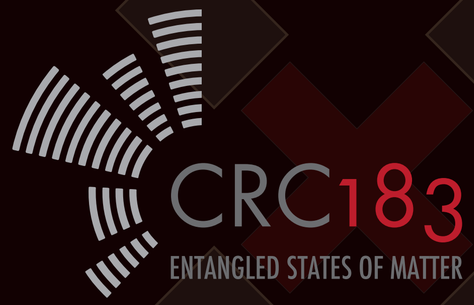
Monitored Kitaev Spin Liquids

Spin Liquids from Measurement-Only Quantum Circuits

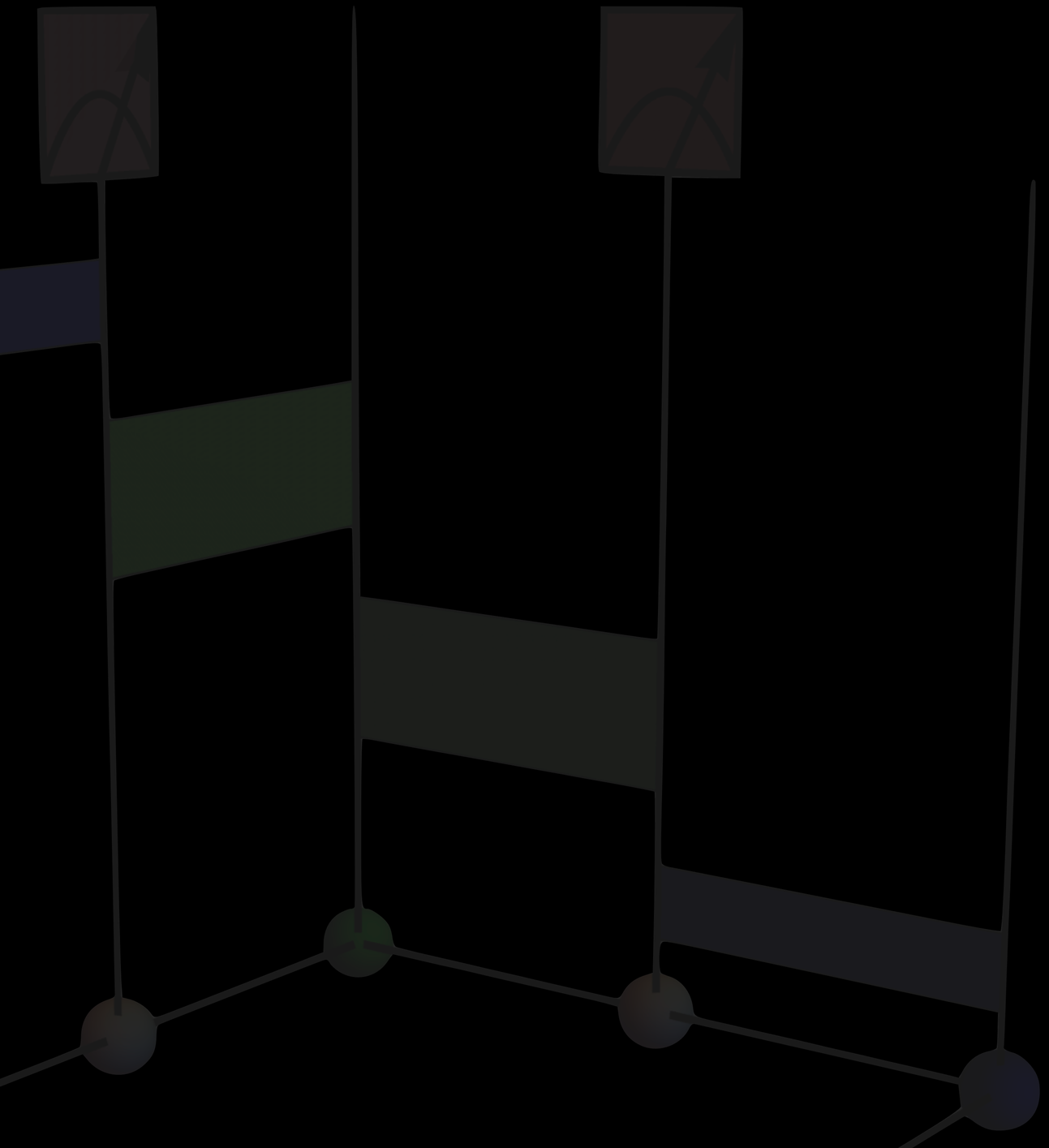


Simon Trebst

University of Cologne

Dynamical Response and Transport in Quantum Magnets

Kavli Institute for Theoretical Physics, August 2023



monitored
dynamics

monitored dynamics

Hamiltonian dynamics

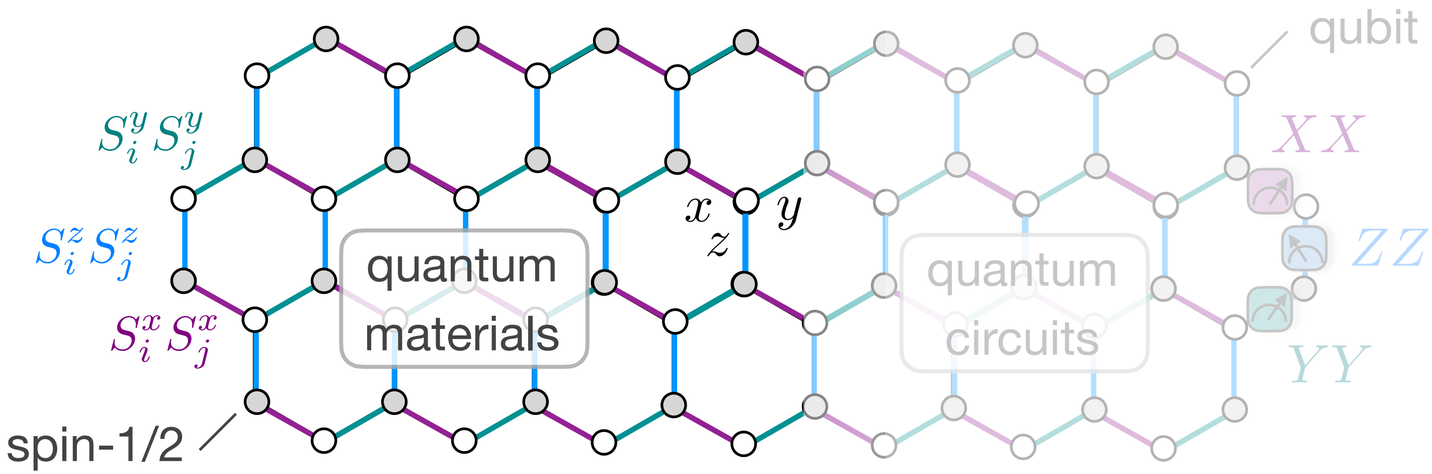
- **equilibrium** dynamics of **isolated** systems
- **unitary** evolution
- energy **conserved**
- quantum **ground states**
- **area-law entanglement** structures
- macroscopic entanglement (spin liquids)

measurement dynamics

- **out-of-equilibrium** dynamics of **open** systems
- **non-unitary** evolution
- energy **not** conserved
- long-time **steady states**
- **plethora of entanglement** structures
- macroscopic entanglement (spin liquids)

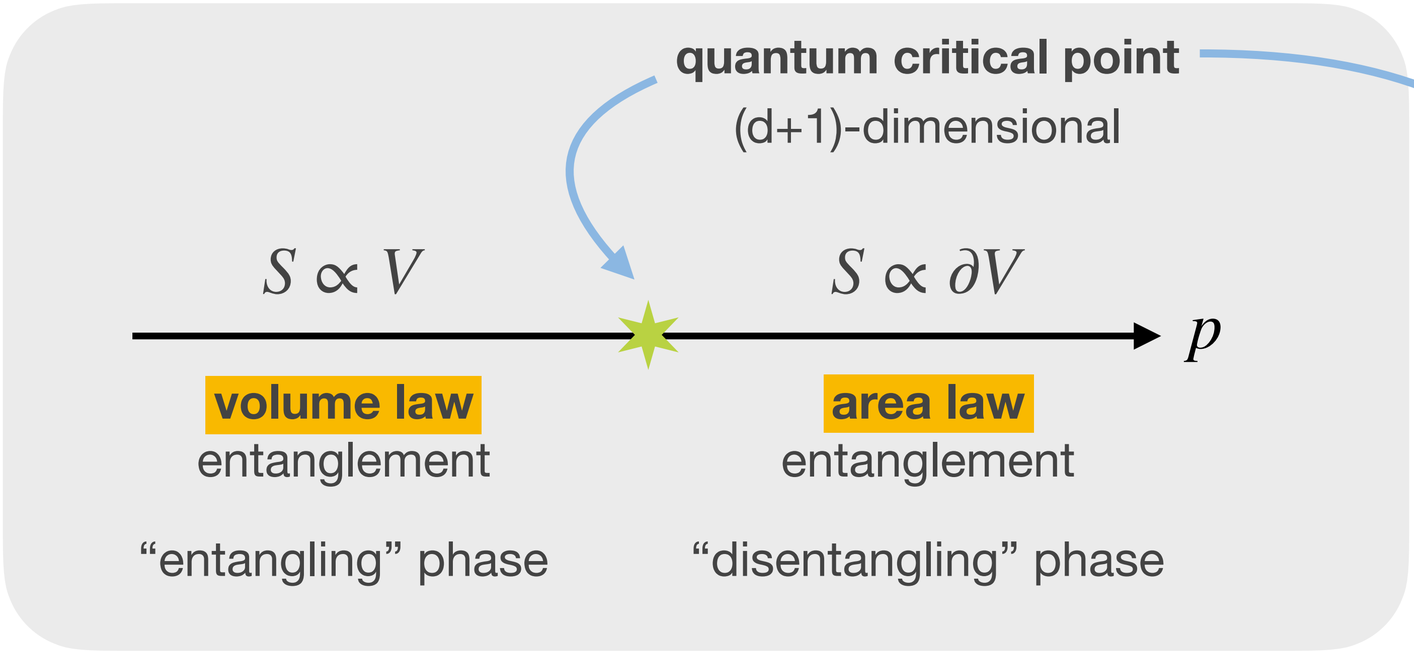
$\mathcal{H}$

magnetic exchange



Clifford gates

entanglement phase transitions



non-equilibrium
statistical physics

non-unitary CFTs
loop models

paradigmatic example: many-body localised (MBL) to chaos transition
logarithmic vs. algebraic entanglement growth in time

M. Fisher, V. Khemani, A. Nahum & S. Vijay, Ann. Rev. Cond. Matt. Phys.**14**, 335 (2023)

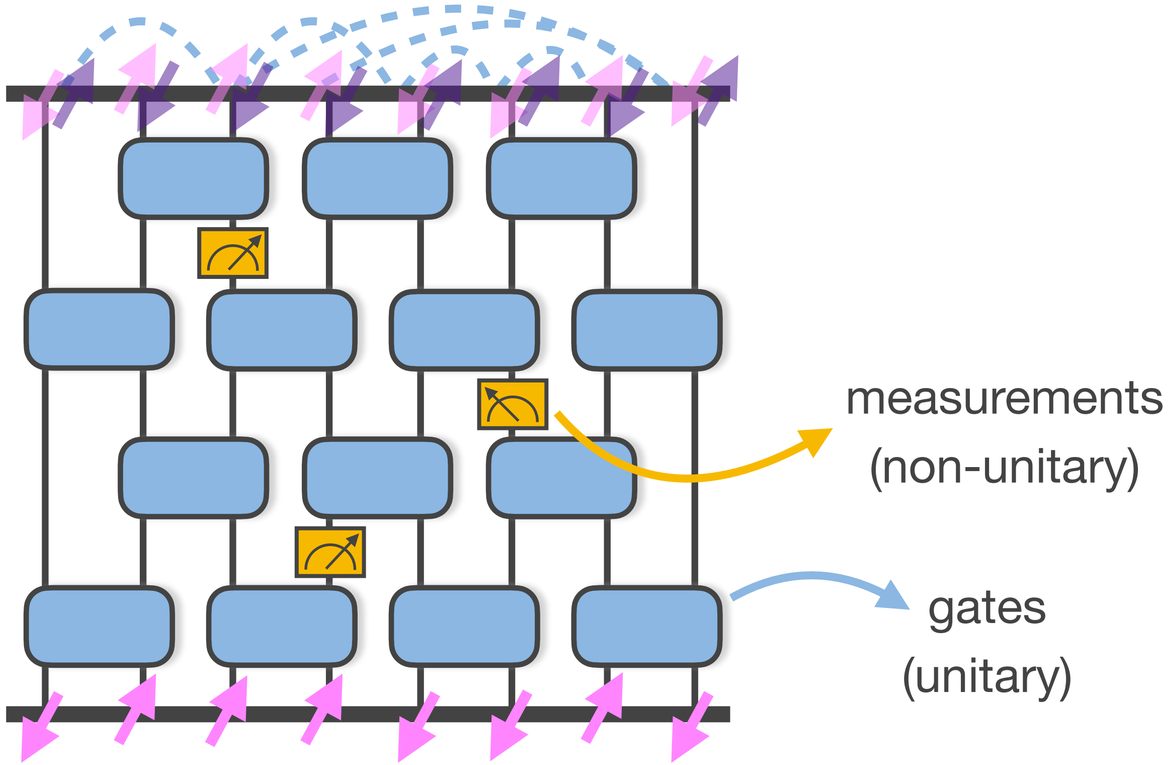
entanglement phase transitions

hybrid **unitary/projective** dynamics

- **competition** between **scrambling** (unitary) and **disentangling** (measurement) dynamics
- entanglement dynamics along *single quantum trajectories*
- **entanglement phase transition** as function of measurement rate

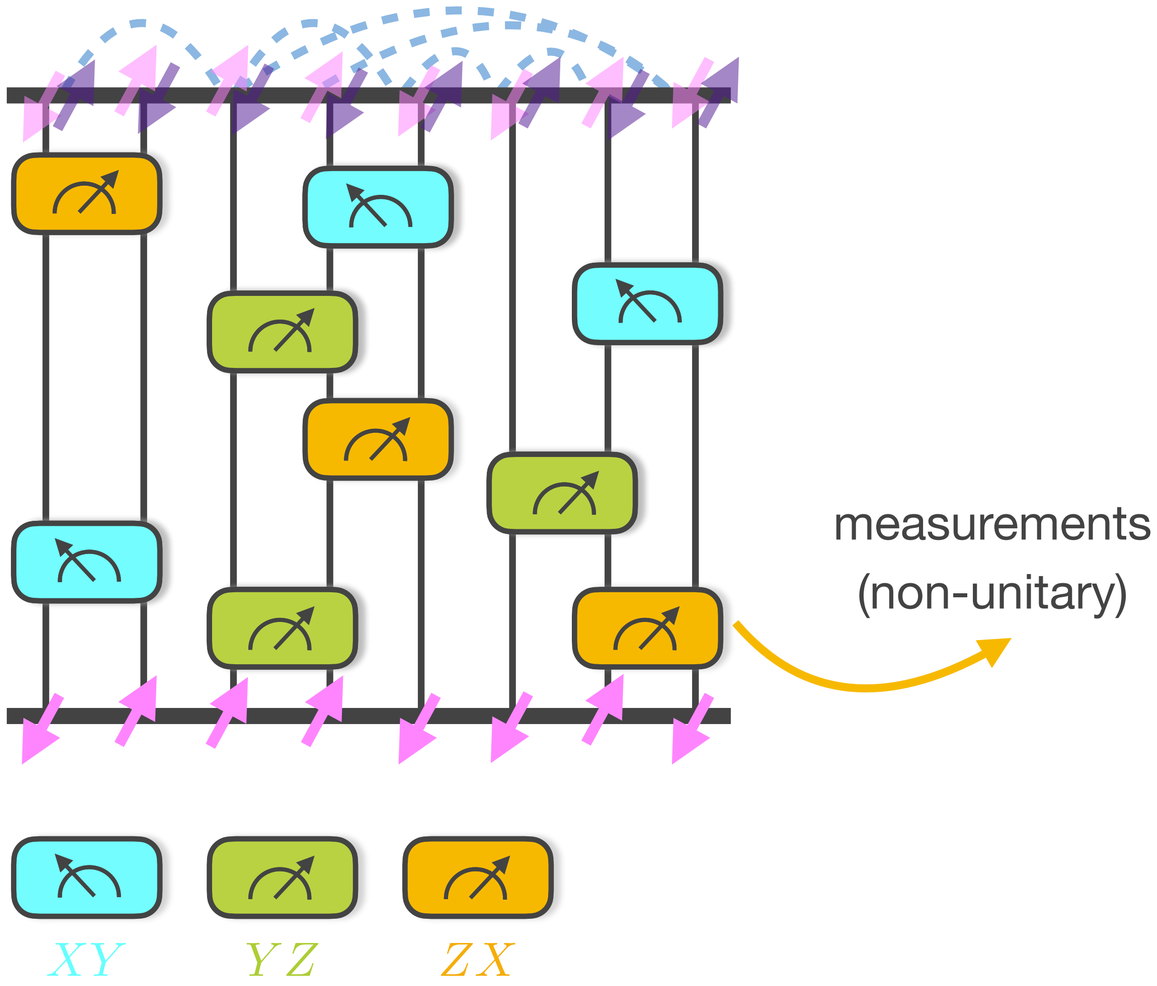
volume-law phase is unexpected

- quantum **coherence is a delicate resource** building up entanglement takes long *O(N)*
- unitary dynamics hides quantum info in **nonlocal correlations** inaccessible to local measurements
- quantum error correcting code



M. Fisher, V. Khemani, A. Nahum & S. Vijay, Ann. Rev. Cond. Matt. Phys.**14**, 335 (2023)

entanglement phase transitions



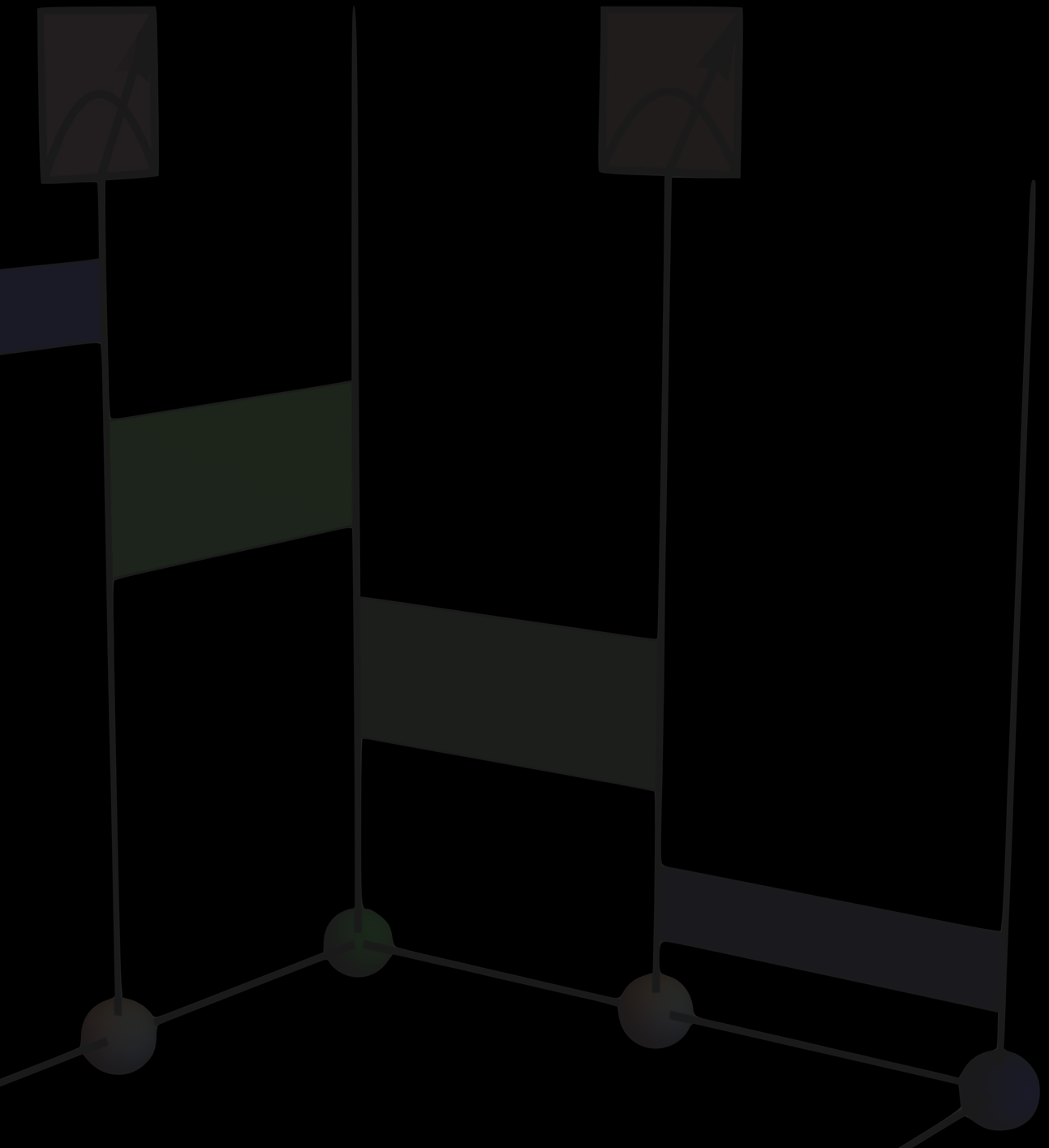
measurement-only dynamics

- **scrambling** and **unscrambling** effects are fundamentally intertwined
- **frustration** by **non-commuting** measurements is key ingredient
- **local** operators alone can induce **volume-law** entanglement

volume-law phase is unexpected

- two-qubit measurements cannot induce volume-law entanglement, but **multi-qubit measurements** can
- **structured** volume-law phases can appear
- quantum error correcting **codes**

M. Ippoliti et al., PRX **11**, 011030 (2021) ; A. Lavasani et al., Nature Physics **17**, 342 (2021); S. Sang & T.H. Hsieh, Phys. Rev. Research **3**, 023200 (2021)



monitored
Kitaev models

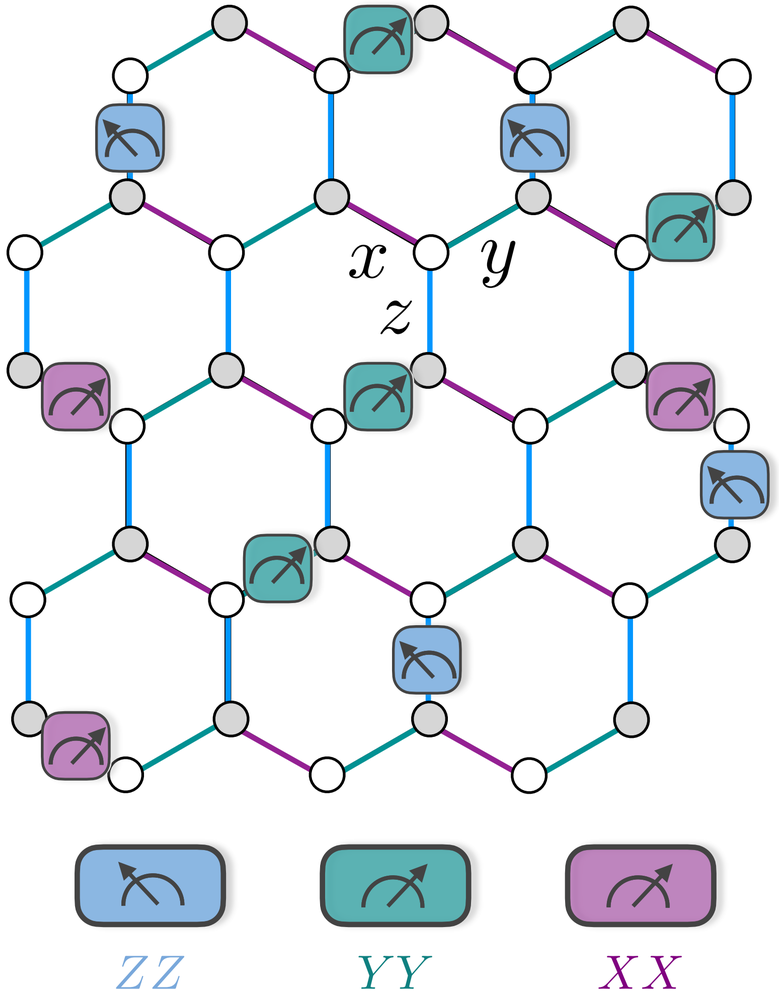
meet the team

G. Zhu & ST, forthcoming preprint

G. Zhu, N. Tantivasadakarn, ST arXiv:2303.17627

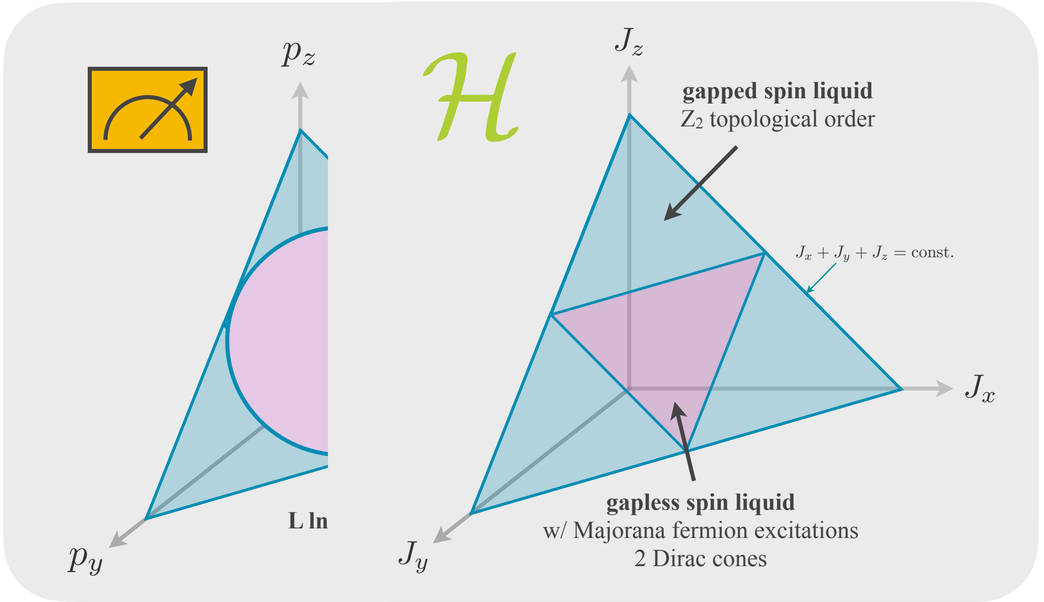
G. Zhu *et al.* arXiv:2208.11136

measurement-only Kitaev circuits



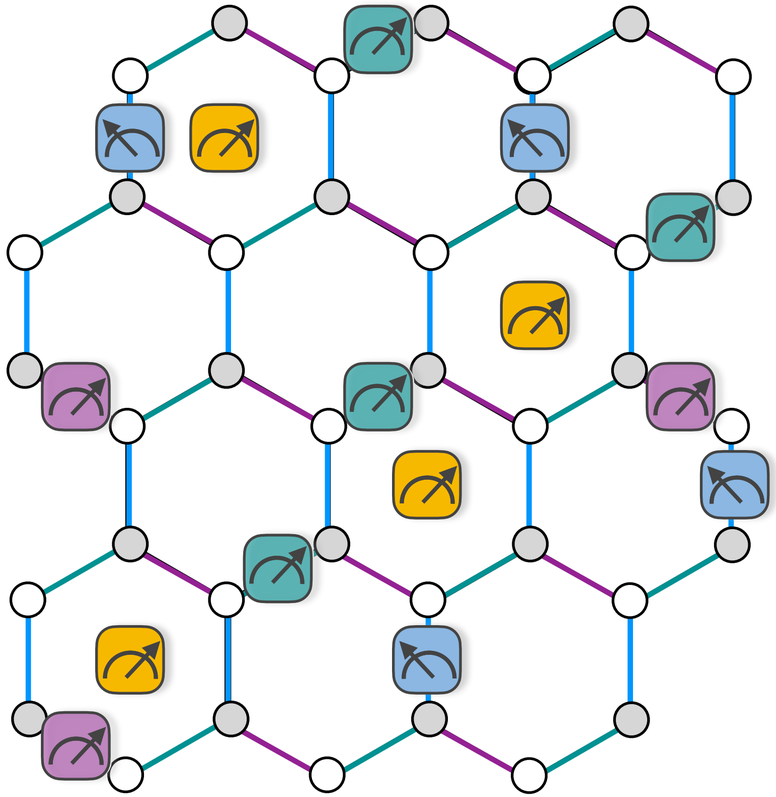
measurement-only dynamics

- non-commuting **two-qubit** measurements
- completely **stochastic**, i.e. no spatial or temporal patterns (such as Floquet)



A. Lavasani, Z.-X. Luo, and S. Vijay, arXiv:2207.02877
A. Sriram, T. Rakovszky, V. Khemani, and M. Ippoliti, arXiv:2207.07096

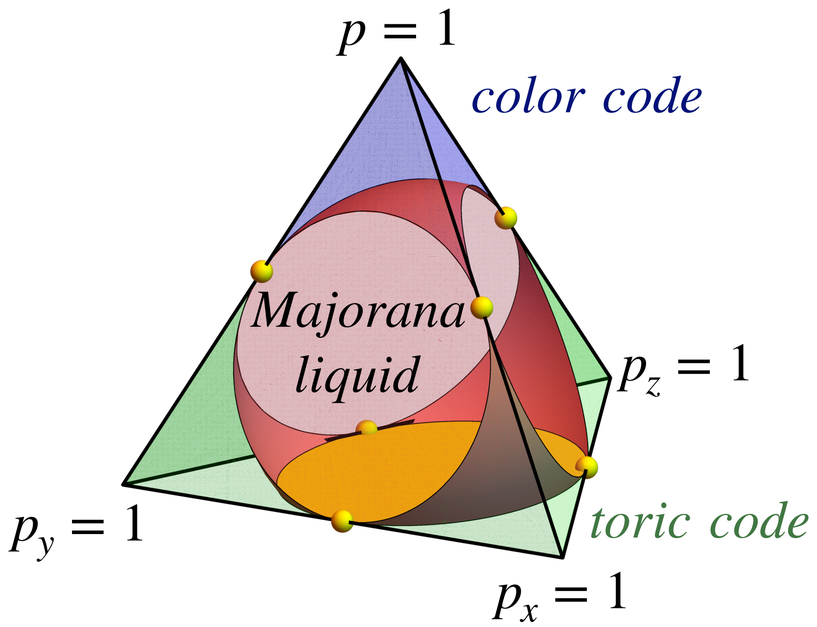
measurement-only Kitaev circuits



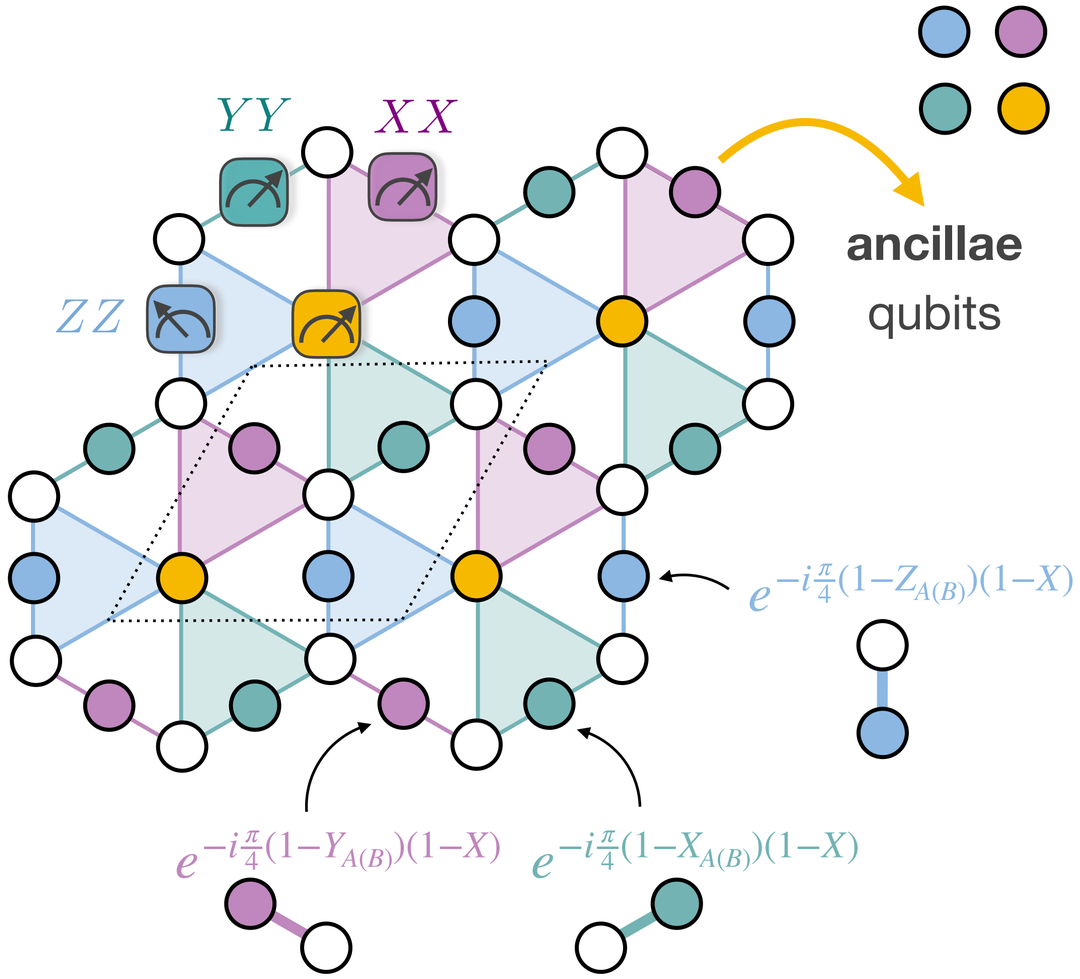
not (commuting) plaquette flux

measurement-only dynamics

- non-commuting **two-qubit** and **six-qubit** measurements
- completely **stochastic**, i.e. no spatial or temporal patterns (such as Floquet)



mid-circuit measurements



Monitored random unitary circuit implementation

- randomly **couple physical to ancillae qubits** by (rotation x CNOT gate)
- **measure ancillae qubits** in Z basis

Realizations

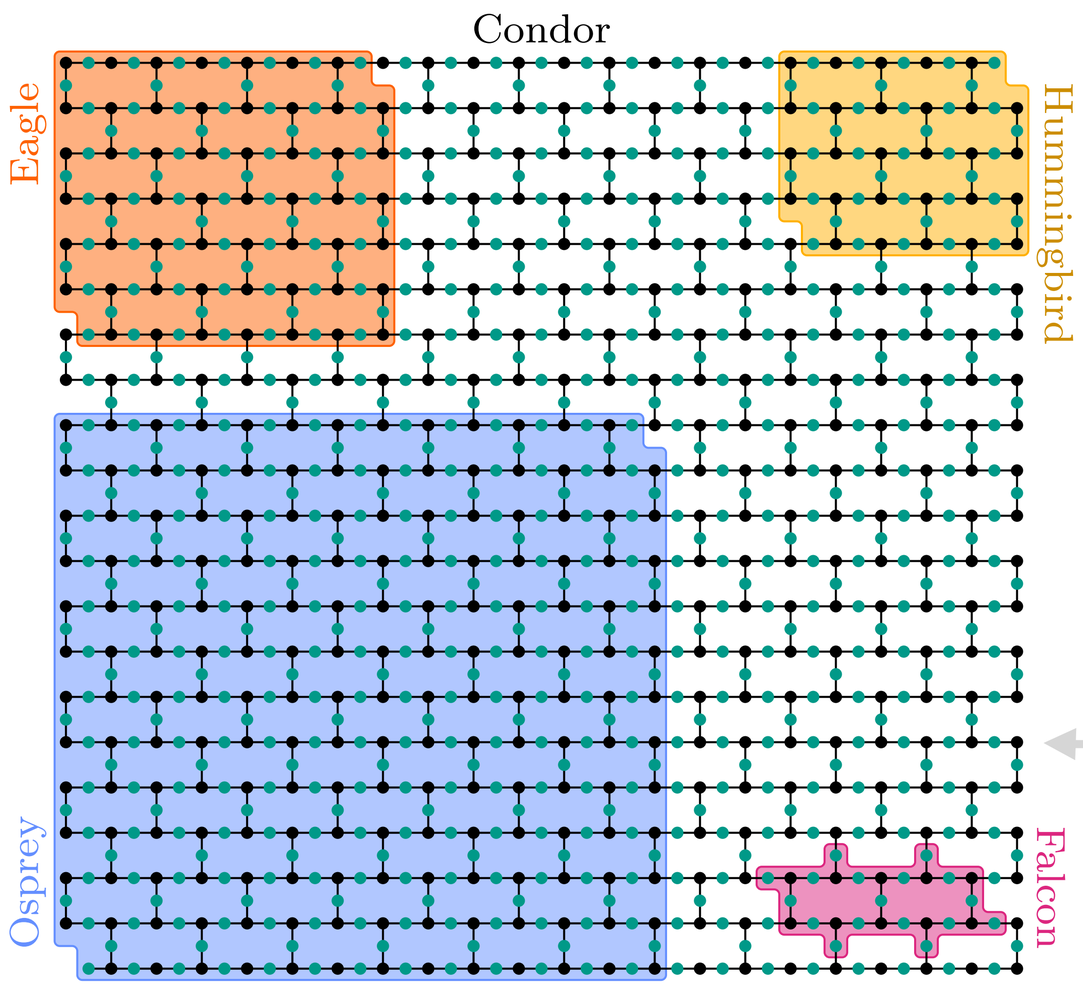
- **transmon processors**

  large number of qubits, mid-circuit measurements, close to IBM's heavy-hexagon geometry, deep circuits?
- Rydberg atoms

  measurement hard?
- trapped ions

  small system sizes?

mid-circuit measurements



heavy-hexagon geometry + Ising evolution gates

Monitored random unitary circuit implementation

- randomly **couple physical to ancillae qubits** by (rotation x CNOT gate)
- **measure ancillae qubits** in Z basis

Realizations

IBM

- **transmon processors**

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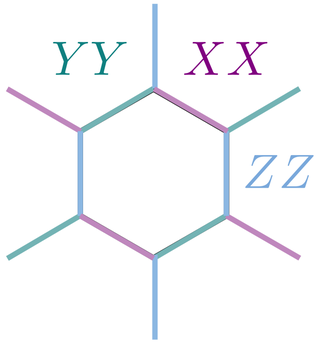
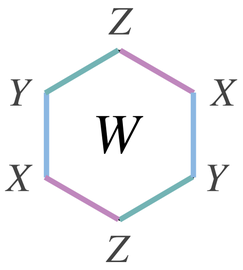
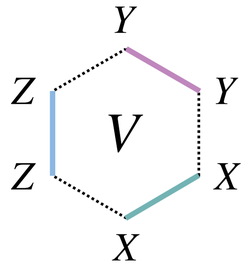
Majorana models (Hamiltonian)





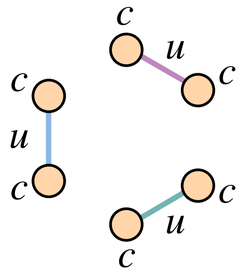
flux

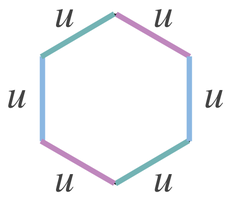
$$H = (1-p)(ZZ + XX + YY) + pV$$



plaquette term

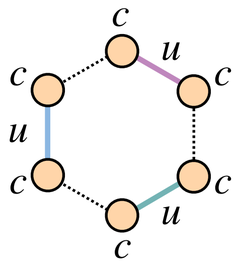
fermionize



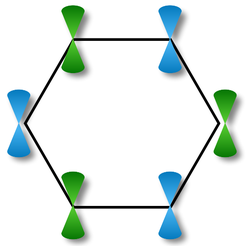


six-Majorana interaction

$$H = (1-p)(iucc) + p(iuuuuccccccc)$$



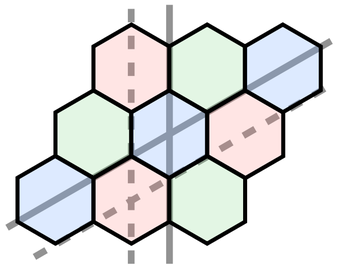
$\mathcal{H}$



gapless Majorana spin liquid

?

gapped Majorana surface code

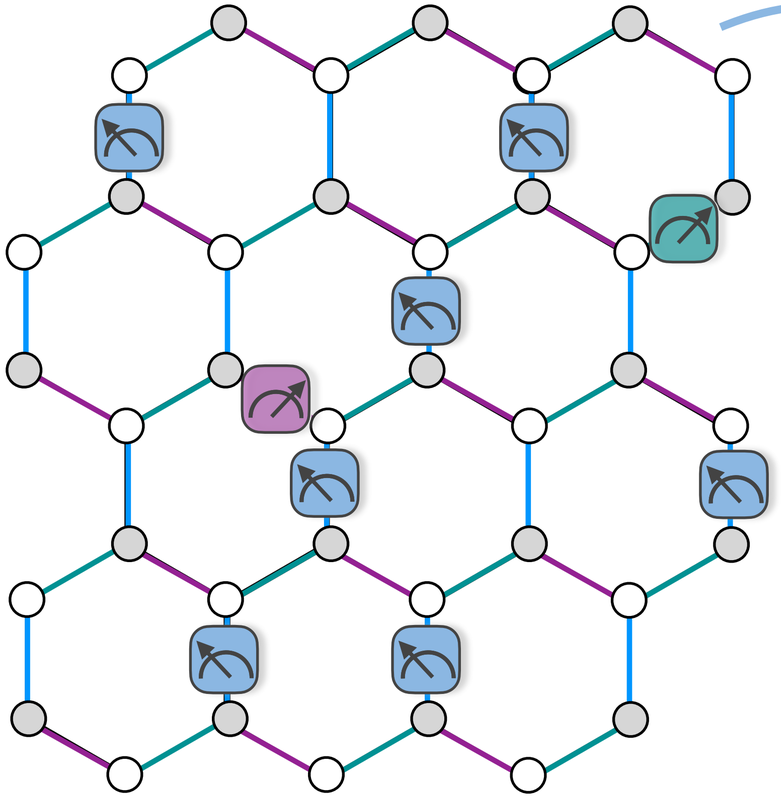


Kitaev 2006

both phases are **long-range entangled**

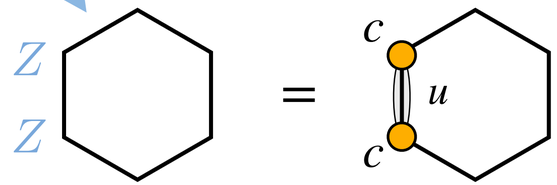
Vijay, Hsieh, Fu 2015

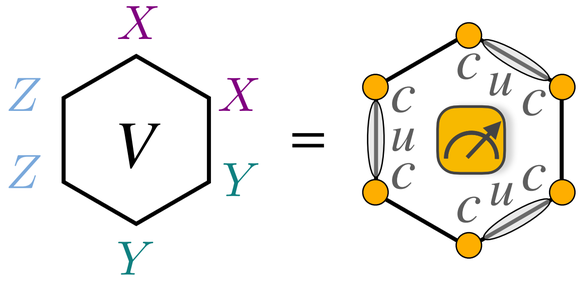
dynamically long-range entangled phases



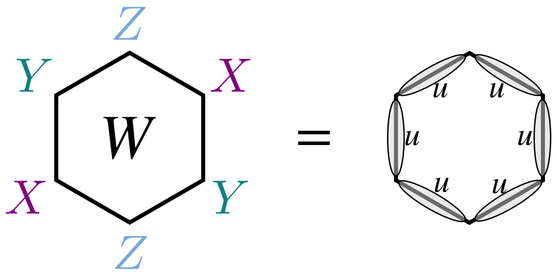
conventional
toric code

short-ranged
Majorana dimer

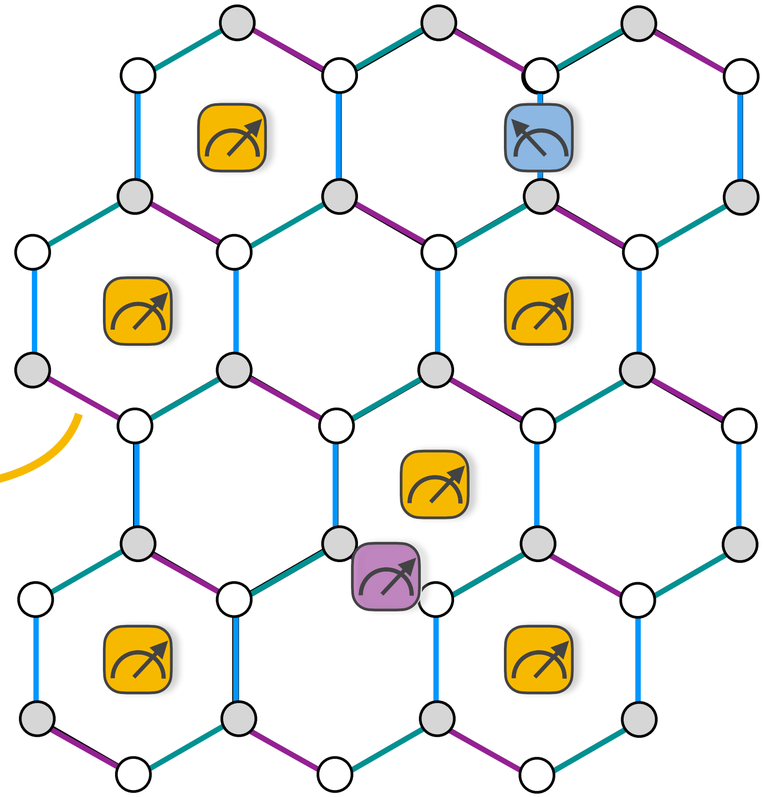




Majorana
surface code

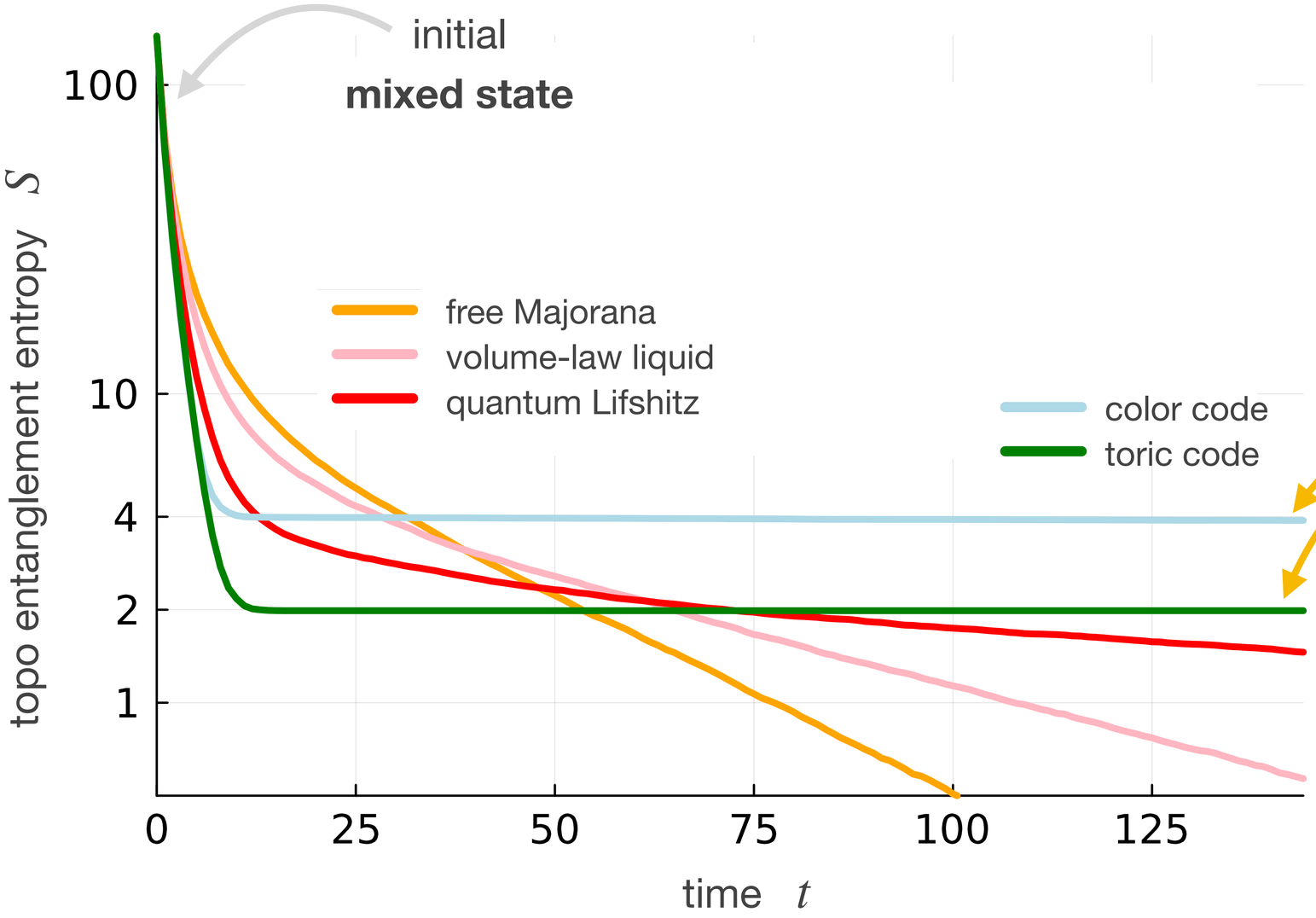


Z_2 gauge field



color
code

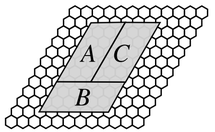
purification dynamics

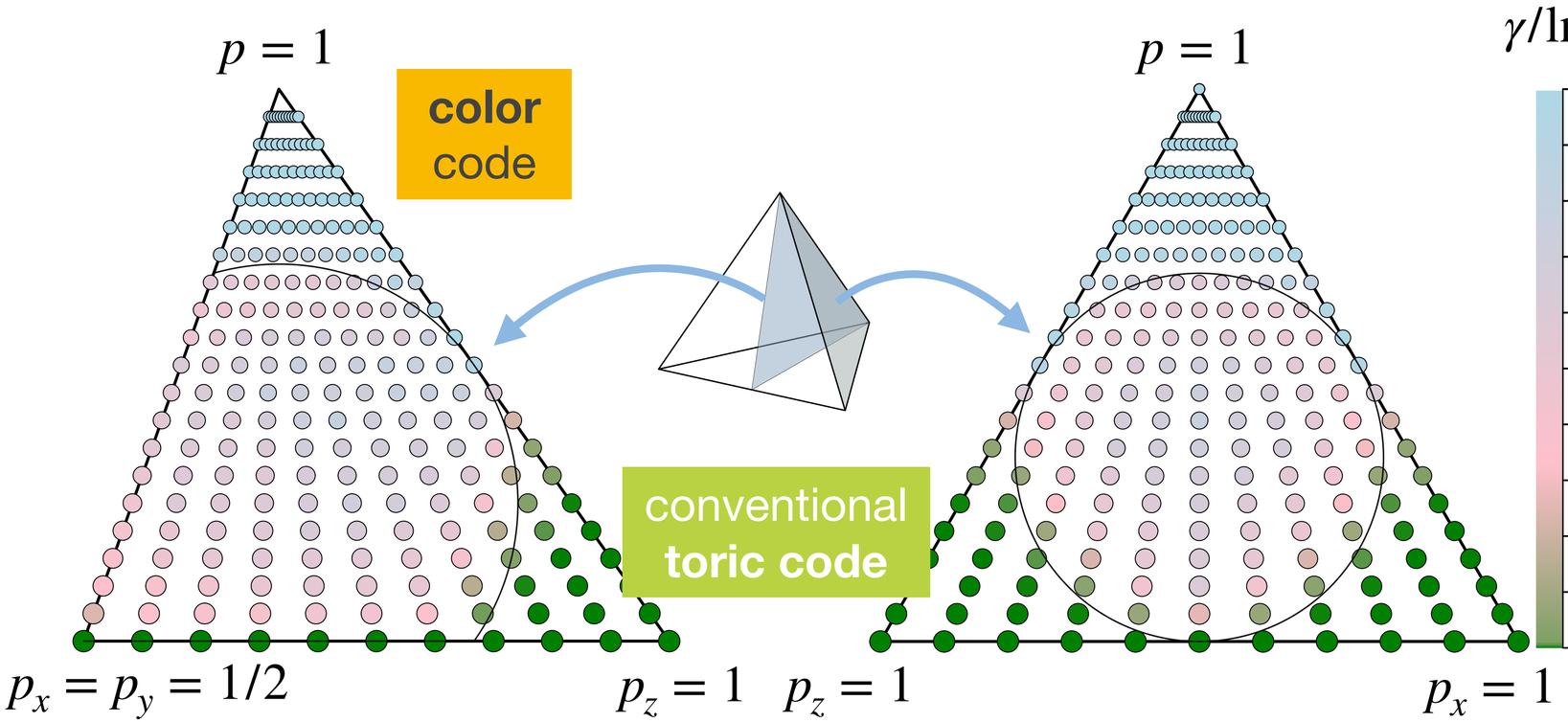


exponentially long-lived
topological quantum memory

long-range entangled phases

topological entanglement entropy





Majorana surface code

four logical qubits

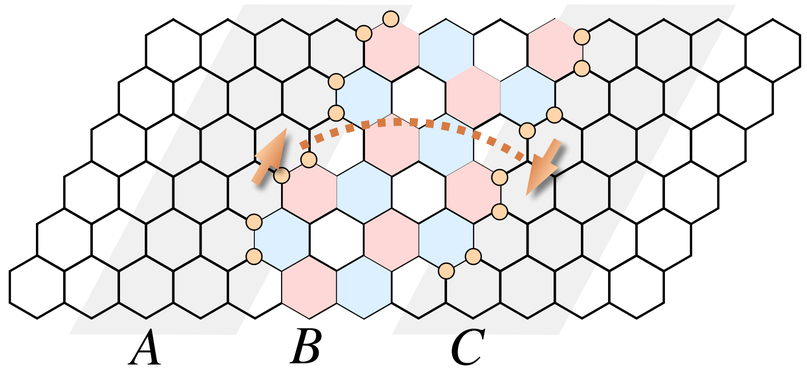
short-ranged Majorana dimer

two logical qubits

error thresholds

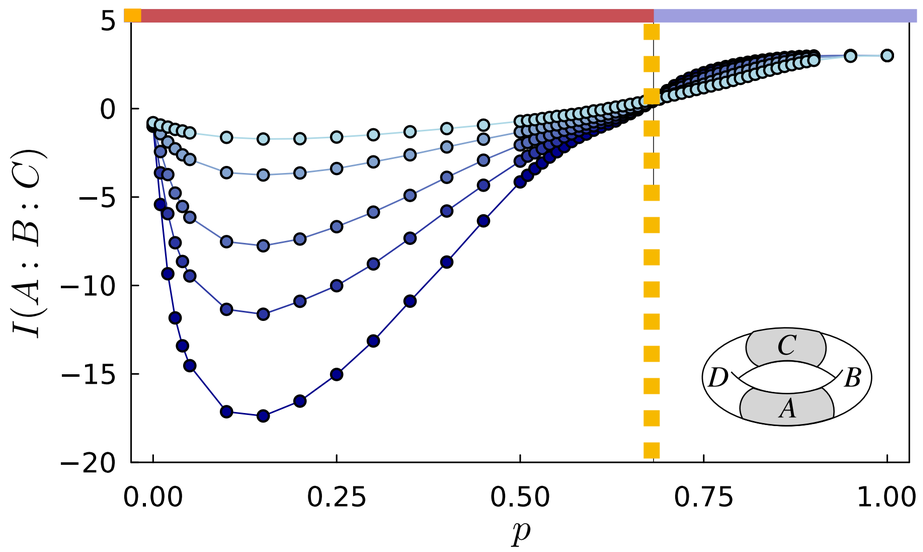
tripartite **mutual information**

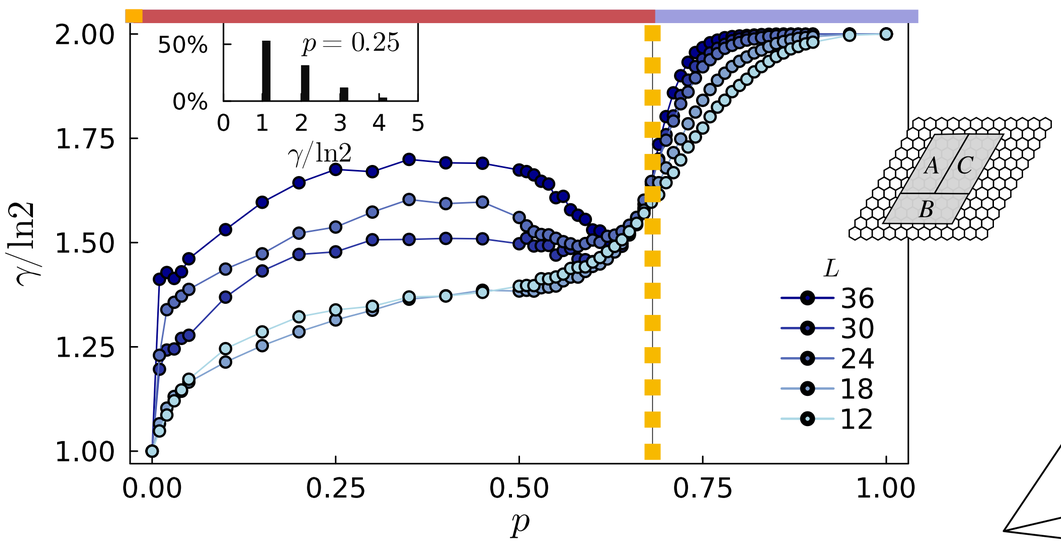
$$I(A:B:C) = S_A + S_B + S_C - S_{AB} - S_{AC} - S_{BC} + S_{ABC}$$

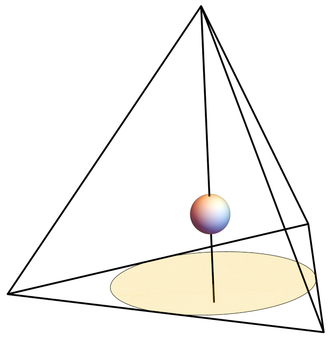


error

threshold

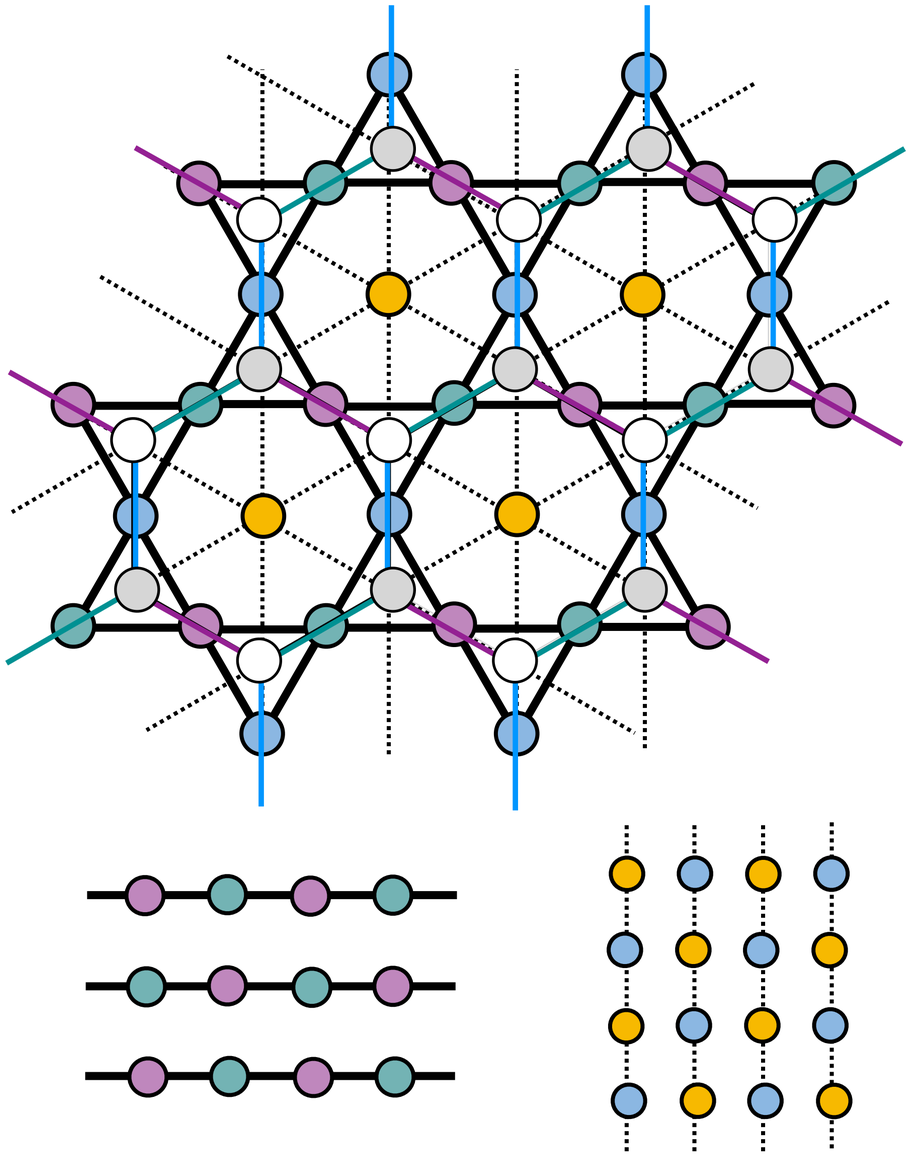






threshold for projective bond errors
+ stochastic syndrome measurements

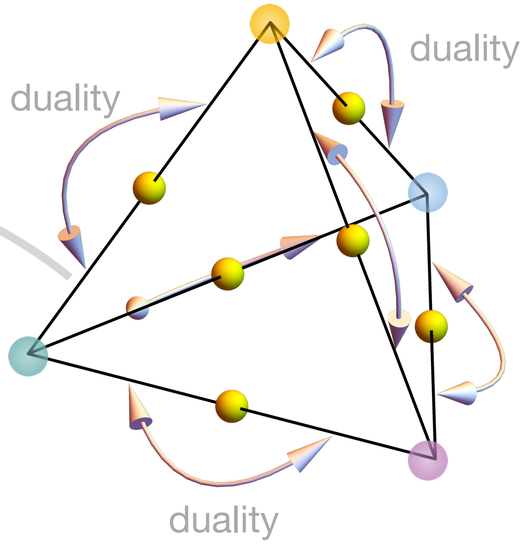
frustration graph



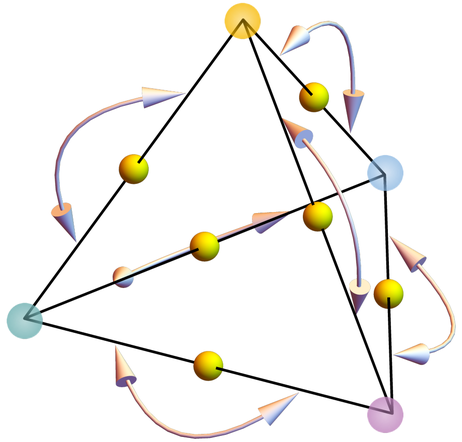
- node = measurement operator
- **link = anti-commutation** (frustration)
- stabilizer = product of certain nodes
- probability of measurement = fugacity

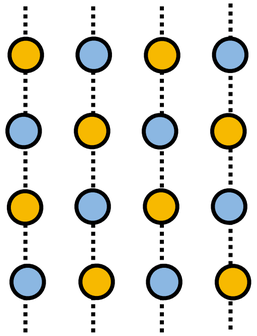
On tetrahedron **edges**

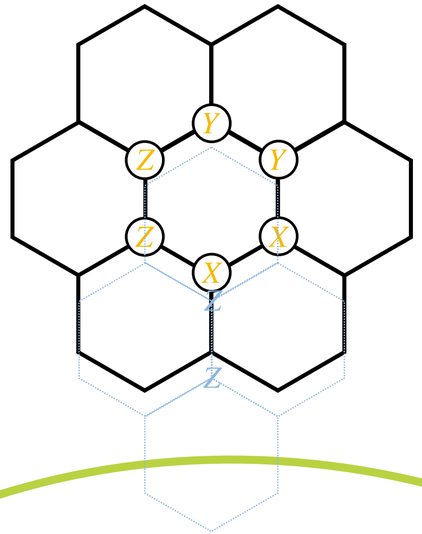
- some frustration lifted
- stacked 1+1D chains
- **self-duality**
- 2D square lattice **percolation**



self-dual points



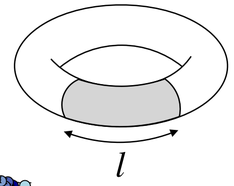




- stacked 1+1D chains (percolation)
- self-dual point: stacked quasi-1D critical

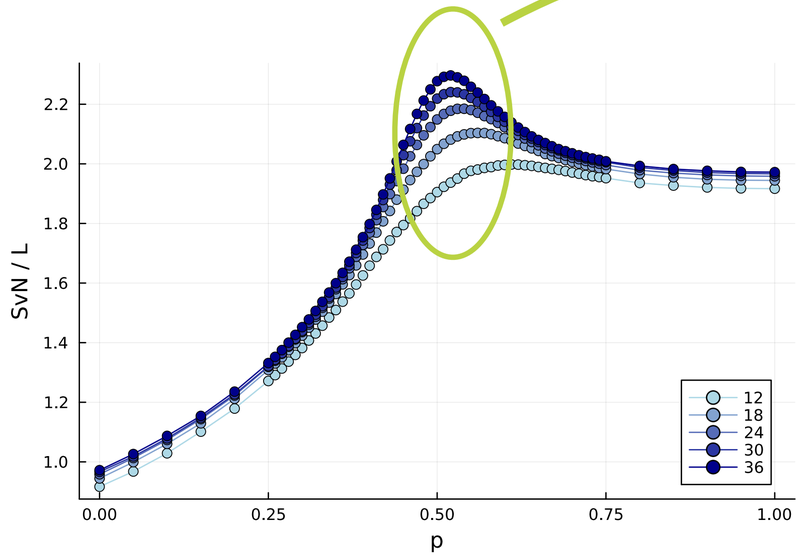
$$S_{vN}(l,L) = \frac{cL}{3}\ln\left(\frac{L}{\pi}\sin\frac{\pi l}{L}\right) + \cdots$$

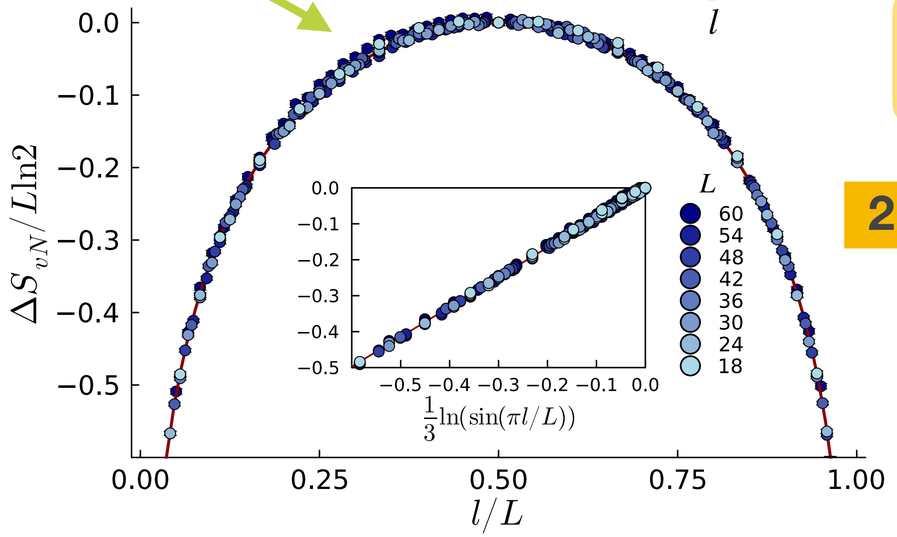
Calabrese & Cardy 2004



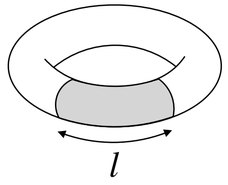
$$c = \frac{3\sqrt{3}}{2\pi}\ln 2$$

2D percolation



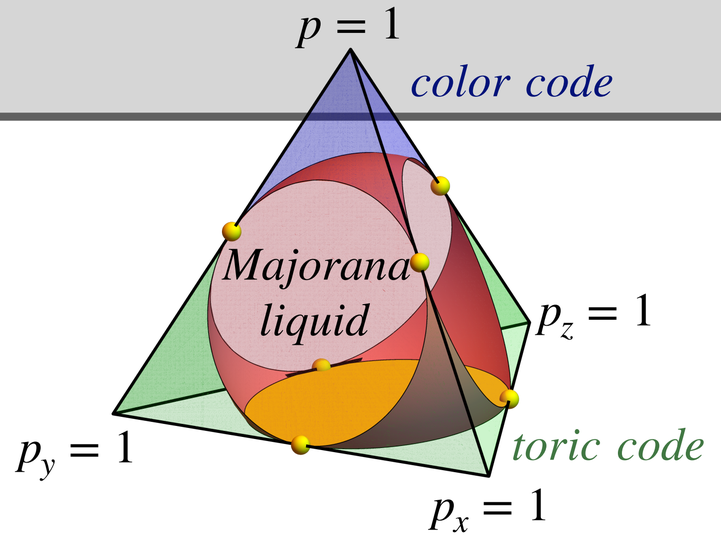


entanglement phase diagram

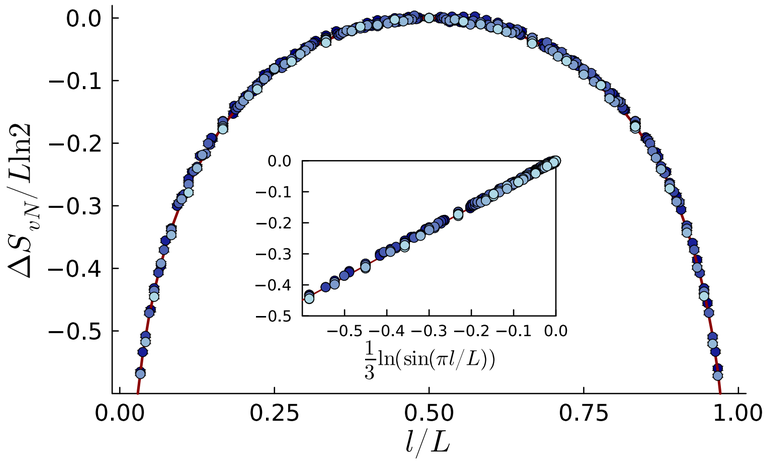


entanglement bipartition

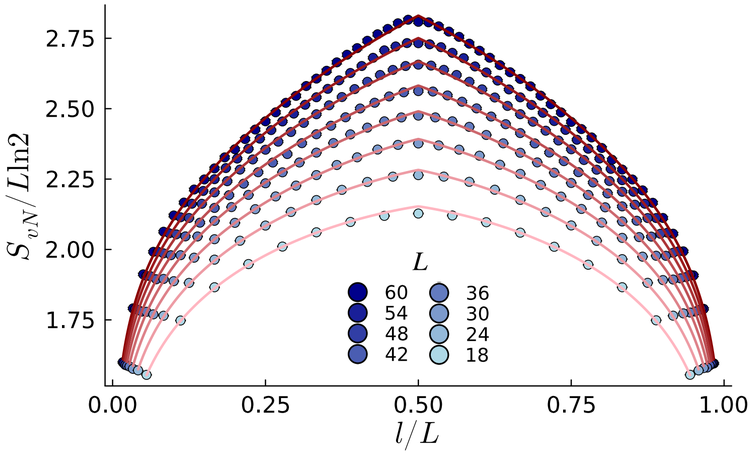
varying the **relative probabilities of measurements** yields a rich entanglement phase diagram



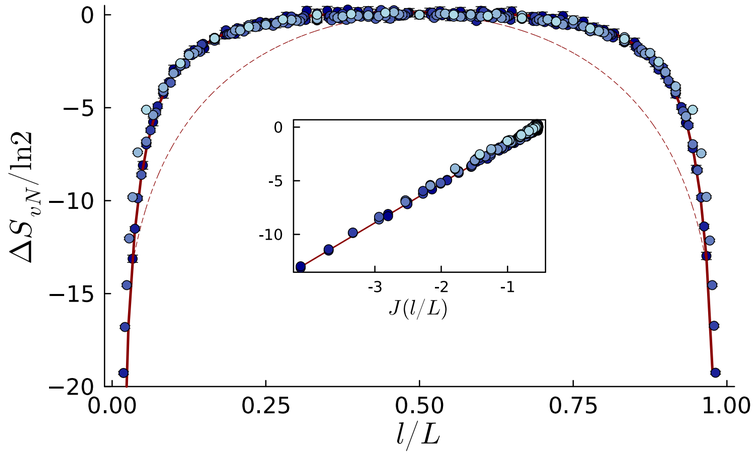
(1+1)D CFT entanglement



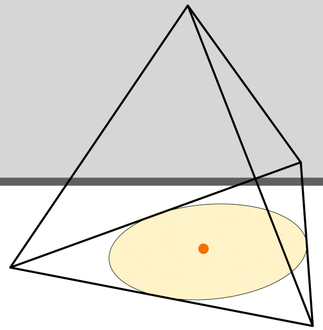
volume-law entanglement



quantum **Lifshitz**



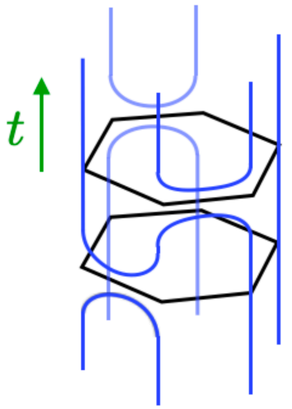
free Majorana liquid

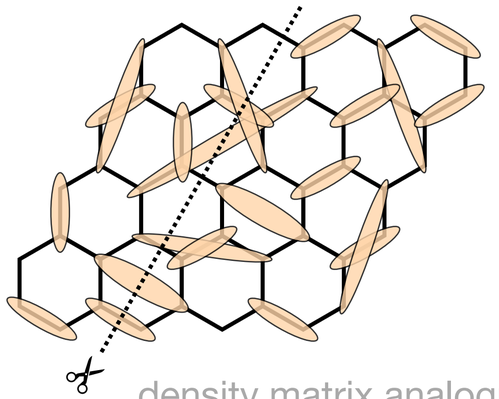


- dynamics: **compact loop model**
  in stacked honeycomb 2+1D lattice
- steady state: **long-range Majorana dimer state**
- entanglement entropy: **1+1D CFT scaling** $(L \ln L)$

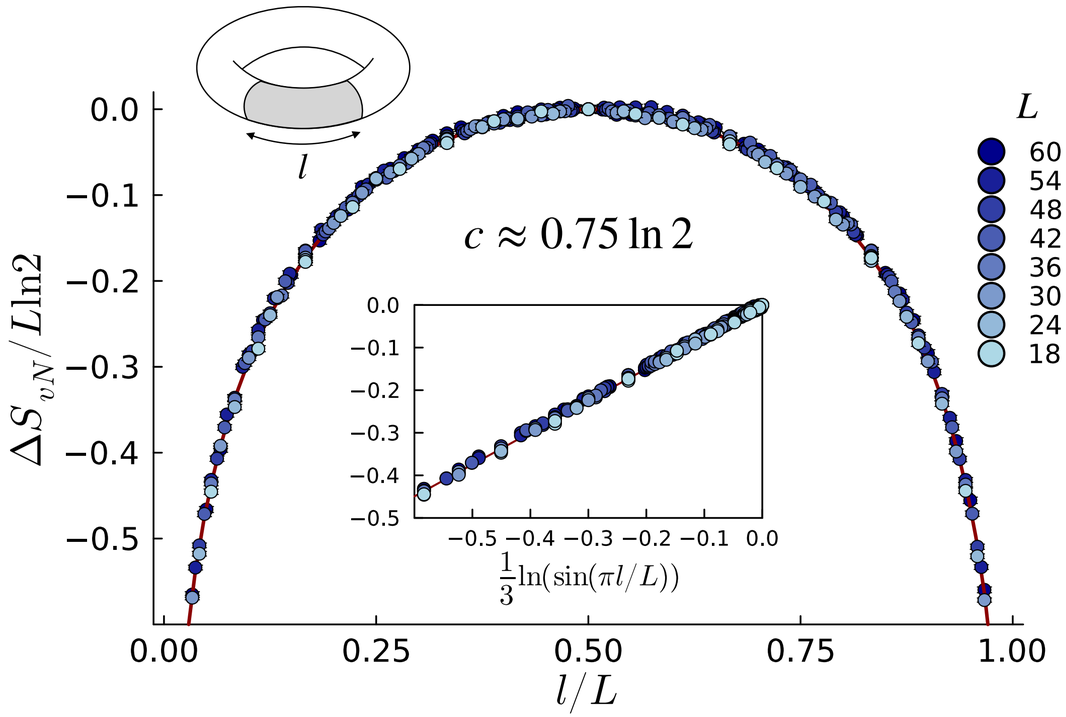
$$S_{vN}(l,L) = \frac{cL}{3} \ln\left(\frac{L}{\pi} \sin\frac{\pi l}{L}\right) + \cdots$$

Fermi-surface scaling





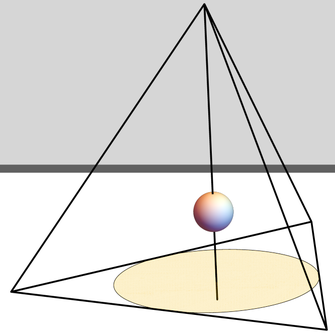
density matrix analogue
of RVB state

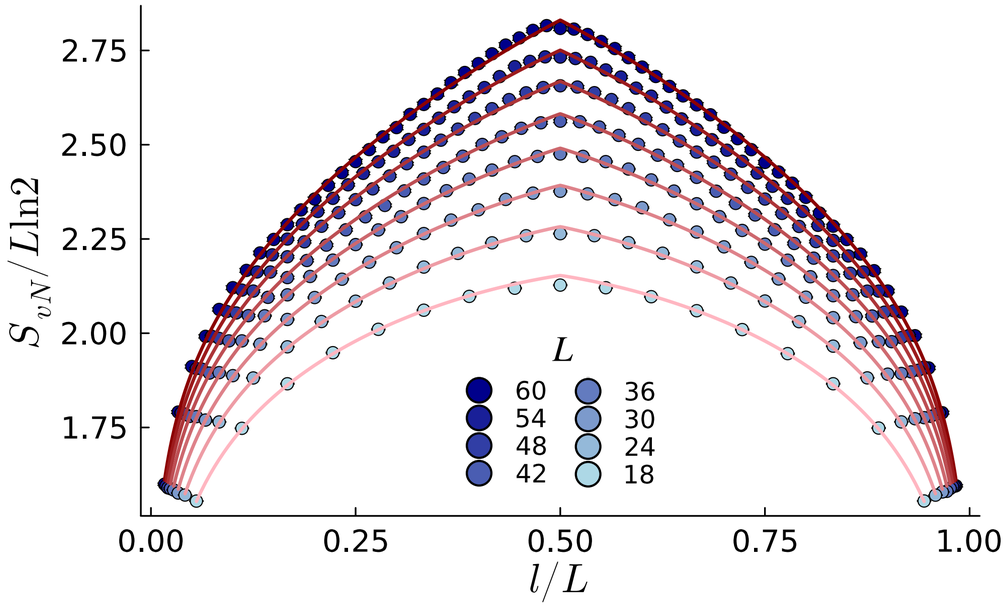


Majorana loop model

A. Lavasani, Z.-X. Luo, and S. Vijay, arXiv:2207.02877
A. Sriram, T. Rakovszky, V. Khemani, and M. Ippoliti, arXiv:2207.07096

interacting Majorana liquid





single-fit for all system sizes

$$S_{vN}(l,L) = v \cdot vol(l,L) + \frac{cL + c'}{3} \ln\left(\frac{L}{\pi}\sin\frac{\pi l}{L}\right) + aL - \gamma$$

volume-law with Page correction

gapless mode ~ 1+1D CFT

area-law with topological correction

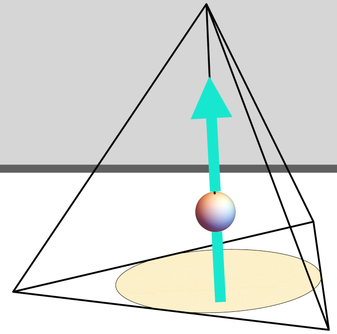
$$v = 0.00951(7)\ln 2$$

weak scrambling

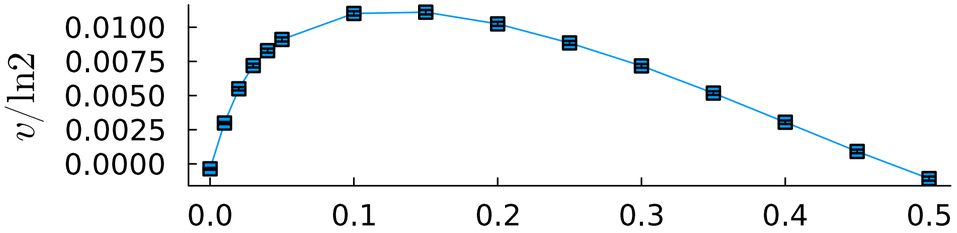
$$c = 0.642(7)\ln 2$$

$$vol(l,L) = 2Ll\ln 2 - 2^{4Ll-N-1}$$

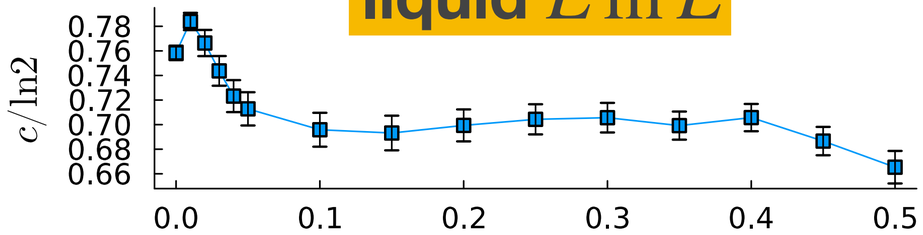
structured volume-law phase



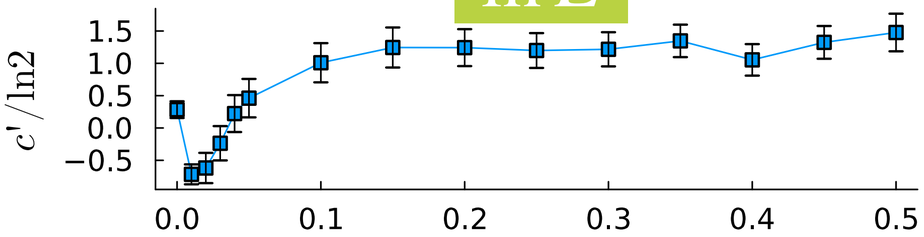
volume-law



liquid $L \ln L$



$\ln L$



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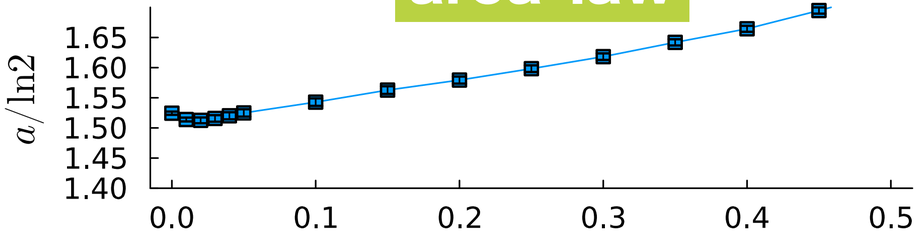
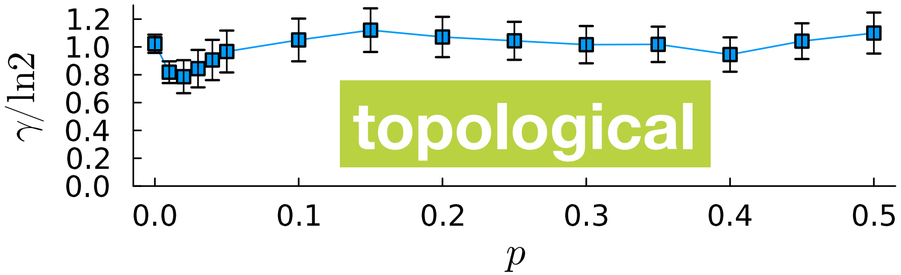
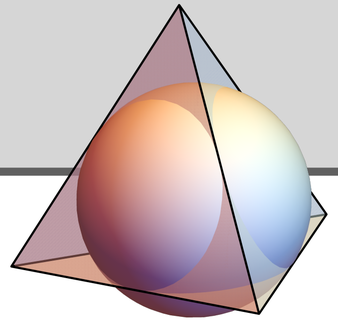
area-law



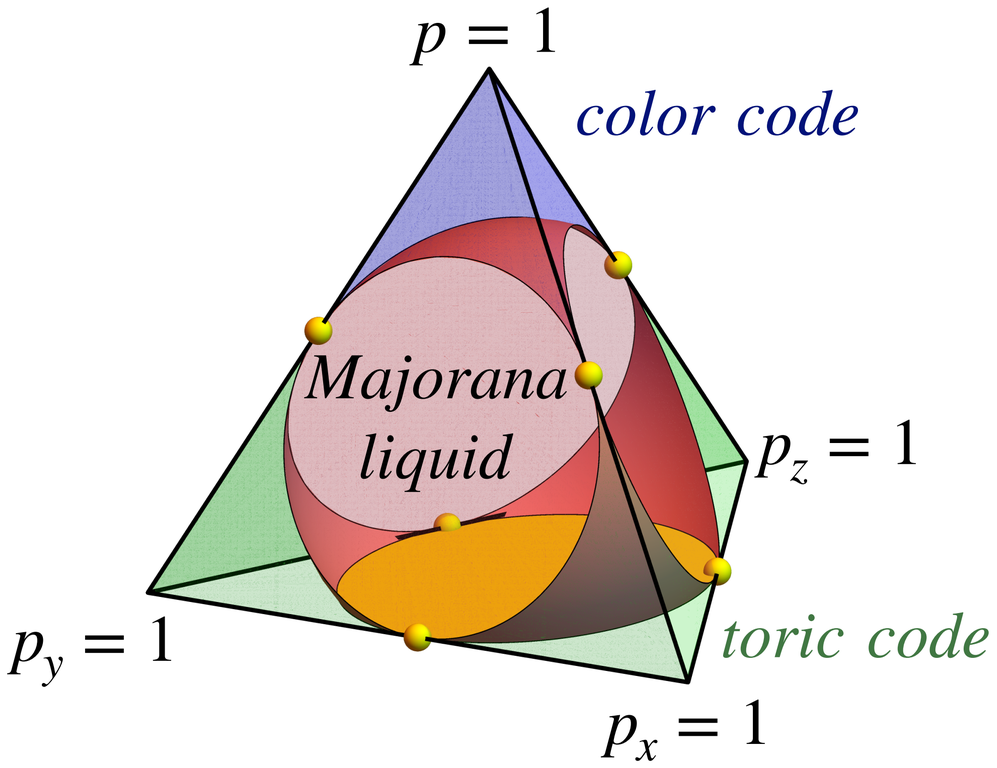
topological

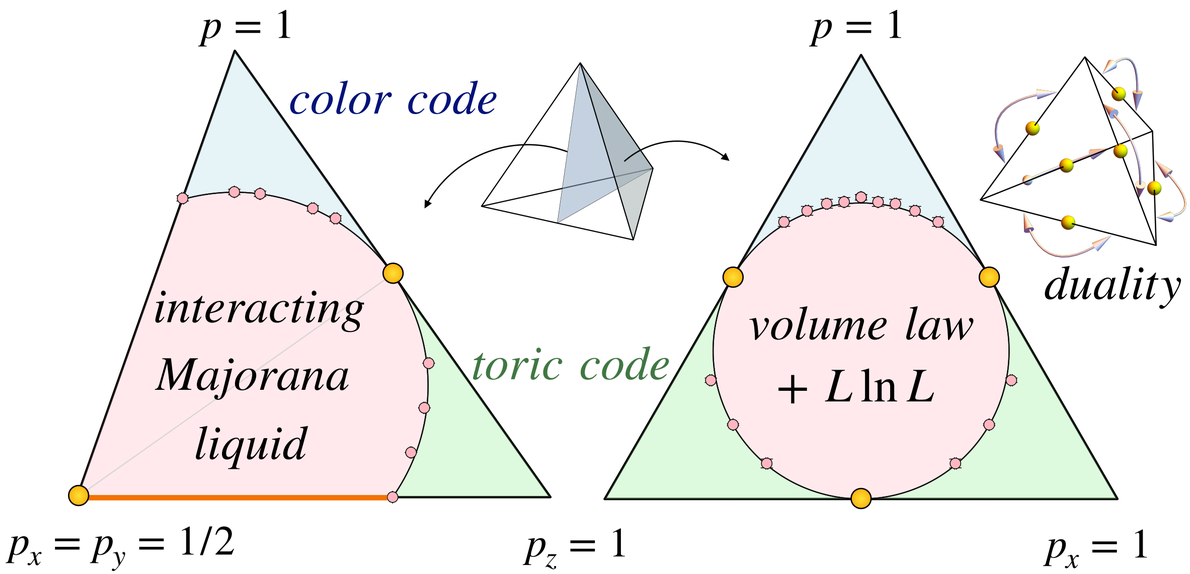


entanglement phase diagram



phase boundaries via
finite-size scaling of **tripartite mutual information**

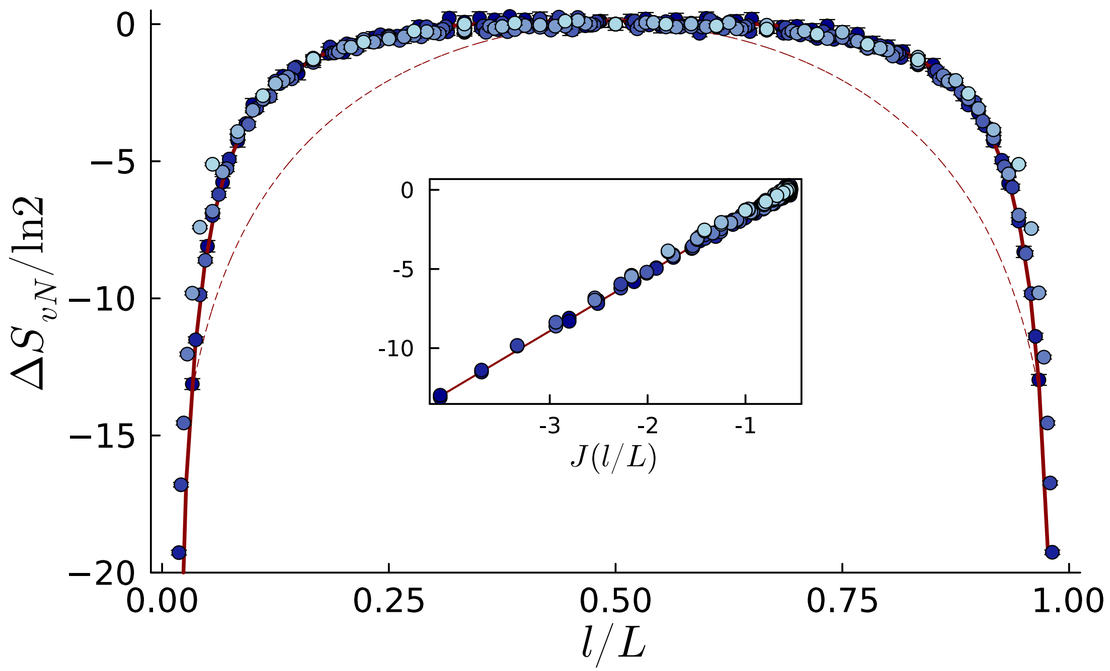




- What gives rise to **spherical symmetry?**

  statistical symmetry, *not* model-specific

entanglement transition

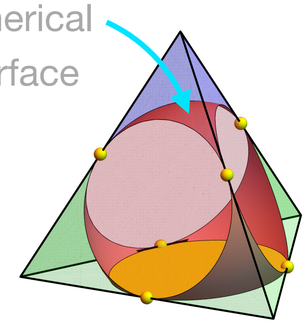


Jacobi theta function

$$J(x) = -\ln \frac{\theta_3(i\lambda x)\theta_3(i\lambda(1-x))}{\eta(2ix)\eta(2i(1-x))} \sim -\frac{\pi}{24x},\ x \ll 1$$

Dedekind eta function

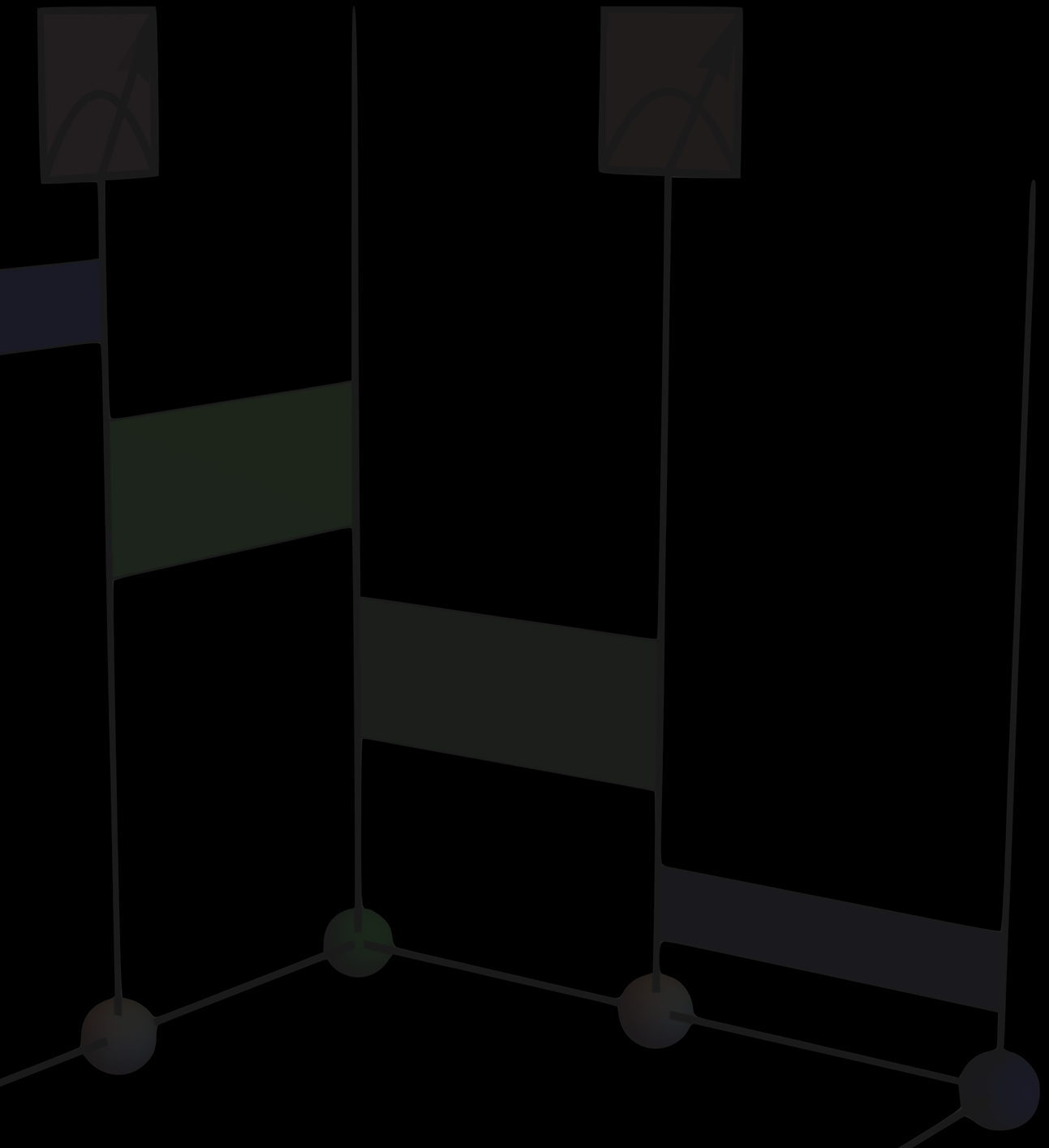
$$S_{vN} = aL + \beta J(l/L)$$



quantum Lifshitz scaling

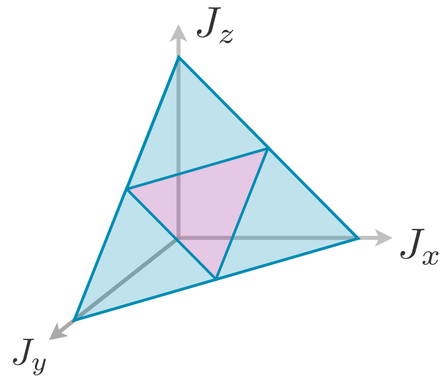
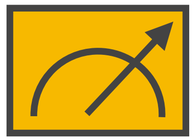
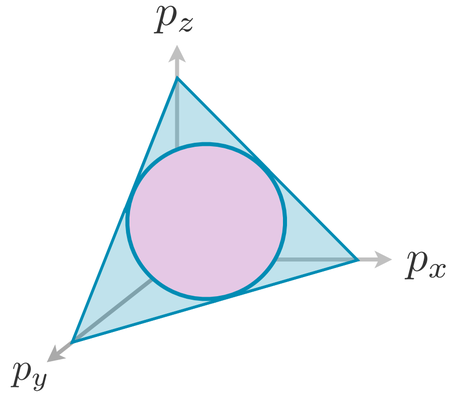
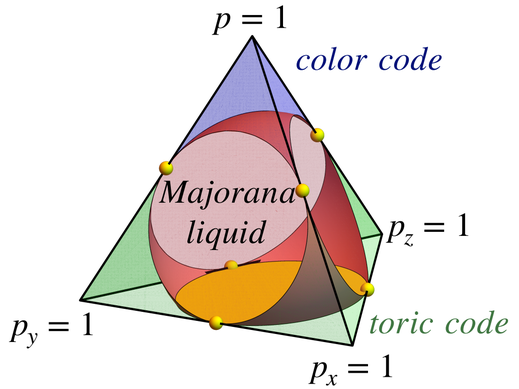
- 2+0D gapless dimer RVB (quantum Lifshitz model) (conformal quantum critical state)
- 2+1D relativistic Dirac fermions
- 2+1D transverse field Ising model
- 2+1D monitored Ising model (Z-monitored toric code)

Swingle 2010; Melko 2012, 2013; Fradkin 2015; Lavasani 2021



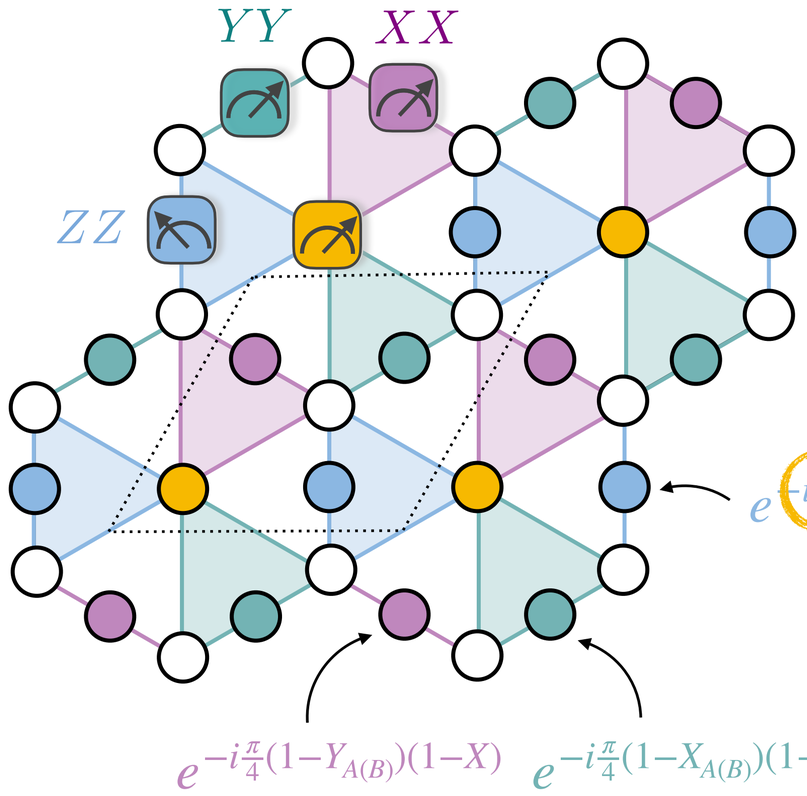
discussion

computational complexity

dynamics	**free** Majorana fermions	**interacting** Majorana fermions
$\mathcal{H}$		? **sign problem**
		**Clifford circuits** 

Gottesman-Knill theorem, quant-ph/9807006

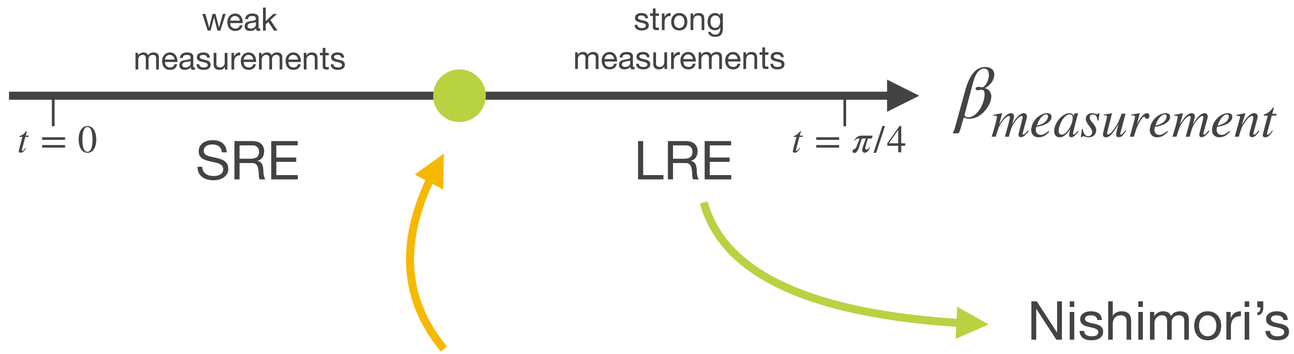
beyond Clifford — **weak measurements**



ancillae qubits

vary coupling
to ancillae qubits

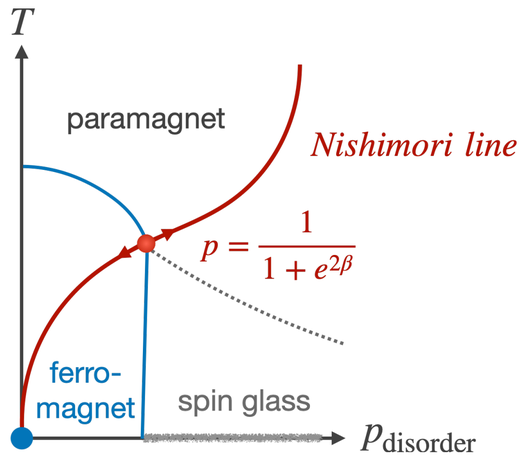
turn strong/projective into
weak measurements



finite stability threshold

Nishimori's cat

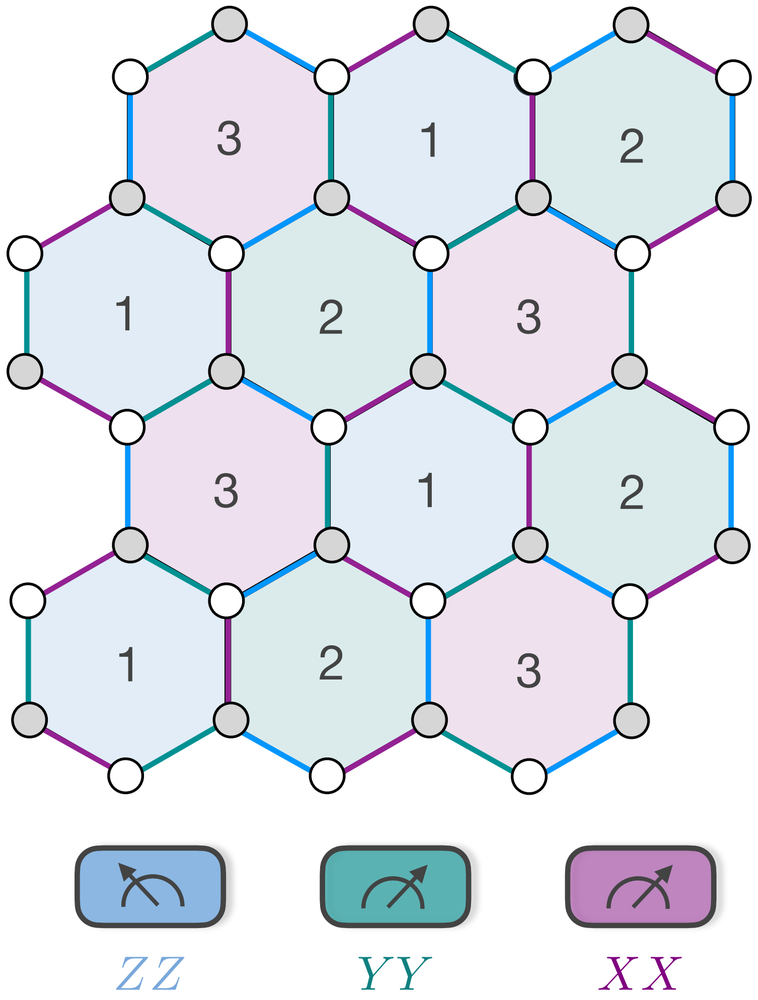
ZZ measurements only



G. Zhu *et al.*
arXiv:2208.11136

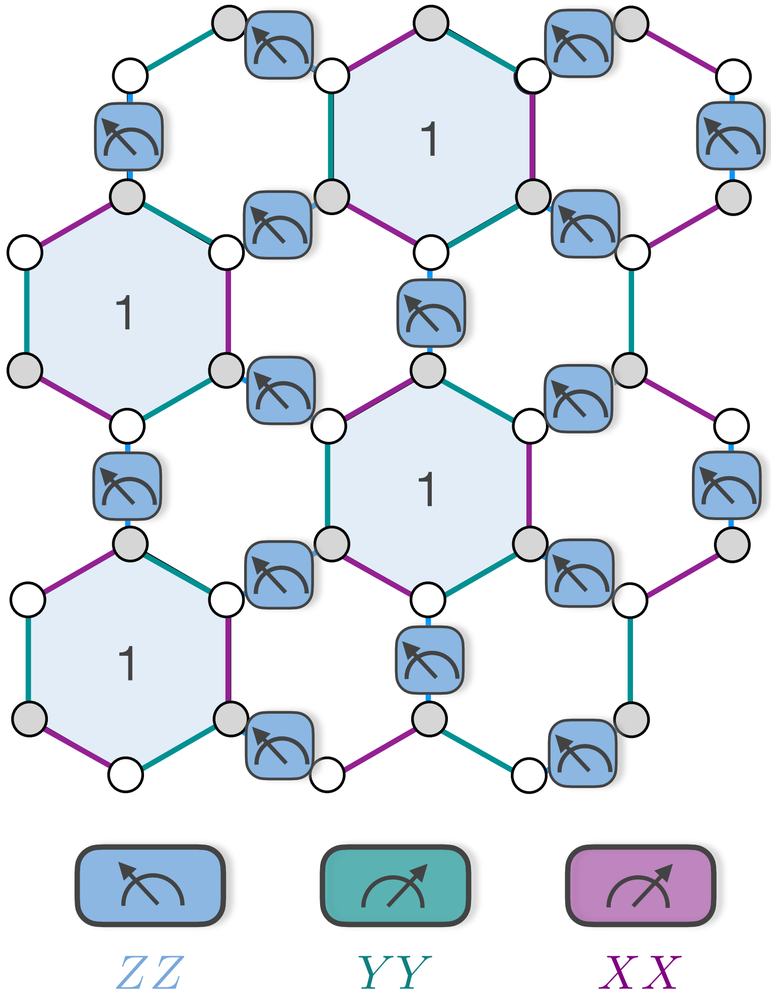
E. Chen *et al.*
forthcoming preprint

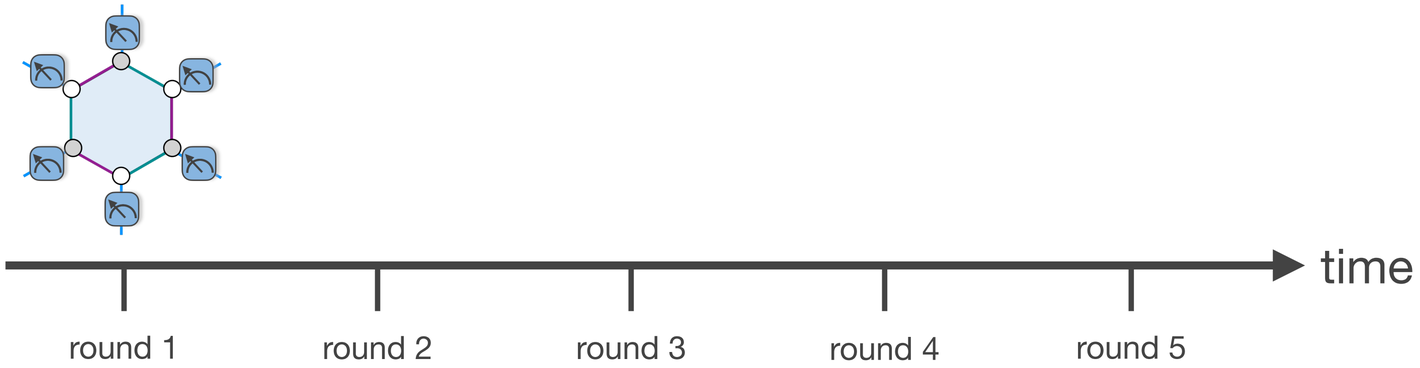
Hastings-Haah Floquet code



Hastings & Haah, Quantum **5**, 564 (2021)

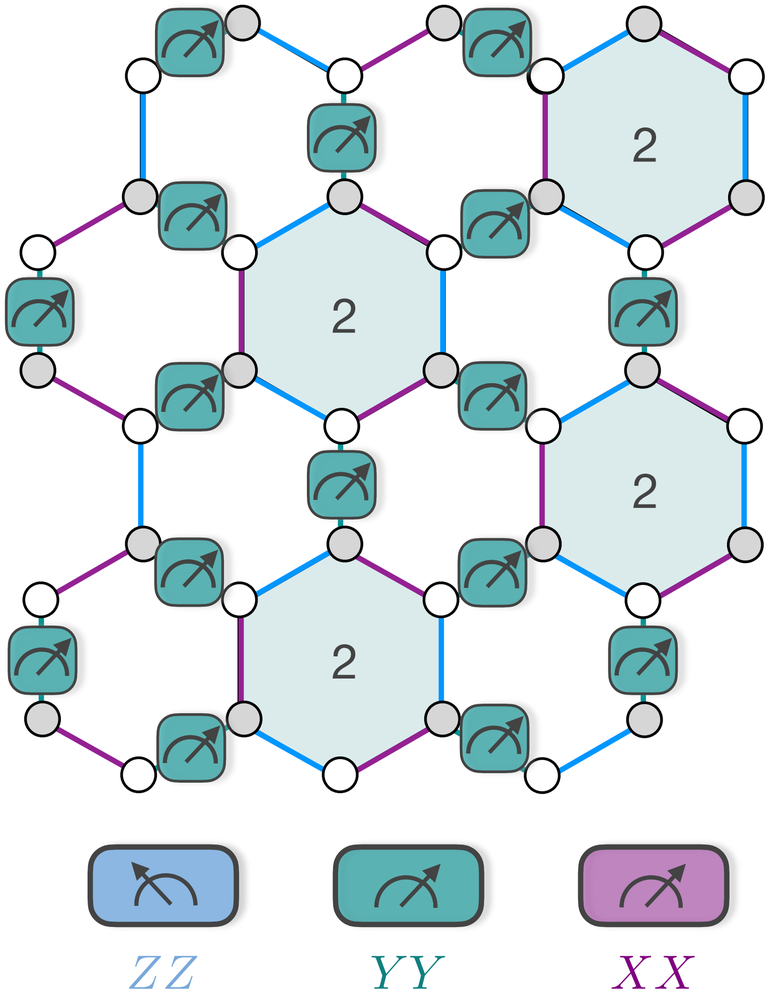
Hastings-Haah Floquet code

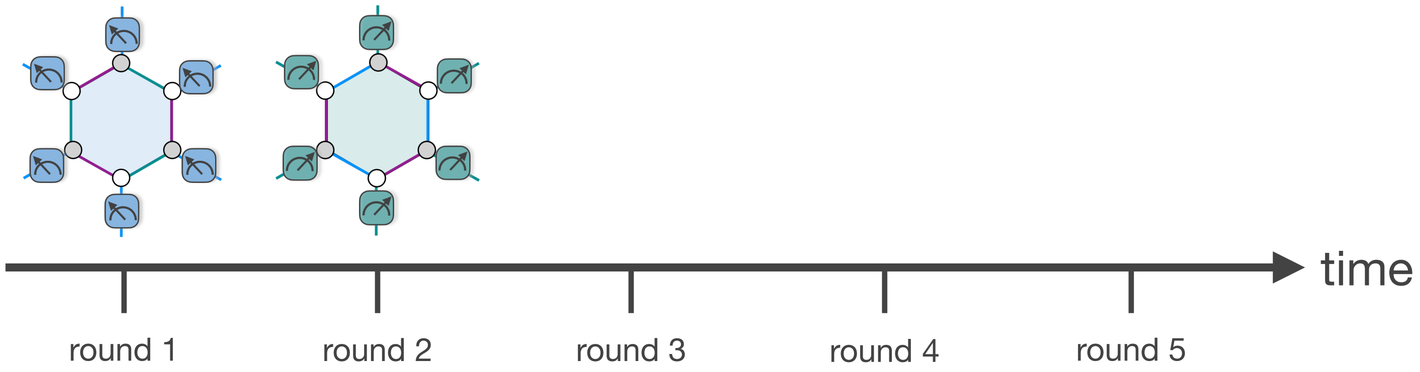




Hastings & Haah, Quantum **5**, 564 (2021)

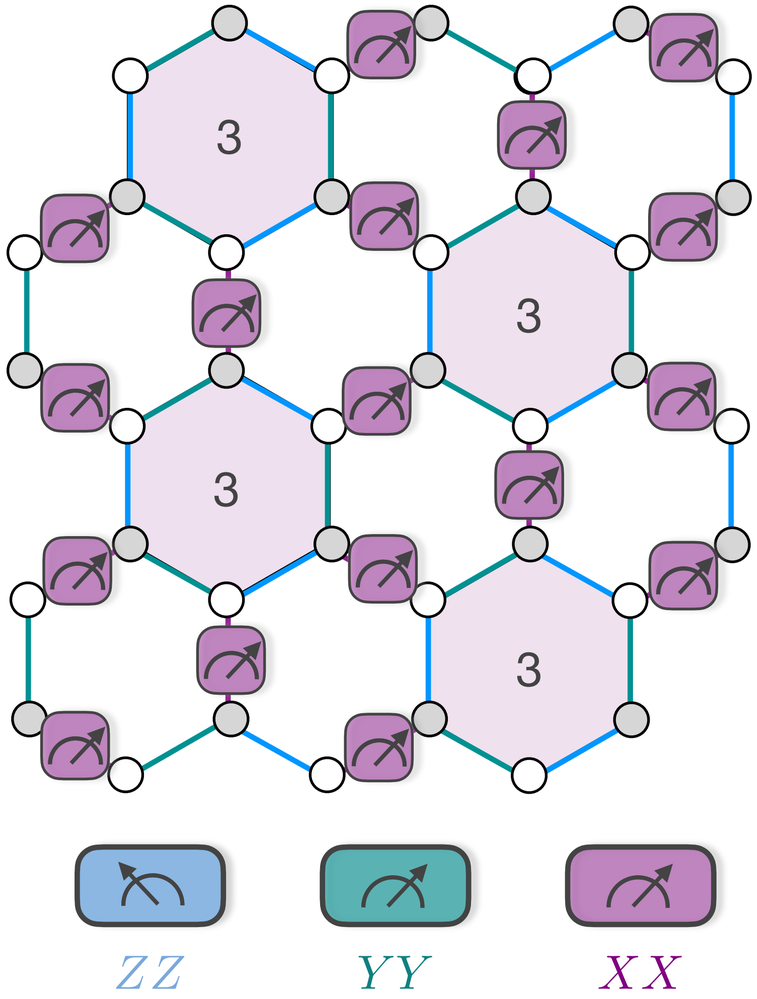
Hastings-Haah Floquet code

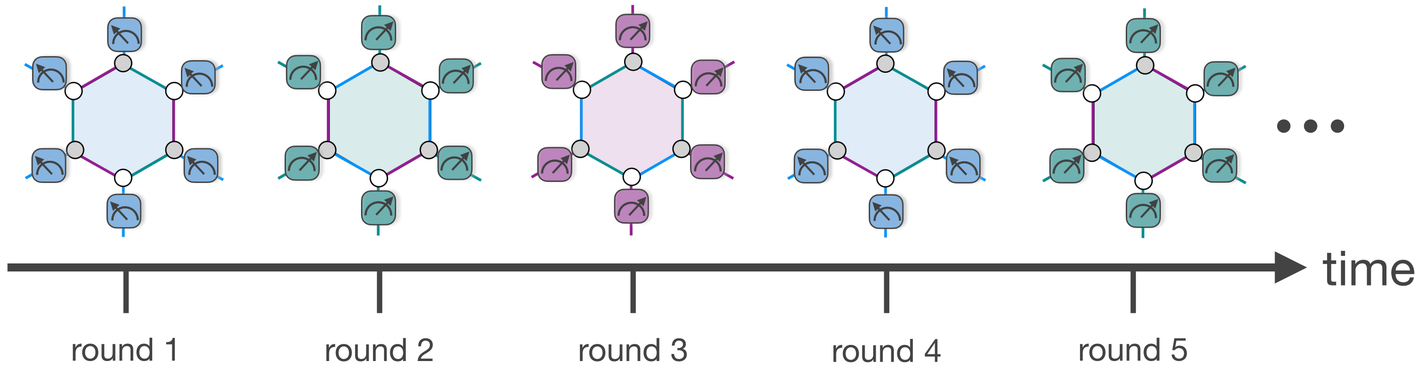




Hastings & Haah, Quantum **5**, 564 (2021)

Hastings-Haah Floquet code





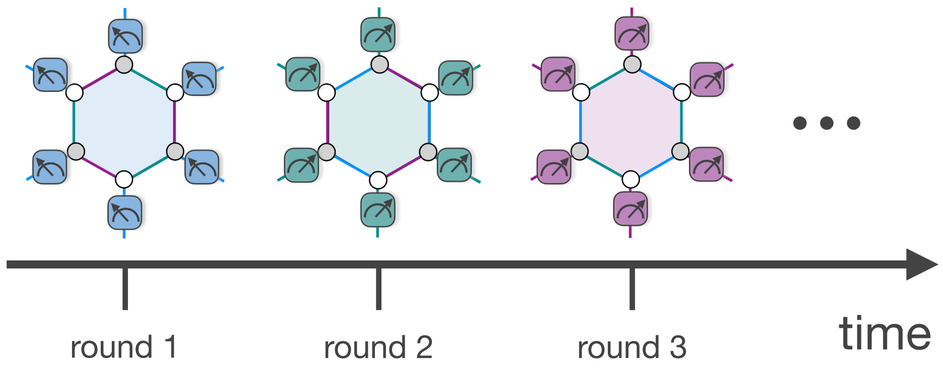
dynamically generated logical qubits

- **Floquet** dynamics
- **two-qubit** Pauli operators
- quantum **error correcting** code
- **two logical qubits**

What happens when you turn stabilizers
from projective measurements into **weak measurements?**

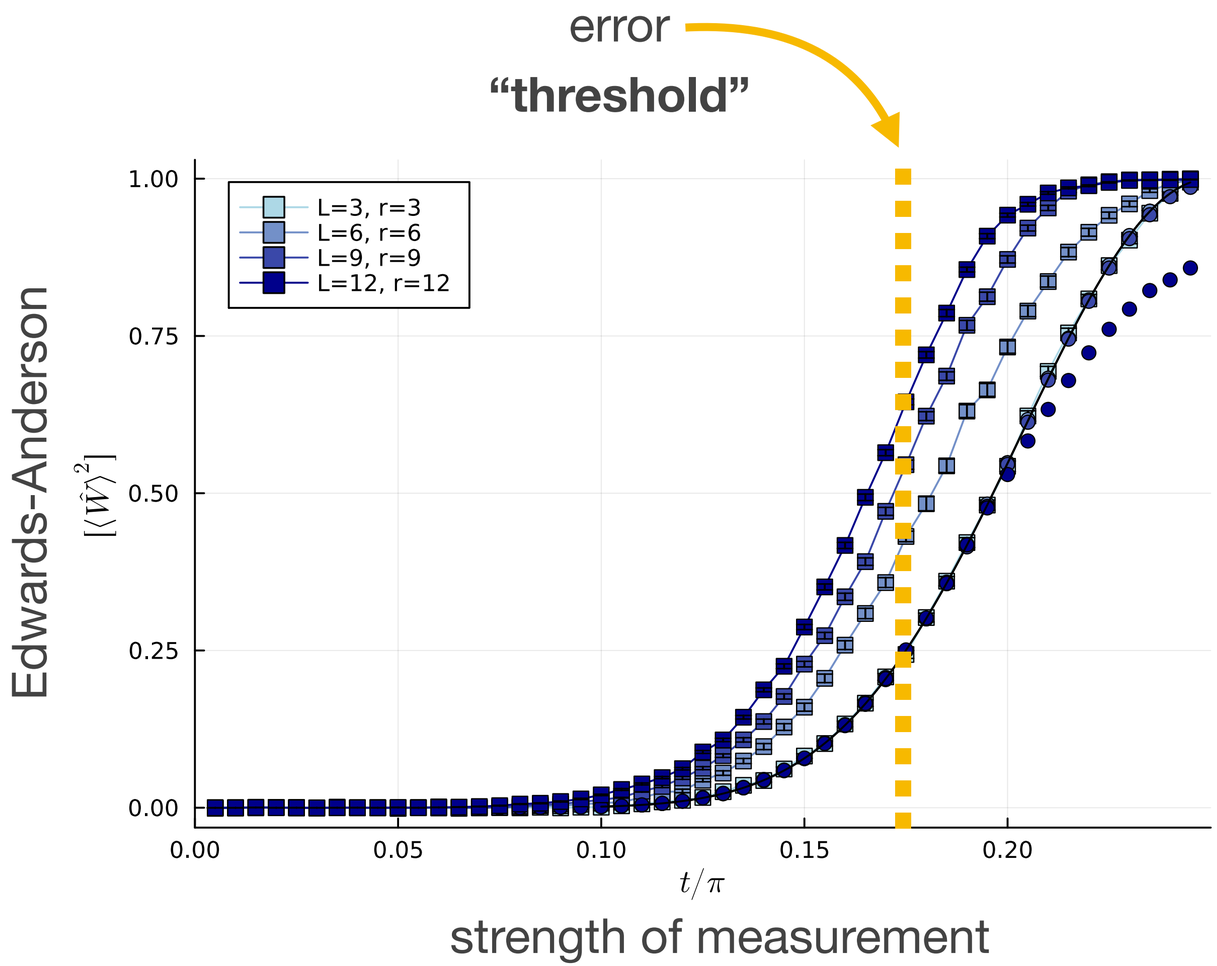
Hastings & Haah, Quantum **5**, 564 (2021)

Hastings-Haah Floquet code



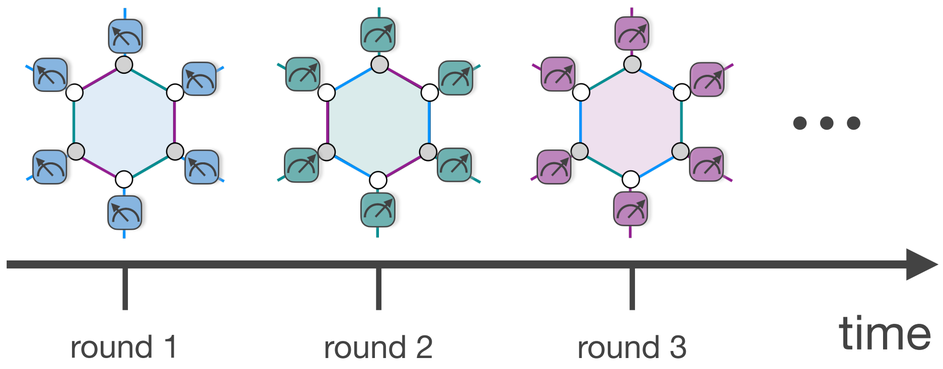
What happens when you turn stabilizers from projective measurements into **weak measurements?**

- finite **error "threshold"**
- **two** crossovers
- **fractionalization** crossover
- intermediate "metallic" phase



Zhu & Trebst, forthcoming preprint (2023)

Hastings-Haah Floquet code



What happens when you turn stabilizers
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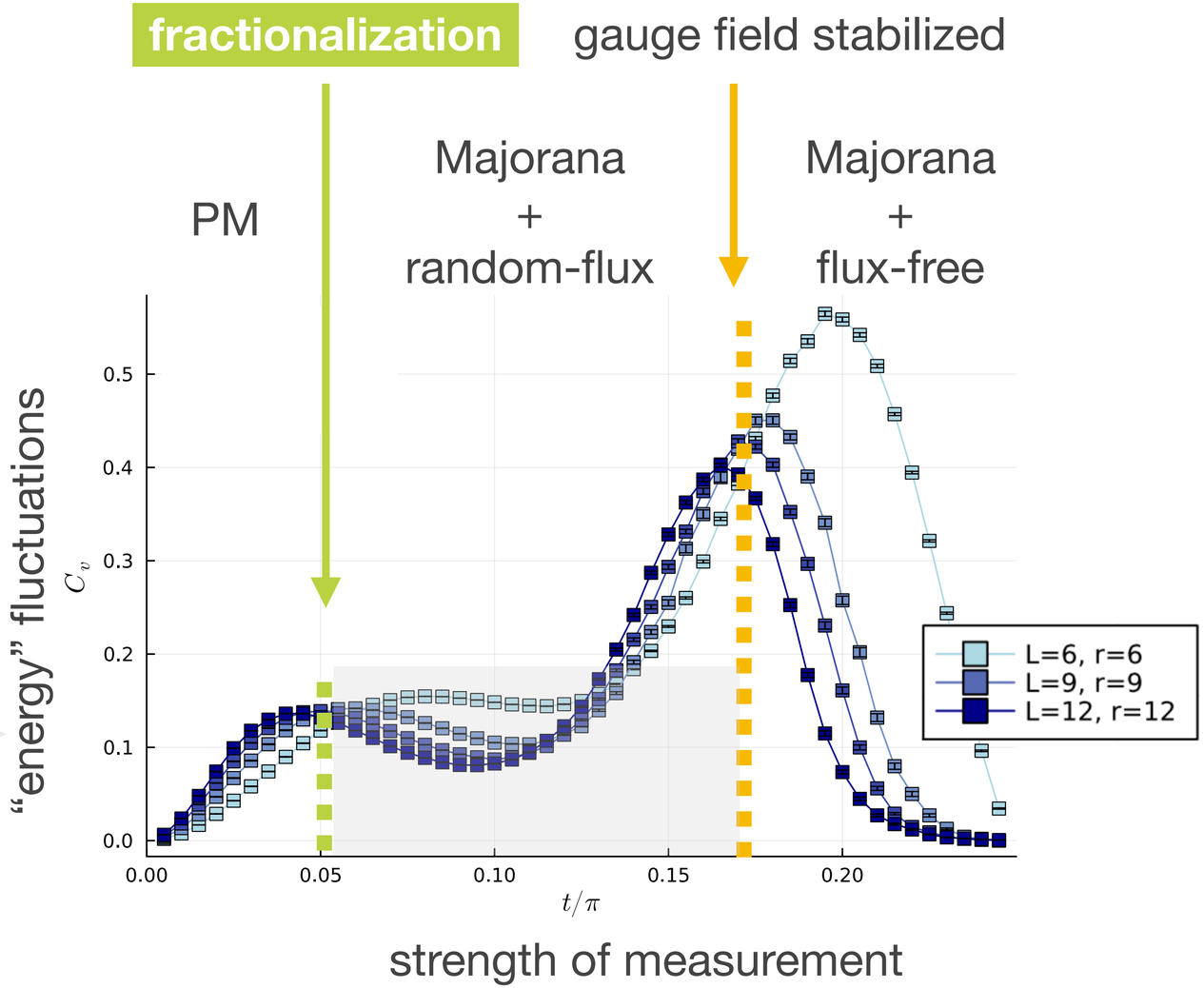
$$\mathcal{H} = -\ln(\rho)$$



Zhu & Trebst, forthcoming preprint (2023)

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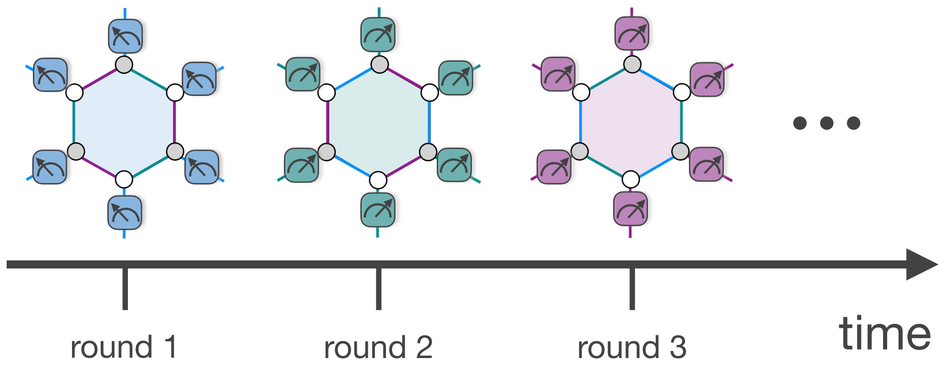

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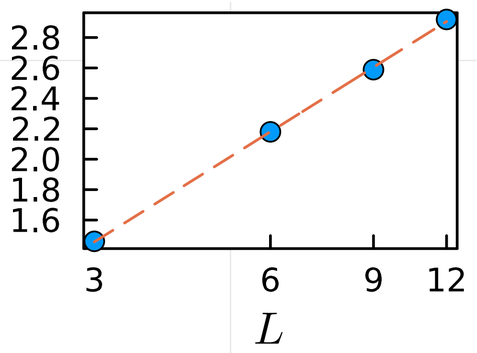
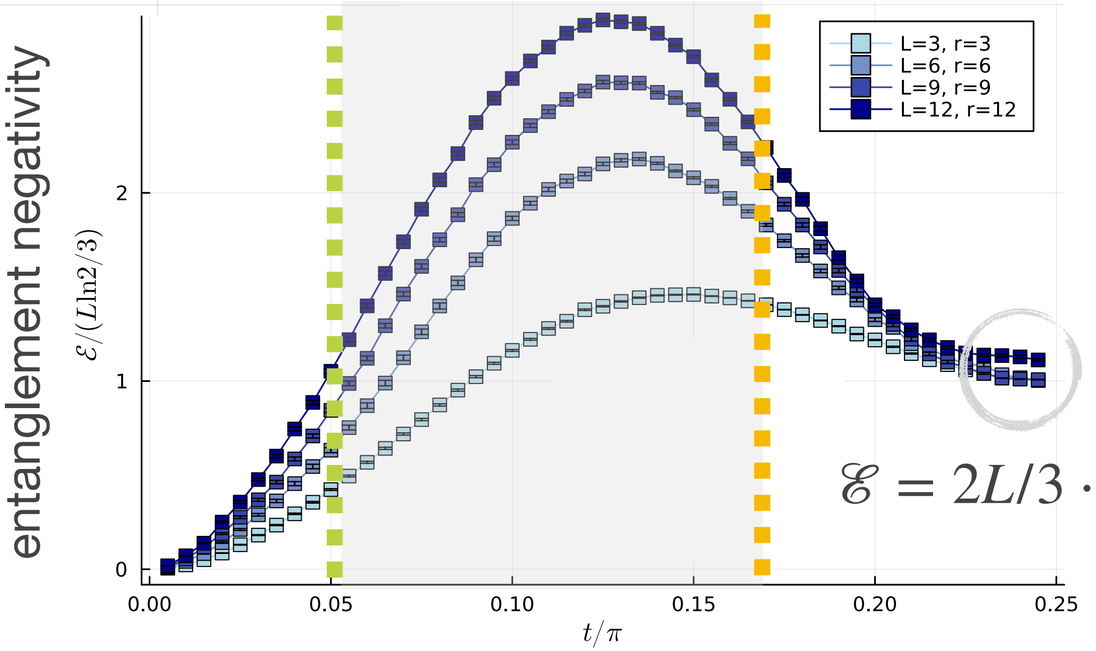


Fermi-surface scaling

scaling $\mathscr{E} \sim L \ln L$

only **quantum** correlations

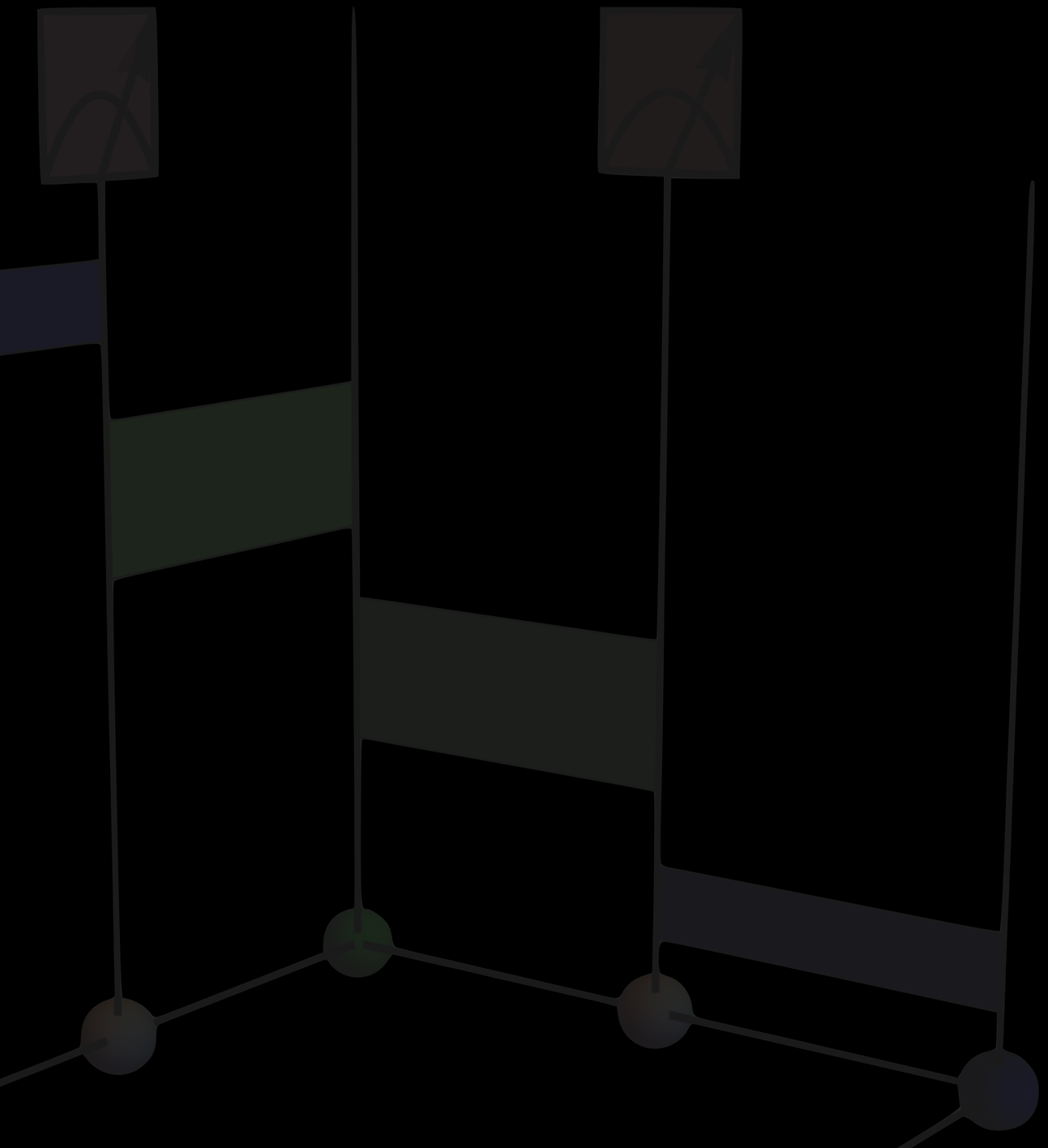
Ryu, 2017



$\mathscr{E} = 2L/3 \cdot \ln 2/2$

strength of measurement

Zhu & Trebst, forthcoming preprint (2023)



summary

summary

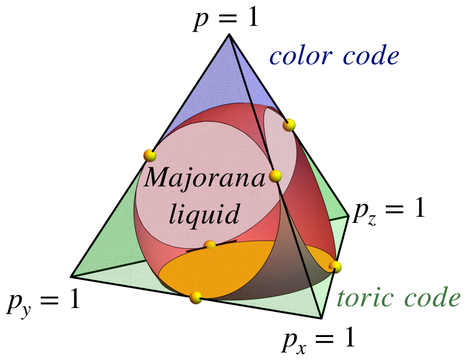

G. Zhu & ST, forthcoming preprint

G. Zhu, N. Tantivasadakarn, ST arXiv:2303.17627

G. Zhu *et al.* arXiv:2208.11136

monitored Kitaev dynamics

- **frustration** by **non-commuting** measurements is key ingredient
- rich **entanglement phase diagram**
- **6-qubit** operators can induce **structured volume-law** entanglement



weak Floquet dynamics

- **error threshold** for Hastings-Haah code
- measurement strength = **effective temperature**
- signature of **fractionalization**
- intermediate **“metallic” phase**

What can we learn from **monitored (Clifford) circuits** about quantum magnetism?

Can we **engineer spin liquid phenomenology** on current quantum computing platforms?


Thanks!