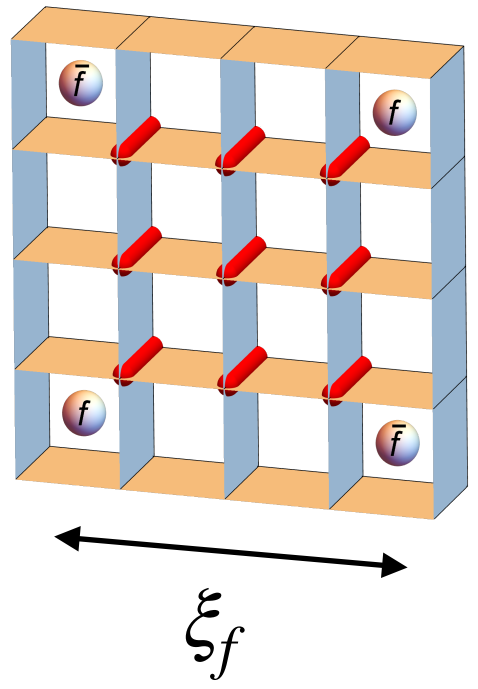
fracton confinement

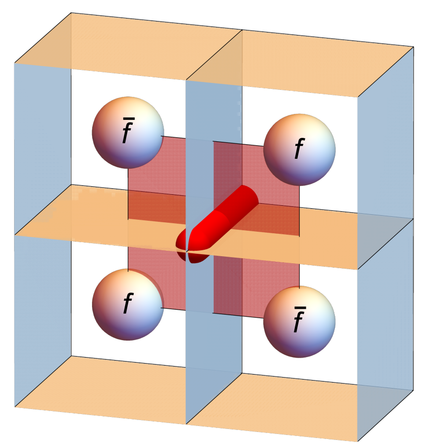
$N \to \infty$

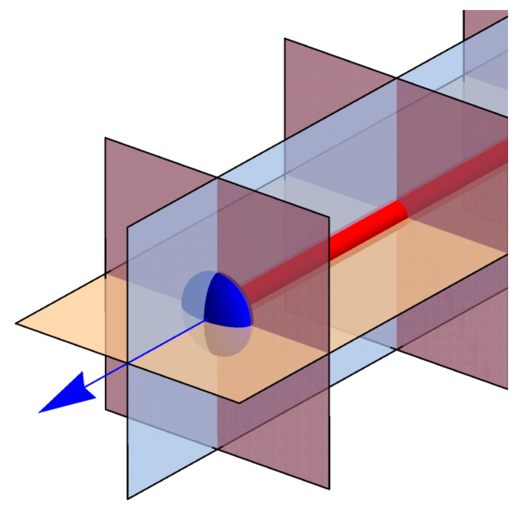


A **finite phase region** for deconfined fracton phase even in limit $N \to \infty$









confinement length scale
diverges linearly
at critical point

fracton quadrupole
vanishes quadratically
at critical point

monopole condensate
keeps jumping
to a finite constant (?)

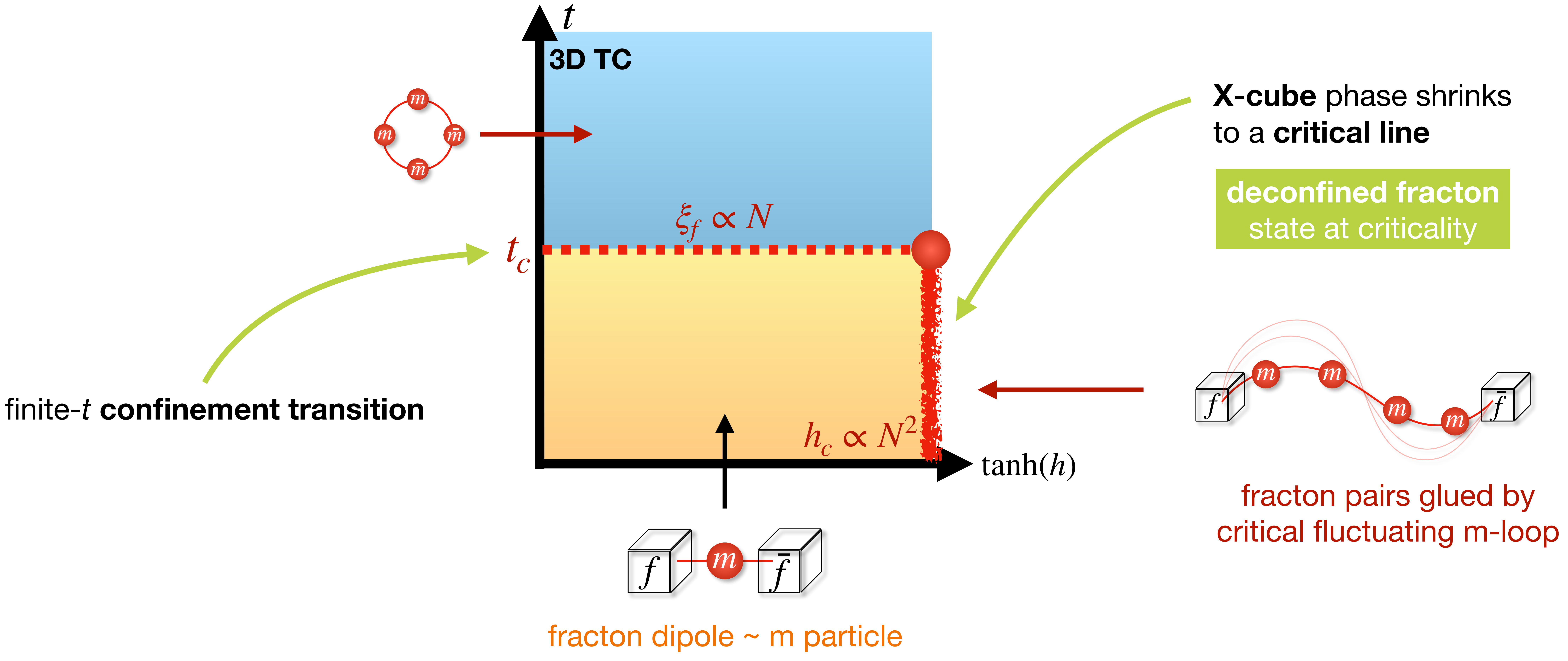
continuous phase transition

non-LGW transition

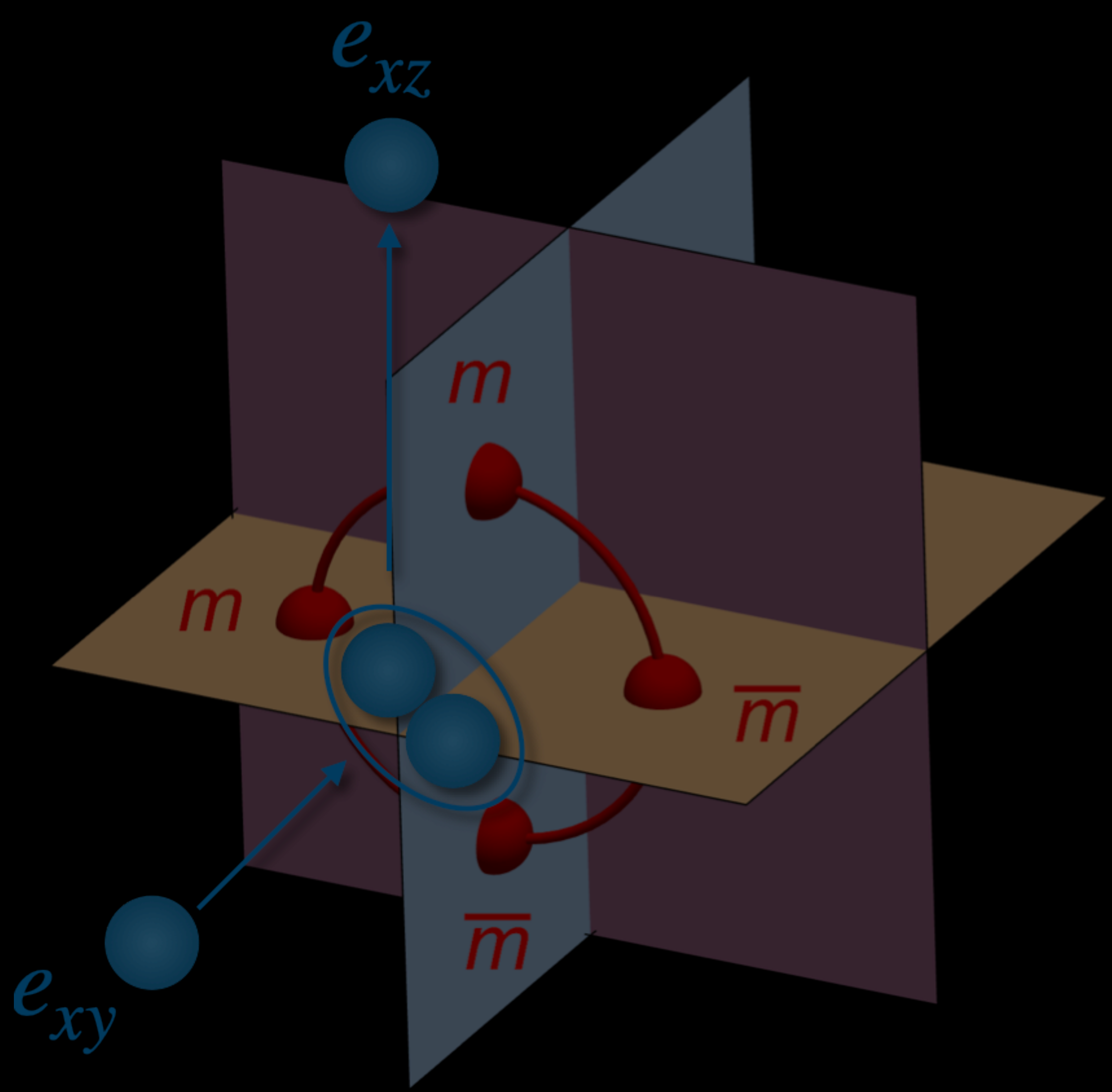
Phase diagram

$Z_N \to U(1)$ $N \to \infty$

- exact tensor network wavefunctions



h-perturbation confines fractons into dipole



summary

summary

arXiv:2203.00015

- **exact** tensor network state phase diagram

  spatial **conformal** quantum critical points

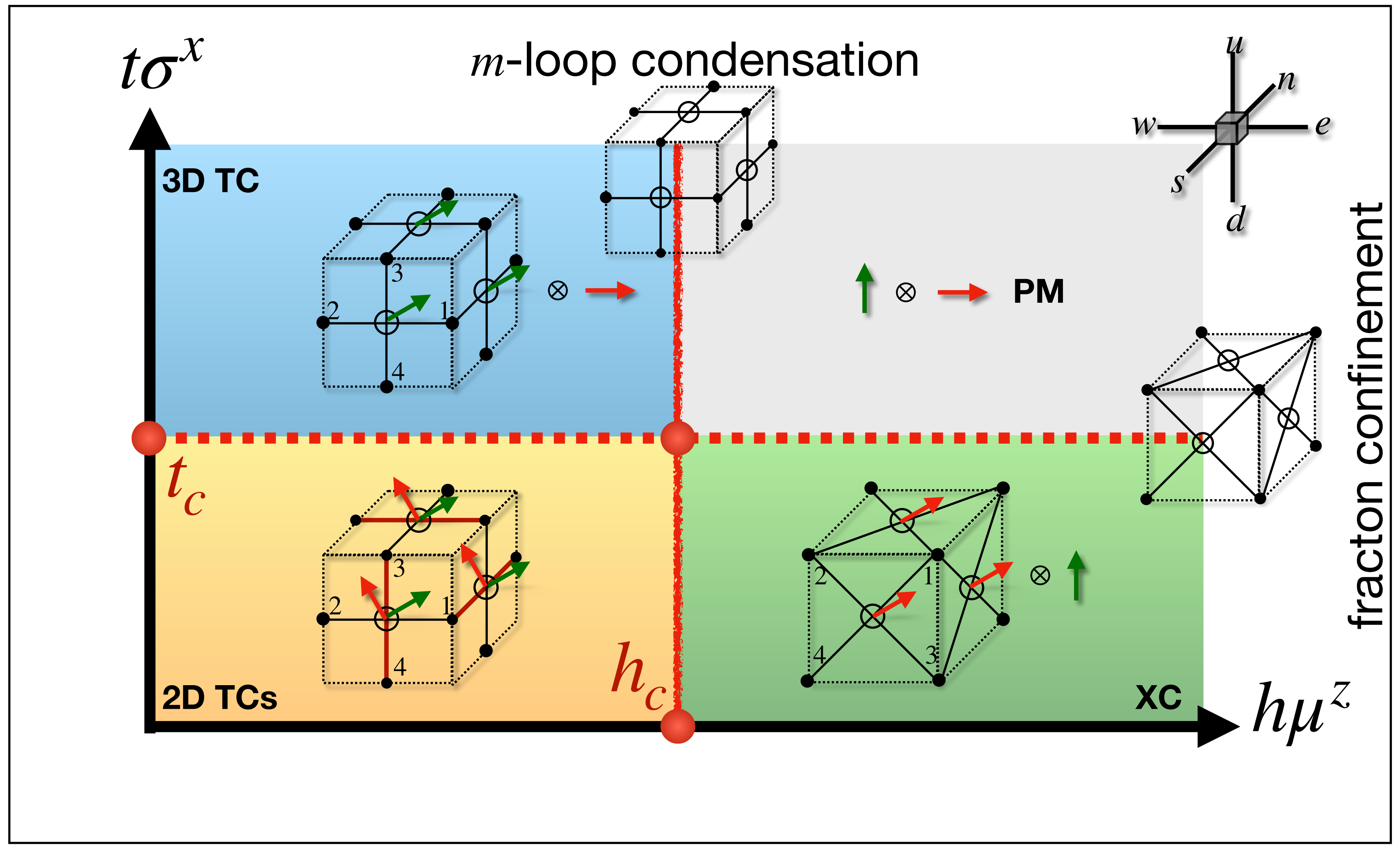
- fracton confinement

  first-order to **continuous** transition **deconfined** QCP

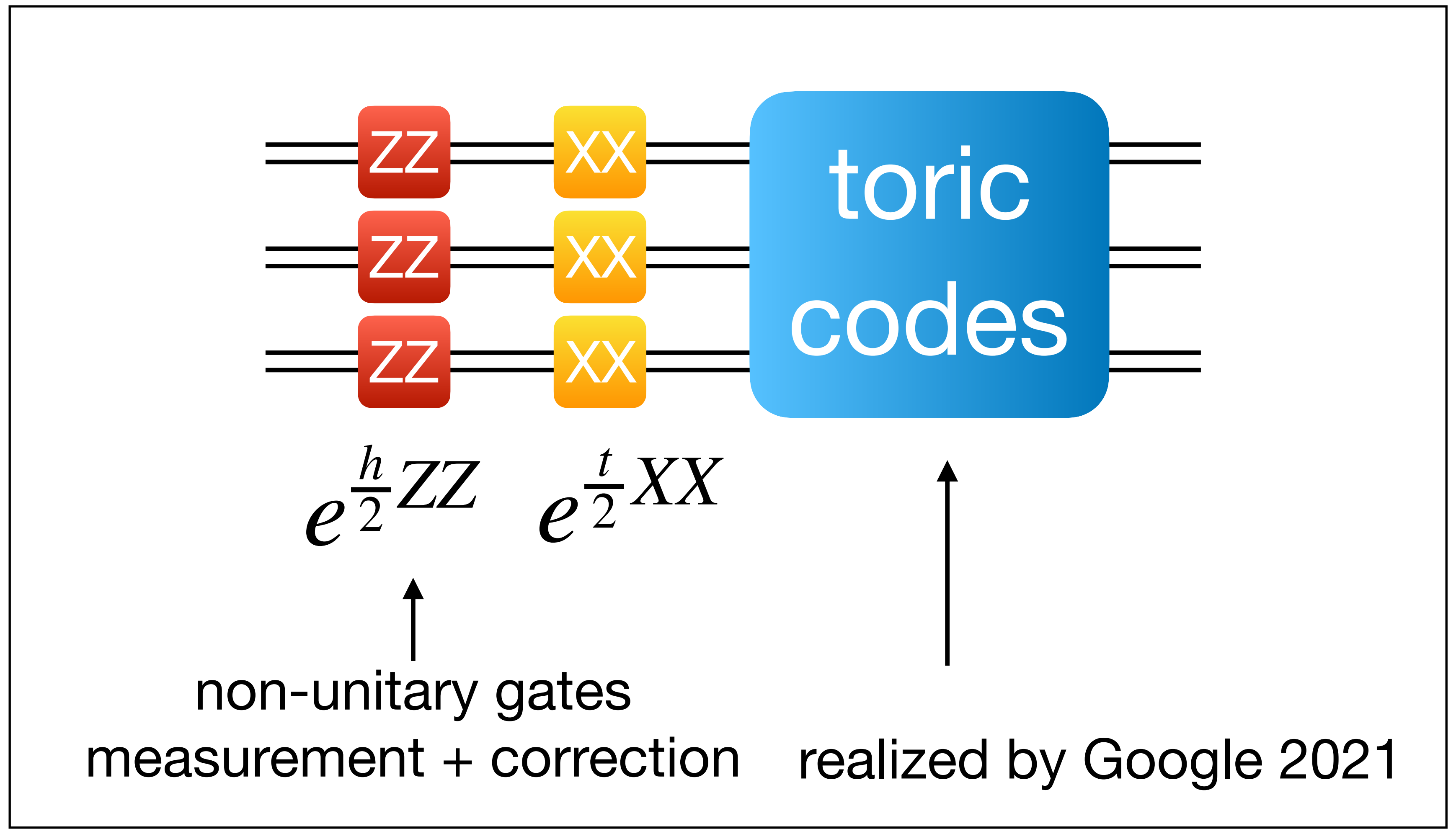
- m-loop condensation

  continuous transition separates **deconfined** fracton & toric codes

  non-LGW transition



- Outlook
  - direct calculation for **U(1) fracton** QPT?
  - generalise to **fractal** liquid or **twisted** fracton order
  - **Hamiltonian** deformation path?
  - realization in **quantum processor**?



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