Monitored Quantum Criticality

Nishimori physics, quantum measurements, and quantum computational physics

CRC183
ENTANGLED STATES OF MATTER

Simon Trebst
University of Cologne

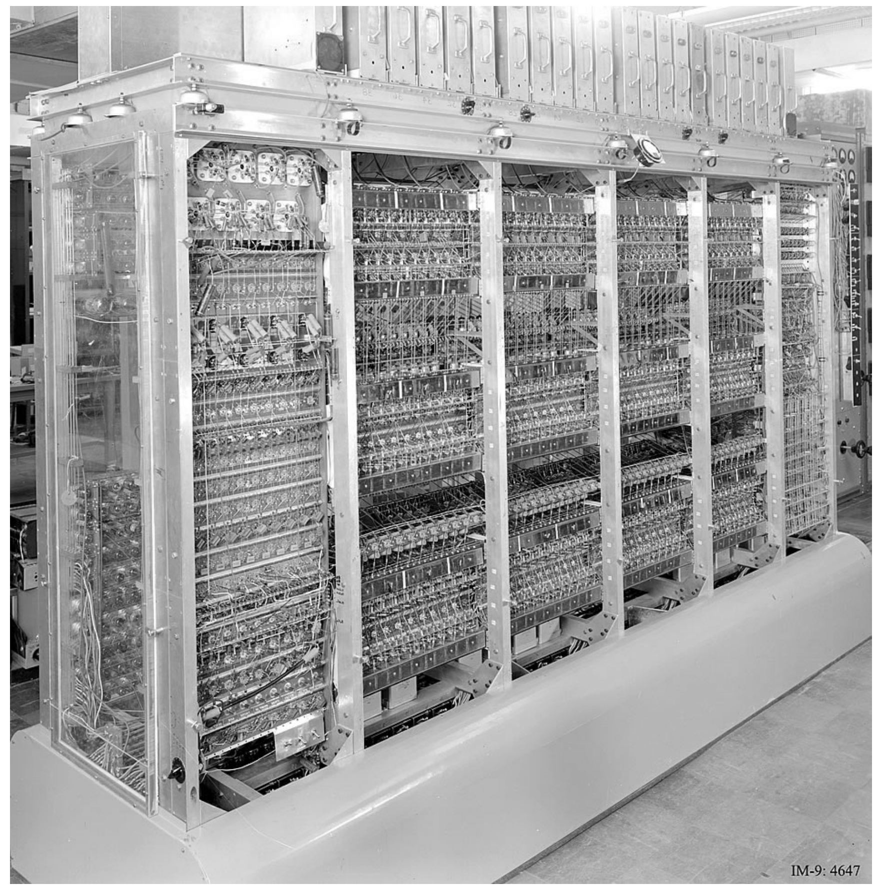
ML4Q
MATTER AND LIGHT FOR
QUANTUM COMPUTING

StablePhases25 program

KITP, September 2025

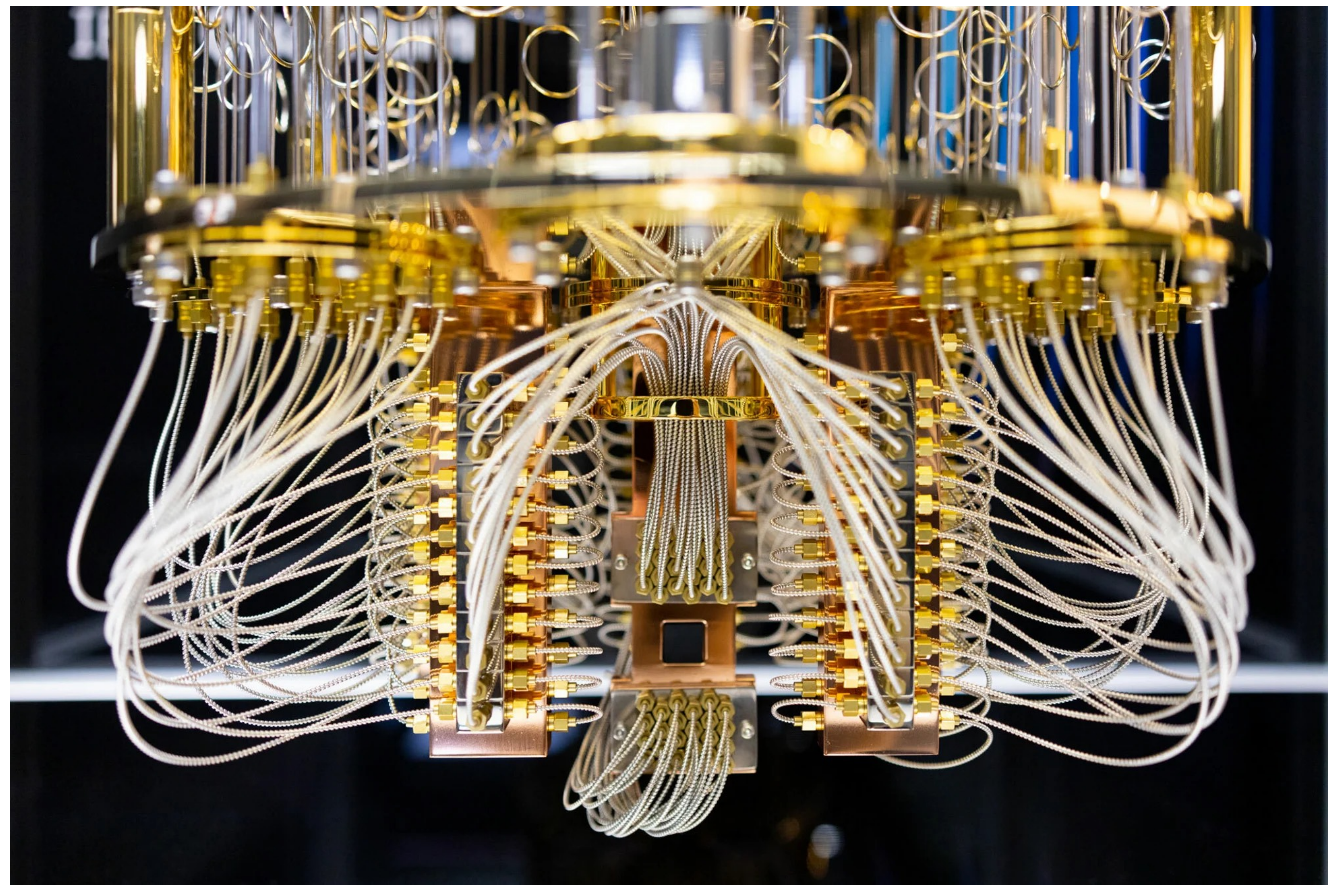
computational physics

Maniac @ Los Alamos



Cray @ ETH Zurich

IBM Quantum (cloud)



(1953) (1992) (2019)

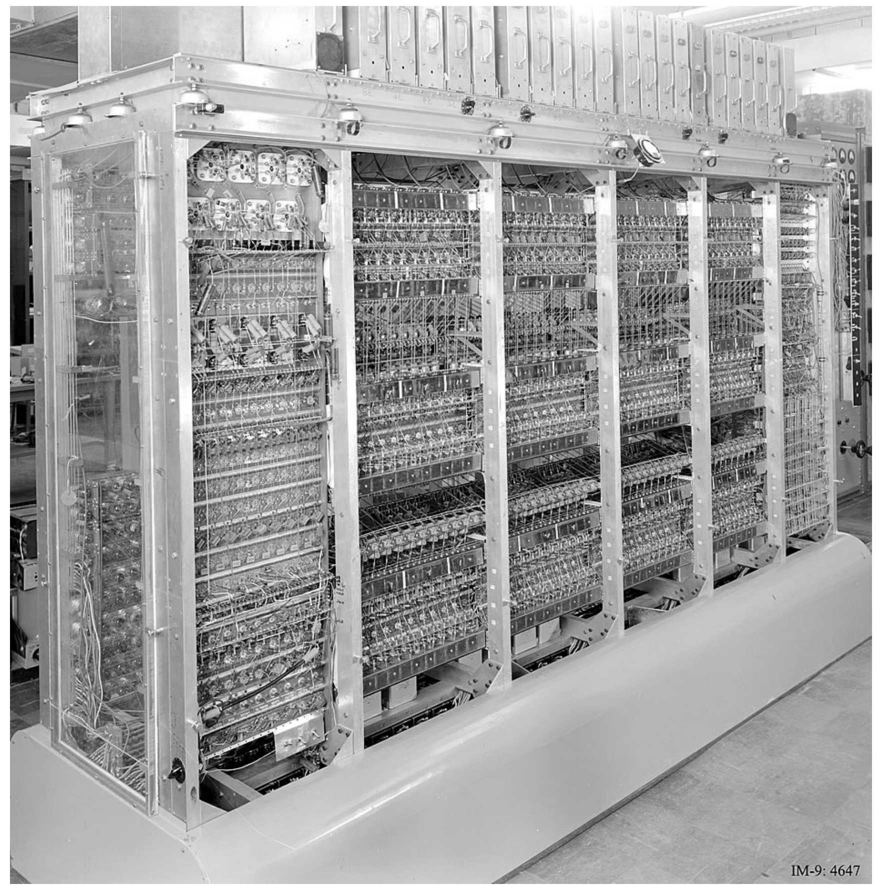
classical many-body

quantum many-body

(quantum)^2 many-body

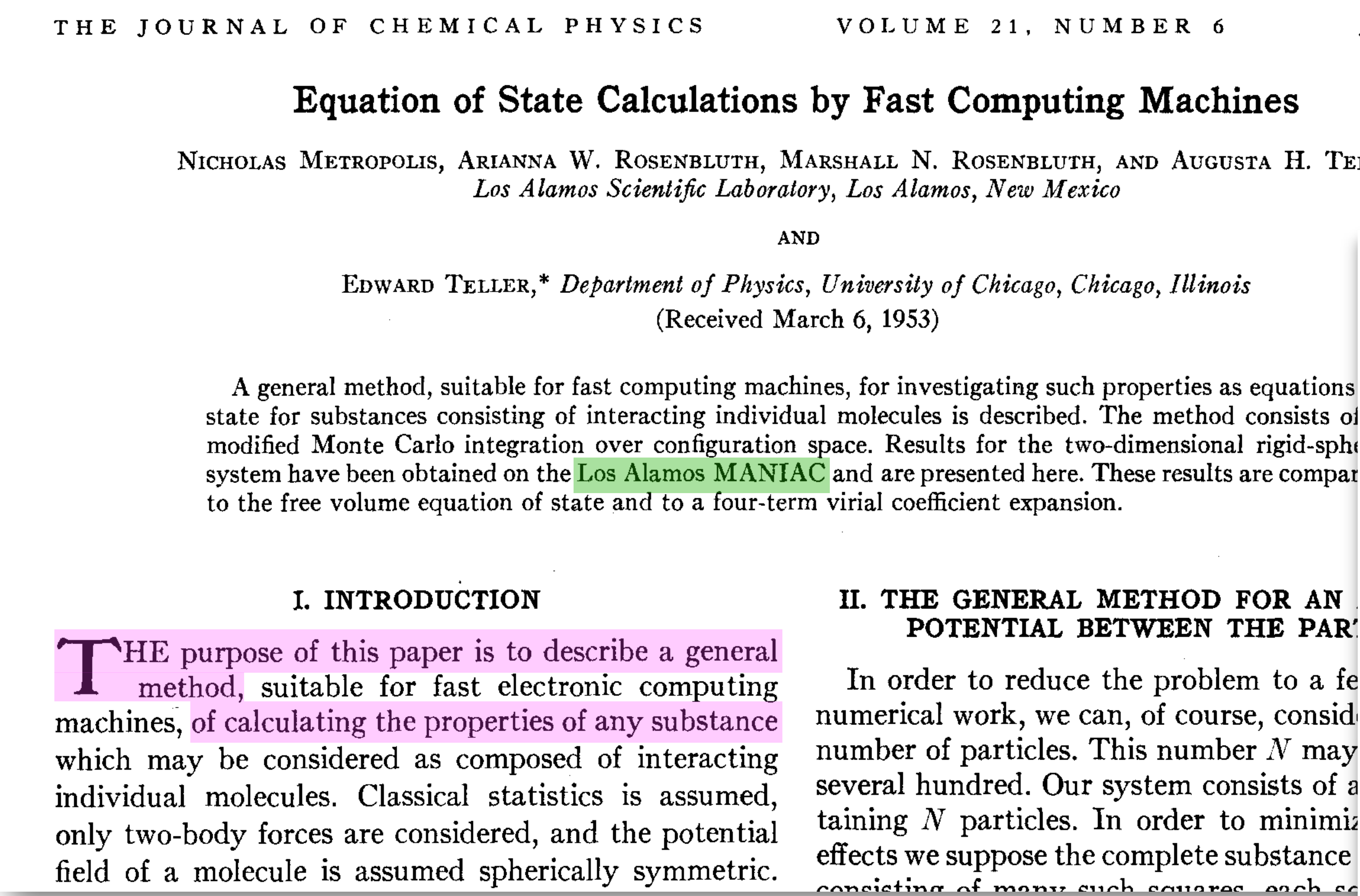
classical many-body physics

Maniac @ Los Alamos



(1953)

classical
many-body



THE JOURNAL OF CHEMICAL PHYSICS VOLUME 21, NUMBER 6 JUNE, 1953

Equation of State Calculations by Fast Computing Machines

Nicholas Metropolis, Arianna W. Rosenbluth, Marshall N. Rosenbluth, and Augusta H. Teller,
Los Alamos Scientific Laboratory, Los Alamos, New Mexico

and

Edward Teller,* *Department of Physics, University of Chicago, Chicago, Illinois*
(Received March 6, 1953)

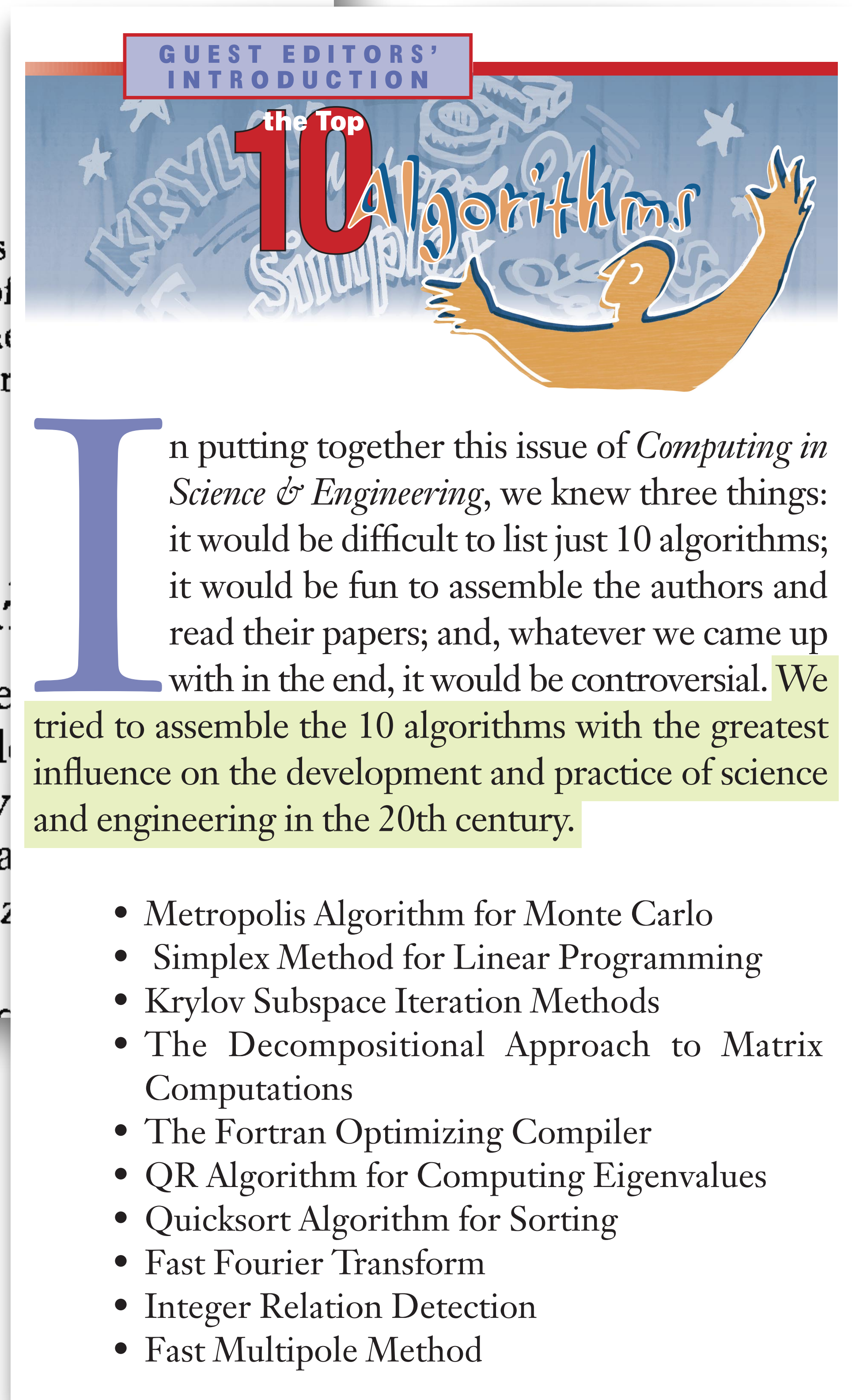
A general method, suitable for fast computing machines, for investigating such properties as equations
state for substances consisting of interacting individual molecules is described. The method consists of
modified Monte Carlo integration over configuration space. Results for the two-dimensional rigid-sphe
system have been obtained on the Los Alamos MANIAC and are presented here. These results are compar
to the free volume equation of state and to a four-term virial coefficient expansion.

I. INTRODUCTION

THE purpose of this paper is to describe a general method, suitable for fast electronic computing machines, of calculating the properties of any substance which may be considered as composed of interacting individual molecules. Classical statistics is assumed, only two-body forces are considered, and the potential field of a molecule is assumed spherically symmetric.

II. THE GENERAL METHOD FOR AN
POTENTIAL BETWEEN THE PAR

In order to reduce the problem to a fe
numerical work, we can, of course, consid
number of particles. This number N may
several hundred. Our system consists of a
taining N particles. In order to minimiz
effects we suppose the complete substance



GUEST EDITORS' INTRODUCTION

the Top 10 Algorithms

In putting together this issue of *Computing in Science & Engineering*, we knew three things: it would be difficult to list just 10 algorithms; it would be fun to assemble the authors and read their papers; and, whatever we came up with in the end, it would be controversial. We tried to assemble the 10 algorithms with the greatest influence on the development and practice of science and engineering in the 20th century.

- Metropolis Algorithm for Monte Carlo
- Simplex Method for Linear Programming
- Krylov Subspace Iteration Methods
- The Decompositional Approach to Matrix Computations
- The Fortran Optimizing Compiler
- QR Algorithm for Computing Eigenvalues
- Quicksort Algorithm for Sorting
- Fast Fourier Transform
- Integer Relation Detection
- Fast Multipole Method

Monte Carlo sampling nevertheless became one of the most widely used algorithms for its ability to sample from arbitrary distributions.

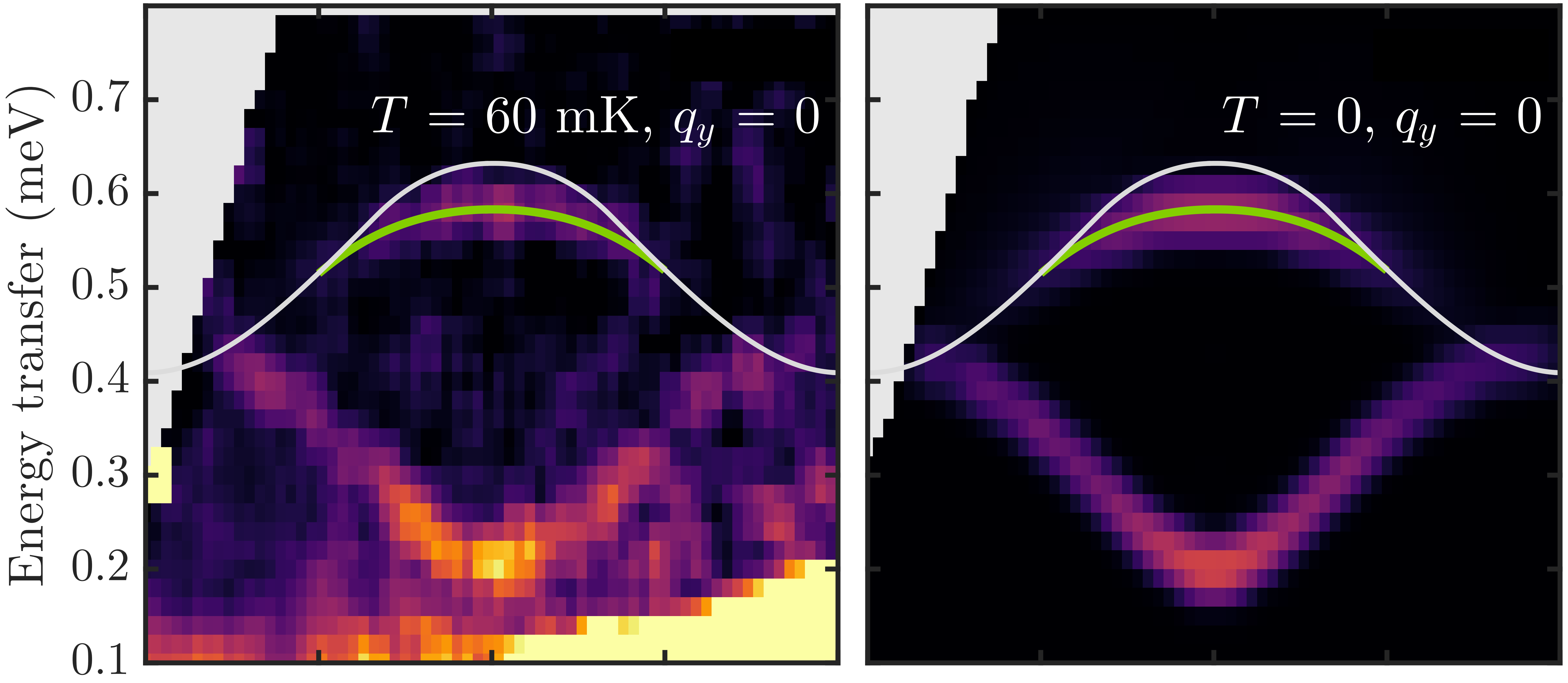
quantum many-body physics

Cray @ ETH Zurich

(1992)

quantum many-body

Collective phenomena in quantum systems become accessible to **quantitative numerical simulations**, such as superconductivity, Bose-Einstein condensation & topological materials.

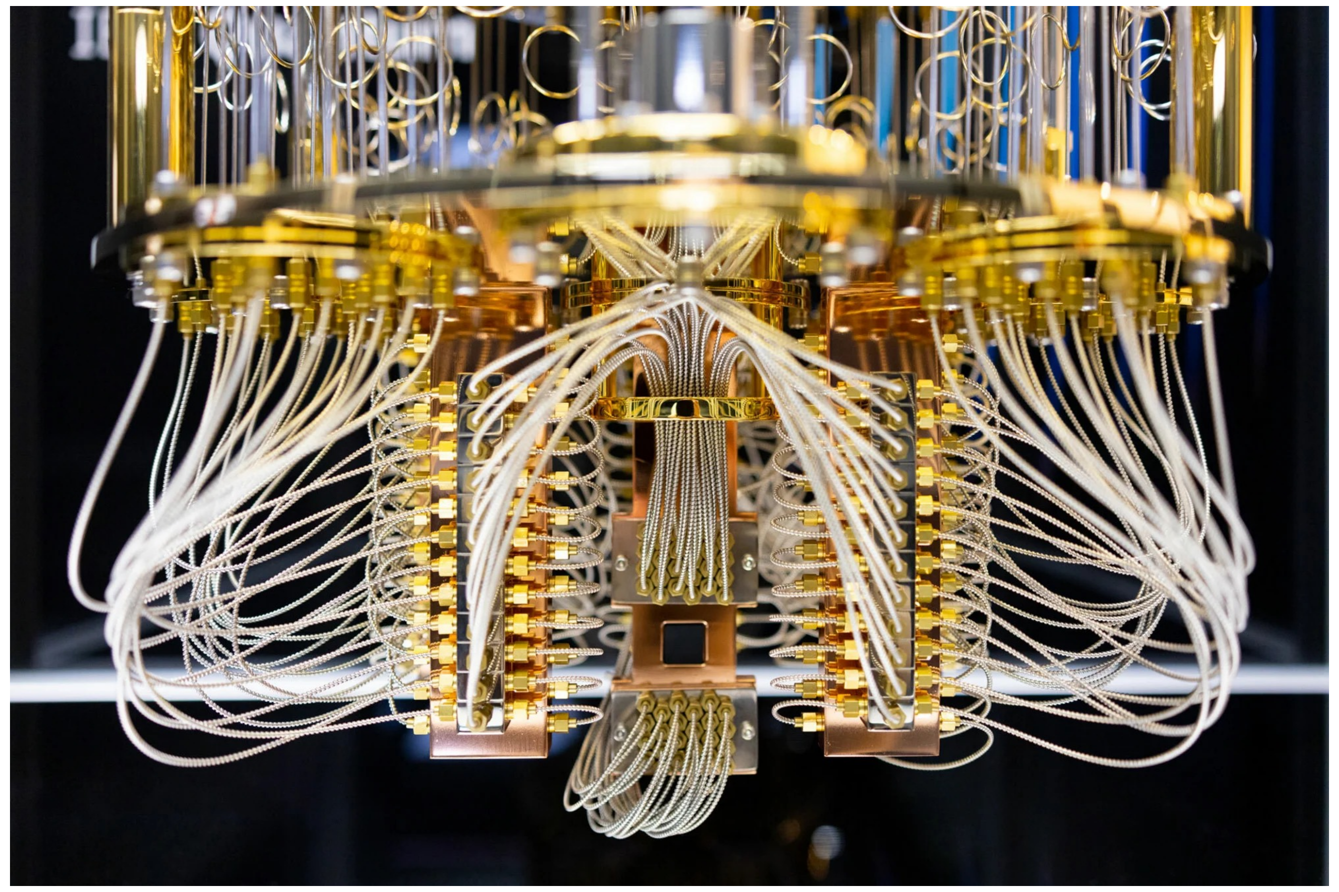


neutron scattering on quantum material

density matrix renormalization group (DMRG) numerical simulation

(quantum)^2 many-body

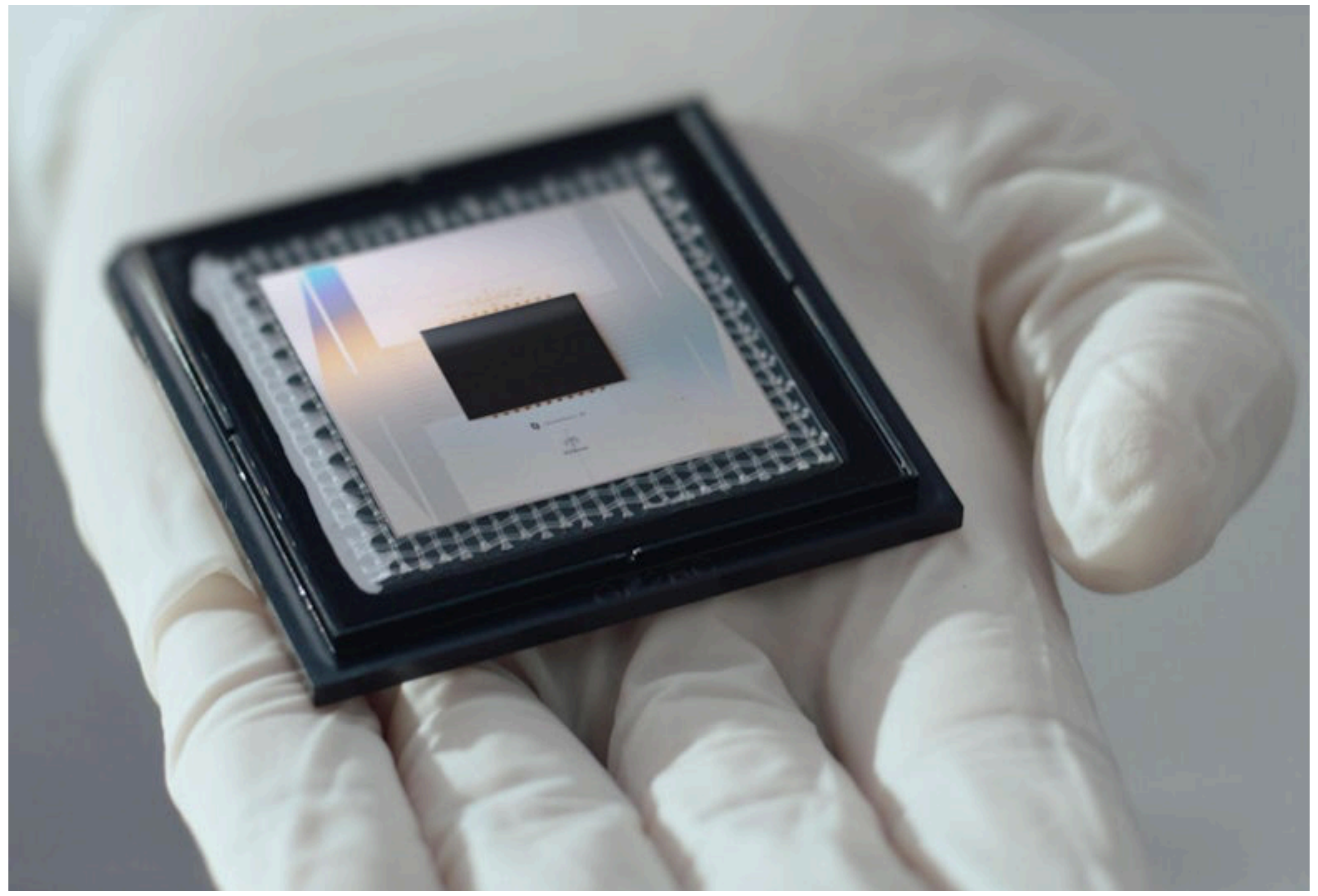
IBM Quantum (cloud)



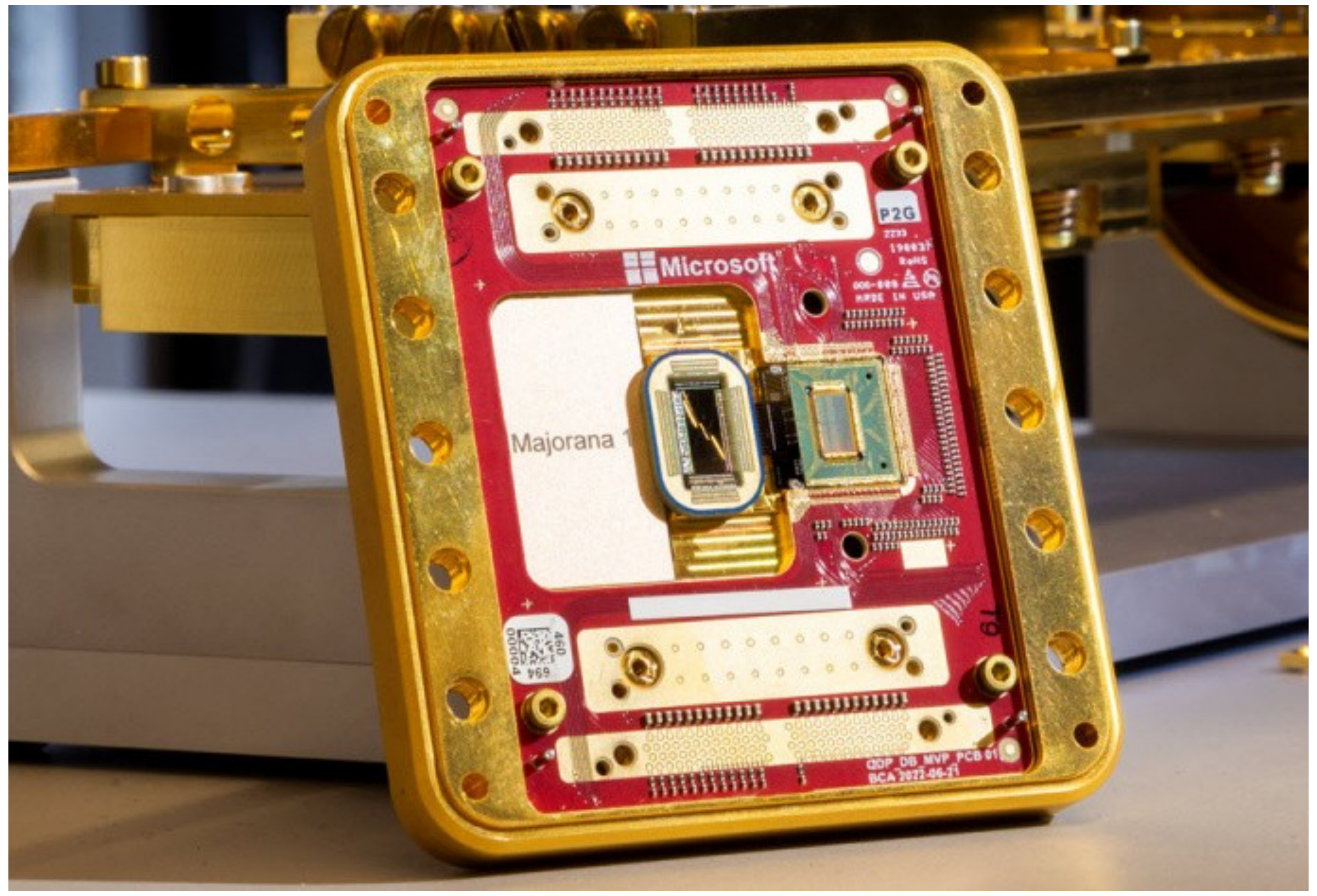
— (2019) —

(quantum)^2
many-body

Quantum processors are emerging as a new experimental platform to simulate **"quantum on quantum"**, i.e. the non-linear **dynamics** of quantum systems with unprecedented control.



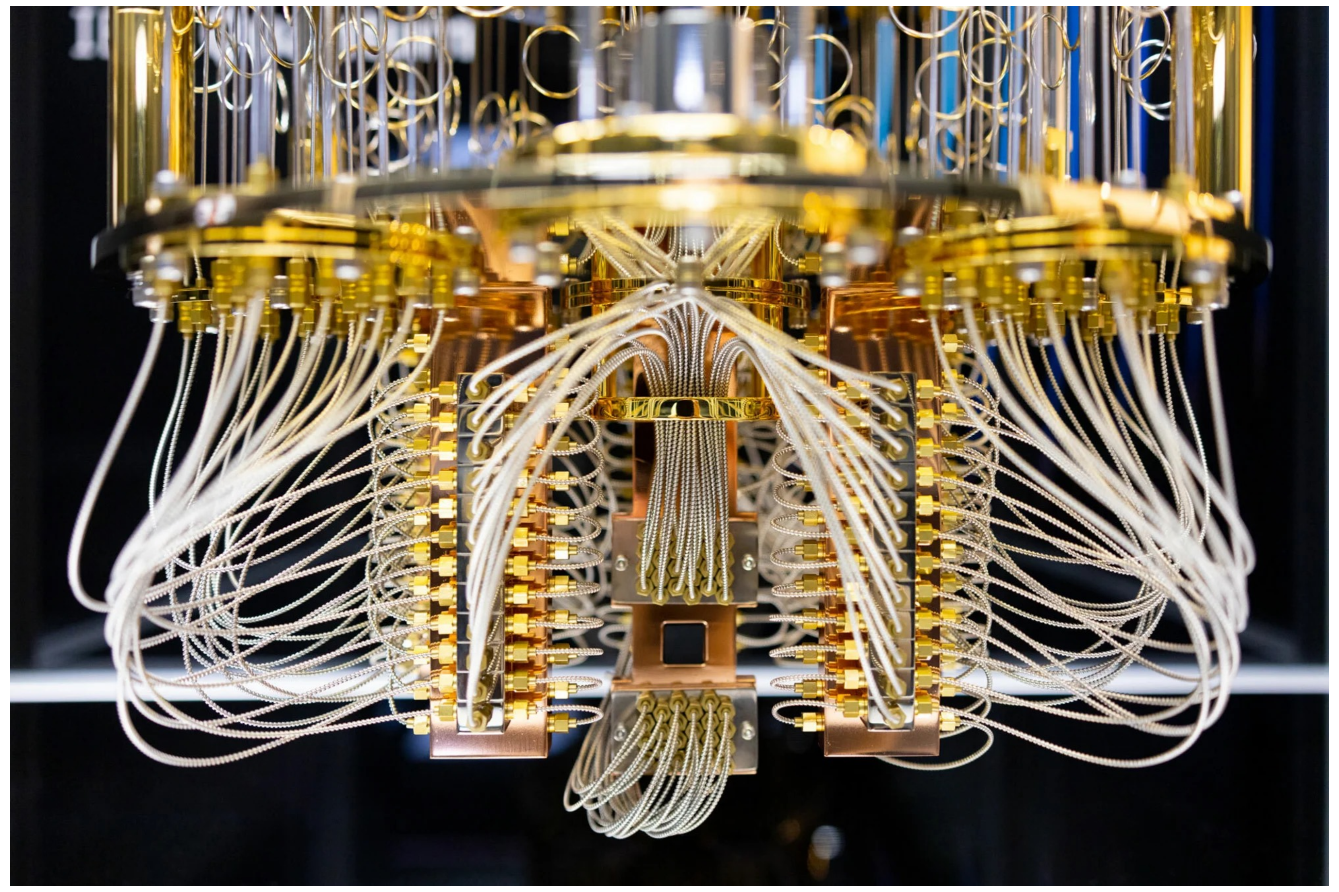
Google's Willow chip
(December 2024)



Microsoft's Majorana 1 chip
(February 2025)

(quantum)^2 many-body

IBM Quantum (cloud)



(2019)

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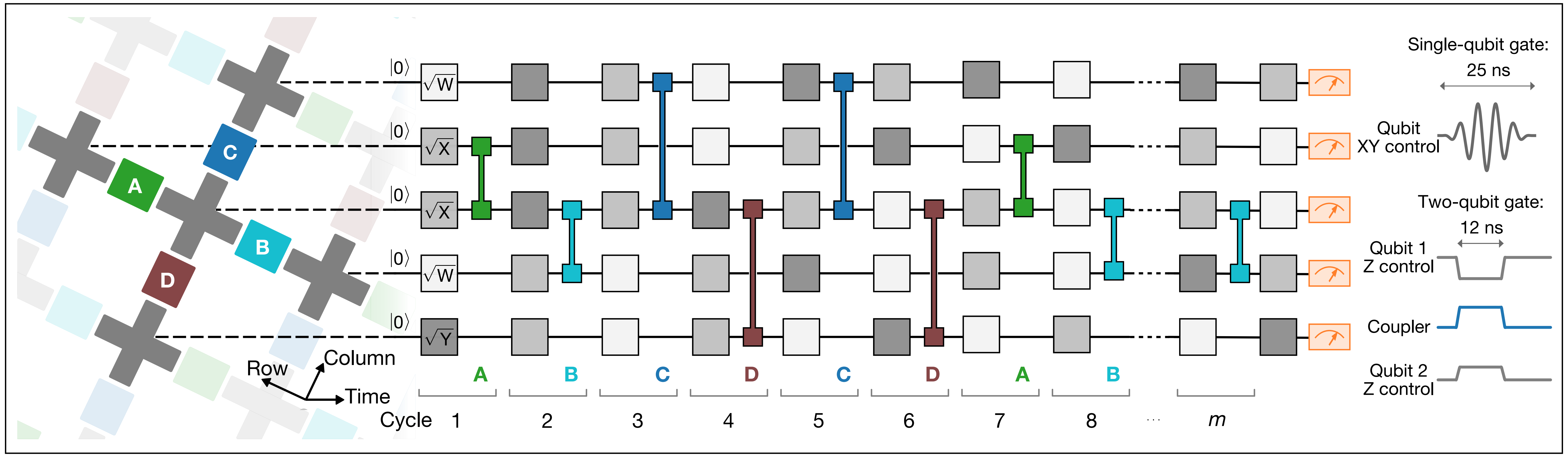
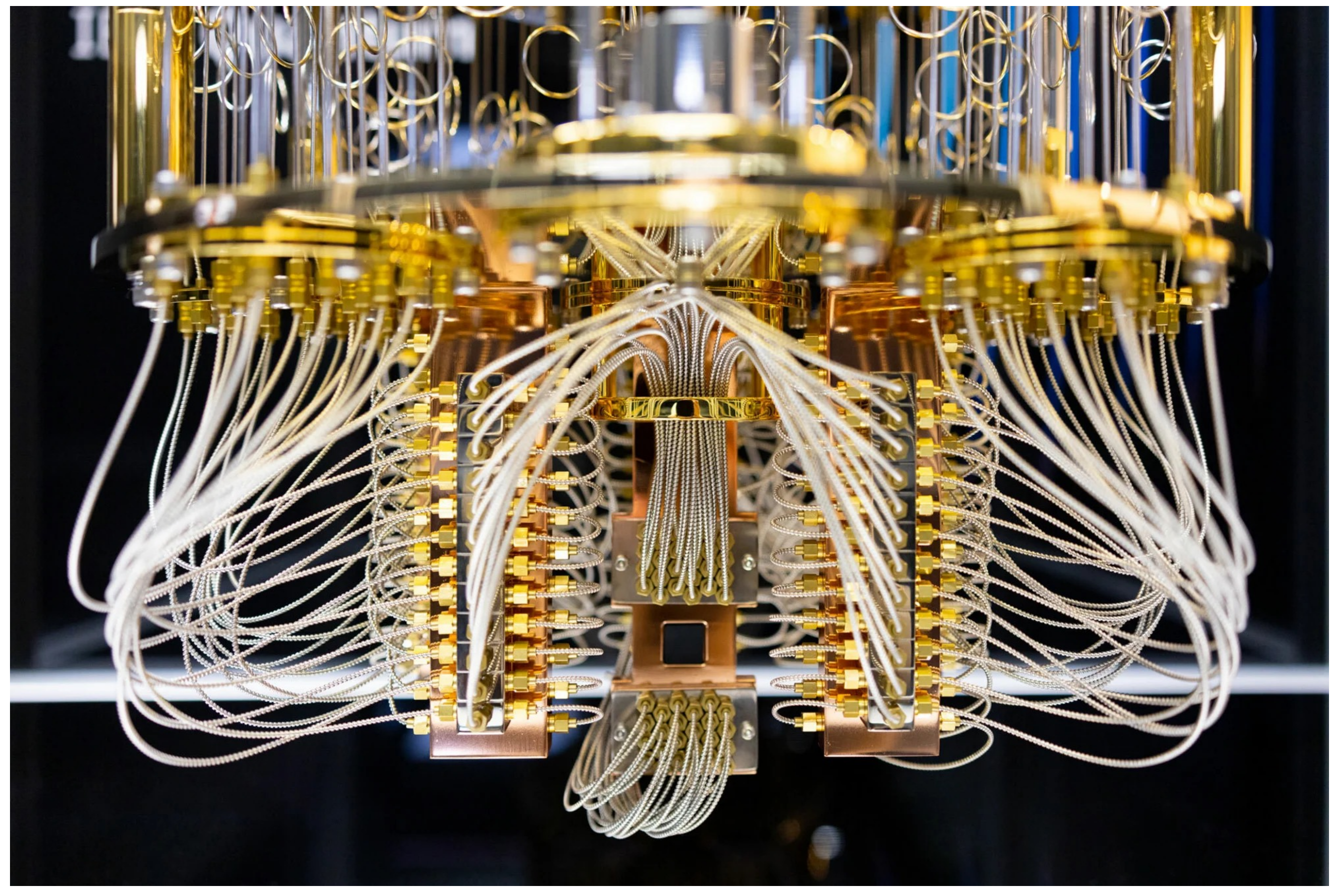


quantum circuits are the “assembler-level” of quantum computing

(quantum)^2
many-body

(quantum)^2 many-body

IBM Quantum (cloud)



— (2019) —

(quantum)^2
many-body

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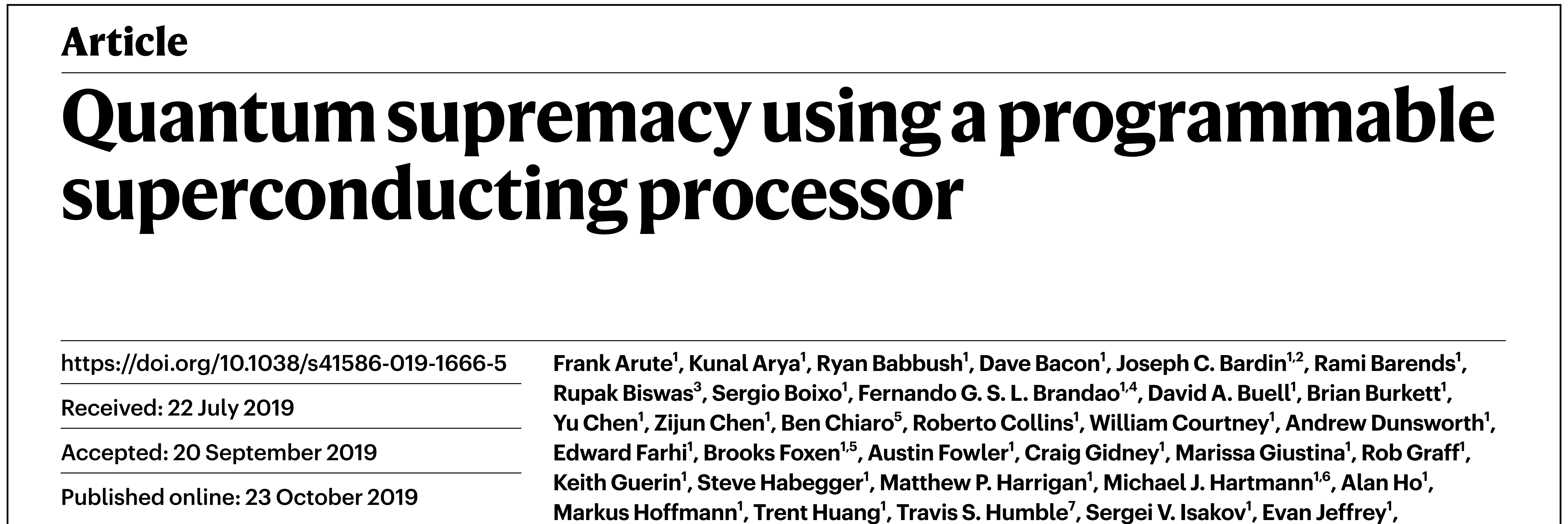

Article

Quantum supremacy using a programmable superconducting processor

https://doi.org/10.1038/s41586-019-1666-5
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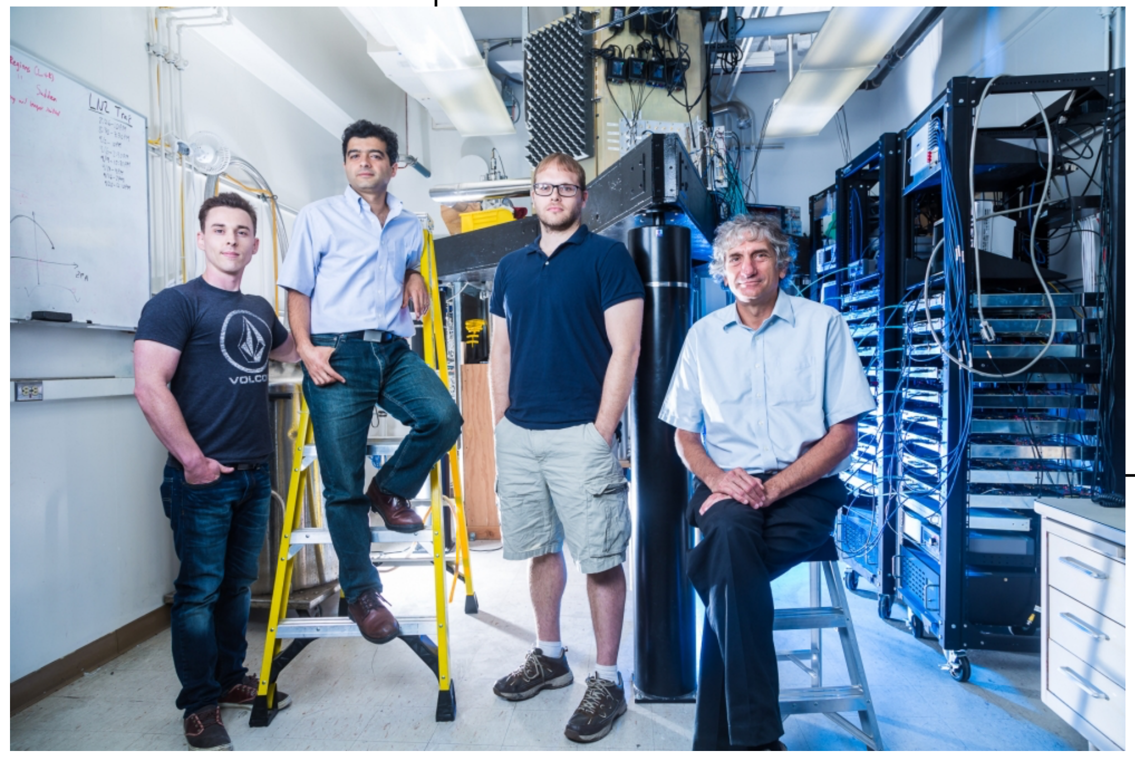
Accepted: 20 September 2019

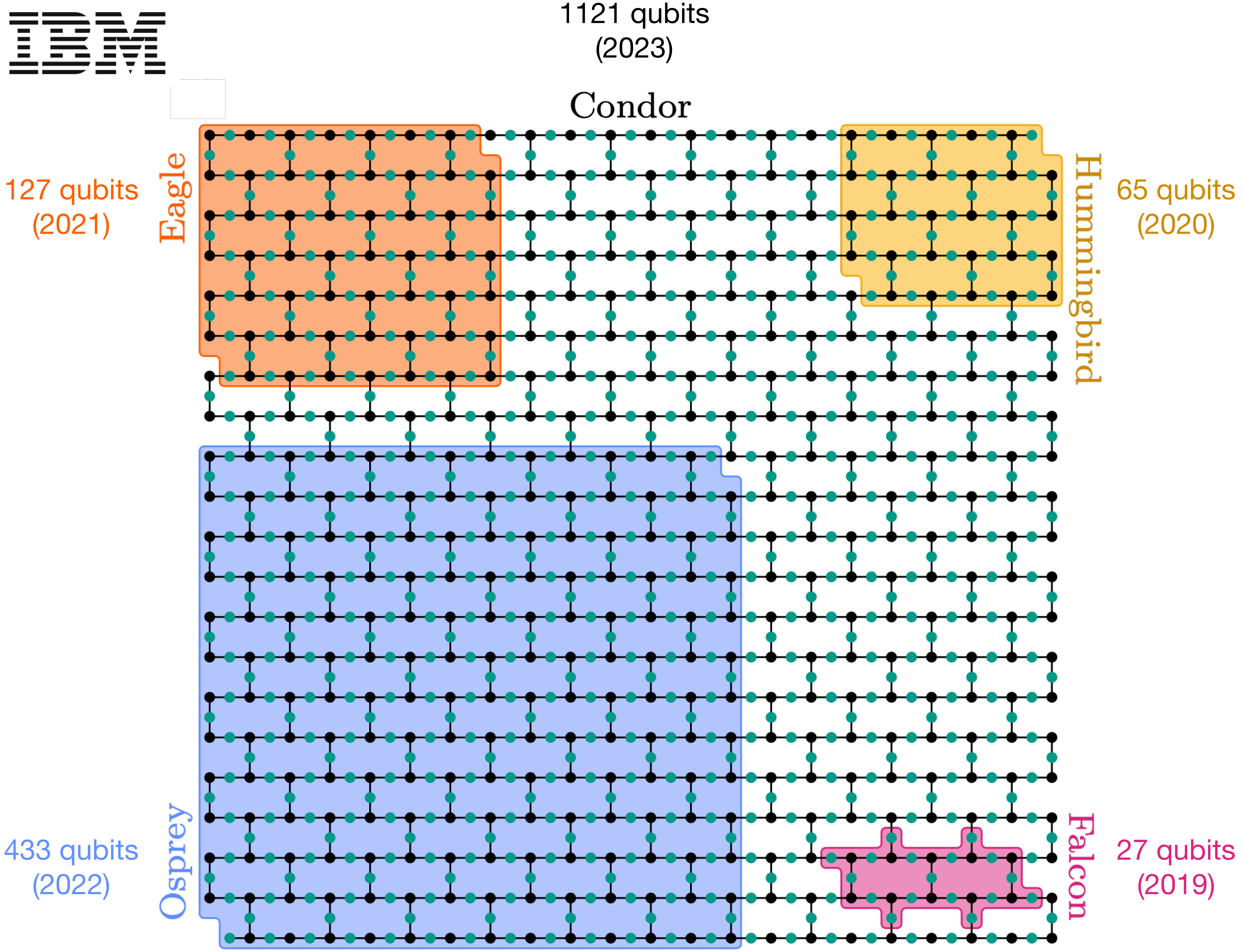
Frank Arute[1], Kunal Arya[1], Ryan Babbush[1], Dave Bacon[1], Joseph C. Bardin[1,2], Rami Barends[1], Rupak Biswas[3], Sergio Boixo[1], Fernando G. S. L. Brandao[1,4], David A. Buell[1], Brian Burkett[1], Yu Chen[1], Zijun Chen[1], Ben Chiaro[5], Roberto Collins[1], William Courtney[1], Andrew Dunsworth[1], Edward Farhi[1], Brooks Foxen[1,5], Austin Fowler[1], Craig Gidney[1], Marissa Giustina[1], Rob Graff[1], Keith Guerin[1], Steve Habegger[1], Matthew P. Harrigan[1], Michael J. Hartmann[1,6], Alan Ho[1], Markus Hoffmann[1], Trent Huang[1], Travis S. Humble[7], Sergei V. Isakov[1], Evan Jeffrey[1], Zhang Jiang[1], Dvir Kafri[1], Kostyantyn Kechedzhi[1], Julian Kelly[1], Paul V. Klimov[1], Sergey Knysh[1], Alexander Korotkov[1,8], Fedor Kostritsa[1], David Landhuis[1], Mike Lindmark[1], Erik Lucero[1], Dmitry Lyakh[9], Salvatore Mandrà[3,10], Jarrod R. McClean[1], Matthew McEwen[5], Anthony Megrant[1], Xiao Mi[1], Kristel Michielsen[11,12], Masoud Mohseni[1], Josh Mutus[1], Ofer Naaman[1], Matthew Neeley[1], Charles Neill[1], Murphy Yuezhen Niu[1], Eric Ostby[1], Andre Petukhov[1], John C. Platt[1], Chris Quintana[1], Eleanor G. Rieffel[3], Pedram Roushan[1], Nicholas C. Rubin[1], Daniel Sank[1], Kevin J. Satzinger[1], Vadim Smelyanskiy[1], Kevin J. Sung[1,13], Matthew D. Trevithick[1], Amit Vainsencher[1], Benjamin Villalonga[1,14], Theodore White[1], Z. Jamie Yao[1], Ping Yeh[1], Adam Zalcman[1], Hartmut Neven[1] & John M. Martinis[1,5]*



John Martinis (2019)

The promise of quantum computers is that **certain computational tasks** might be executed **exponentially faster** on a quantum processor than on a classical processor.

quantum hardware & simulations



Article | Published: 23 October 2019

Quantum supremacy using a programmable superconducting processor

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Nature **574**, 505–510 (2019) | Cite this article

RESEARCH ARTICLE | TOPOLOGICAL MATTER

Realizing topologically ordered states on a quantum processor

K. J. SATZINGER, Y.-J LIU, A. SMITH, C. KNAPP, […], AND P. ROUSHAN +93 authors Authors Info & Affiliations

SCIENCE • 2 Dec 2021 • Vol 374, Issue 6572 • pp. 1237-1241 • DOI: 10.1126/science.abi8378

Article | Open access | Published: 11 May 2023

Non-Abelian braiding of graph vertices in a superconducting processor

Google Quantum AI and Collaborators

Nature **618**, 264–269 (2023) | Cite th

nature physics

Article https://doi.org/10.1038/s41567-024-02696-6

Nishimori transition across the error threshold for constant-depth quantum circuits

Received: 13 October 2023

Accepted: 7 October 2024

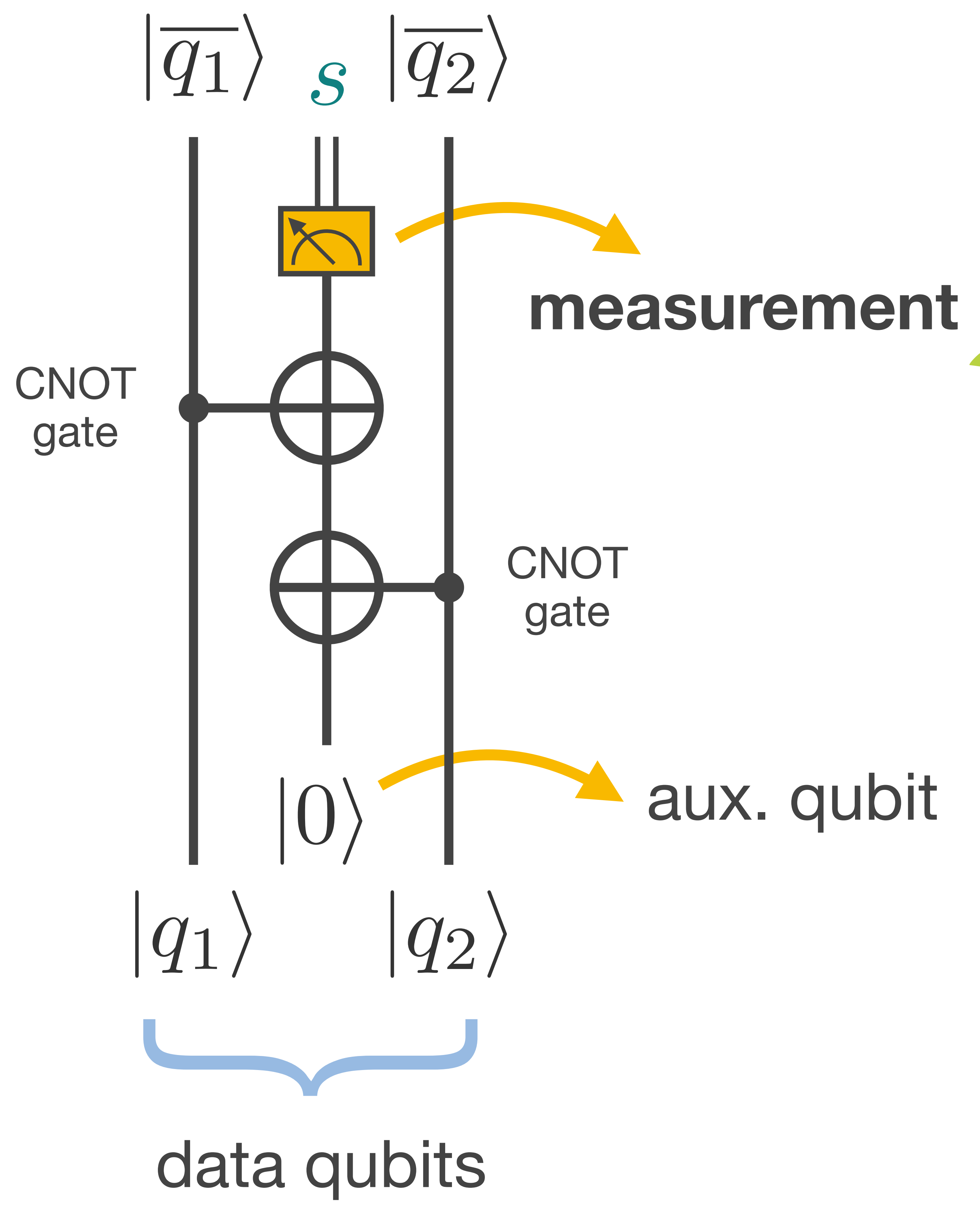
Published online: 16 December 2024

Edward H. Chen[1,2] ✉, Guo-Yi Zhu[3] ✉, Ruben Verresen[4] ✉, Alireza Seif[5], Elisa Bäumer[6], David Layden[1,5], Nathanan Tantivasadakarn[4,7], Guanyu Zhu[5], Sarah Sheldon[5], Ashvin Vishwanath[4], Simon Trebst[3] & Abhinav Kandala[5]

quantum measurements

joint measurements & entanglement

quantum circuit for 2-qubit
joint measurement



extract information

$\lvert q_1\rangle\lvert q_2\rangle$	$\lvert\overline{q_1}\rangle\lvert\overline{q_2}\rangle$	s
$\lvert 0\,0\rangle$	$\lvert 0\,0\rangle$	0
$\lvert 0\,1\rangle$	$\lvert 0\,1\rangle$	1
$\lvert 1\,0\rangle$	$\lvert 1\,0\rangle$	1
$\lvert 1\,1\rangle$	$\lvert 1\,1\rangle$	0

syndrome measurement

ZZ **parity check**

shape entanglement

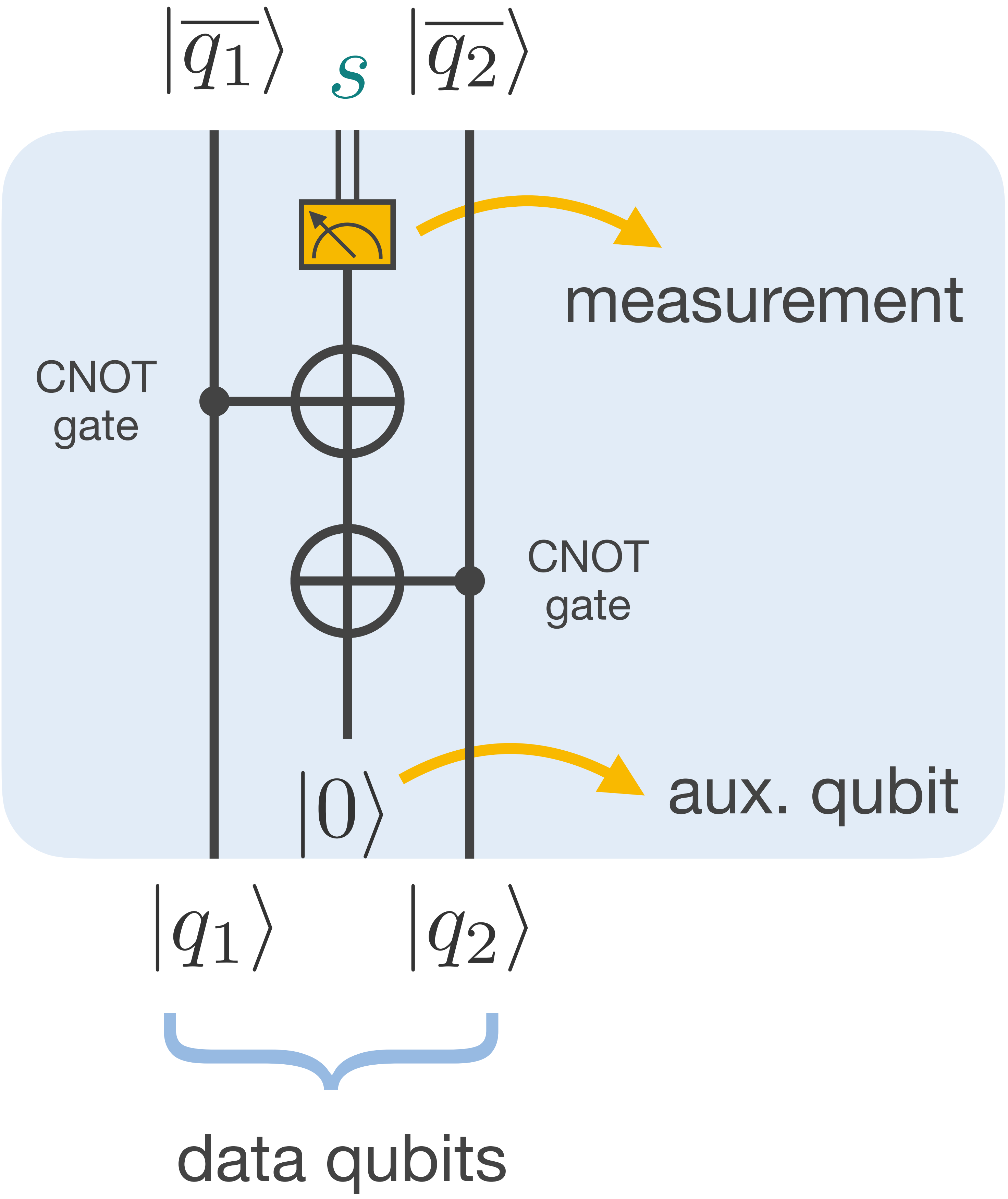
product state

$\lvert q_1\rangle\lvert q_2\rangle$	$\lvert\overline{q_1}\rangle\lvert\overline{q_2}\rangle$	s
$\lvert + +\rangle$	p = 1/2 $\left(\lvert 00\rangle + \lvert 11\rangle\right)/\sqrt{2}$	0
	p = 1/2 $\left(\lvert 01\rangle + \lvert 10\rangle\right)/\sqrt{2}$	1

Bell state

$$|++\rangle = \frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)$$

joint measurements & entanglement



$|\overline{q_1}\rangle$ s $|\overline{q_2}\rangle$

measurement

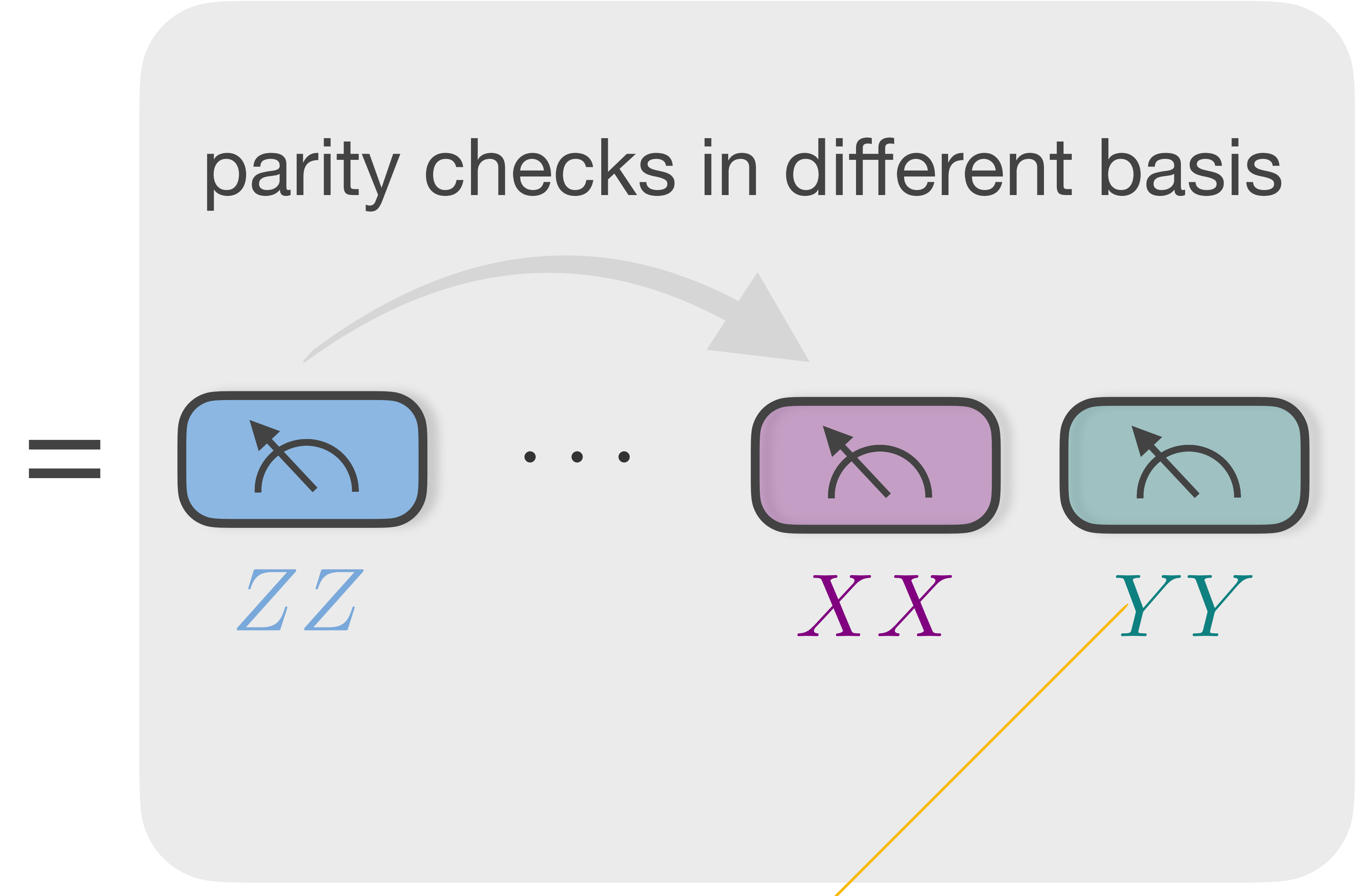
CNOT gate

CNOT gate

$|0\rangle$ aux. qubit

$|q_1\rangle$ $|q_2\rangle$

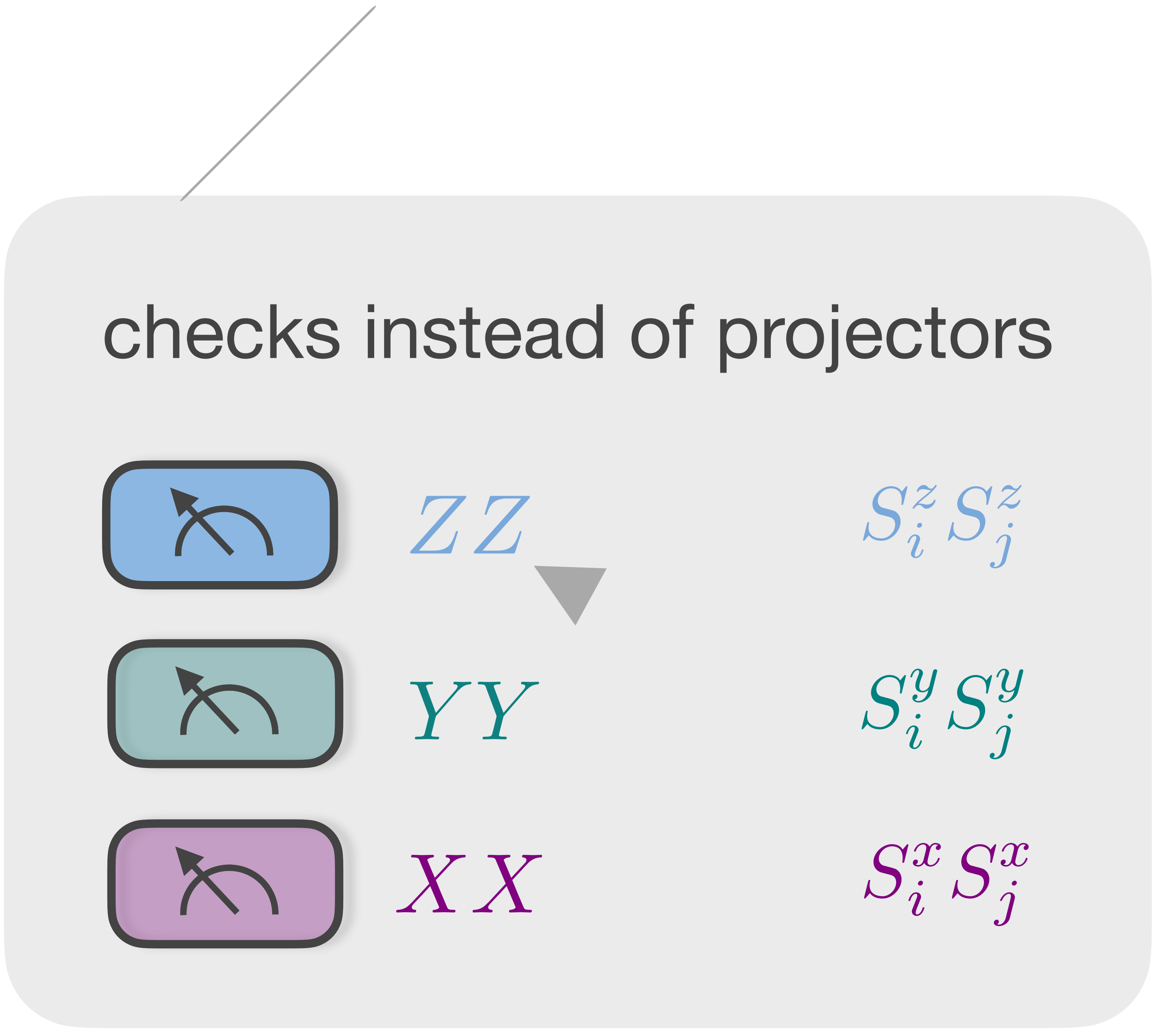
data qubits



$=$

parity checks in different basis

ZZ … XX YY



checks instead of projectors

ZZ $S_i^z S_j^z$

YY $S_i^y S_j^y$

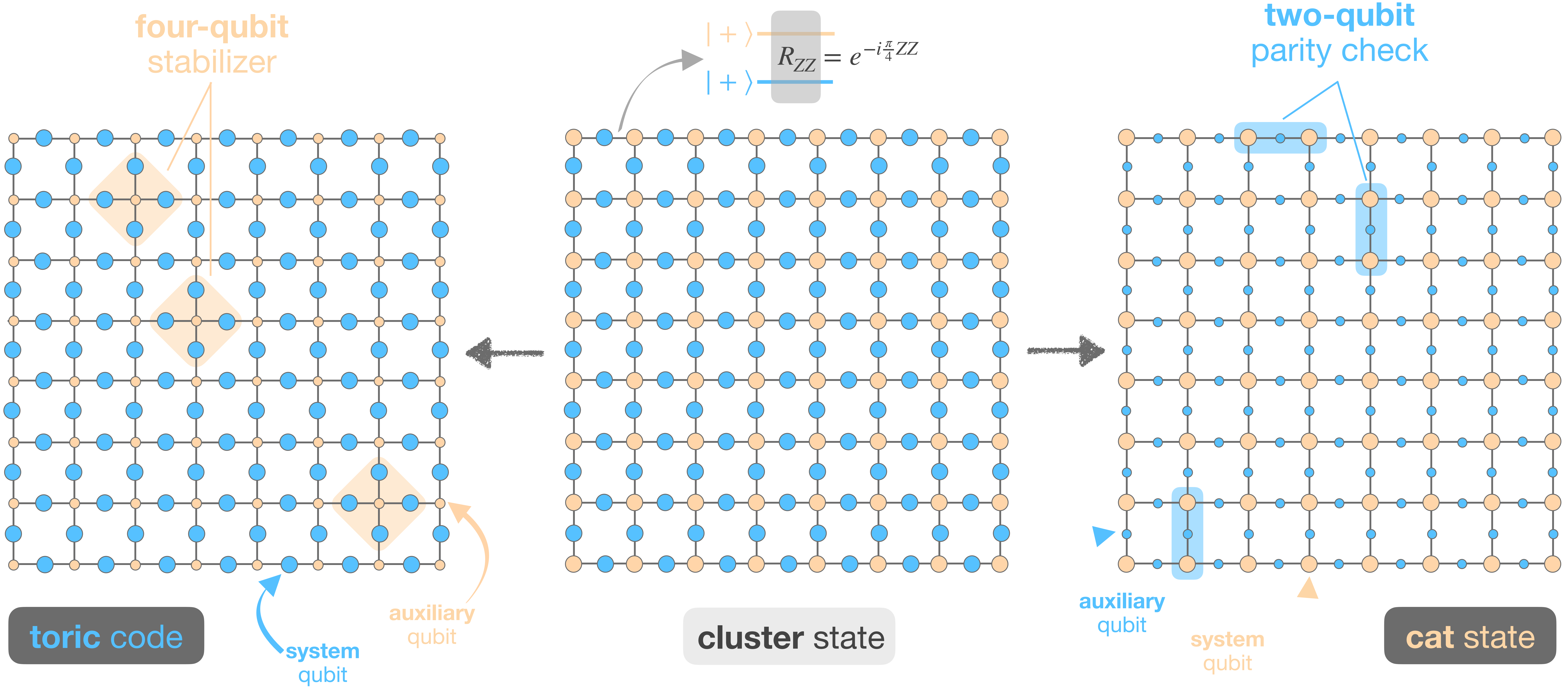
XX $S_i^x S_j^x$

measurement dynamics

Hamiltonian dynamics

new toolbox for quantum many-body physics

joint measurements & entanglement



four-qubit stabilizer

$|+\rangle$

$|+\rangle$

$R_{ZZ} = e^{-i\frac{\pi}{4}ZZ}$

two-qubit parity check

toric code

system qubit

auxiliary qubit

cluster state

auxiliary qubit

system qubit

cat state

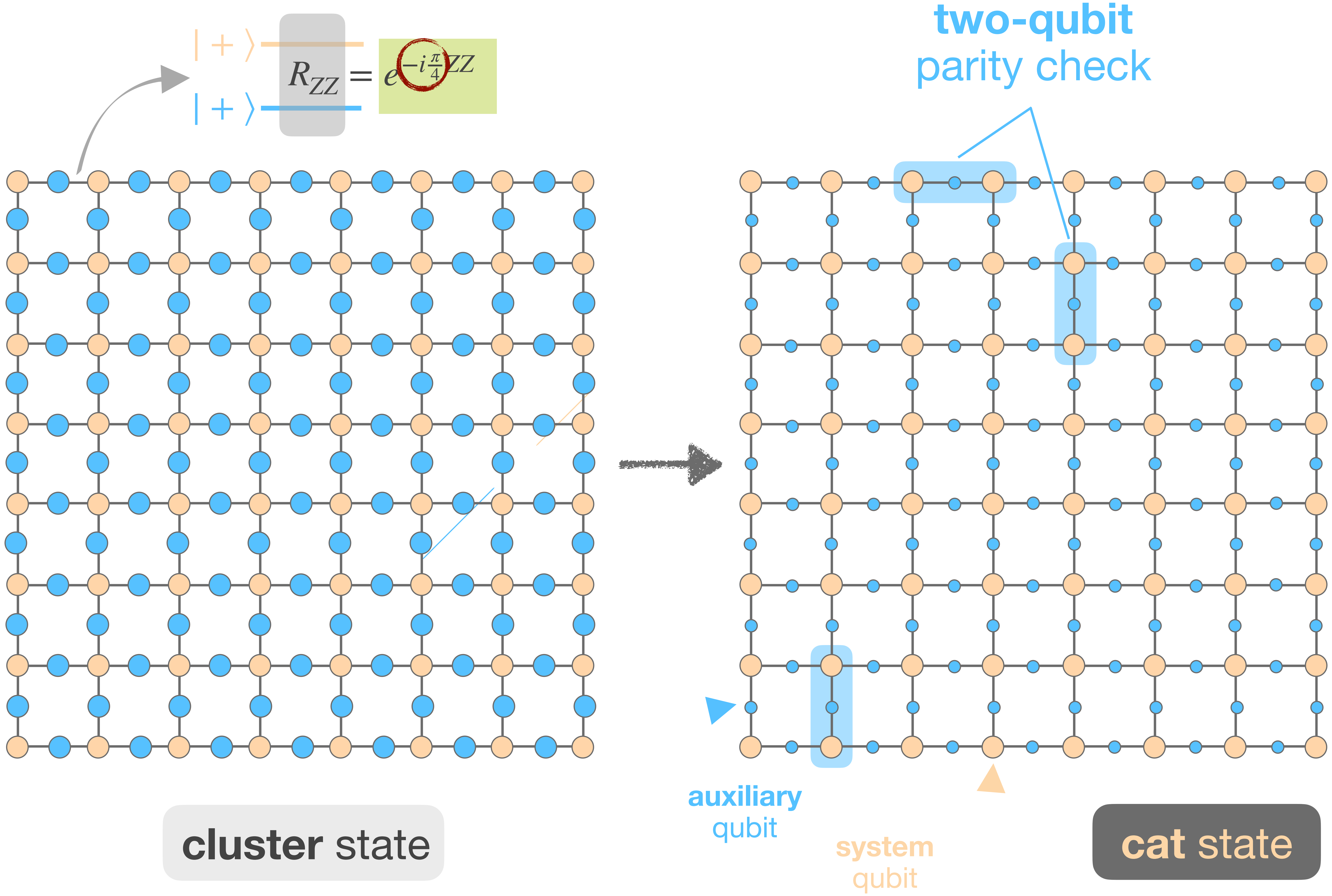
measurement-induced phase transitions

What happens if our **resource** — the cluster state — is **imperfectly prepared**?

$$0 \leq t \leq \frac{\pi}{4}$$

Is the prepared **cat state stable** to this coherent deformation? Is there a **threshold**?

Nishimori transition



$|+\rangle$

$|+\rangle$

$R_{ZZ} = e^{-i\frac{\pi}{4}ZZ}$

two-qubit
parity check

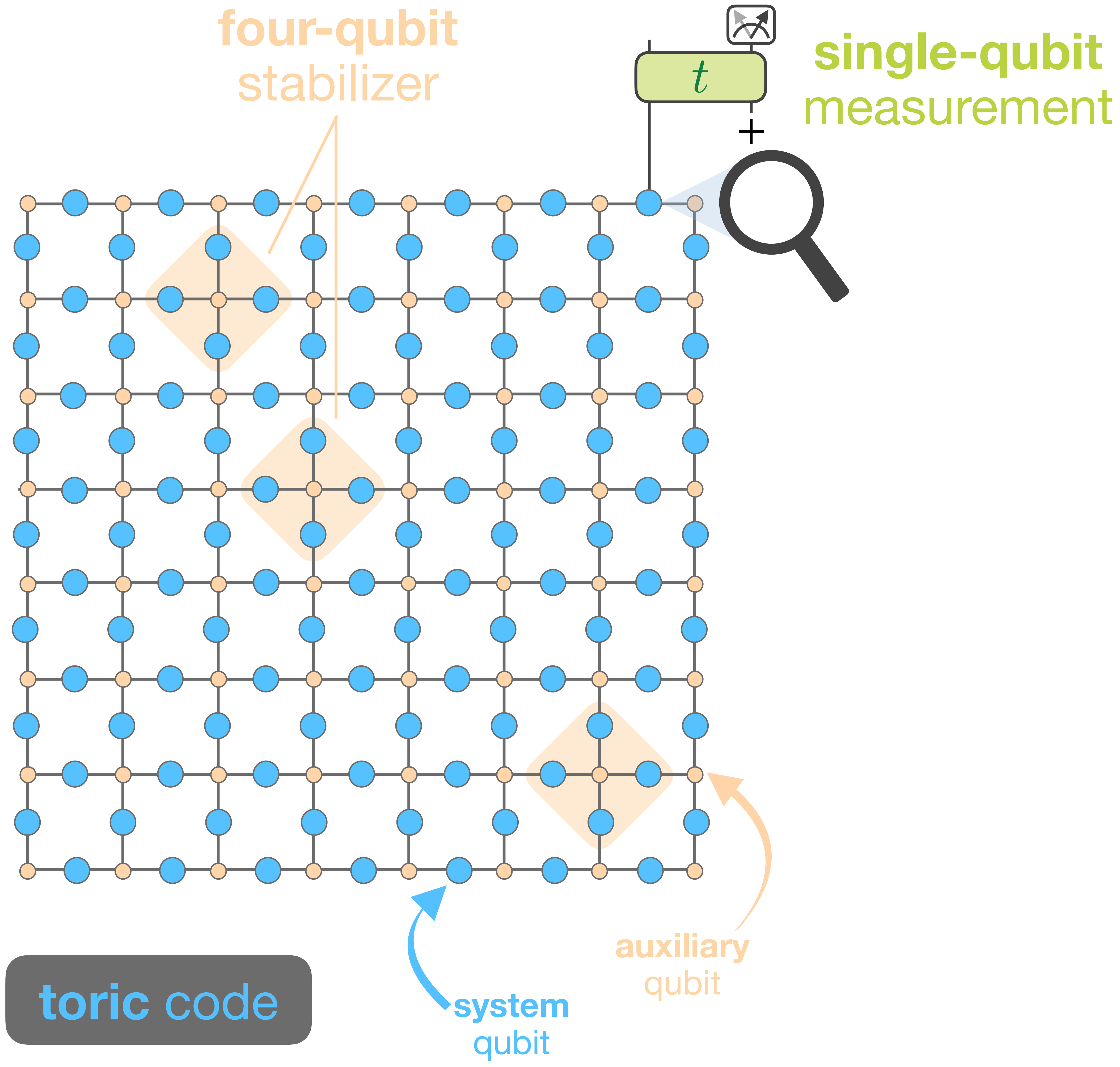
cluster state

auxiliary
qubit

system
qubit

cat state

measurement-induced phase transitions



What happens if, for the toric code, we weakly **monitor all system qubits**?

$$0 \leq t \leq \frac{\pi}{4}$$

Is the monitored **toric code stable** to this coherent deformation? Is there a **threshold**?

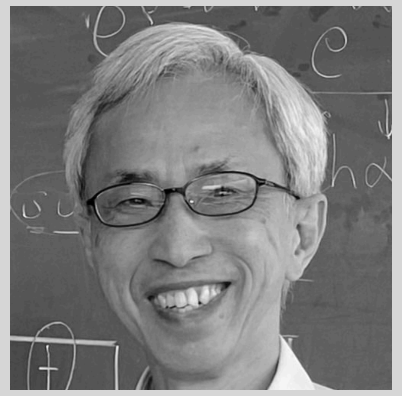
Nishimori transition

Nishimori physics

Phys. Rev. Lett. 131, 200201 (2023)

G-Y. Zhu, N. Tantivasadakarn, A. Vishwanath, R. Verresen

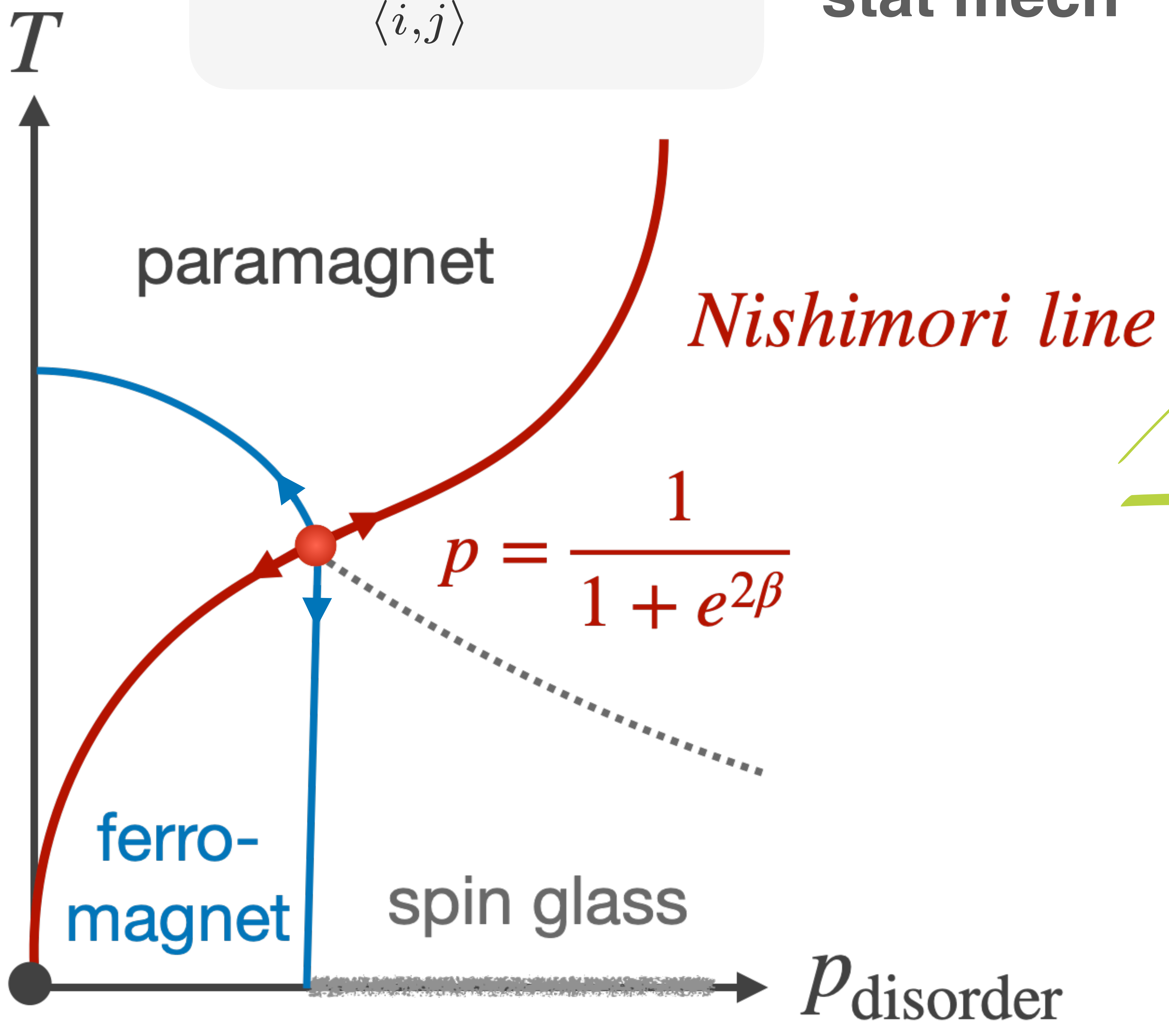
Nishimori physics



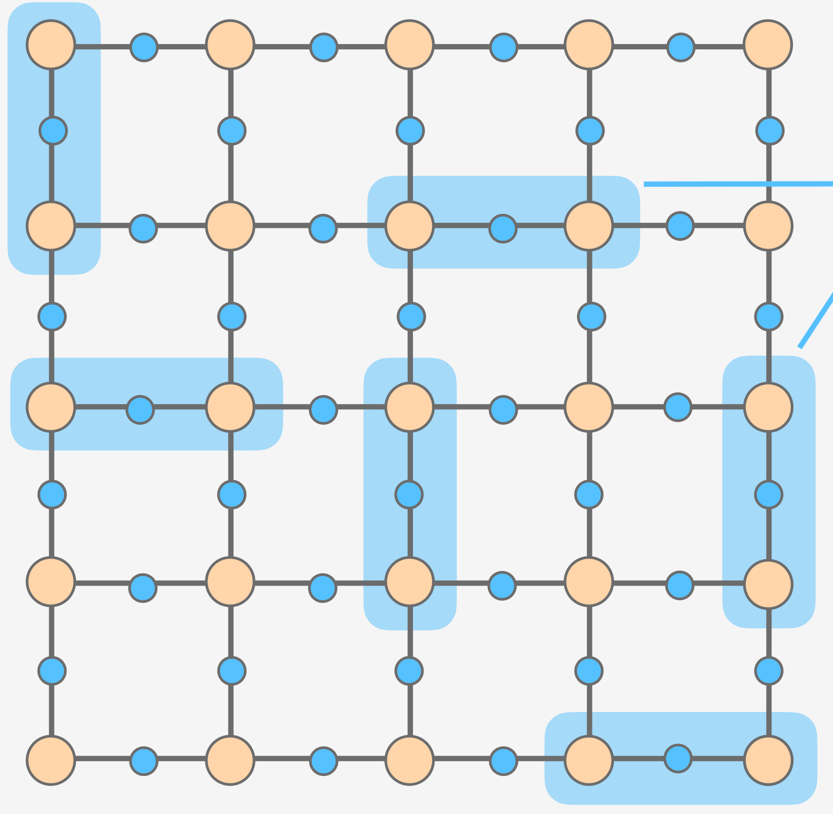
Nishimori (1981)

$$\mathcal{H} = \sum_{\langle i,j \rangle} J_{ij} \; S_i^z S_j^z$$

classical
stat mech



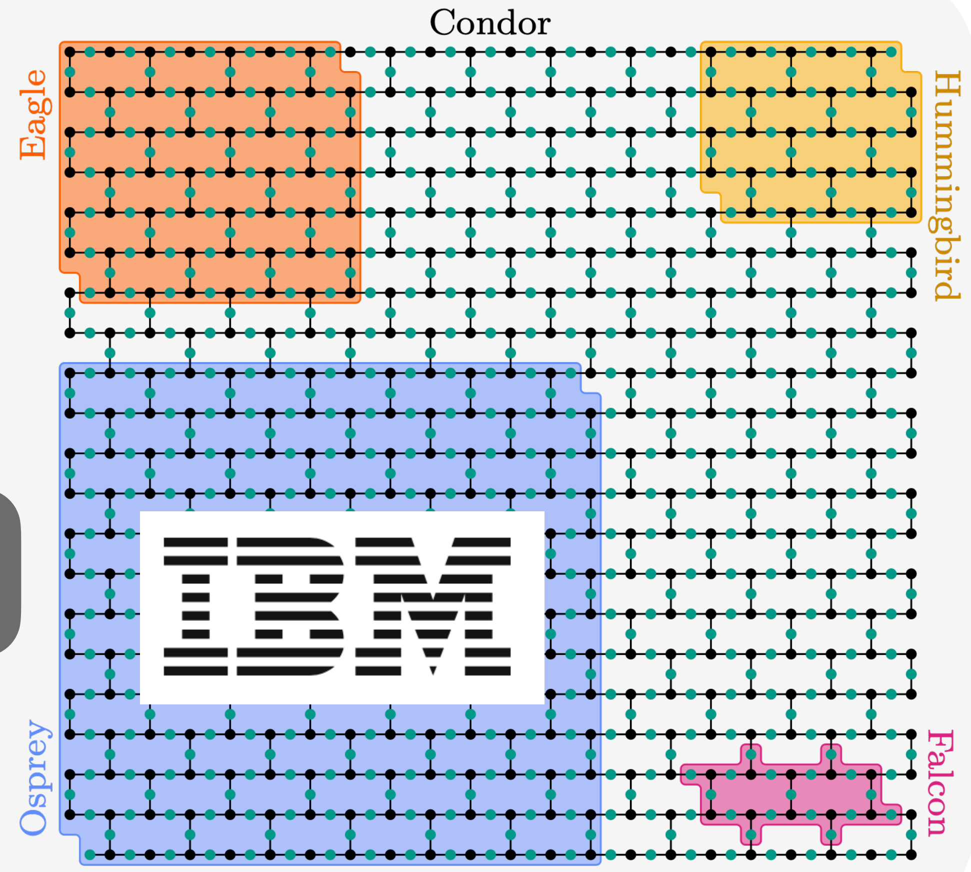
monitored
circuits



two-qubit
parity check

cat state

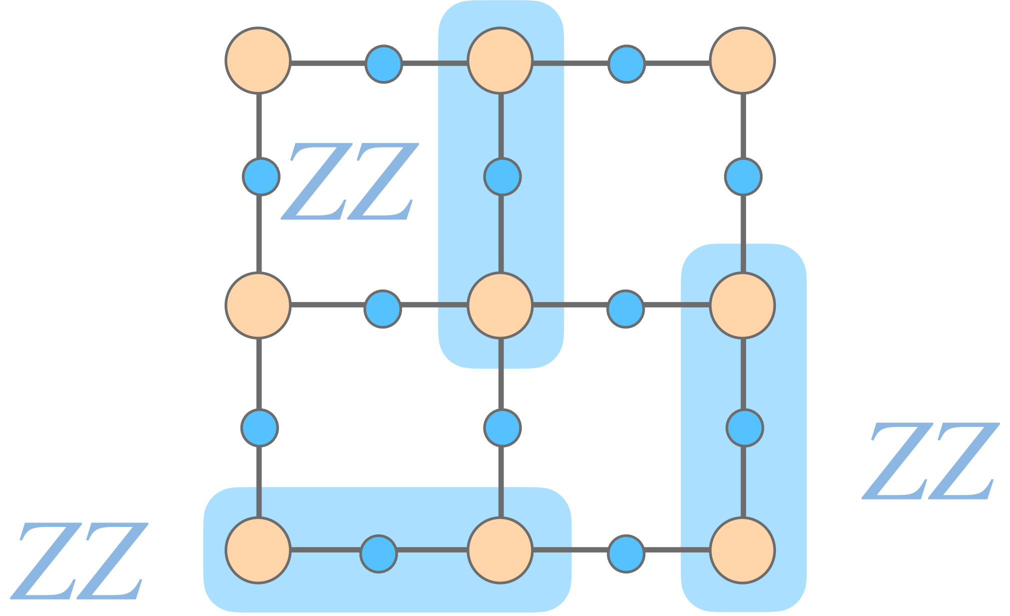
NISQ
devices



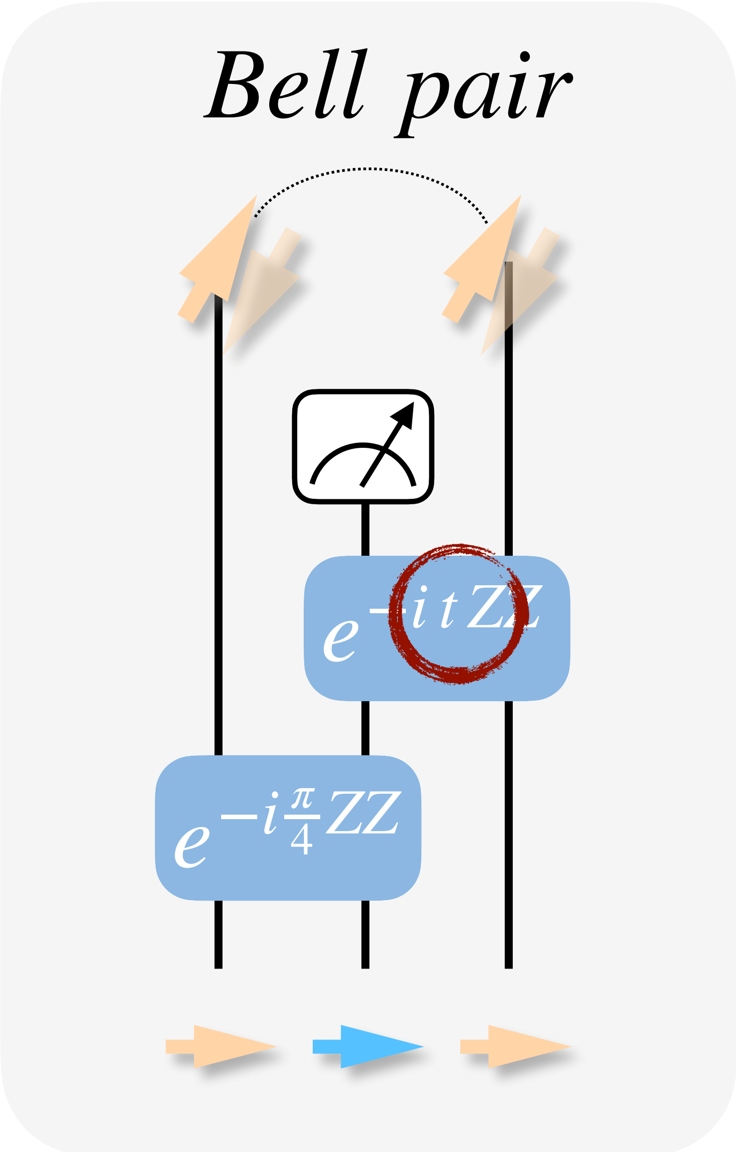
127-qubit
device

Sherbrooke

Nishimori’s cat



Nishimori’s cat



$$|\psi\{s\}\rangle = e^{-\frac{\beta}{2}\sum_{ij} J_{s_{ij}} \sigma_i^z \sigma_j^z} |+\rangle^{\otimes N}$$

interpret as
classical
stat mech model

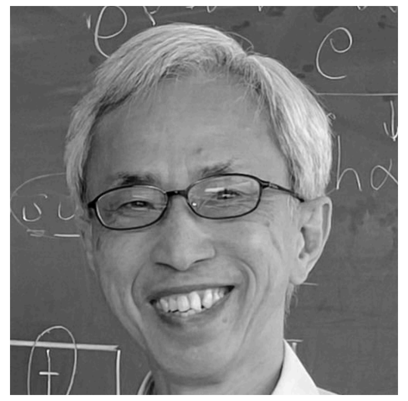
$$\tanh\frac{\beta J_\pm}{2} = \pm\tan t$$

Born’s rule

thermal **fluctuations** and
disorder are **locked**

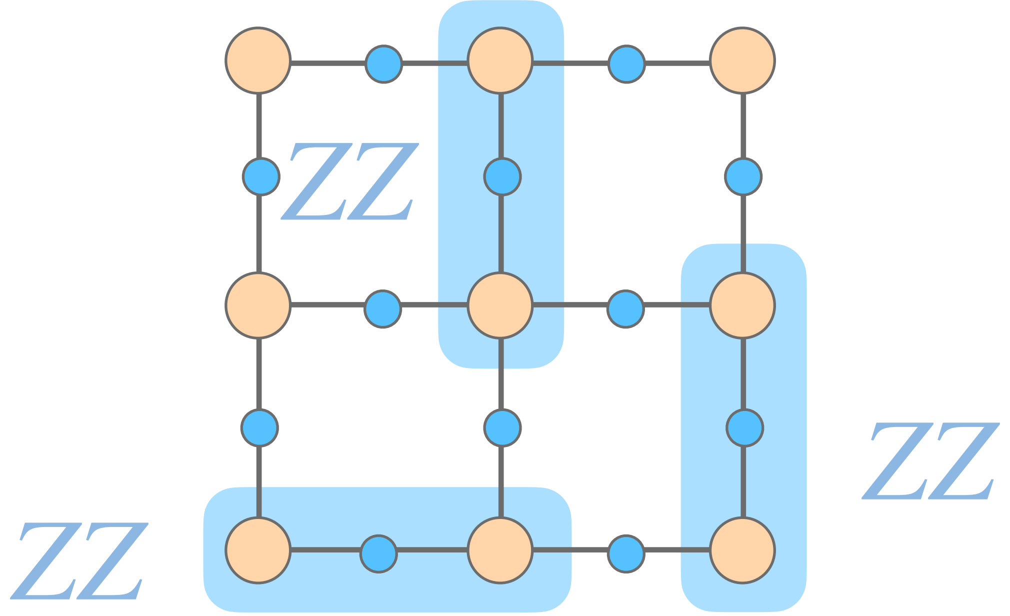
$$Z_{\{s\}} = \sum_{\{\sigma\}} e^{-\beta\sum_{ij} J_{s_{ij}} \sigma_i \sigma_j}$$

random bond Ising model

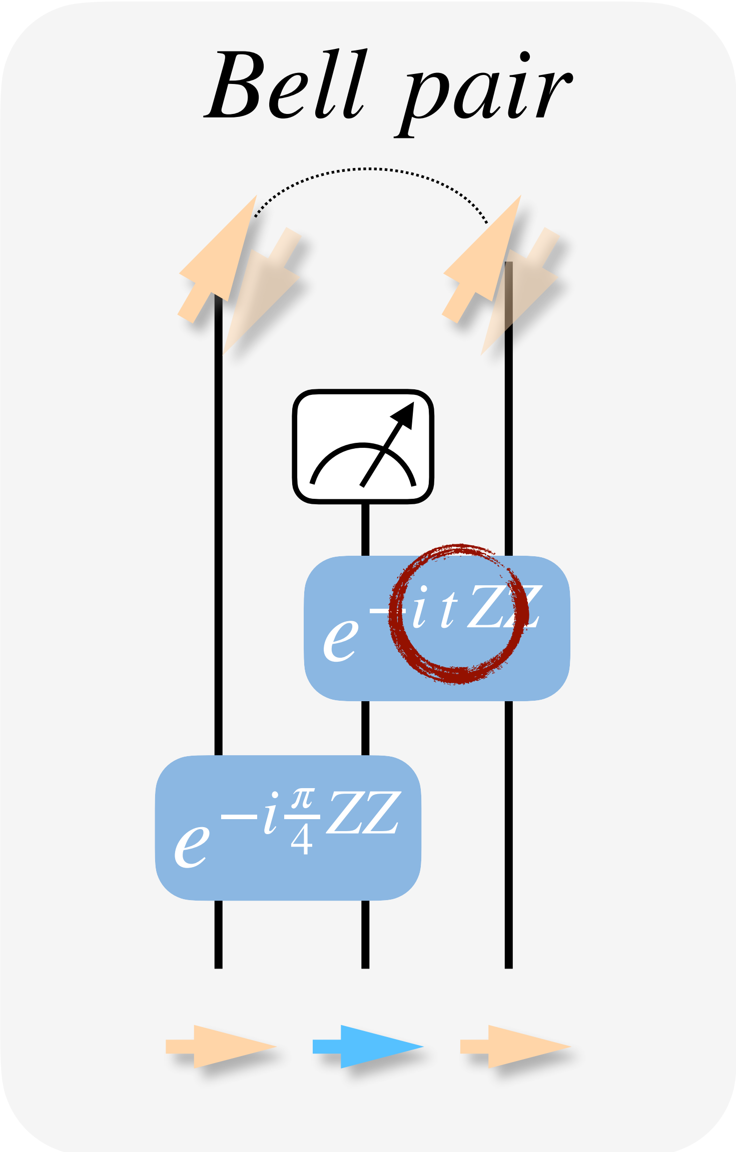


Nishimori (1981)

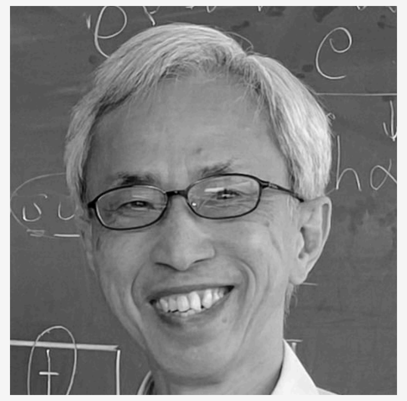
Nishimori’s cat



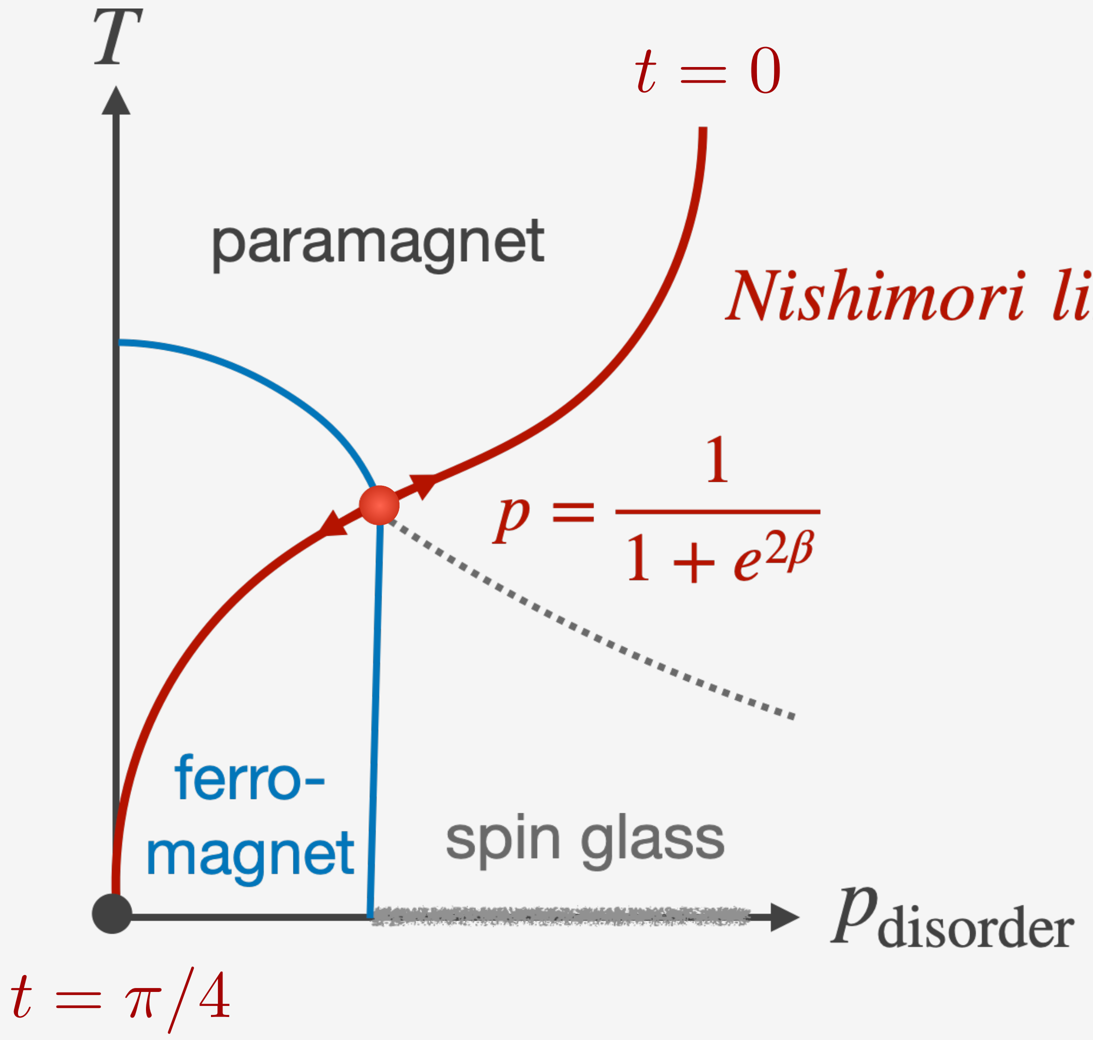
Nishimori’s cat



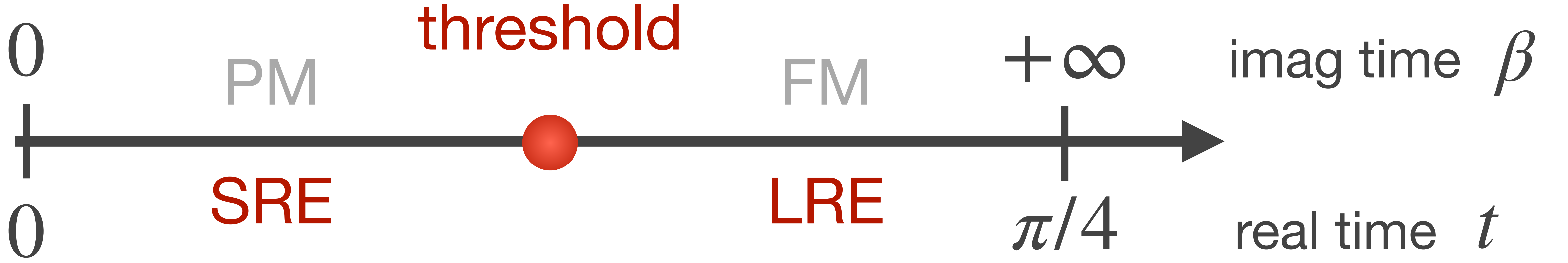
thermal fluctuations and disorder are **locked**



Nishimori (1981)



theory – Phys. Rev. Lett. 131, 200201 (2023)
experiment (IBM) – Nature Physics (2025)



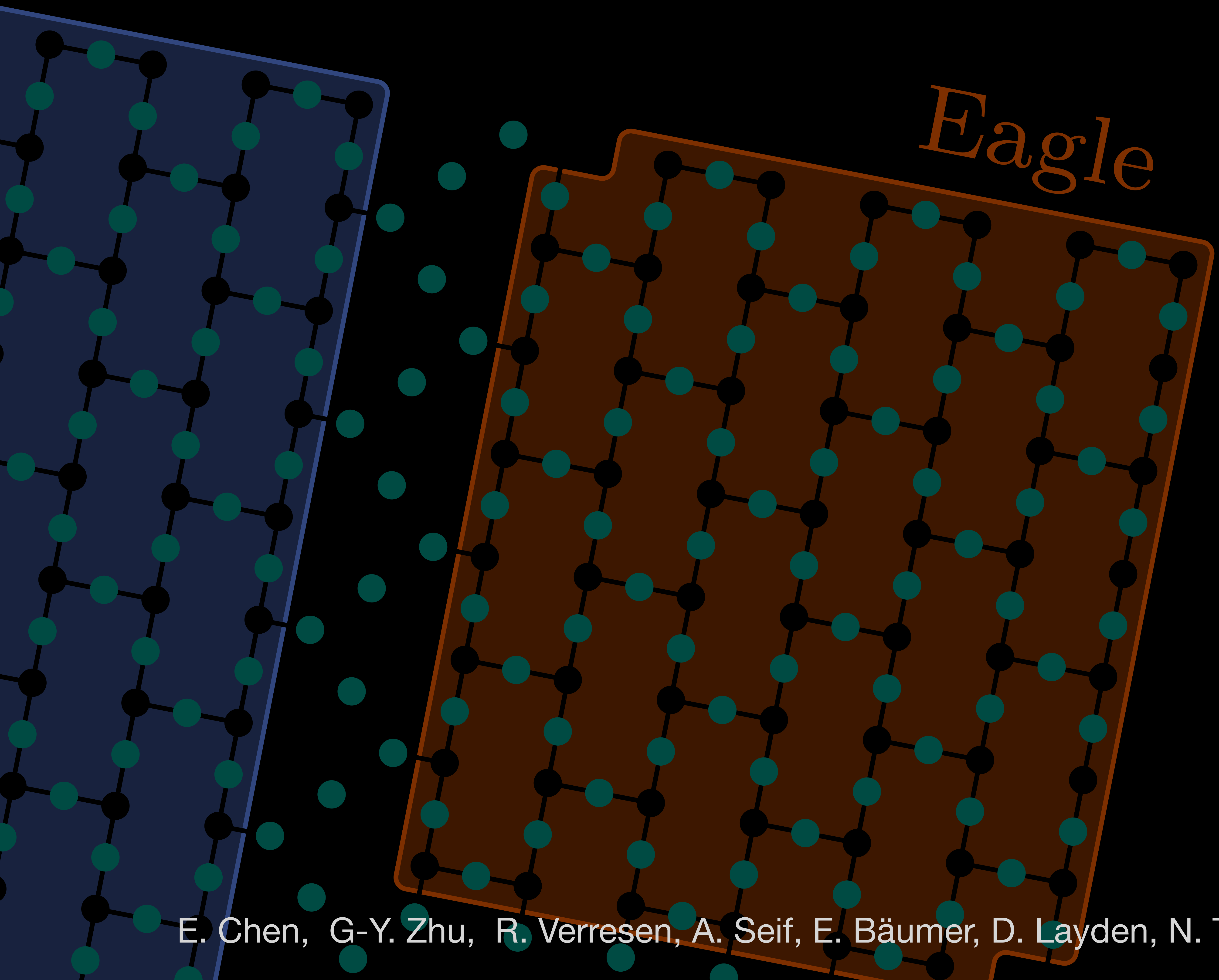
“high temperature” finite threshold “low temperature”

0 PM FM $+\infty$ imag time β

0 SRE LRE $\pi/4$ real time t

weak measurement

strong measurement

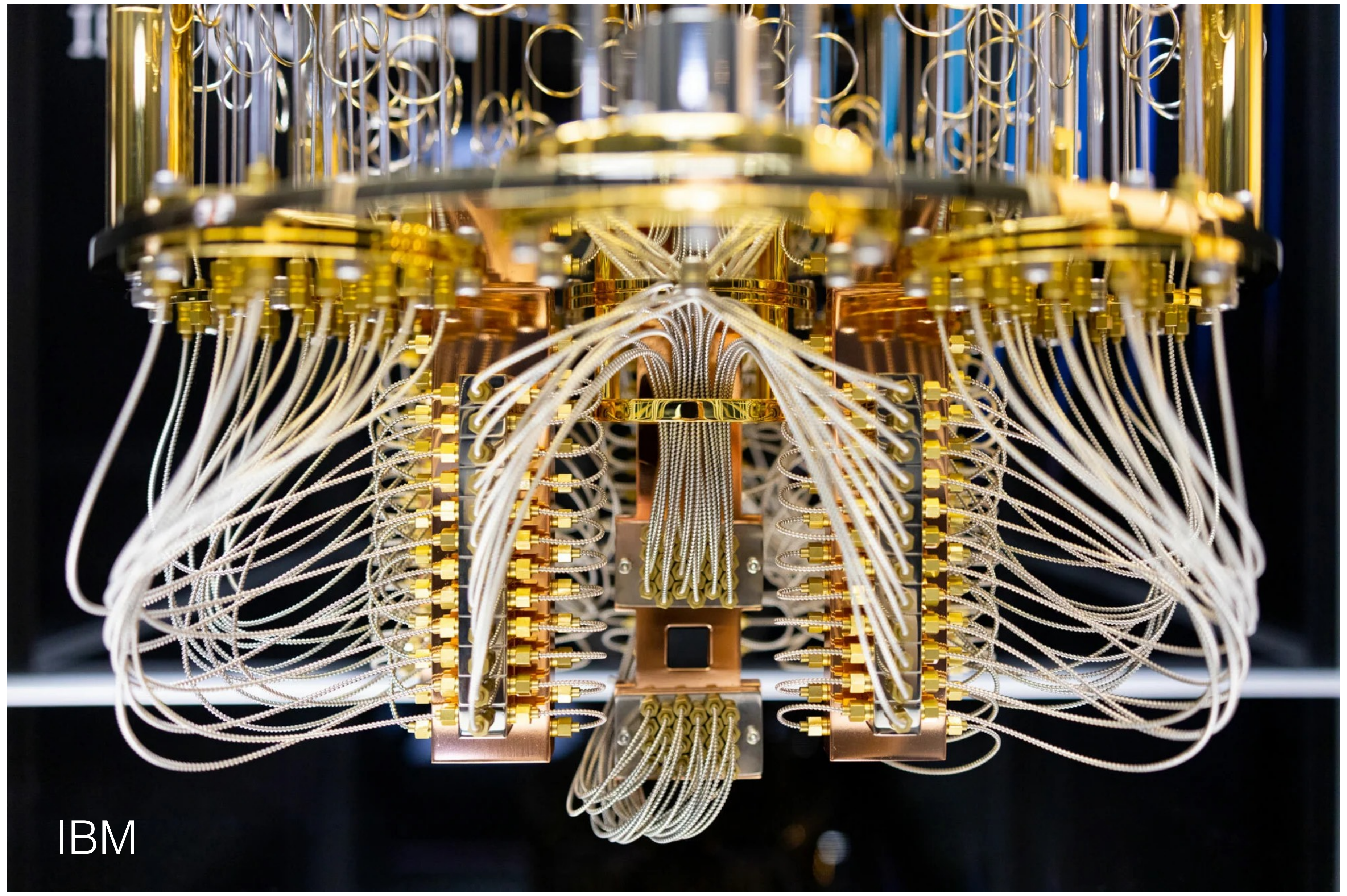


experiment

Nature Physics 21, 161 (2025)

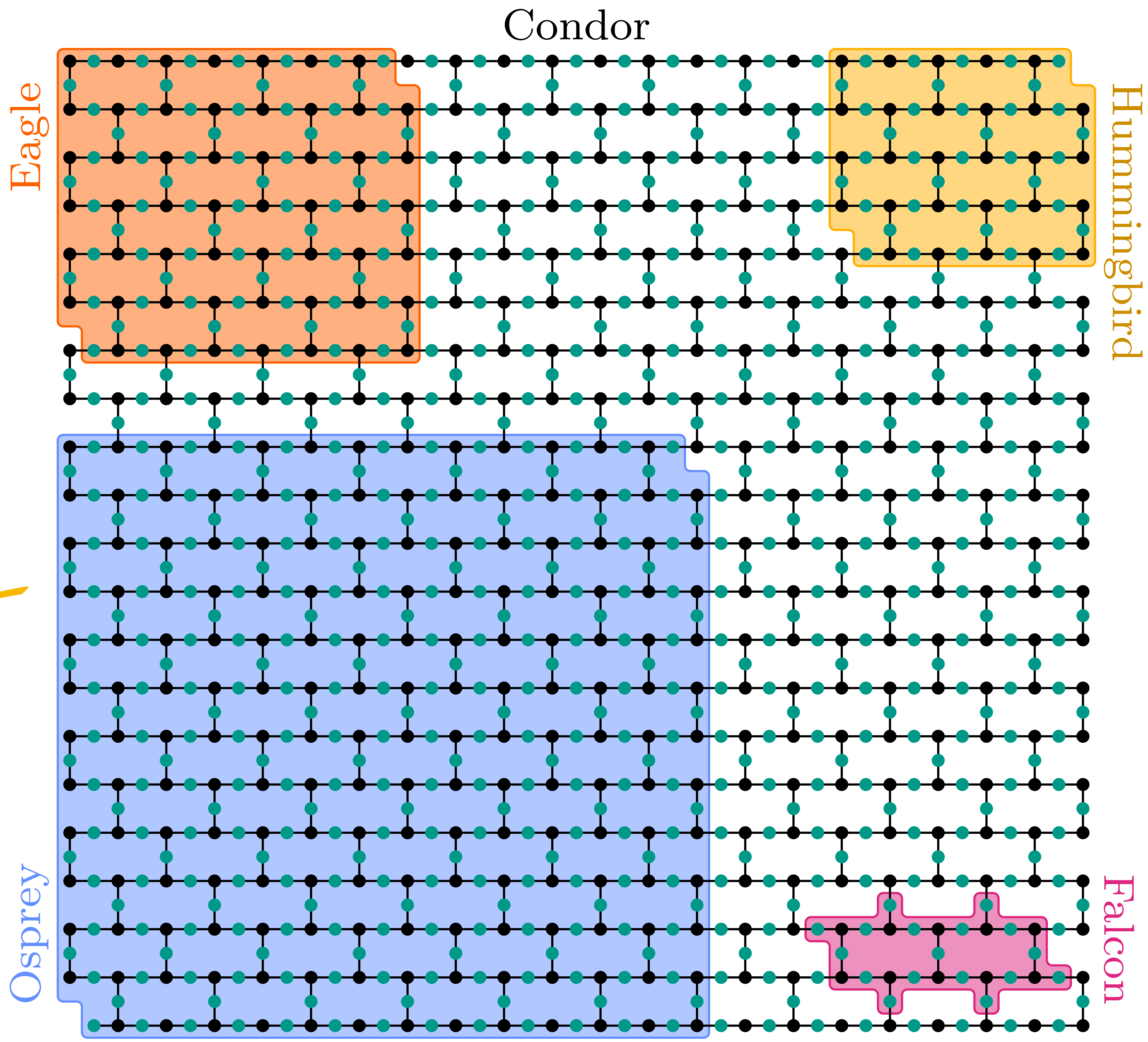
E. Chen, G-Y. Zhu, R. Verresen, A. Seif, E. Bäumer, D. Layden, N. Tantivasadakarn, G. Zhu, S. Sheldon, A. Vishwanath, A. Kandala

IBM quantum cloud devices



NISQ devices built on transmon qubits

noisy intermediate scale quantum devices



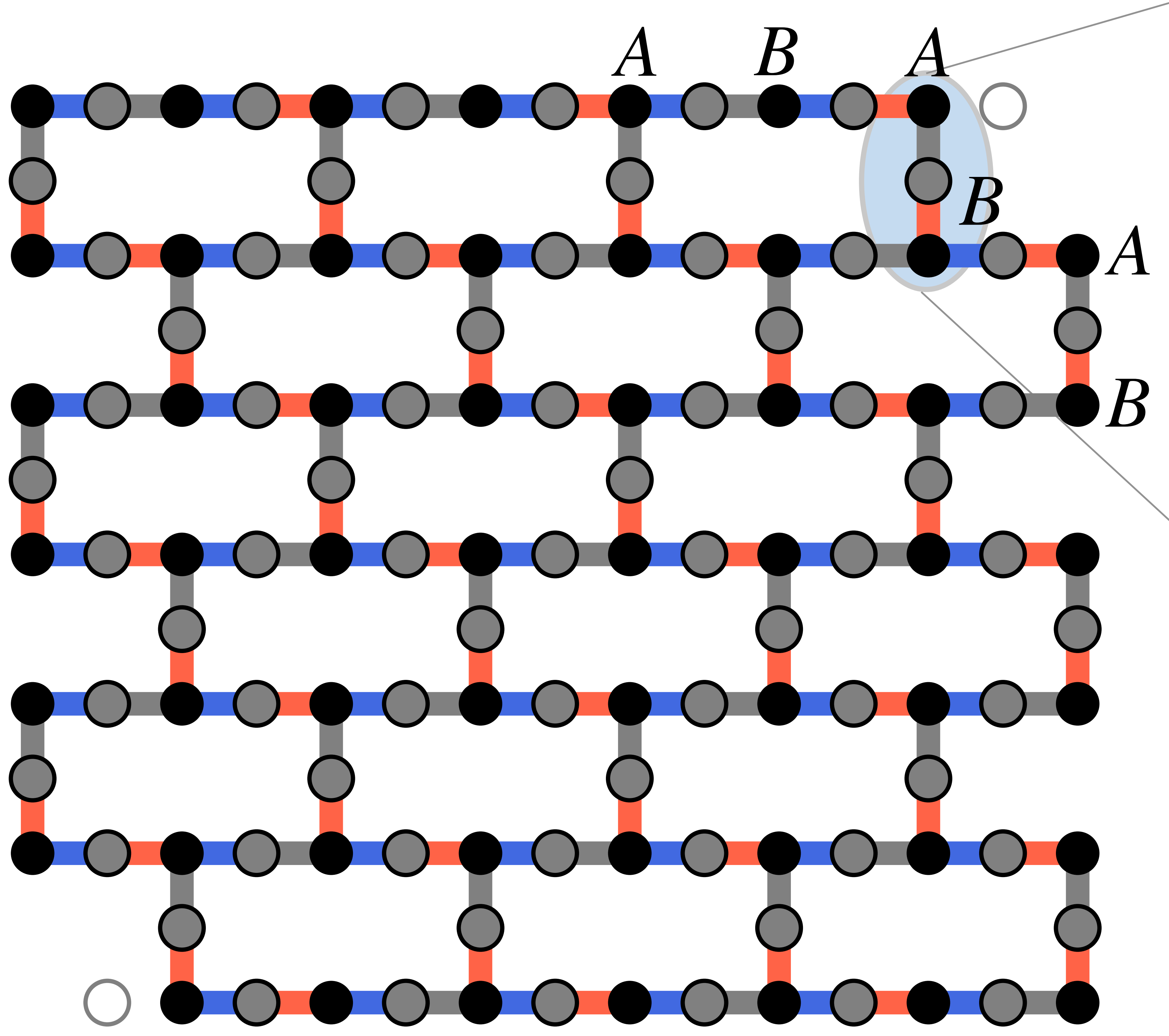
heavy-hexagon geometry

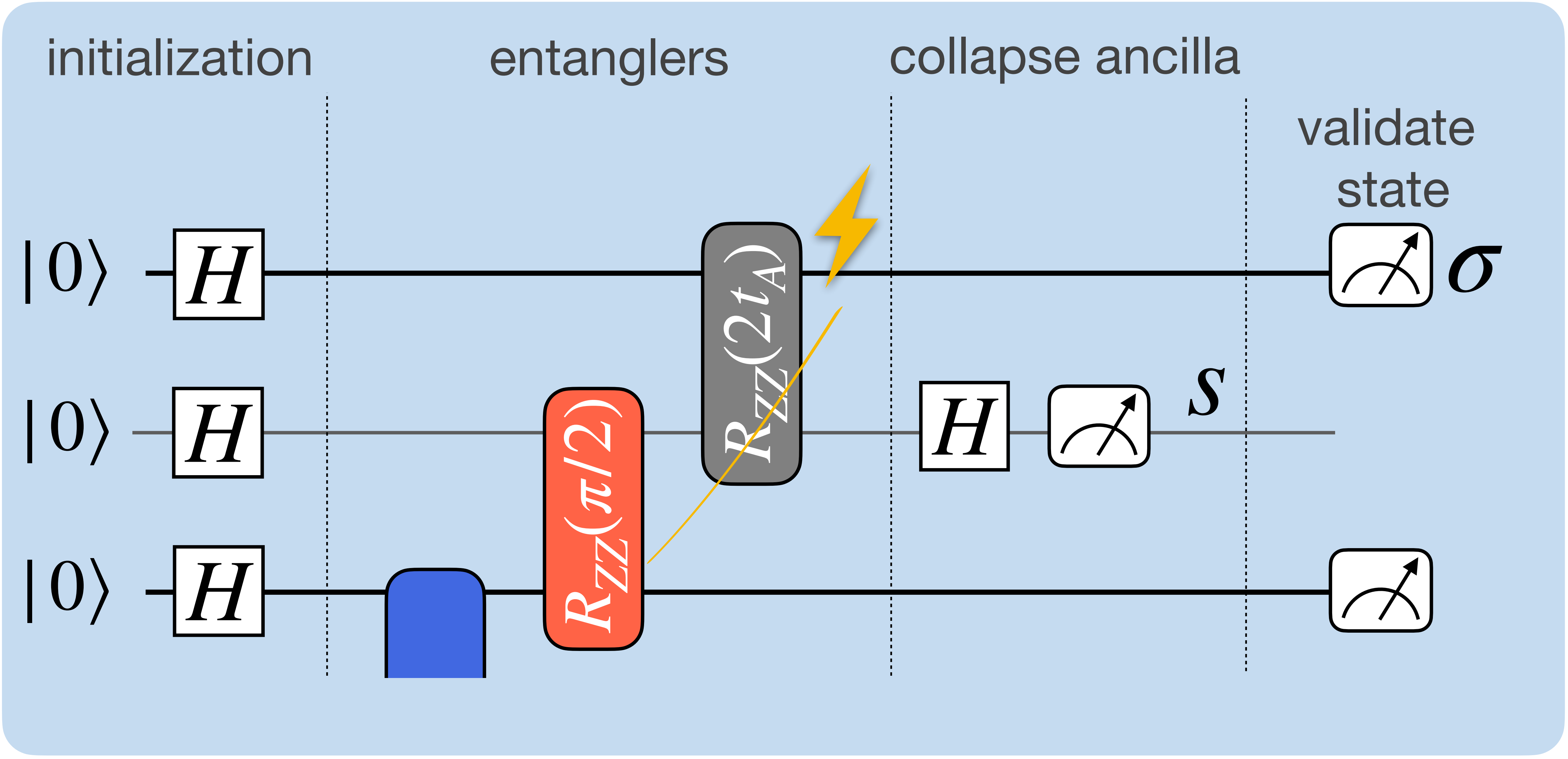
+

Ising evolution gates

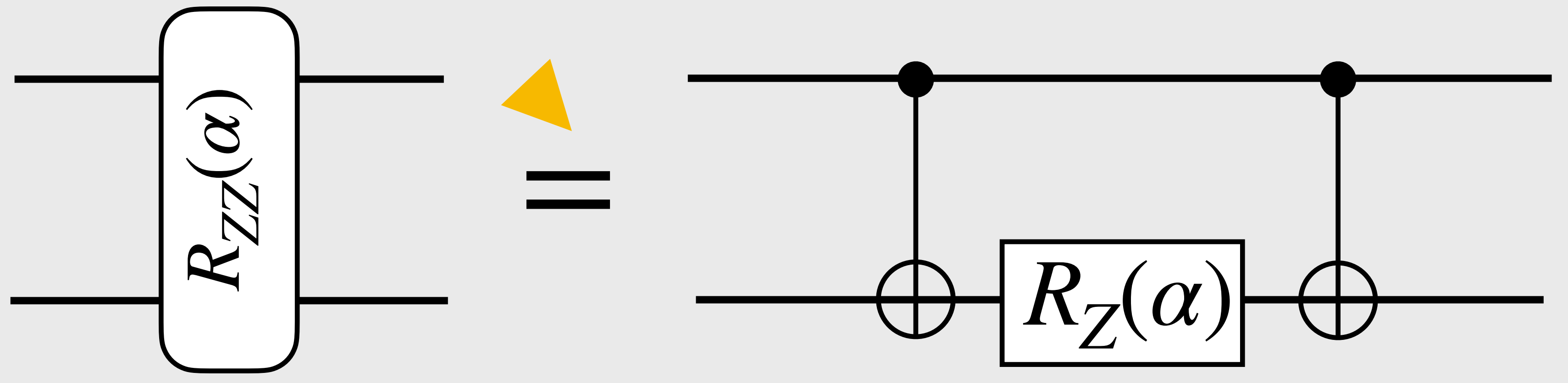
protocol on IBM heavy-hexagon lattice

IBM_Sherbrooke





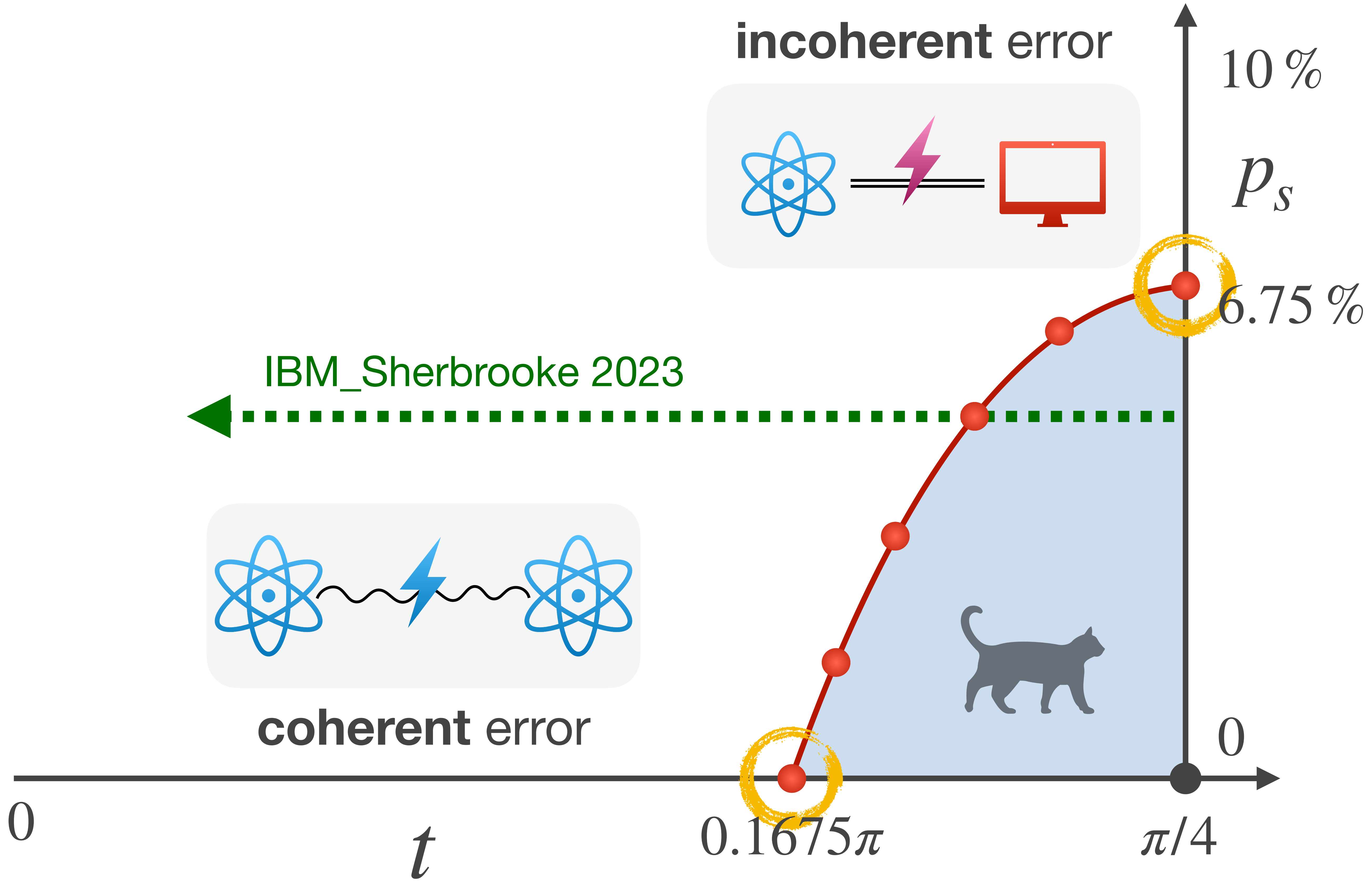
- ● **system** qubit: site of honeycomb lattice
- ● **auxiliary** qubit: bond of honeycomb lattice

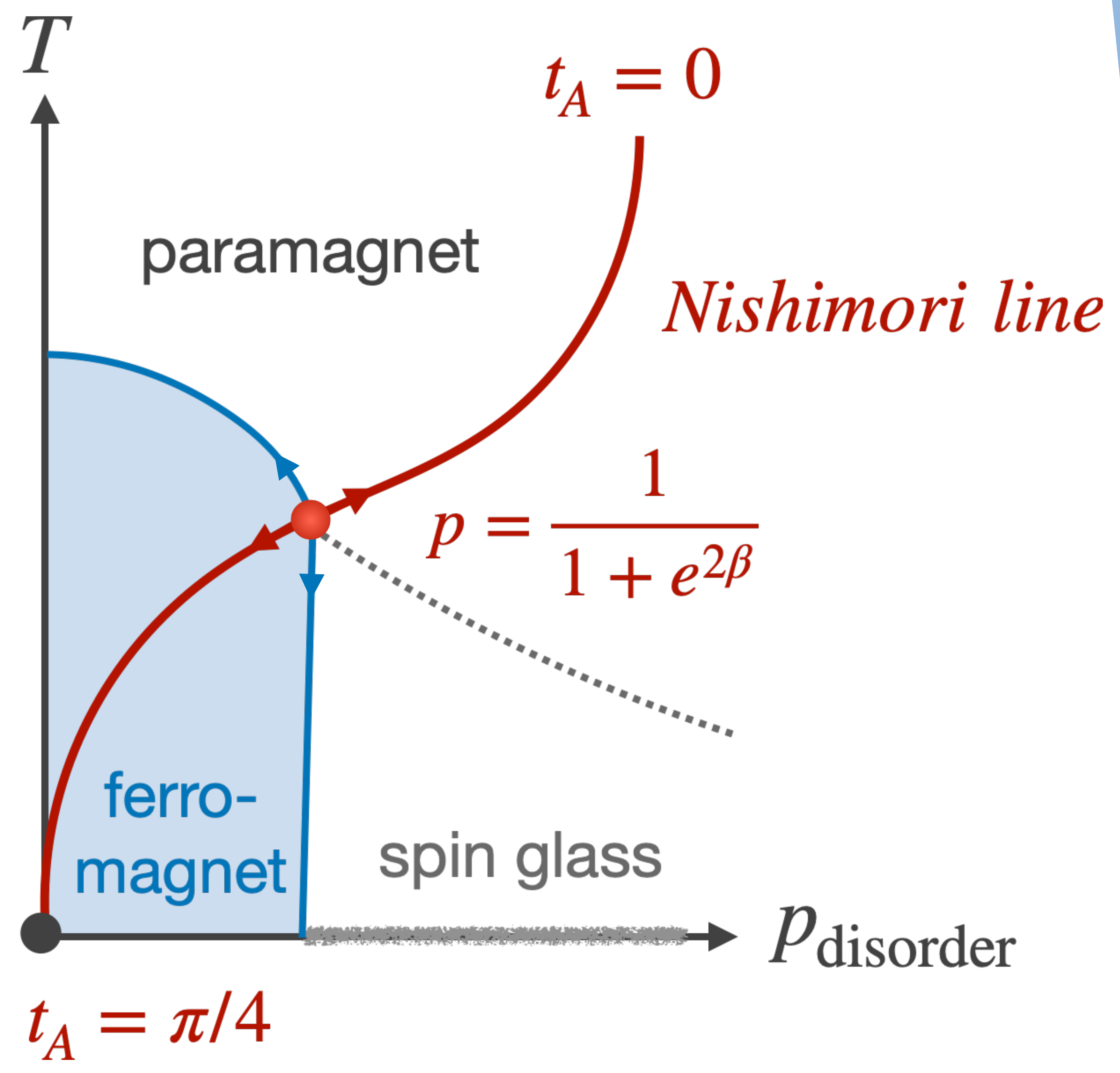


- $\alpha < \pi/4$ injects **tunable coherent error**
- generically a **non-Clifford** gate

incoherent & coherent errors

Dennis, Kitaev, Landahl, Preskill 2002

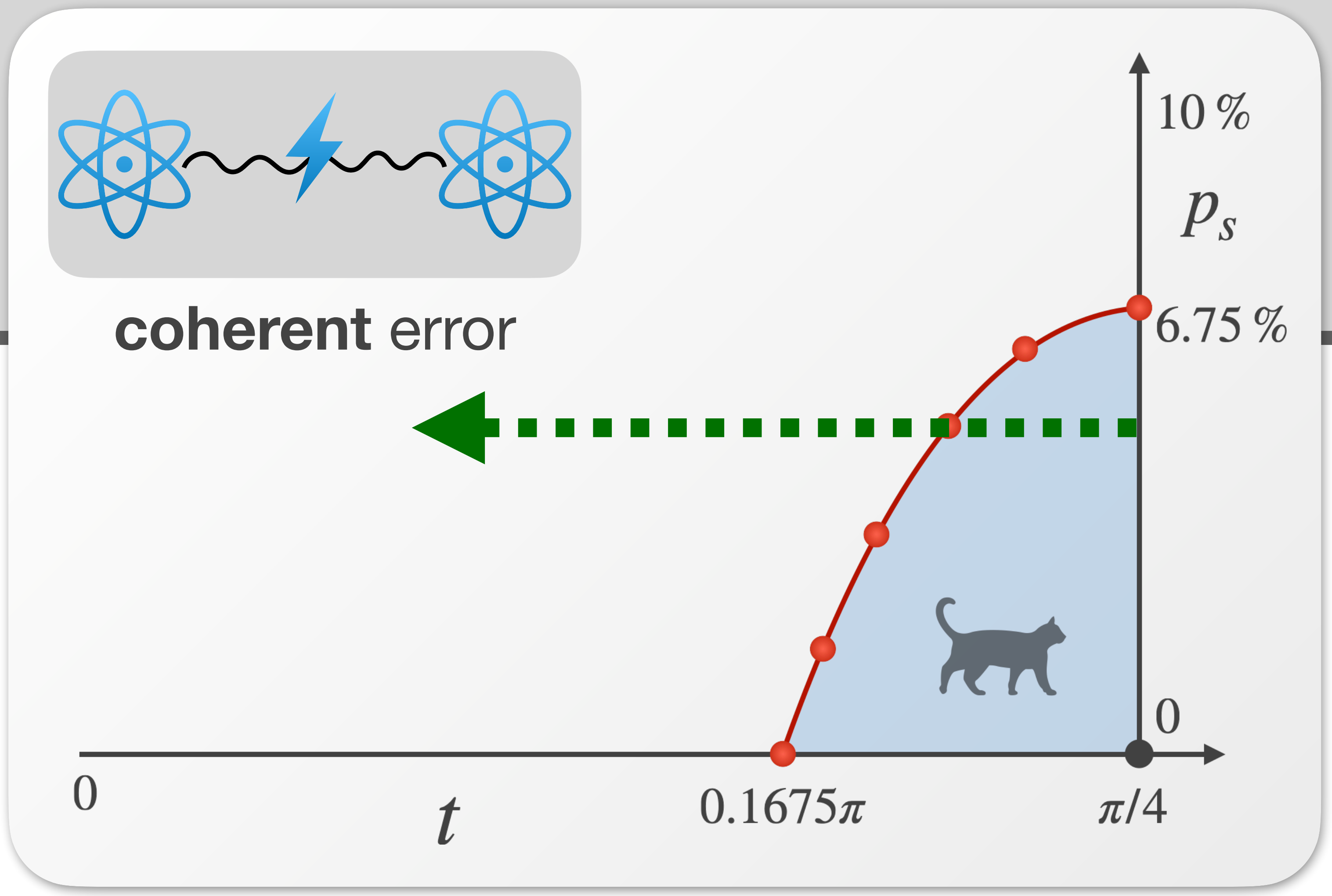


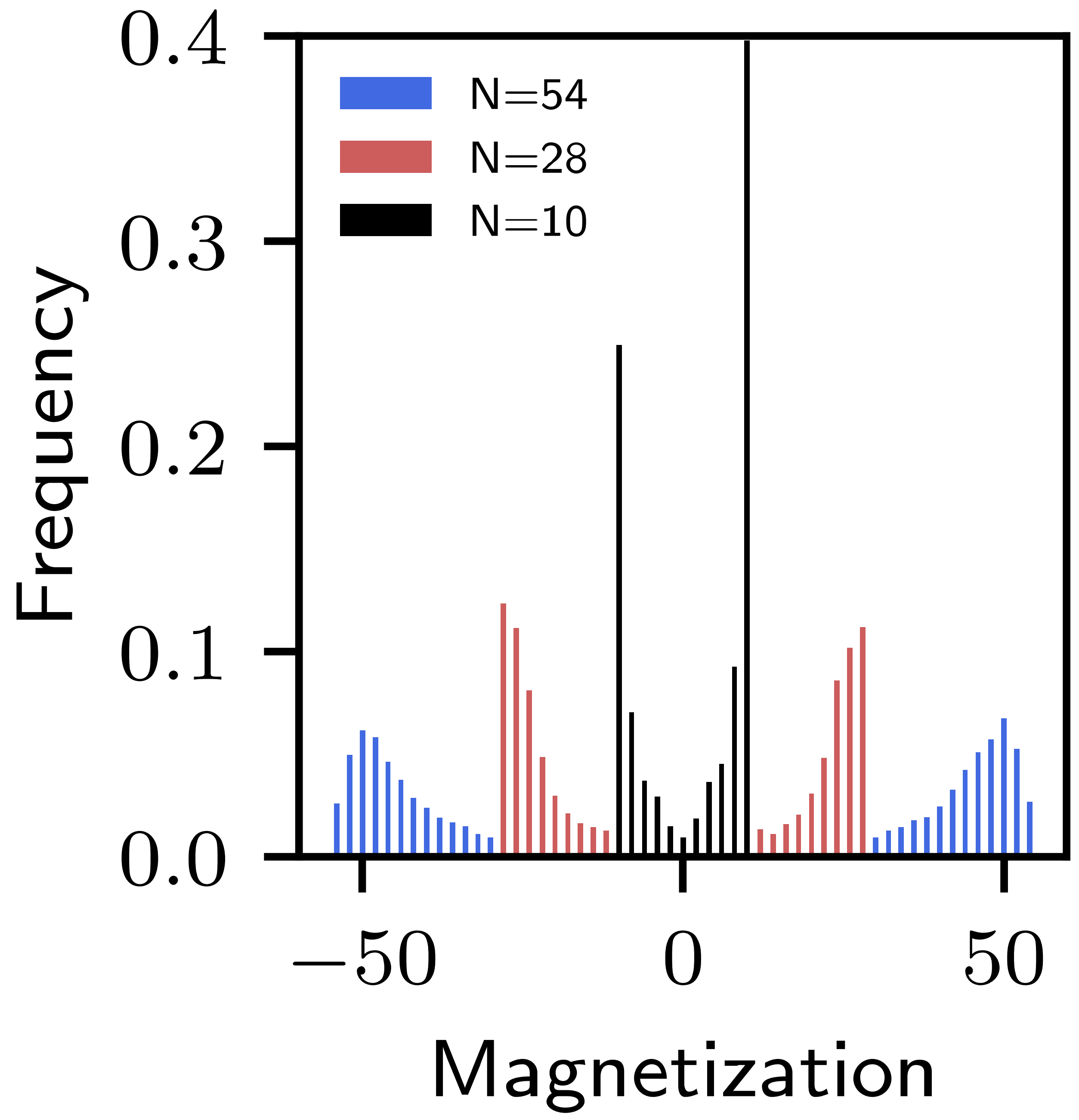


entire phase diagram is mapped to **Nishimori line**

$$\tilde{p} = \frac{1-(1-2p_s)\sin(2t_A)}{2}$$

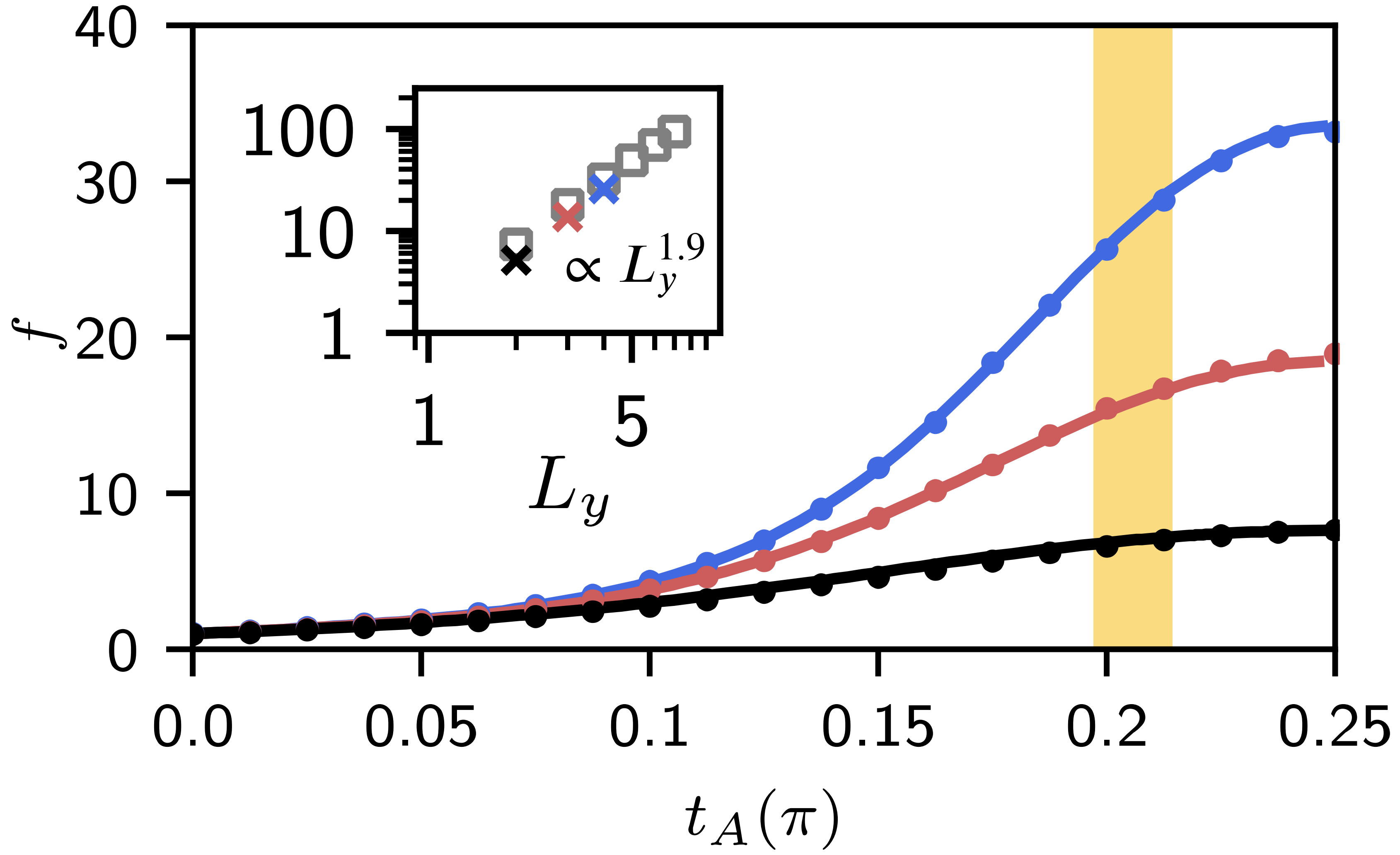
coherent error transition





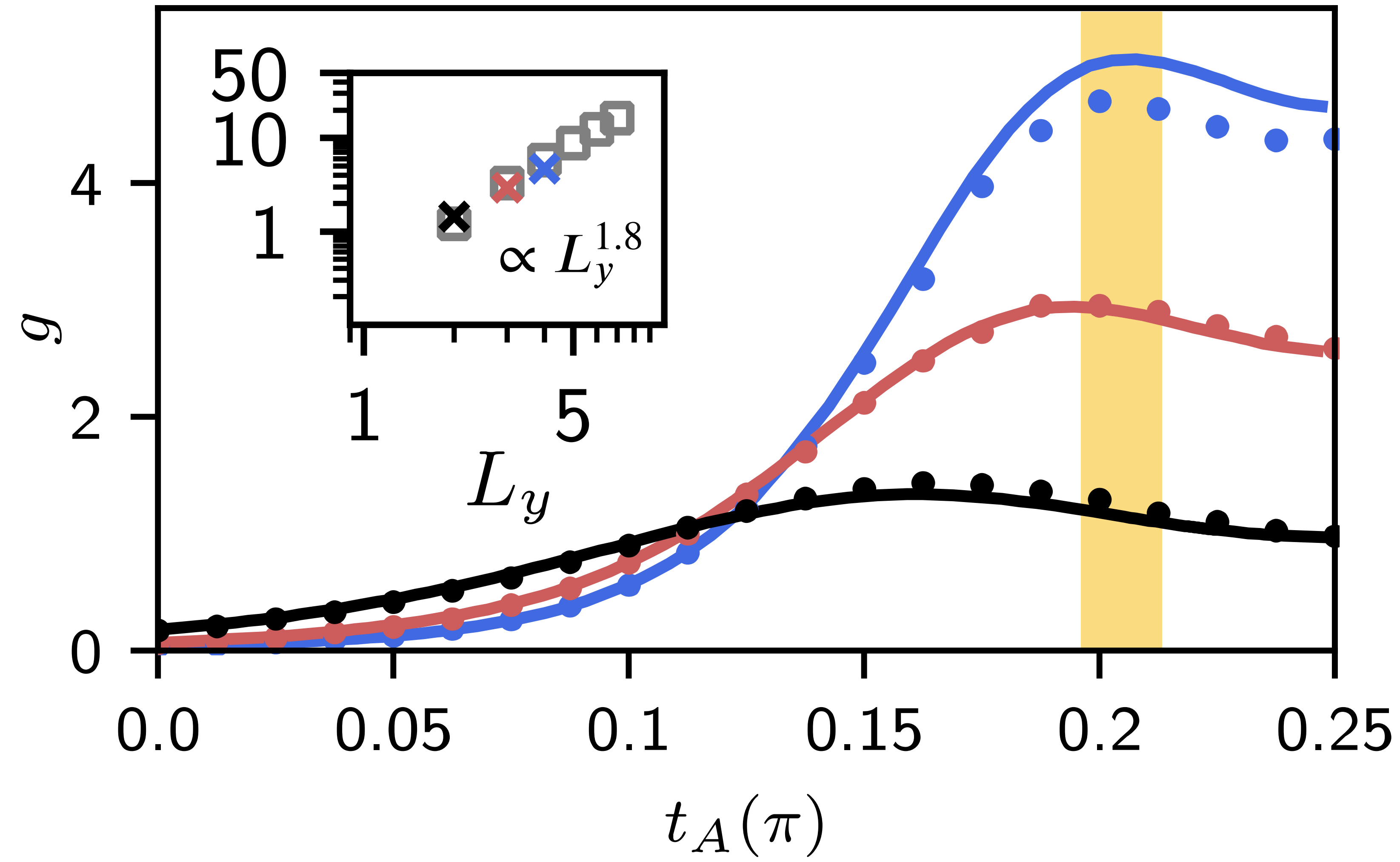
$$f = \frac{1}{N}\left(\langle M^2\rangle - \langle M\rangle^2\right)$$

classical
correlations

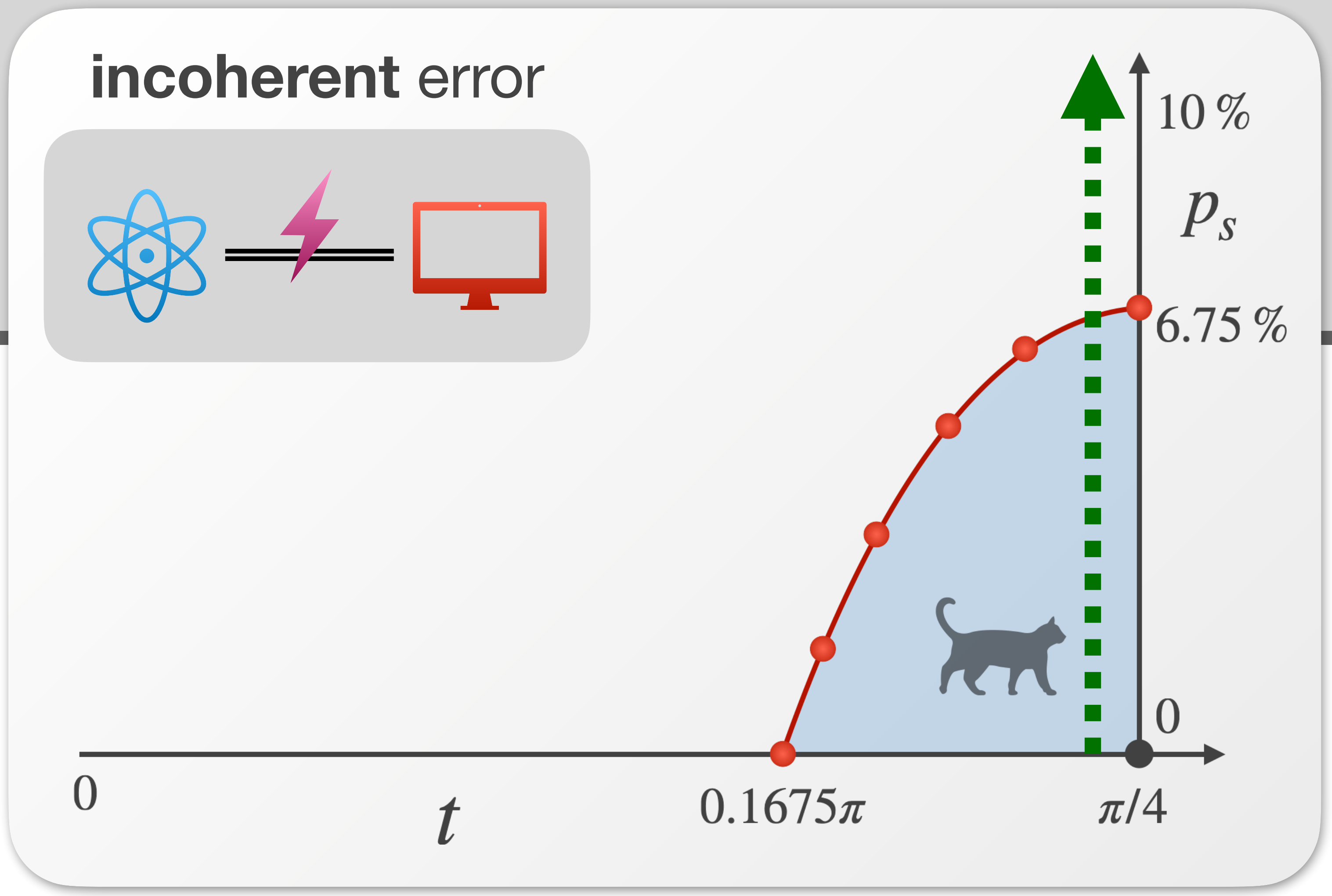


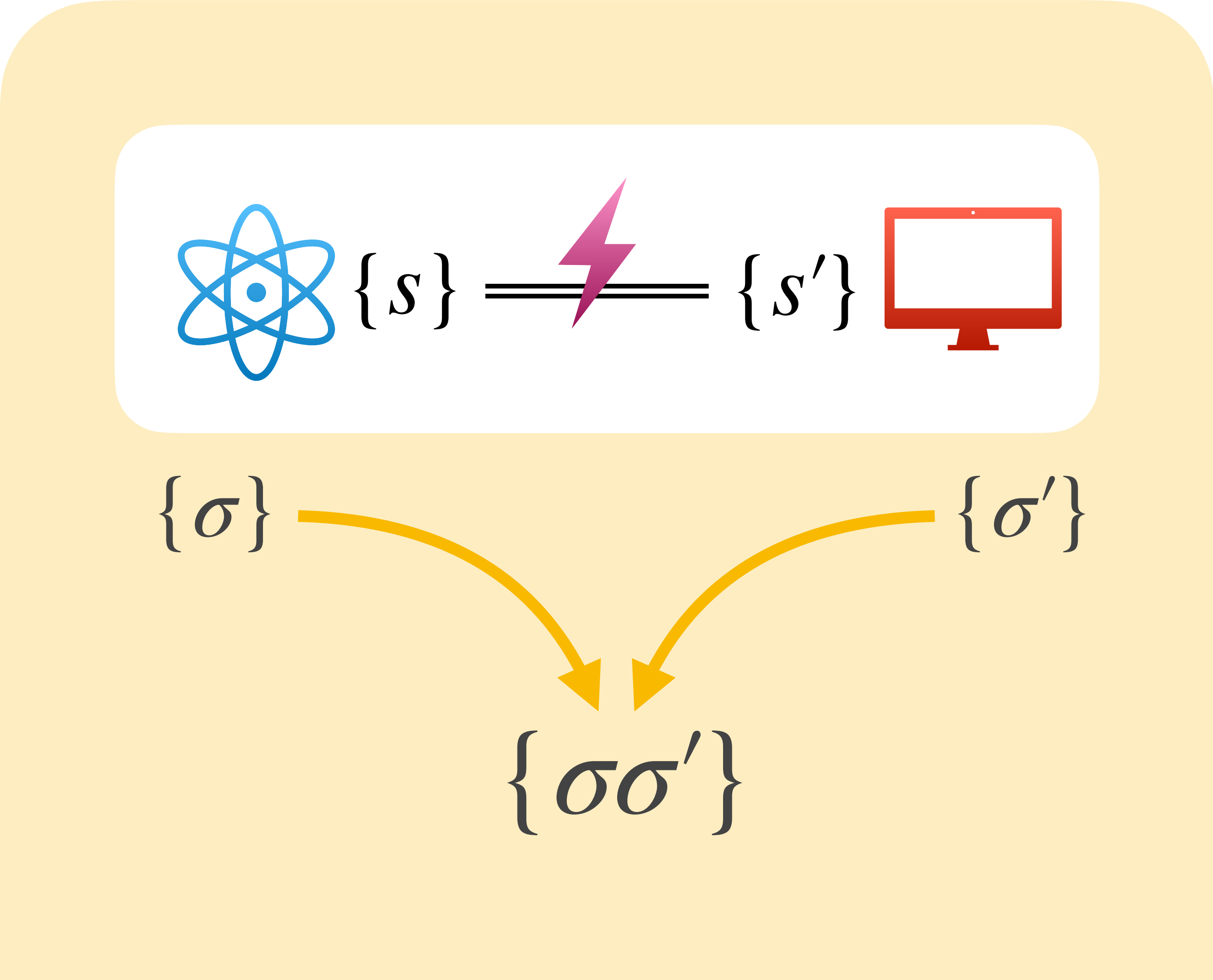
normalized
variance

$$g = \frac{1}{N^3}\left(\langle M^4\rangle - \langle M^2\rangle^2\right)$$



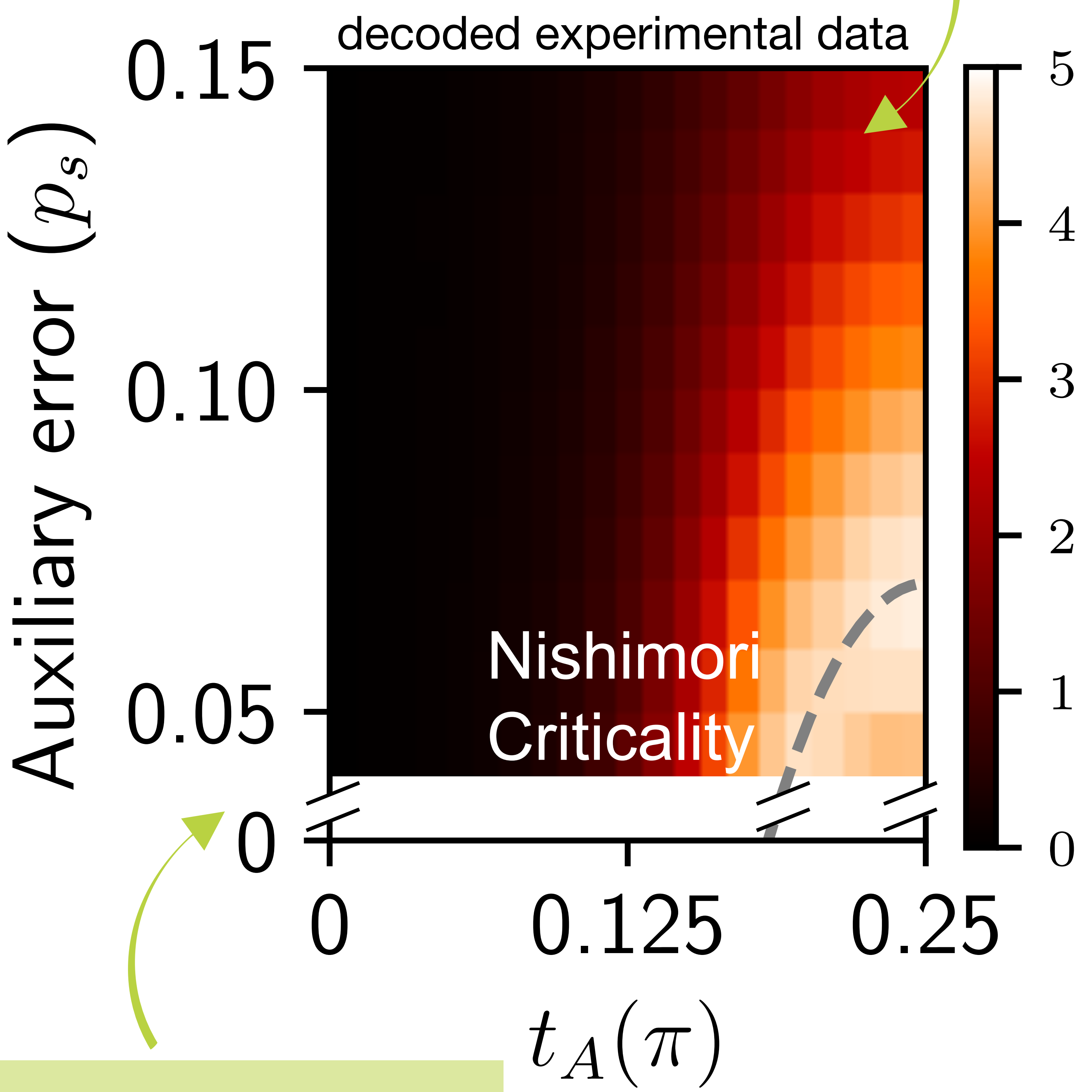
incoherent error transition



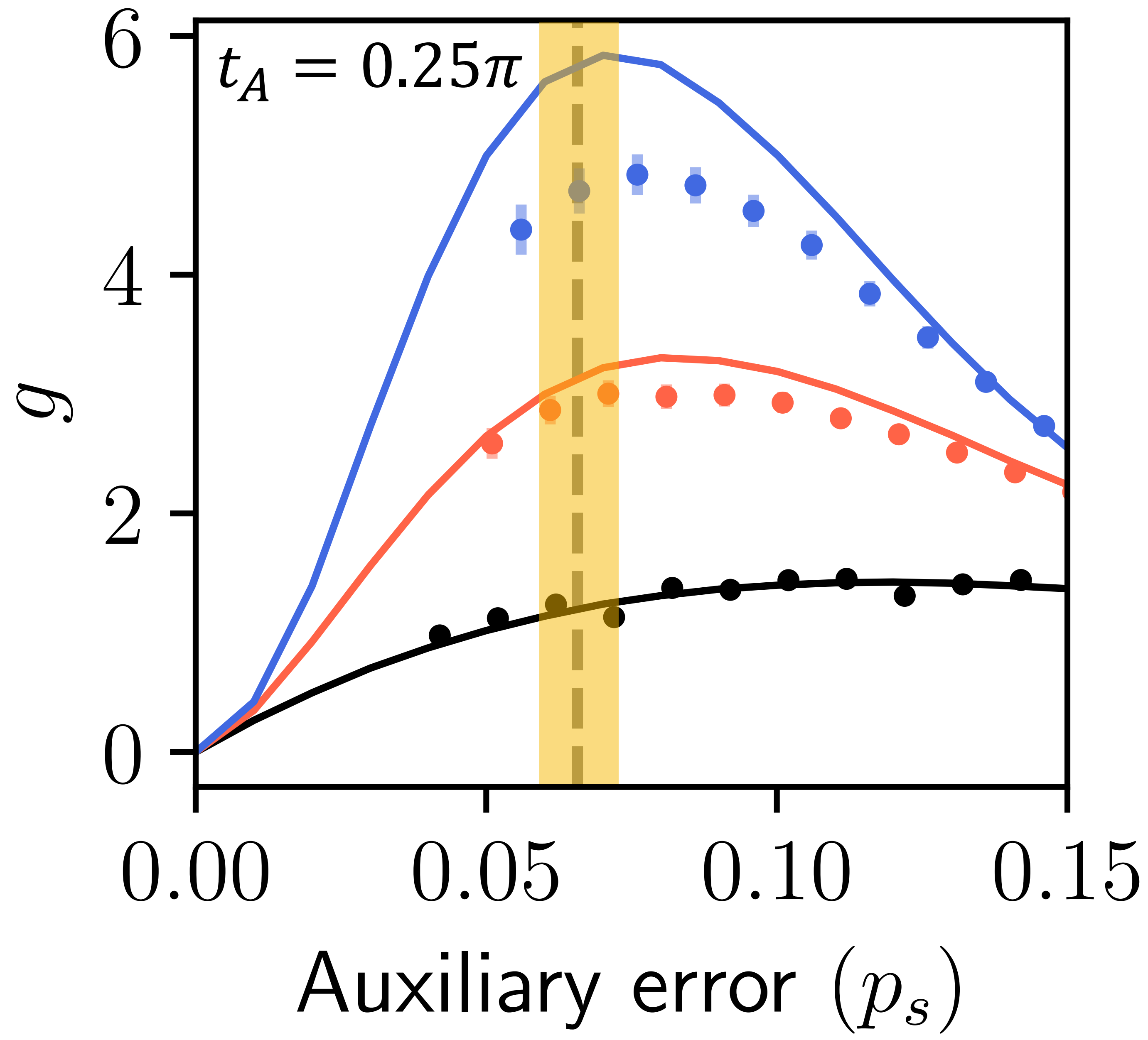


decoding transition

injected error
into classical computer



intrinsic error
of quantum device



⋯ experimental data on IBM Sherbrooke
— theoretical benchmark with noise

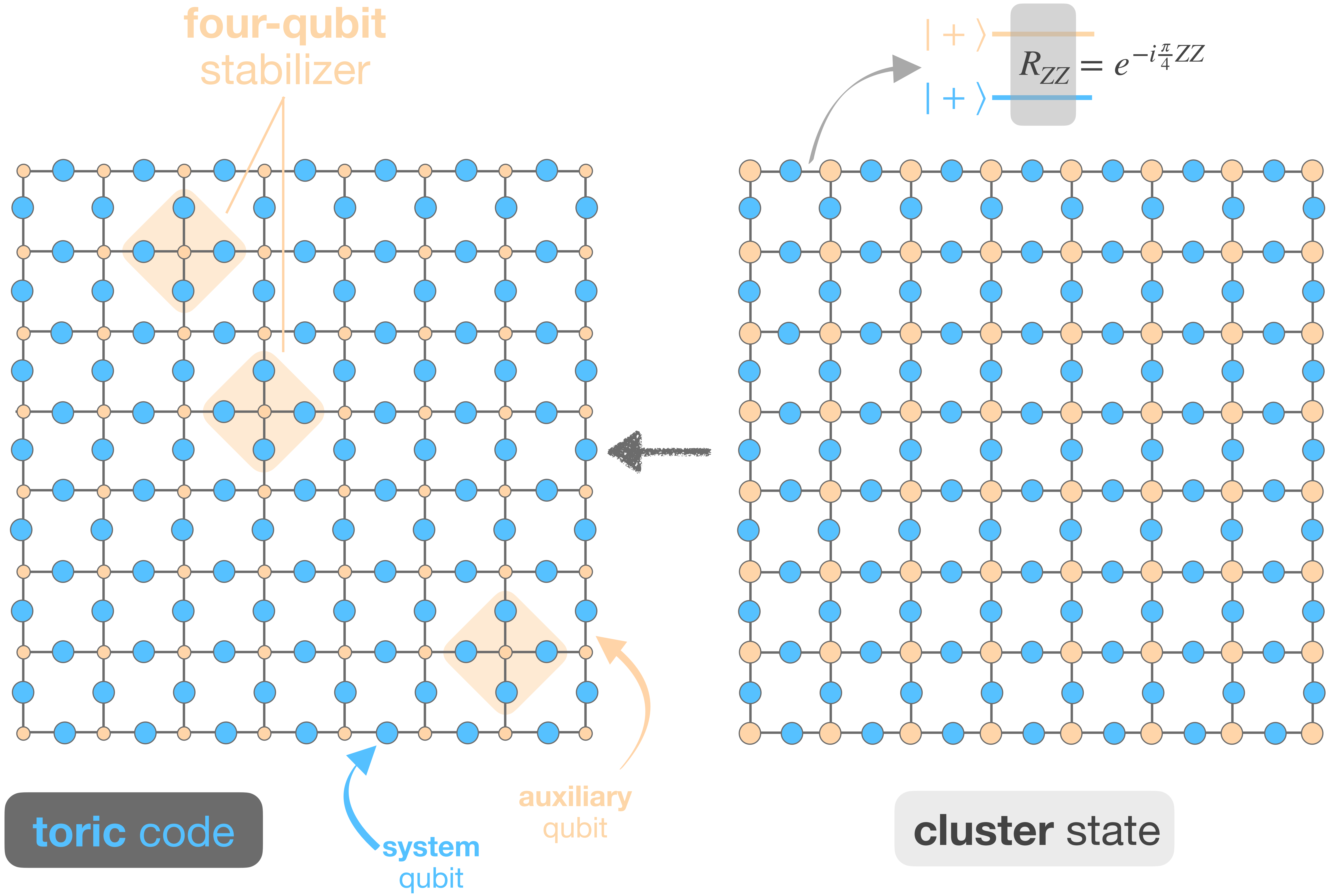
monitored toric codes

arXiv:2502.14034
PRX Quantum **5**, 040313 (2024)

F. Eckstein, B. Han, Q. Wang, R. Vasseur, A. Ludwig, G-Y. Zhu

entanglement by measurement: **surface code**

Kitaev (1997)



four-qubit stabilizer

$|+\rangle$

$|+\rangle$

$R_{ZZ} = e^{-i\frac{\pi}{4}ZZ}$

toric code

system qubit

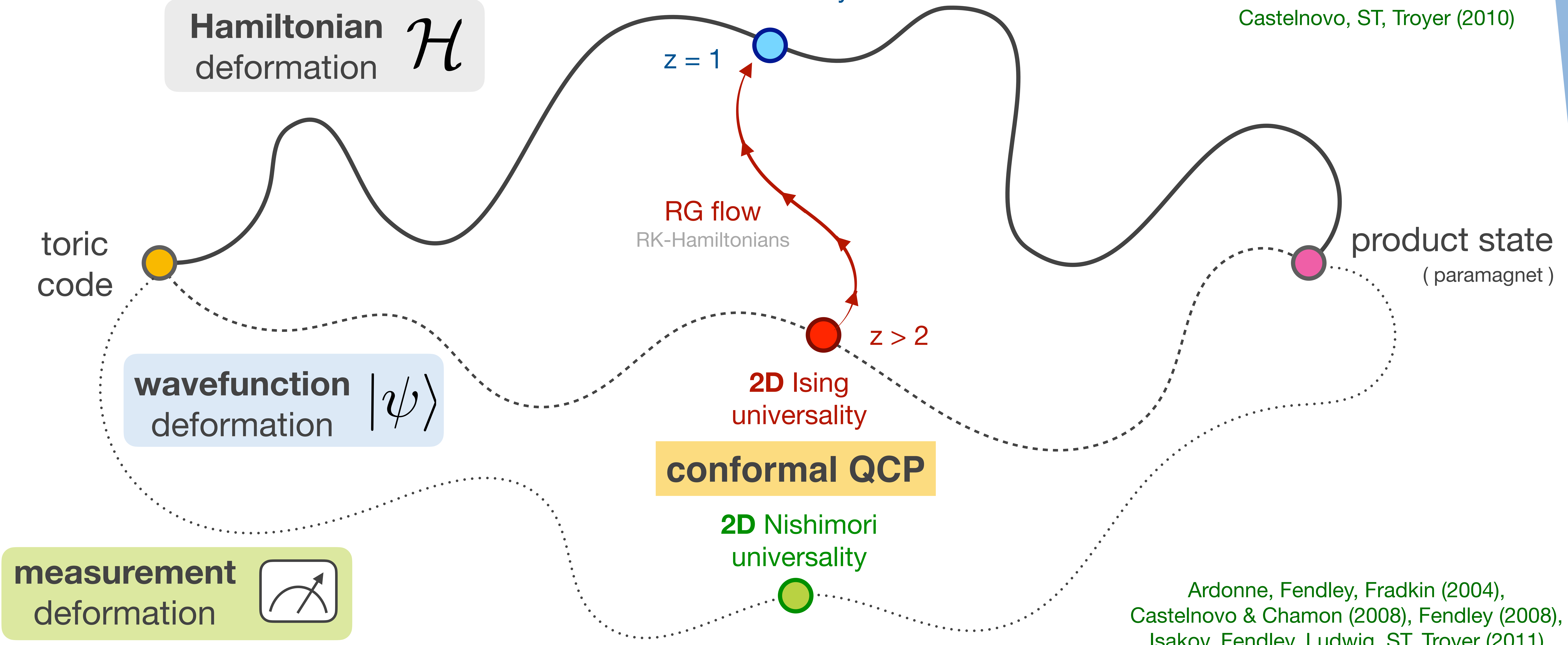
auxiliary qubit

cluster state

The toric/surface code is an ingenious **measurement protocol.**

The toric/surface code is an exactly solvable **Hamiltonian model.**

phase transitions & deformations



3D Ising*
universality

Castelnovo, ST, Troyer (2010)

Hamiltonian deformation $\mathcal{H}$

z = 1

RG flow

RK-Hamiltonians

toric code

product state
(paramagnet)

z > 2

wavefunction deformation $|\psi\rangle$

2D Ising
universality

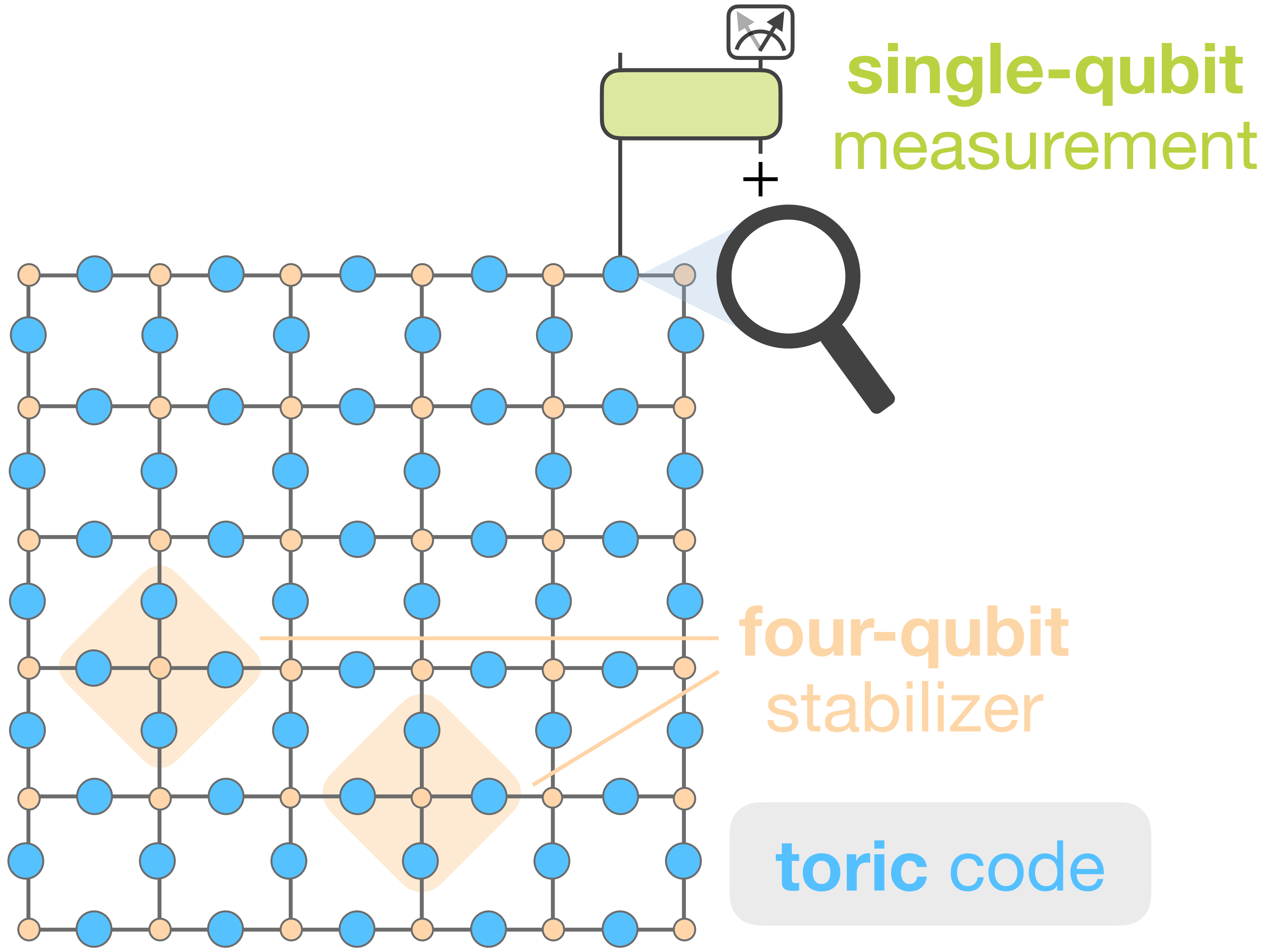
conformal QCP

2D Nishimori
universality

measurement deformation

Ardonne, Fendley, Fradkin (2004),
Castelnovo & Chamon (2008), Fendley (2008),
Isakov, Fendley, Ludwig, ST, Troyer (2011)

toric code under **weak measurement**

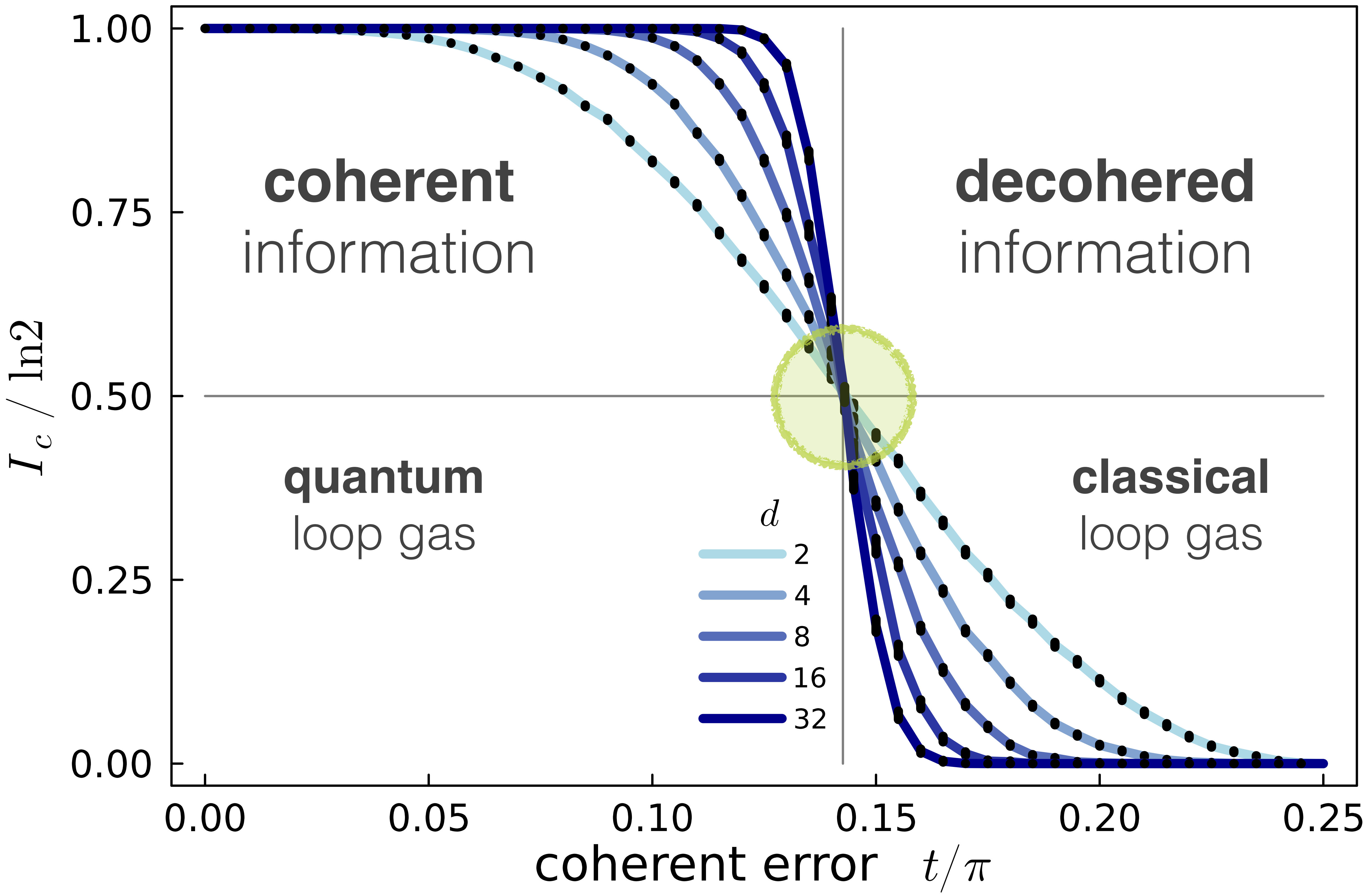


single-qubit measurement

four-qubit stabilizer

toric code

$$|\psi\rangle \mapsto \begin{cases} \left(1+\tanh\frac{\beta}{2}Z_{ij}\right)|\psi\rangle \\ \left(1-\tanh\frac{\beta}{2}Z_{ij}\right)|\psi\rangle \end{cases}$$



Nishimori transition

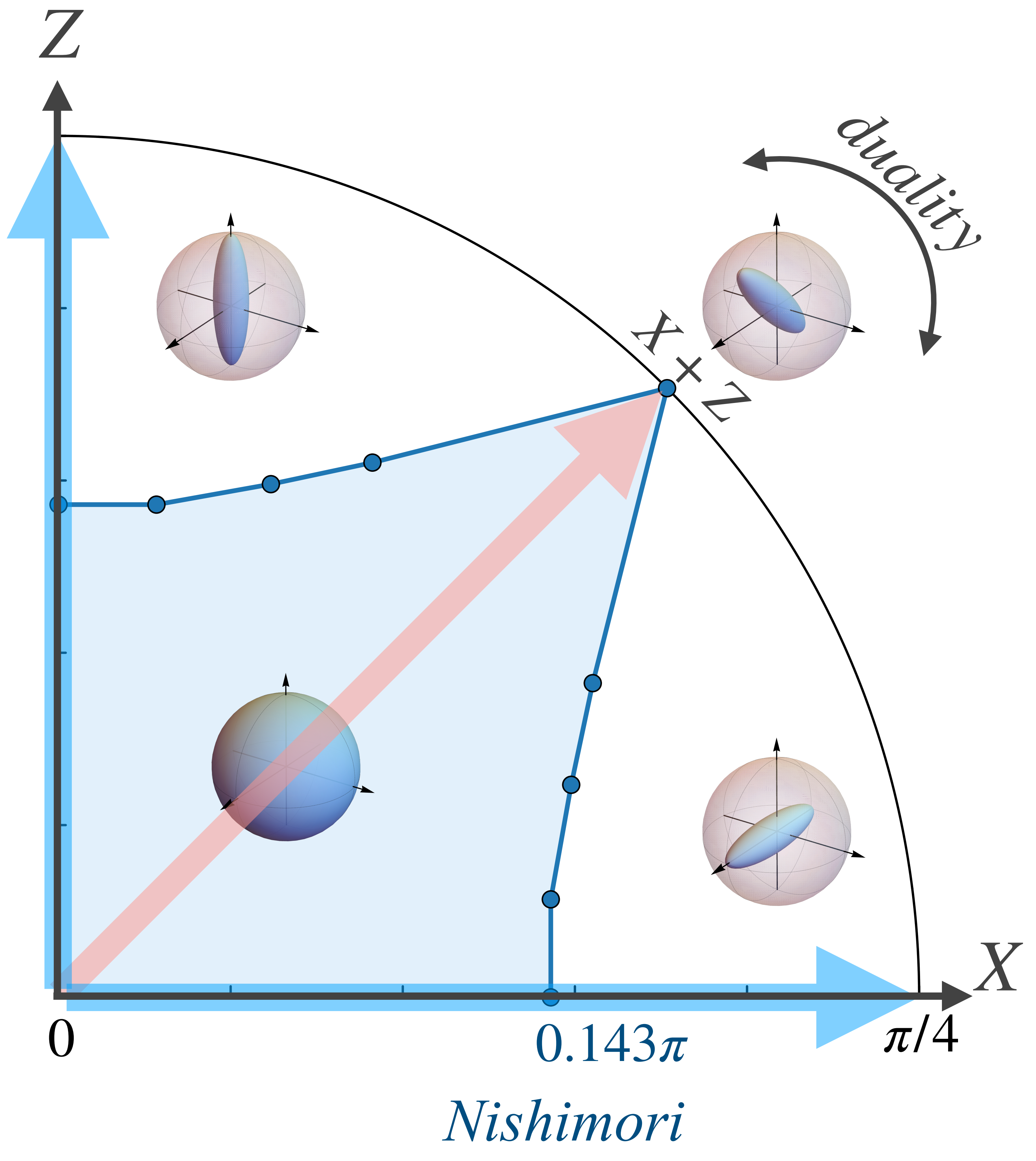
F. Eckstein, B. Han, ST, G.Y. Zhu
PRX Quantum **5**, 040313 (2024)

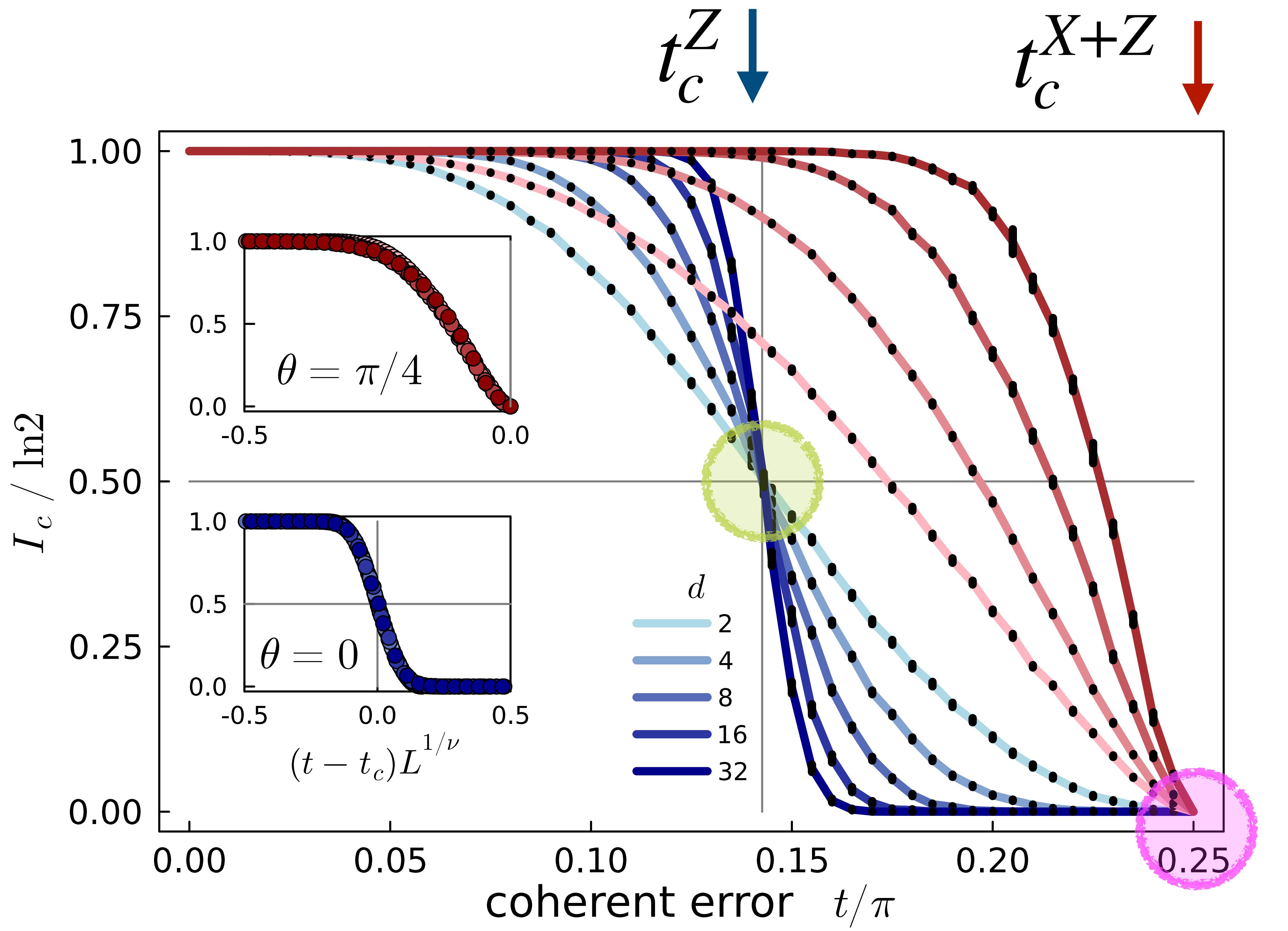
toric code under **weak measurement**

$$\Pi_{ij}[1 \pm \tanh\frac{\beta}{2}\,(\boxed{\cos\theta}\, Z_{ij} + \boxed{\sin\theta}\, X_{ij})]|\psi\rangle$$

measurement **angle**

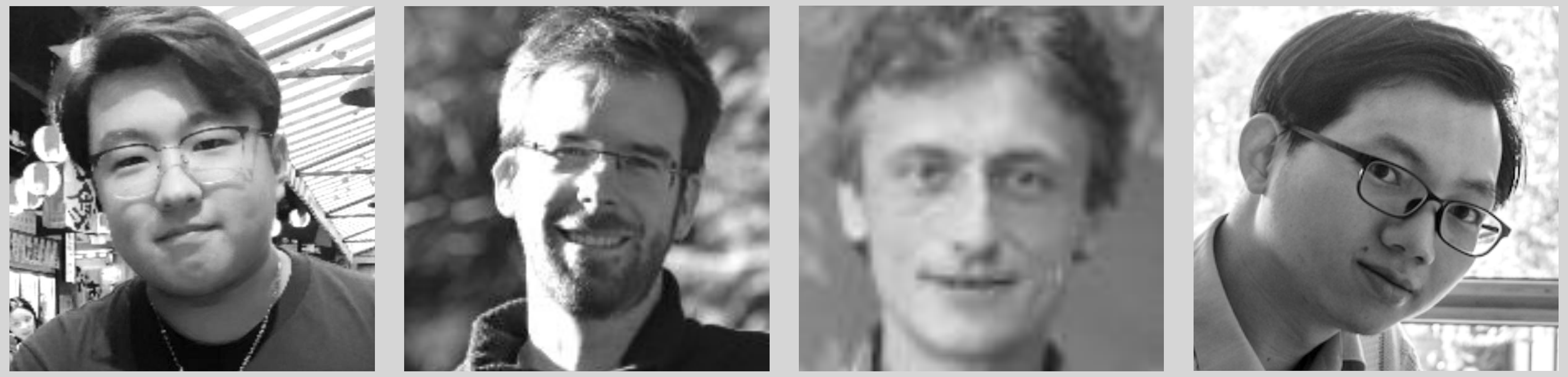
Self-duality makes the toric code absolutely **stable** against decoherence.





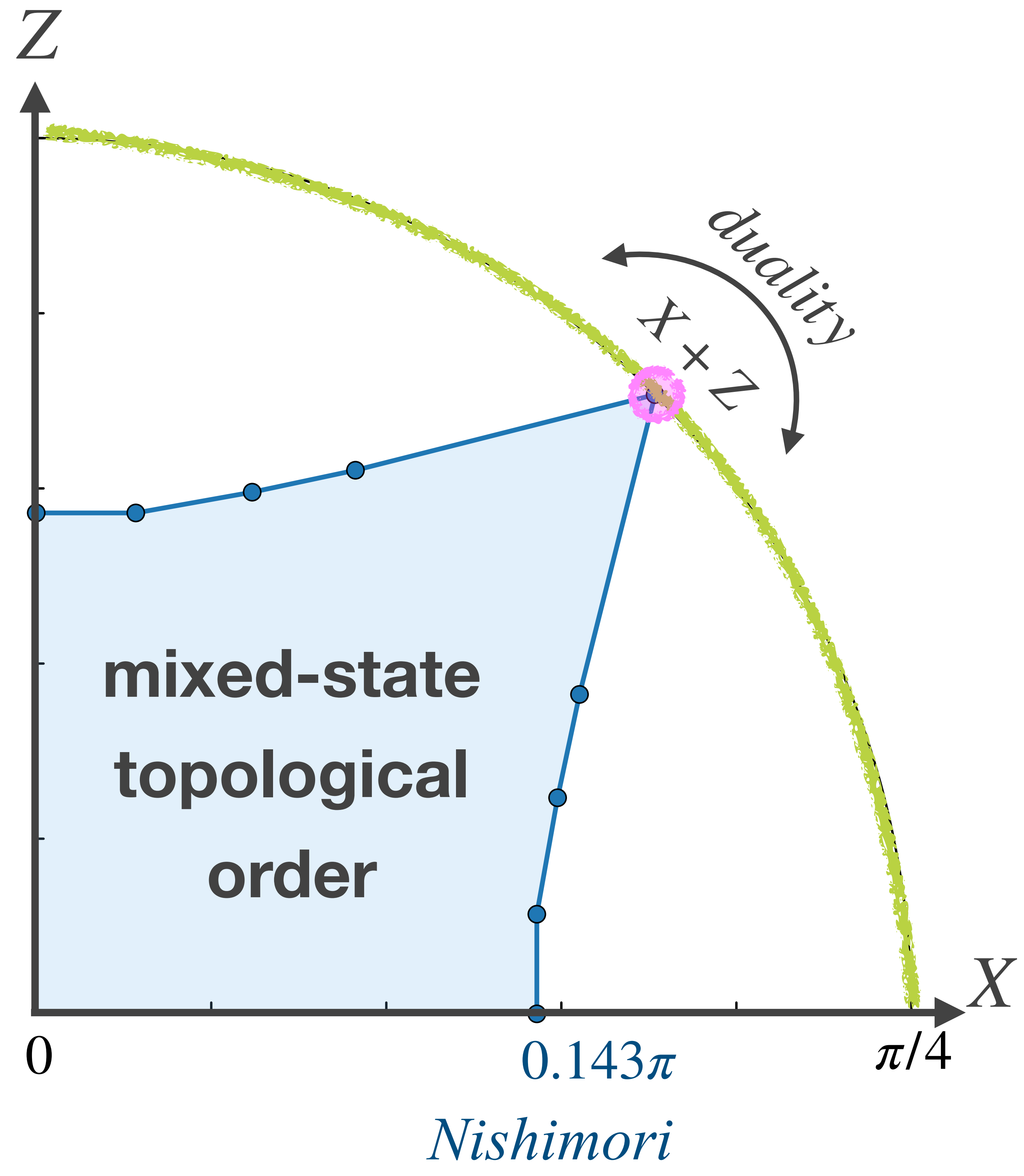
$t_c = 0.14\pi$ $t_c = 0.25\pi$

toric code & **strong measurement**

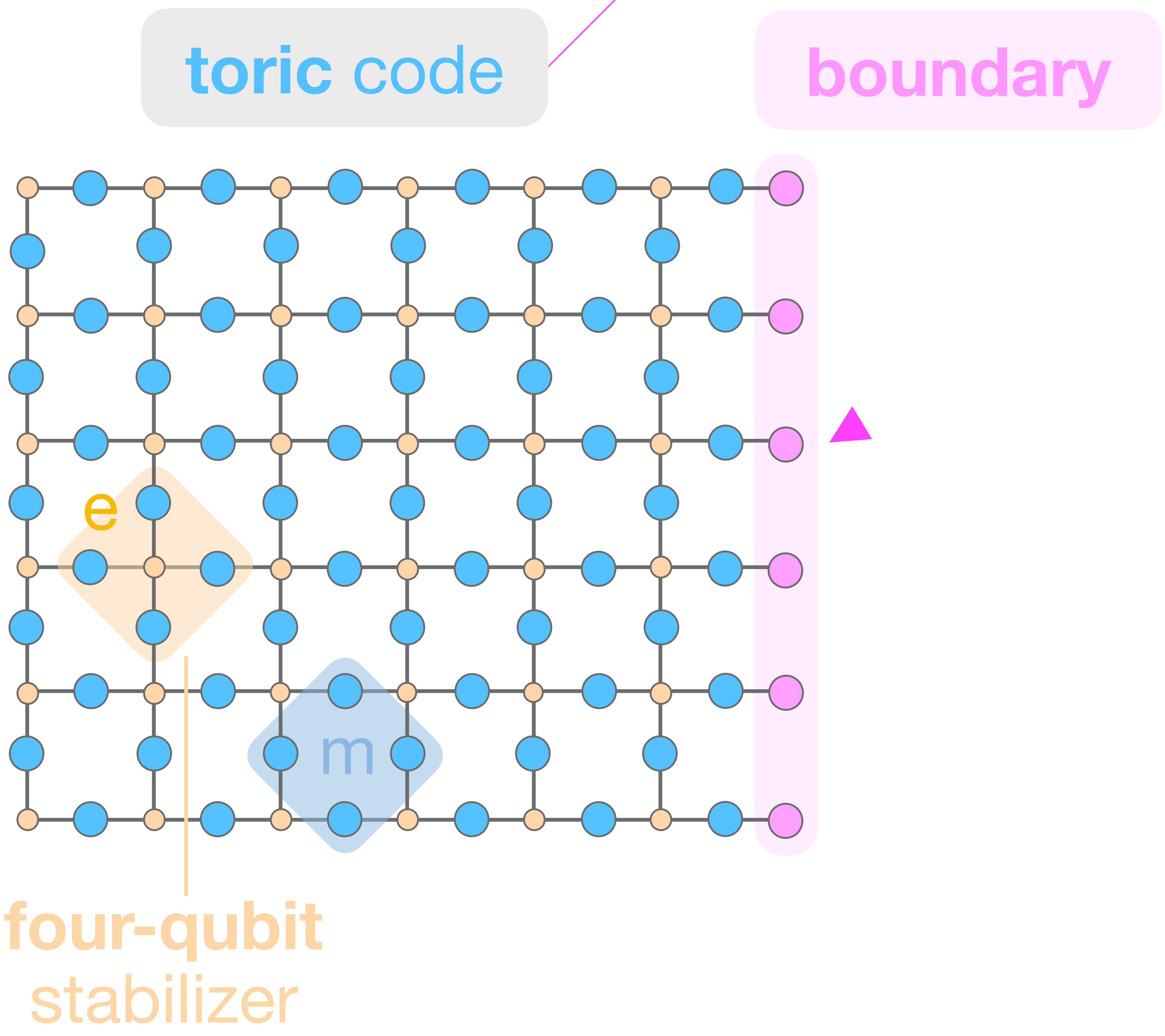


$$\Pi_{ij}[1 \pm \tanh\frac{\beta}{2}\,(\cos\theta\, Z_{ij} + \sin\theta\, X_{ij})]|\psi\rangle$$

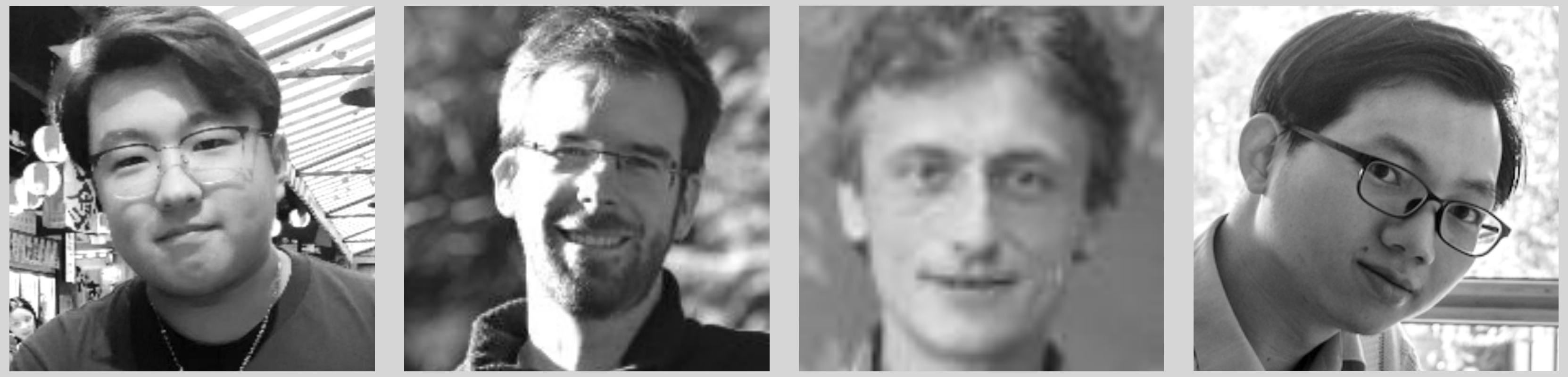
measurement **angle**



Why is there a **critical point** when all the bulk qubits **collapse**?

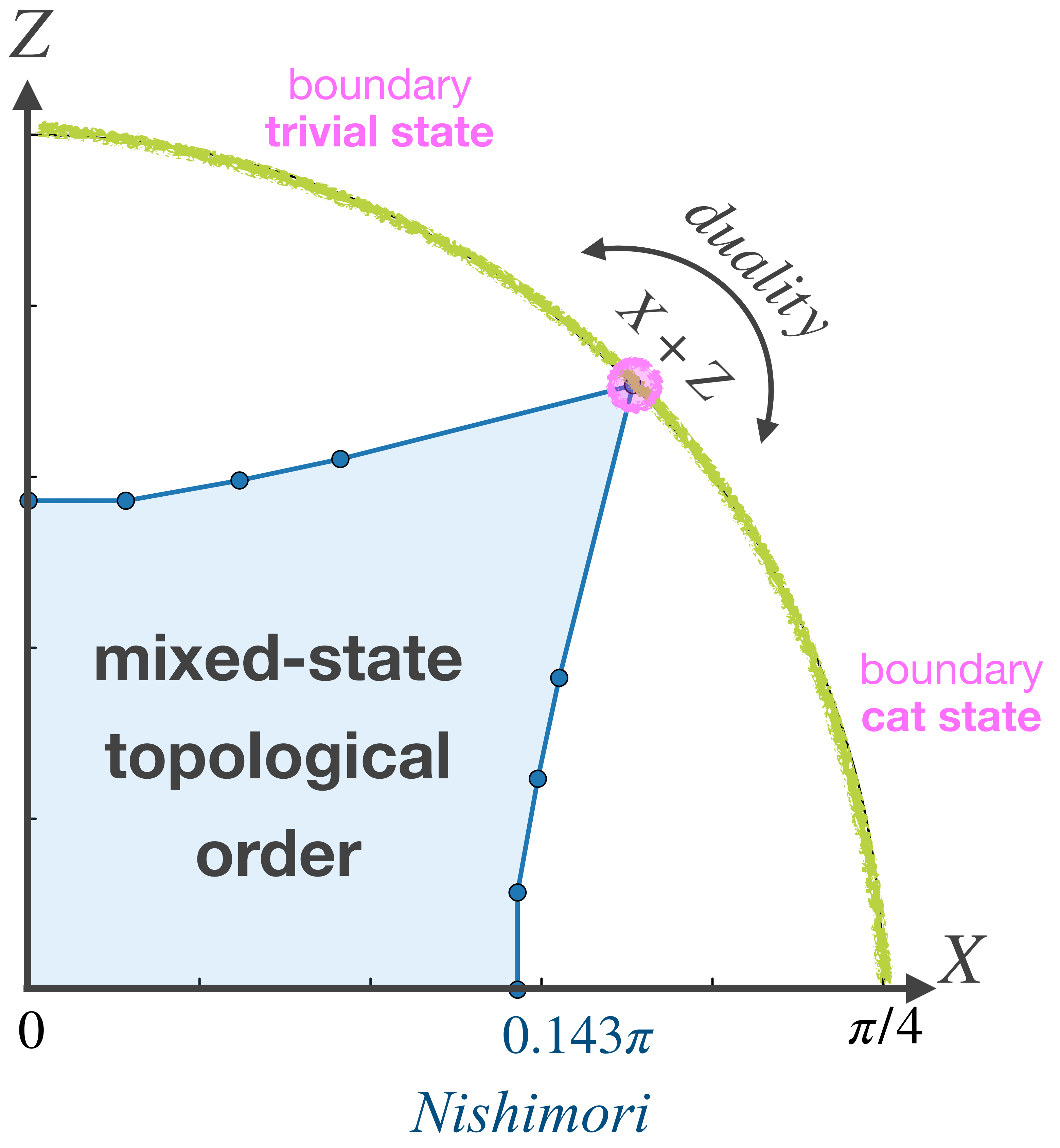


bulk-boundary correspondence

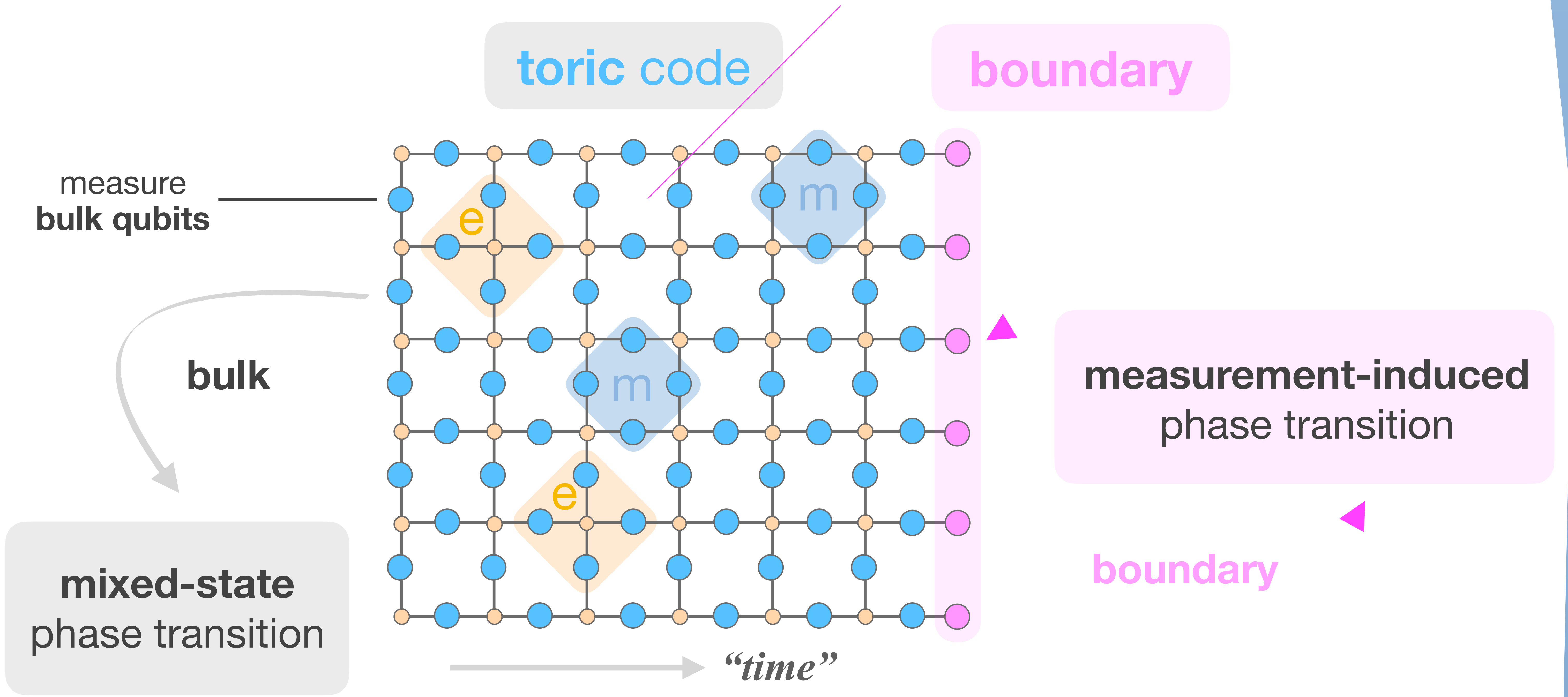


$$\Pi_{ij}[1 \pm \tanh\frac{\beta}{2}(\cos\theta\, Z_{ij} + \sin\theta\, X_{ij})]|\psi\rangle$$

measurement **angle**



Why is there a **critical point** when all the bulk qubits **collapse**?



toric code

boundary

measure **bulk qubits**

bulk

mixed-state phase transition

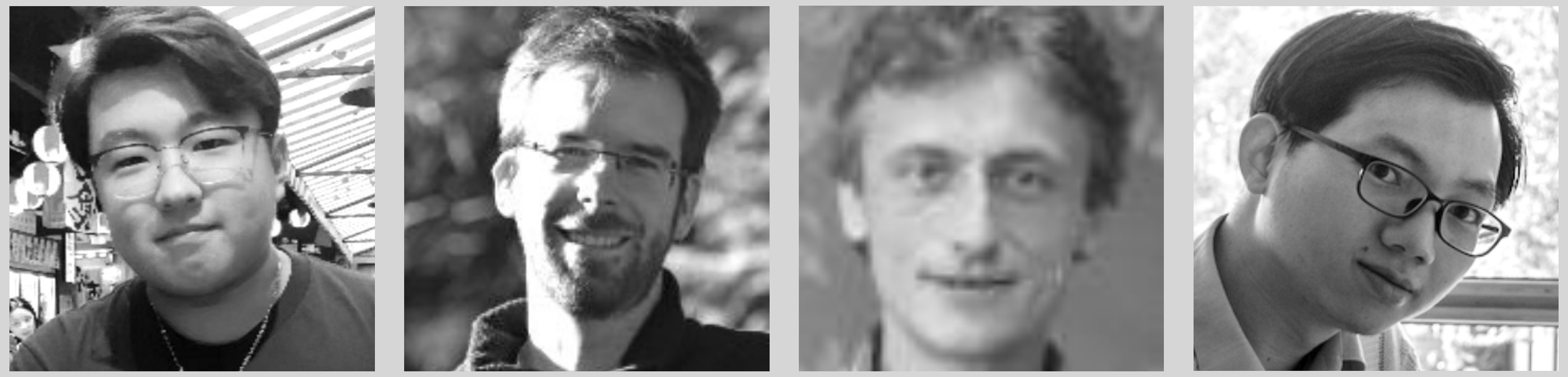
measurement-induced phase transition

boundary

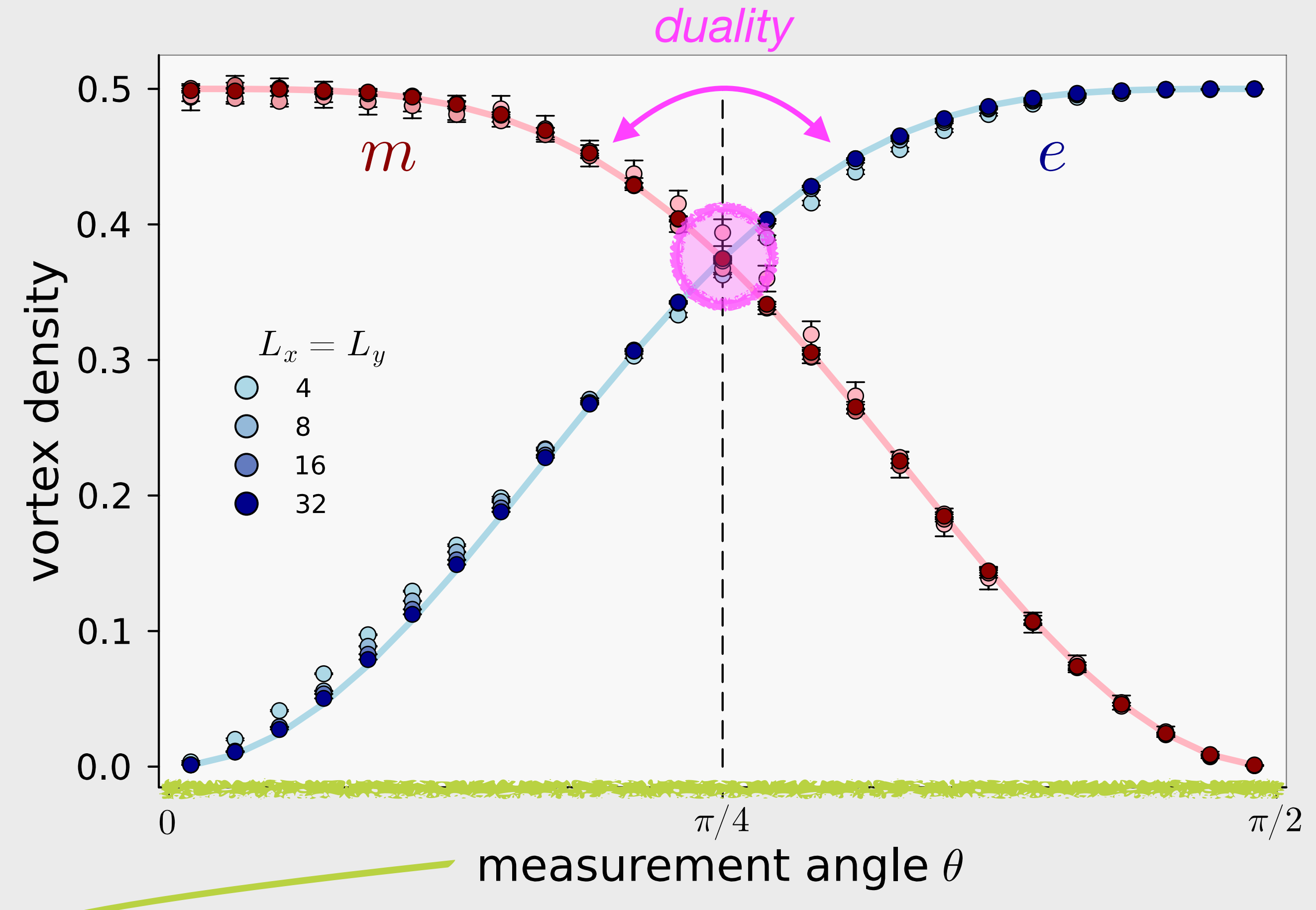
“time”

$$\rho = \sum_{em} \overset{\text{classical}}{|em\rangle\langle em|_C} \otimes \overset{\text{quantum}}{|\psi(em)\rangle\langle\psi(em)|_Q}$$

bulk-boundary correspondence



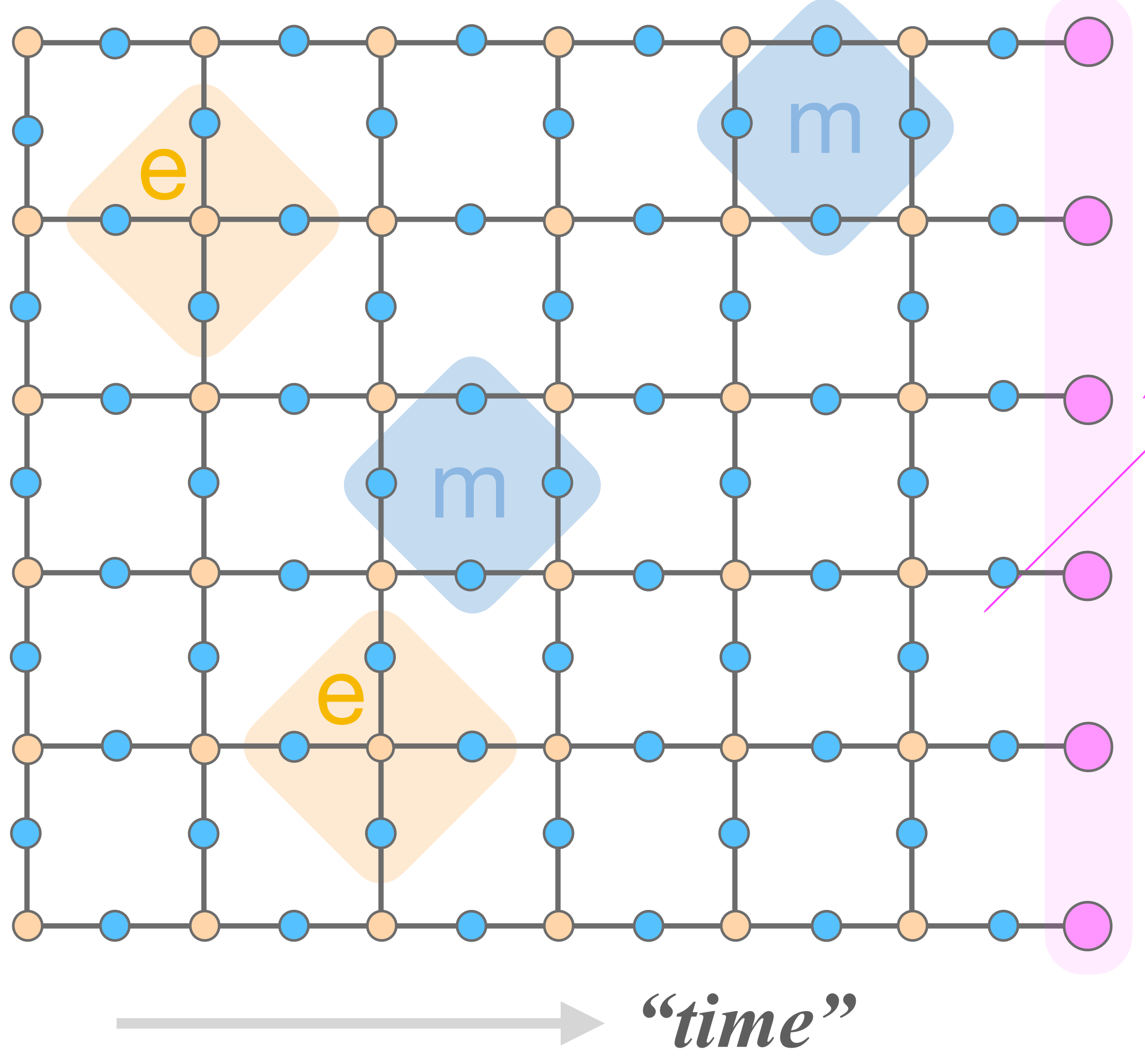
criticality is dictated by **self-duality**



weak Kramers-Wannier duality at self-dual point

toric code

boundary



bulk

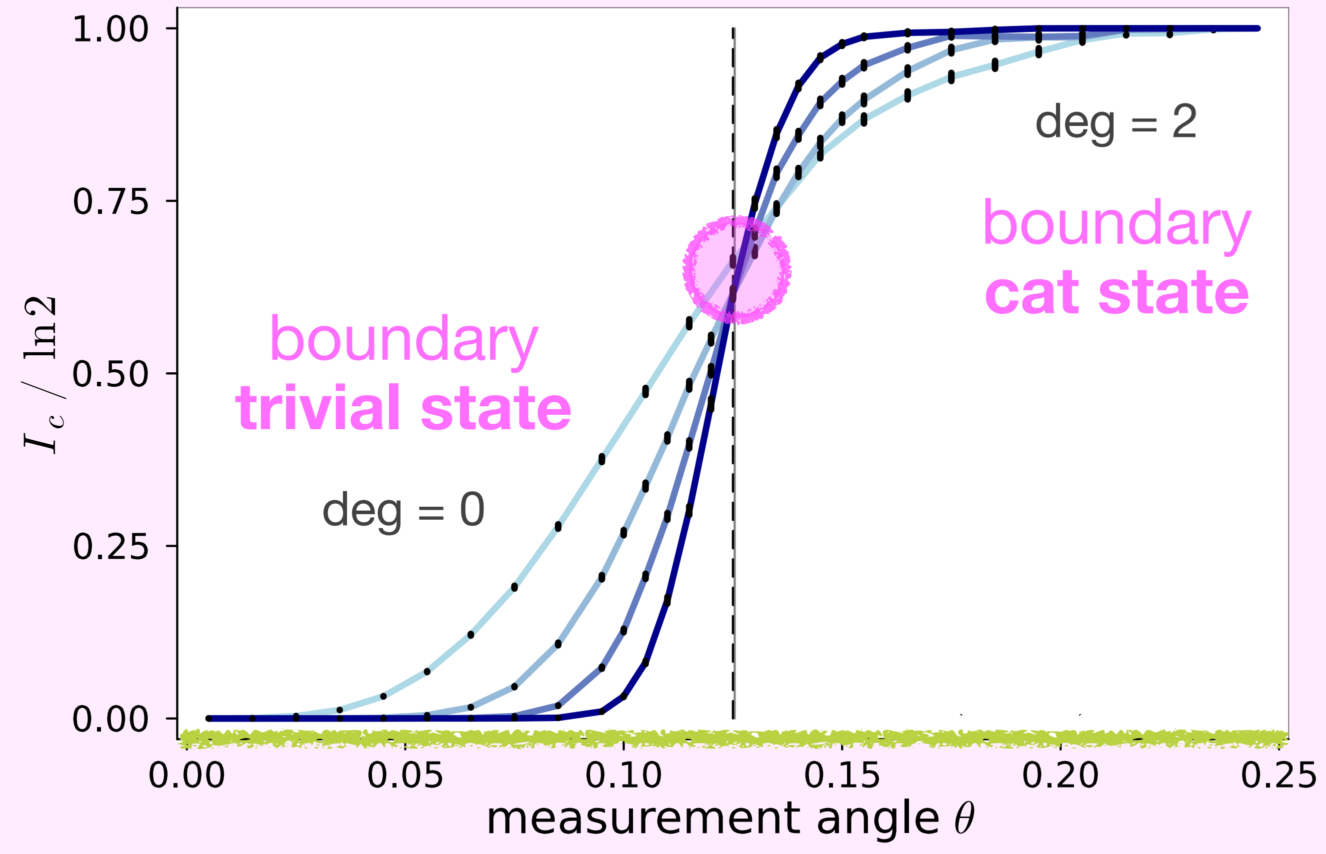
mixed-state phase transition

$$\rho = \sum_{em} \overset{\text{classical}}{|em\rangle\langle em|_C} \otimes \overset{\text{quantum}}{|\psi(em)\rangle\langle\psi(em)|_Q}$$

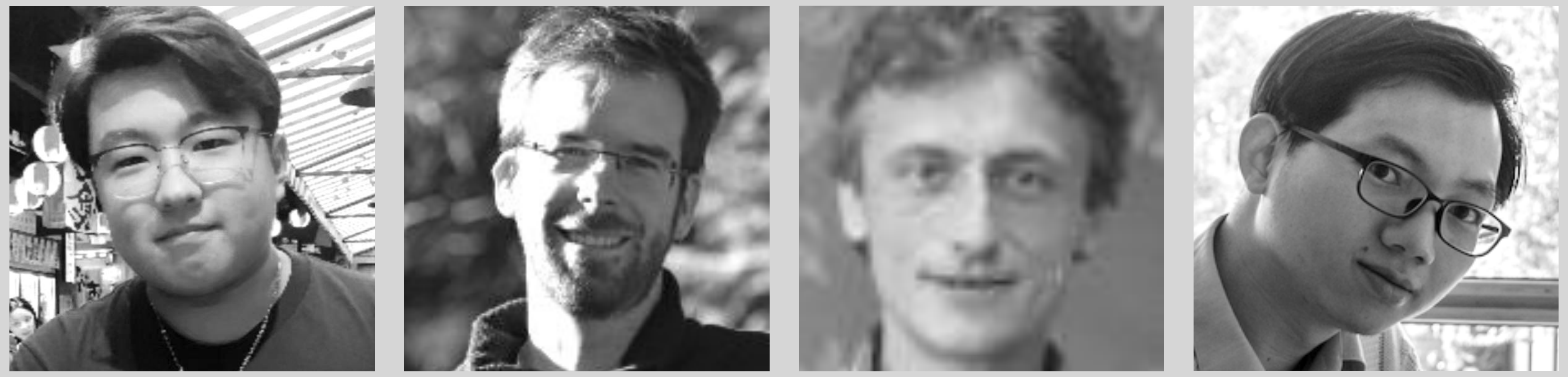
measurement-induced phase transition

boundary

coherent information evaluates the size of the **degenerate space** of the boundary state

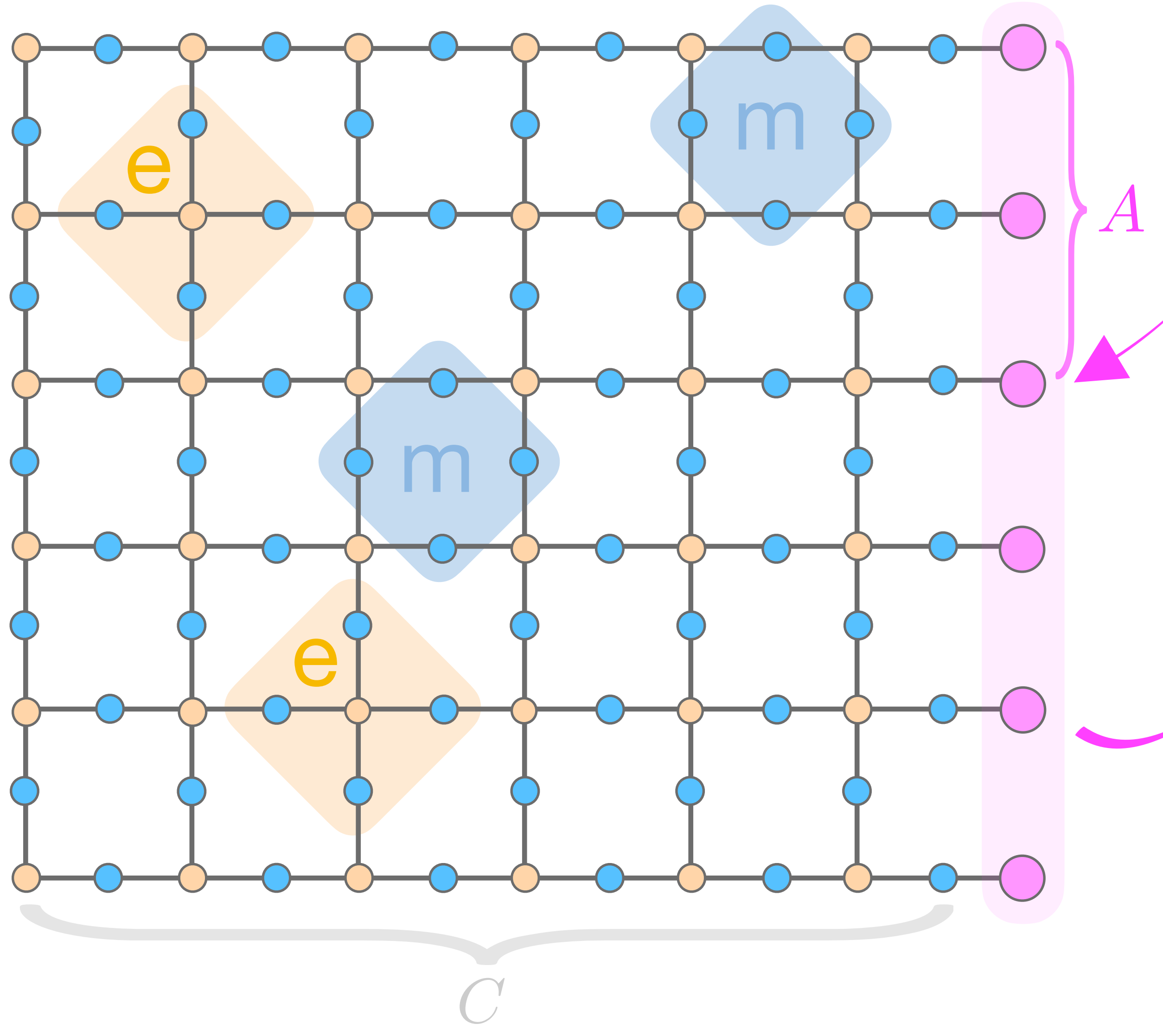


bulk-boundary correspondence



toric code

boundary



bulk

measurement-induced phase transition

entanglement entropy of critical mixed state

mixed-state phase transition

boundary

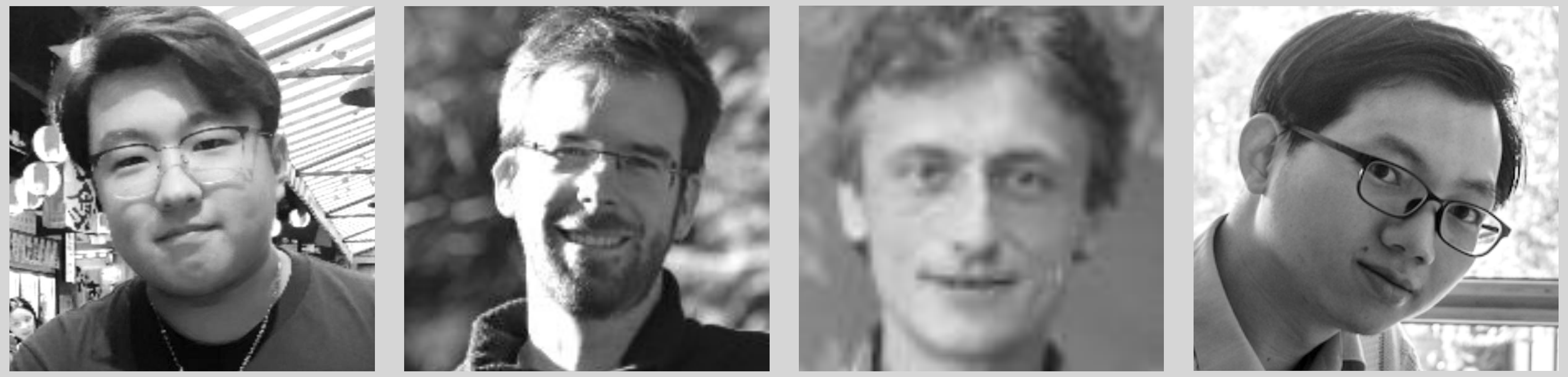
Shannon entropy of meas. record

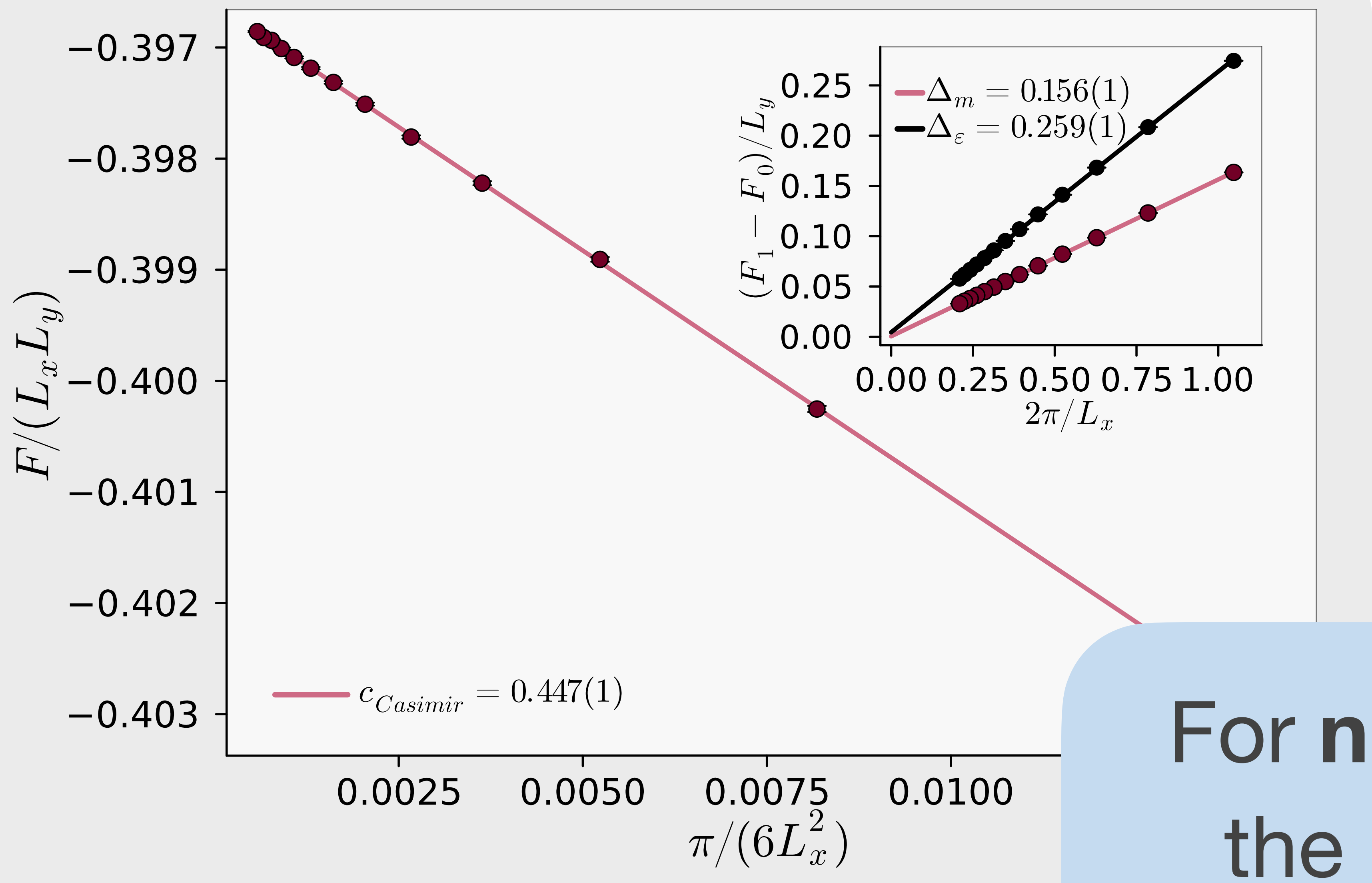
$$S_{AC} = S_C + S_{A|C}$$

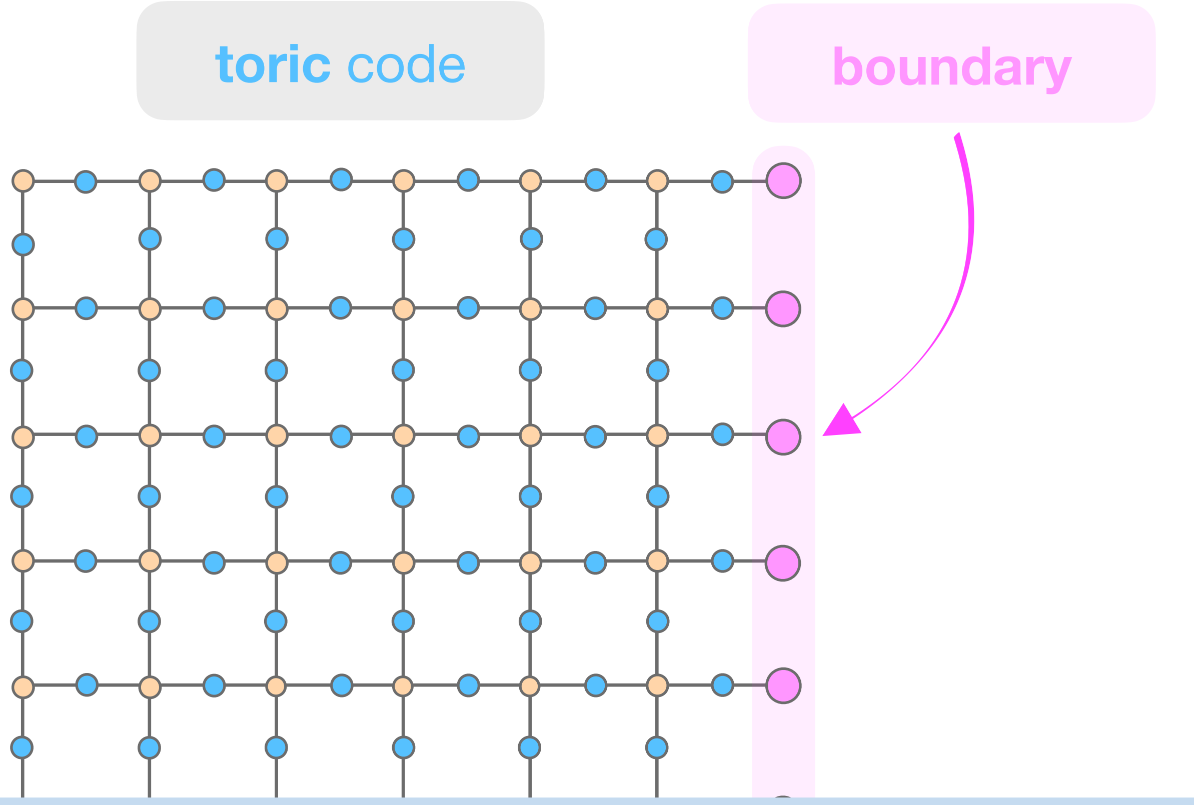
$$= -\frac{\pi c_{\text{Casimir}} L_y}{6L_x} + \frac{c_{\text{ent}}}{3}\ln\left(\frac{L_x}{\pi}\sin\frac{\pi l}{L_x}\right) + \dots$$

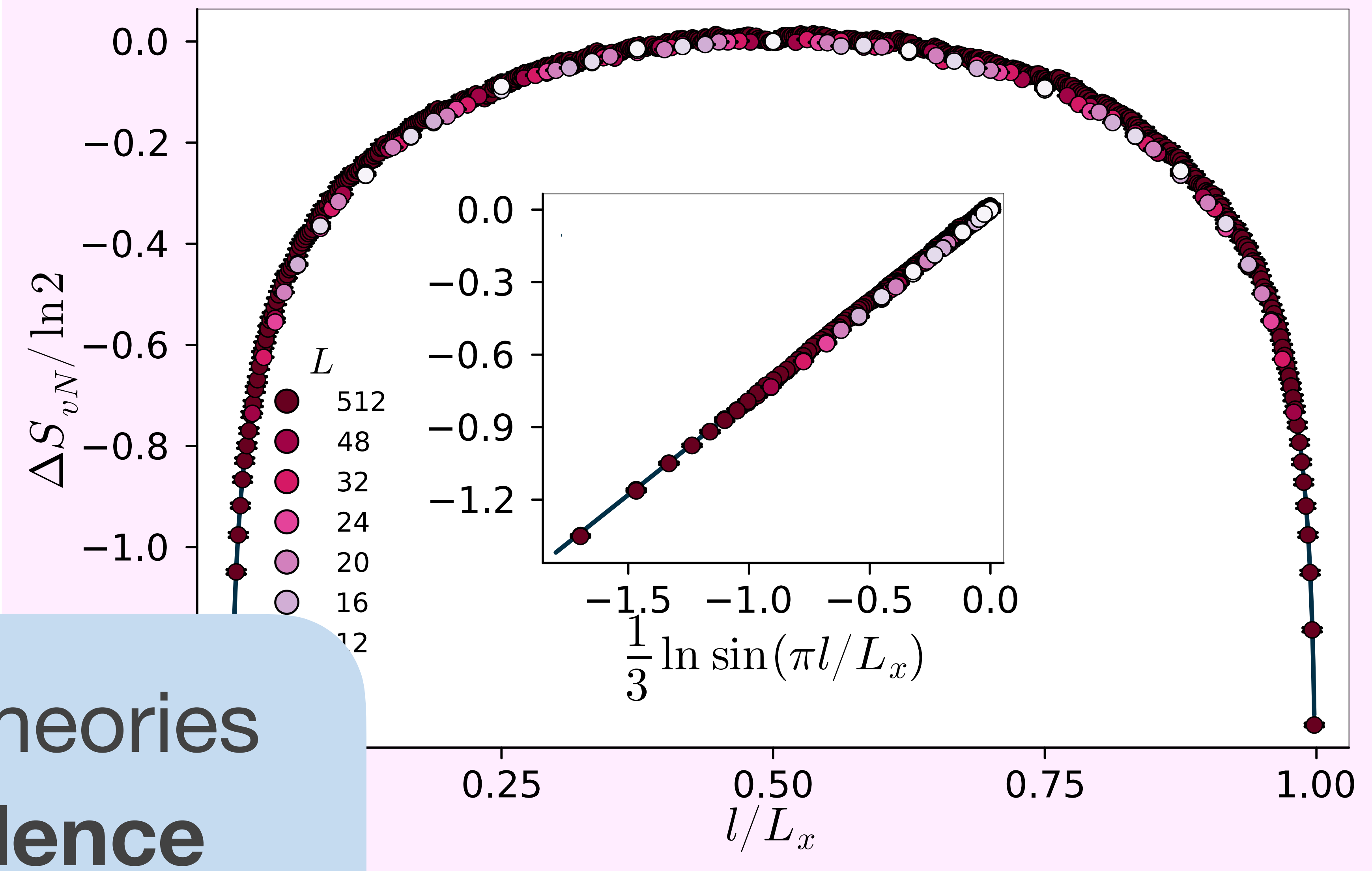
entanglement entropy of boundary state

self-dual criticality









For **non-unitary** conformal field theories the **bulk-boundary correspondence** generally allows c_{Casimir} and c_{ent} to differ.

$c_{\text{Casimir}} = 0.447(1)$

$c_{\text{ent}}^{\text{vN}} = 0.795(1)$

Shannon entropy
of meas. record

entanglement entropy
of boundary state

$$S_{AC} = S_C + S_{A|C}$$

$$= -\frac{\pi c_{\text{Casimir}} L_y}{6L_x} + \frac{c_{\text{ent}}}{3}\ln\left(\frac{L_x}{\pi}\sin\frac{\pi l}{L_x}\right) + \dots$$

universality classes

Ising

no disorder

$\mathbb{Z}_2$ **SSB**

spontaneous
symmetry breaking

$\mathrm{KW}\rho \propto \rho$

Kramers-Wannier
duality

Nishimori

Born-rule disorder
(1-replica limit)

$\mathbb{Z}_2$ **SW-SSB**

strong-to-weak spontaneous
symmetry breaking

broken KW
duality

$c_{\mathrm{Casimir}} = 0.464(4)$ $c_{\mathrm{ent}}^{\mathrm{vN}} = 0.41956(3)$

weak self-dual

Born-rule disorder
(1-replica limit)

$\mathbb{Z}_2$ **SW-SSB**

strong-to-weak spontaneous
symmetry breaking

weak KW
duality

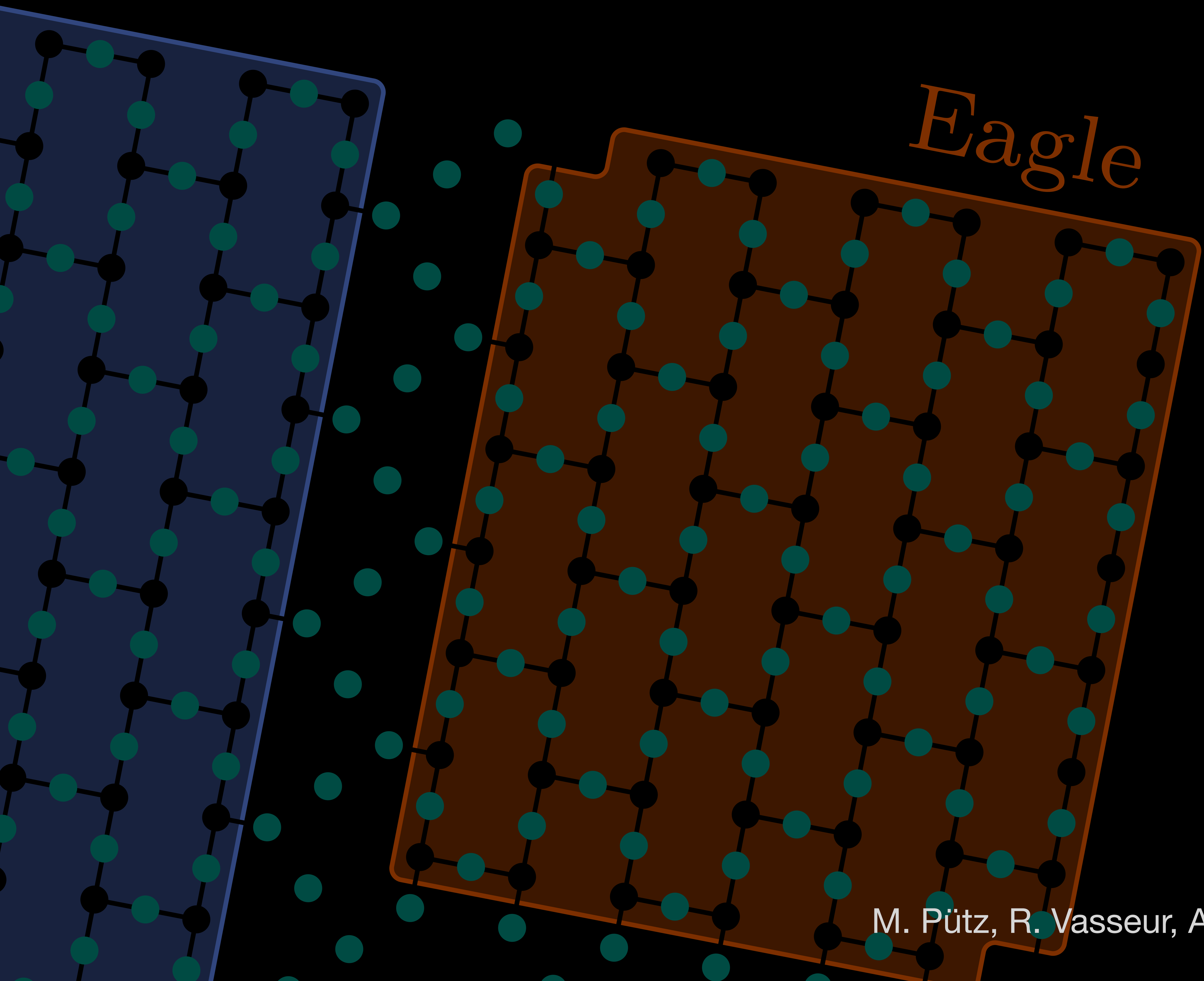
$\mathrm{KW}\,\rho\,\mathrm{KW} = \rho$

$c_{\mathrm{Casimir}} = 0.447(1)$ $c_{\mathrm{ent}}^{\mathrm{vN}} = 0.795(1)$

universality classes

Ising	**Nishimori**	**weak self-dual**	**tricritical**
no disorder	Born-rule disorder (1-replica limit)	Born-rule disorder (1-replica limit)	Born-rule disorder (1-replica limit)
Z_2 **SSB** spontaneous symmetry breaking	Z_2 **SW-SSB** strong-to-weak spontaneous symmetry breaking	Z_2 **SW-SSB** strong-to-weak spontaneous symmetry breaking	**strong/weak/broken** Z_2 symmetry phases meet
Kramers-Wannier duality	**broken KW** duality	**weak KW** duality	learning transitions

arXiv:2504.12385



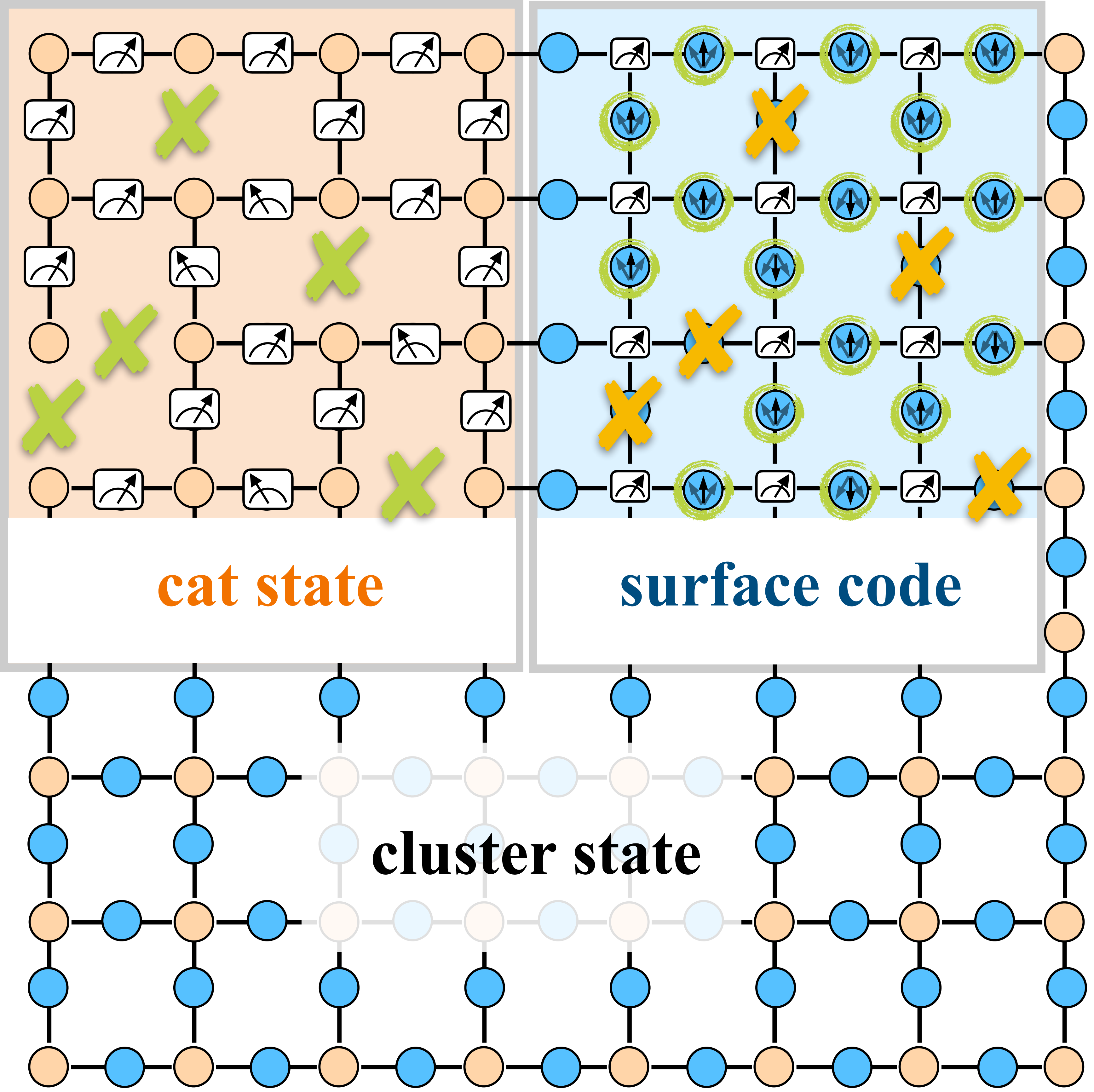
qubit loss

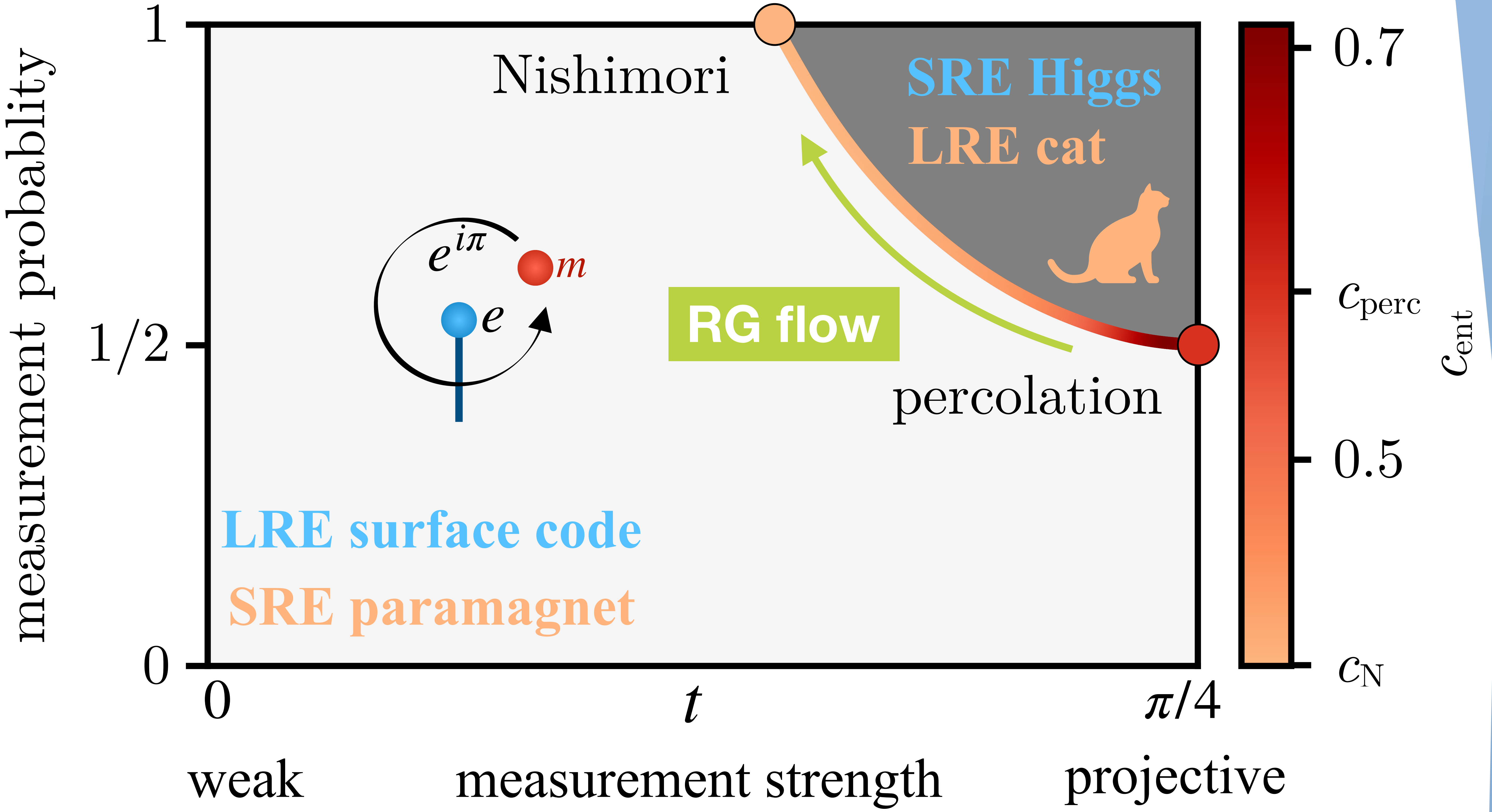
arXiv:2505.22720

M. Pütz, R. Vasseur, A. Ludwig, G-Y. Zhu

Nishimori / percolation & **RG flows**

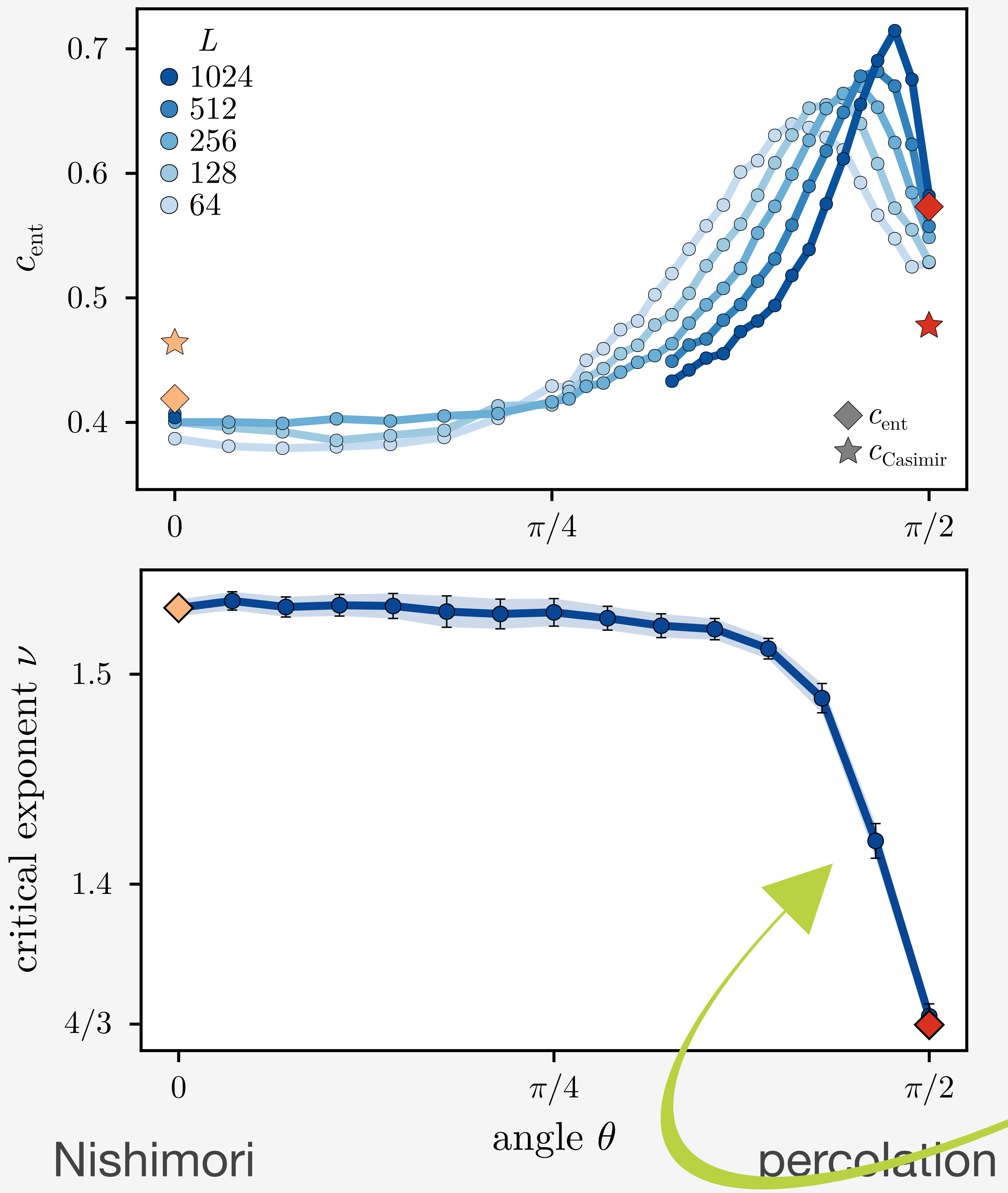
qubit loss

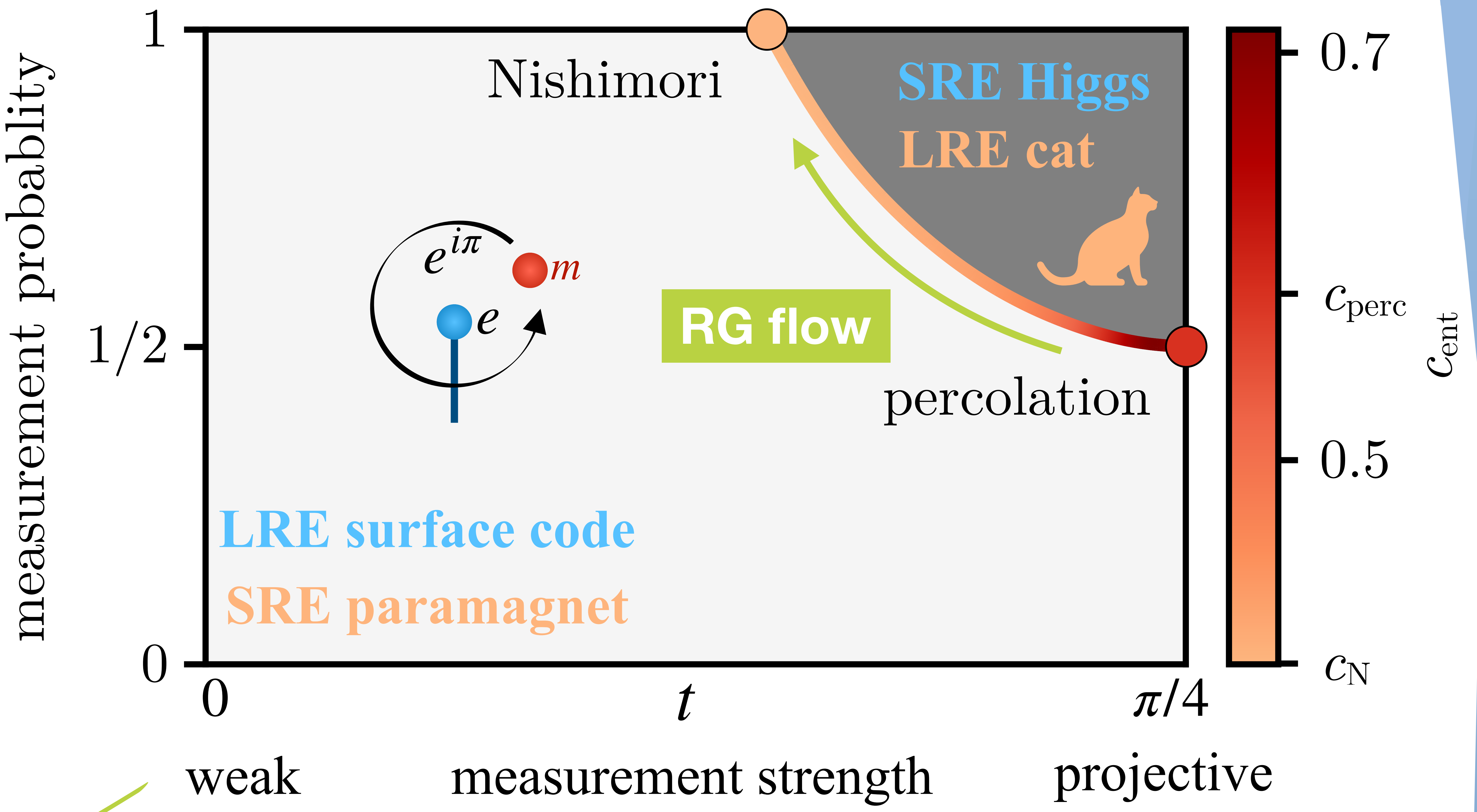




RG flow from percolation
to Nishimori criticality

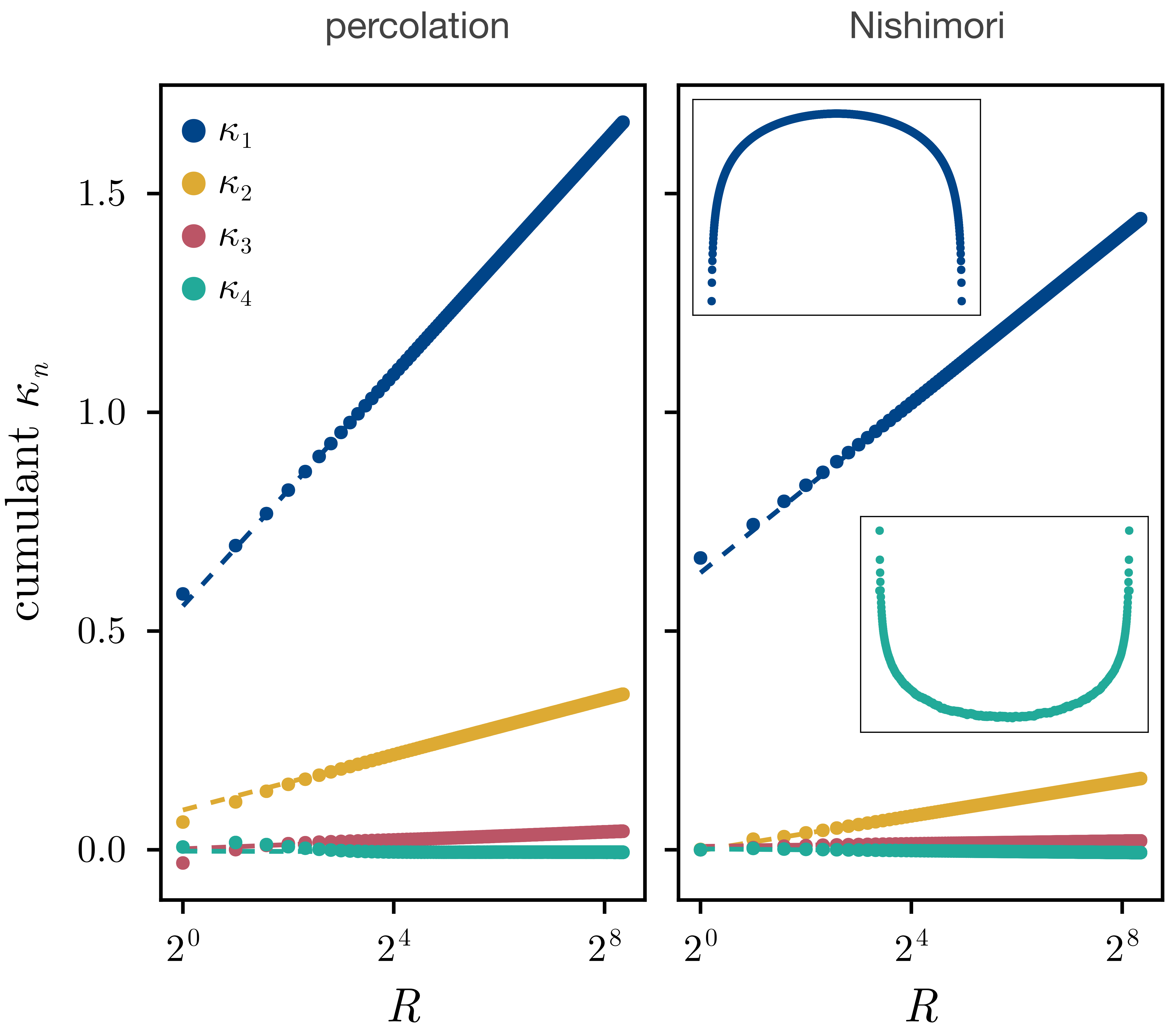
Nishimori / percolation & **RG flows**



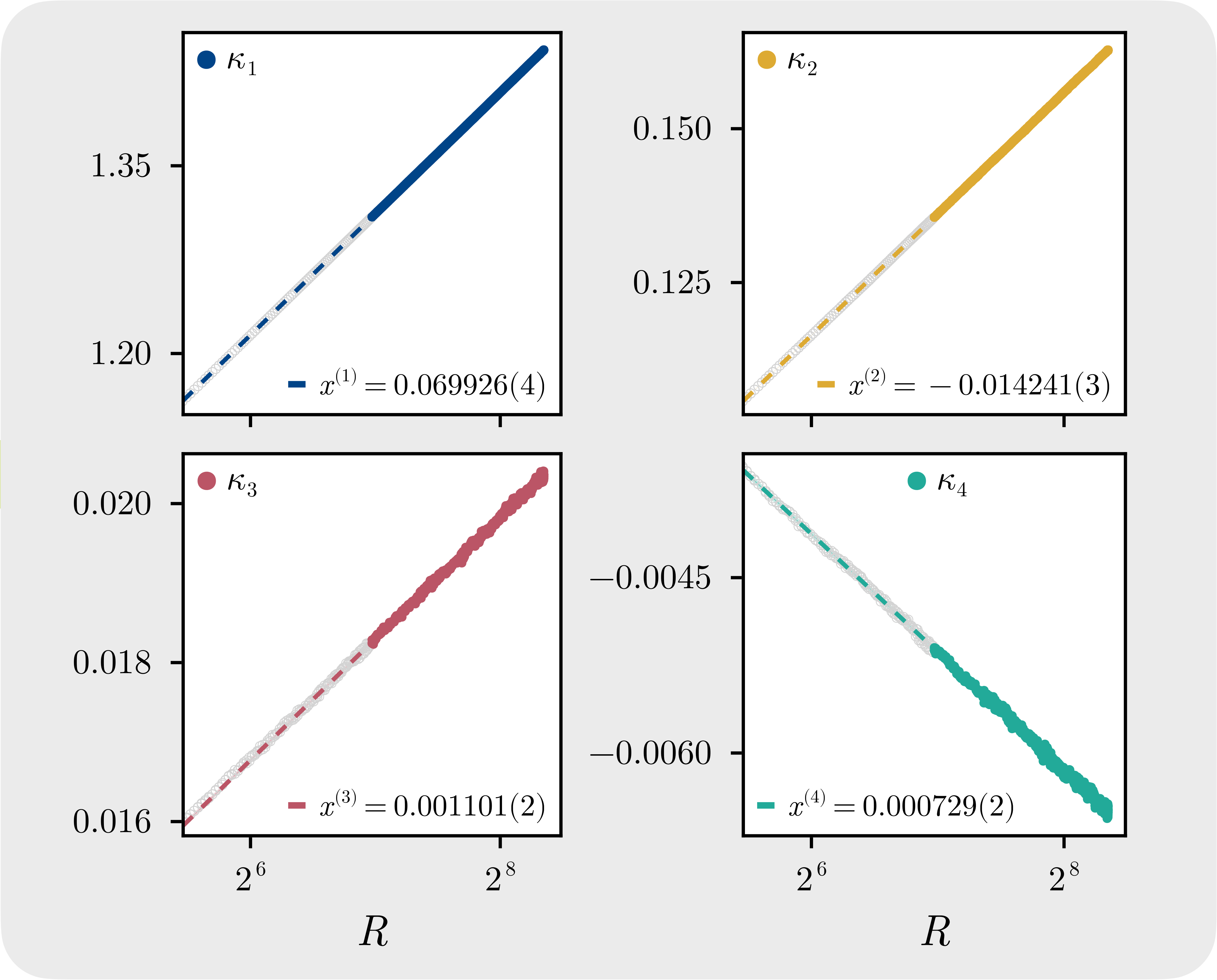


RG flow from percolation
to Nishimori criticality

Nishimori / percolation & **multifractality**



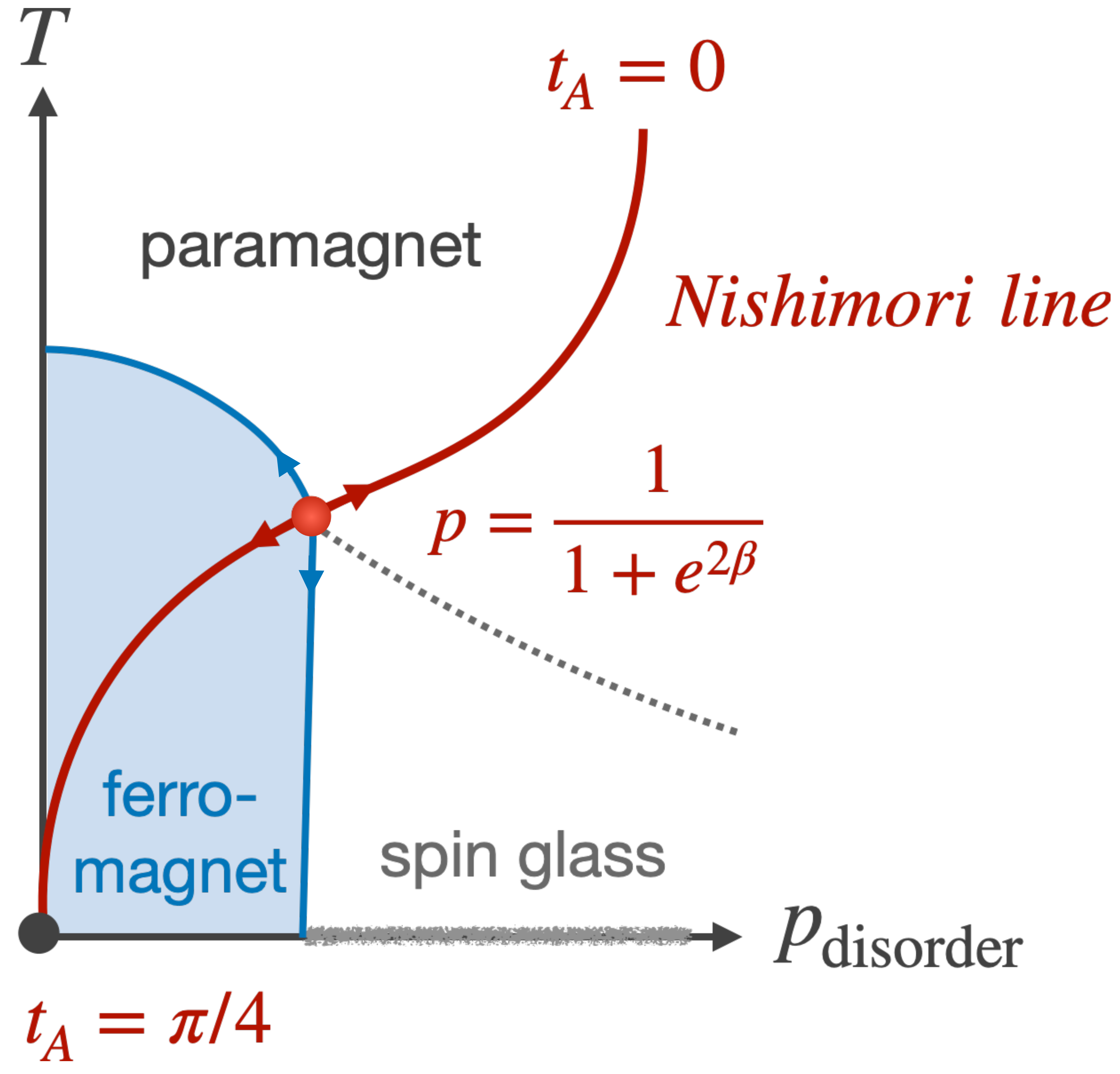
Both percolation and Nishimori criticality exhibit **multifractal spectra** of scaling dimensions.



perspectives

Nishimori physics

- a staple of classical **statistical physics**
- but **ubiquitous in quantum** physics
  - enforced by **Born's rule**
  - induced by **coherent** and **incoherent** errors
  - **RG flow** form percolation, self-dual, tricriticality
  - an emerging **fixed point universality class** for **non-unitary** conformal QCPs?



theory – Phys. Rev. Lett. **131**, 200201 (2023)
experiment (IBM) – Nature Physics **21**, 161 (2025)

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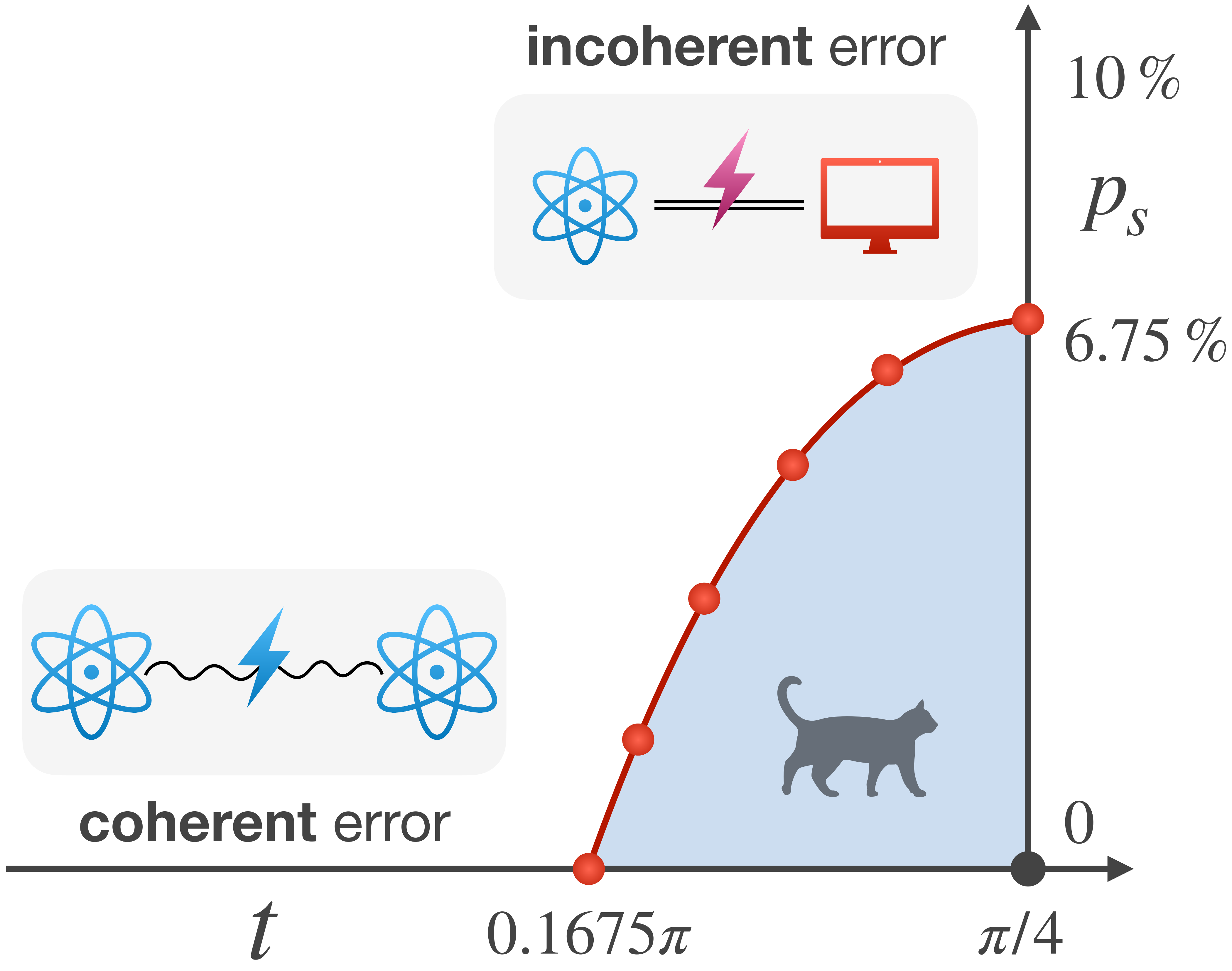

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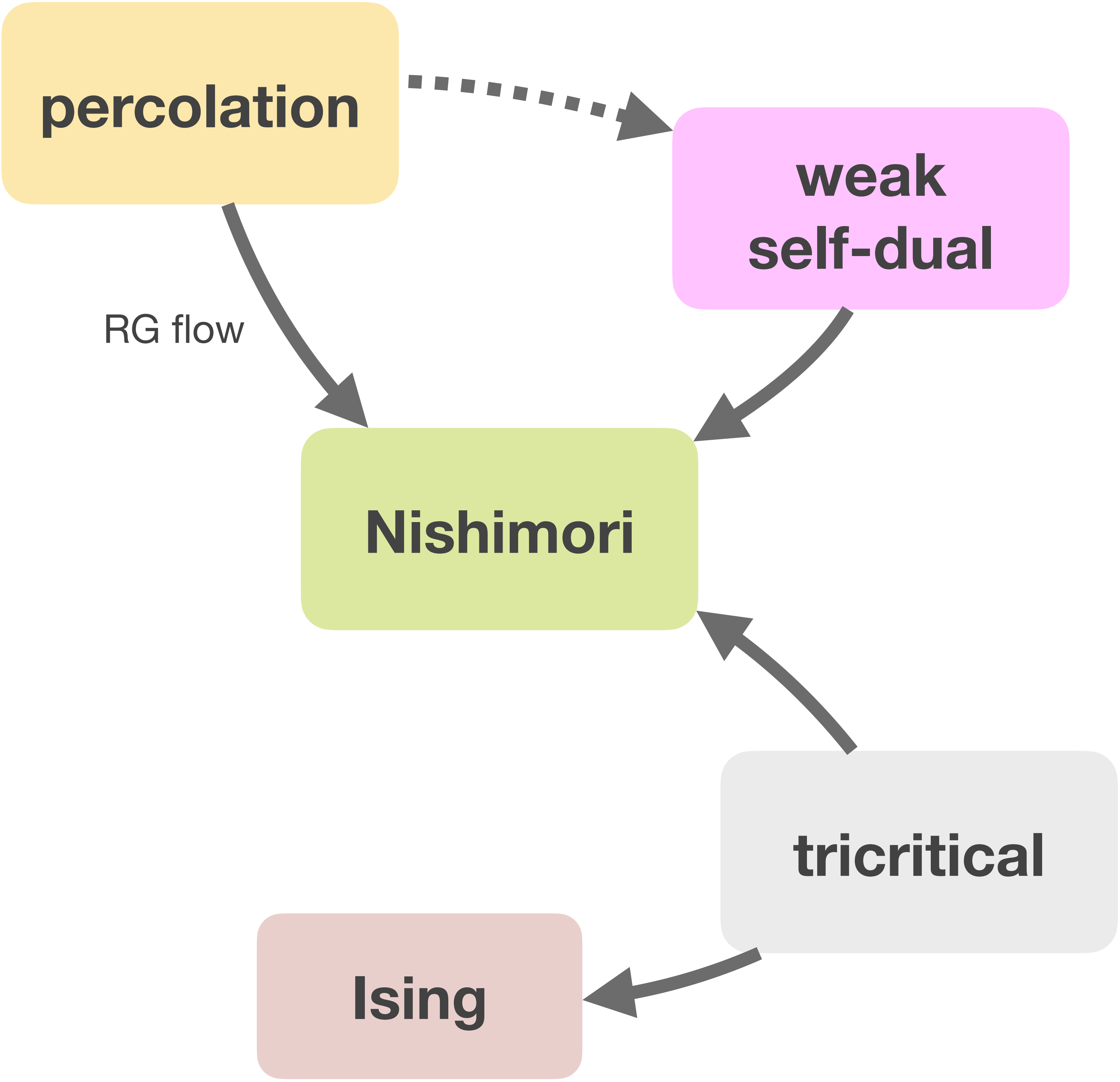

theory – Phys. Rev. Lett. **131**, 200201 (2023)
experiment (IBM) – Nature Physics **21**, 161 (2025)

percolation – arXiv:2505.22720
self-dual – arXiv:2502.14034, PRX Quantum 5, 040313 (2024)
tricritical – arXiv:2504.12385

Thanks!